Components of Volatility and their Empirical Measures: A Note

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Abstract: A descriptive decomposition of the observed volatility of a variable into three components is proposed here. These components have been named the *Strength*, *Duration* and *Persistence* of volatility. This decomposition is unique and is such that measurement and analysis of these components will facilitate both a better understanding of the nature of volatility of a variable and, more importantly, a comparison of the patterns of volatility of two or more variables. The proposed methodology is illustrated here by applying it to the time series of daily observations on three variables, viz., stock return, inter-bank call money rate and foreign institutional investment, pertaining to India.

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I. INTRODUCTION

In common financial parlance, volatility of a variable is understood to reflect the degree of fluctuation that the value of the variable is likely to show in its over time movements. For example, if the price of a stock is capable of large swings, it is said to have a high volatility. Formal models of *stochastic volatility* relate volatility of a variable to the *autocorrelated* nature of its *conditional variance*. A basic observation about most (high frequency) time series data on financial variables like asset return is that a large value (of either sign) tend to be followed by a large value (of either sign), thus suggesting a strong *temporal clustering* of the high and low fluctuations of the variable concerned. Following Engle (1982) and Bollerslev (1986), this feature of a given set of time series data on a (financial) variable is sought to be explained using an appropriate form of ARCH or GARCH model (Campbell, Lo and MacKinlay, 1997).

Given the notion of volatility as mentioned above, it is only reasonable to expect that the pattern and intensity of volatility of a variable may change over time, smoothly in some cases and in a discrete manner in others. For example, policy intervention may result in changes in volatility of macroeconomic or financial variables (see, e.g., Eichengreen and Tong (2003) for an analysis of the effect of monetary policy on the stock market volatility based on historical data, Cecchetti and Ehrmann (1999) for a discussion on the effect of inflation targeting on output volatility and Valachy and Kocenda (2003) for a comparison of volatility of exchange rate in different exchange rate regimes for exchange rates of European countries; see also Watkins and McAleer (2002)). The volatility pattern of a variable that varies continuously over time may be modeled as a *rolling-sample* GARCH and analysed by examining the over time movements of the estimated parameters of the variance equation of the GARCH. An alternative is to use data-driven non-parametric rolling sample estimators of spot or integrated volatility (see, Andreou and Ghysels (2000) for a comprehensive discussion on this methodology for analysis of volatility of stock returns based on high frequency stock price data).

Given a time series data on a variable which is subject to volatile movements, three different aspects of observed volatility are implicit in the data set - viz., the excess of the average amplitude of fluctuations in volatile states over that in non-volatile states, the fraction of the total sample period the variable is observed to be in volatile states and the average duration (i.e., the average length of time) of a volatile state. These aspects may be called the *strength*, *duration* and *persistence* of volatility, respectively.

It may be noted that these three components/aspects completely characterize the nature/pattern of volatility of a given variable as contained in a given set of time series data on the variable. Also, the patterns of volatility of a variable in two or more situations or those of two or more variables may be compared in terms of these components/aspects of volatility. Needless to mention, a decomposition of volatility as mentioned above should help get a deeper insight in to the nature of volatility on the basis of historical data. In Coondoo and Mukherjee (2004) this approach to the study of volatility has been used on the Indian data on foreign institutional investment (FII) and related variables. The suggested procedure of volatility decomposition is being formally presented here. In what follows, the proposed methodology of estimation of the volatility components is explained in Section II; Section III presents the results of an illustrative application; and finally, the paper is concluded in Section IV.

II. THE METHODOLOGY OF DECOMPOSITION

Consider an observed time series data $(x_t , t = 1,T)$ of a variable *X*, which is known to contain significant volatile movements. Without loss of generality, suppose $\{X_t\}$ is non-stationary in mean such that an ARIMA of appropriate order fitted to the given data would give residuals $(e_t, t = 1, T)$ that might be modeled as a stationary GARCH (*p,q*) process as

$e_t = \eta_t \sqrt{h_t}$, $h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j e_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$ and $\eta_t \sim iid\, N(0,1)$ with appropriate restrictions on the parameters of the GARCH process. Let *s* be the sample standard deviation of the residuals. Define the standardised[1] variable $w_t = |e_t / s|$ for $t=1,T$ and denote the empirical *pdf* of *w* by $f(w)$, where $w \in [0,\infty)$. Typically, $f(w)$ will be unimodal and positively skewed with a thick right hand tail.

Let w_m denote the mode of $f(w)$ and $\overline{w}_m = \int_{w_m}^{\infty} wf(w)dw/(1 - F(w_m))$ be the mean value of $w \geq w_m$, where $F(w_m) = \int_{0}^{w_m} f(w)dw$ is the cumulative density up to w_m. Now, w_m and $\overline{w}_m$ may be regarded as measures of average amplitude of variation of *X* in non-volatile *normal* period and volatile period, respectively. Thus, $S = \overline{w}_m - w_m \geq 0$ may be taken as an empirical measure of *excess amplitude due to volatility*. Clearly, a larger value of *S* will indicate a stronger volatility and hence here we call *S* a measure of *Strength of Volatility*.

Next, consider $D = 1 - F(w_m) \geq 0$ - i.e., the area under the pdf to the right of w_m. Evidently, *D* is an indicator of the portion of the total sample period the variable is observed to be in the volatile state and larger the value of *D*, the more enduring is the volatile state. We therefore call *D* a measure of duration of volatility.

Finally, we consider a measure of autocorrelation of $w_t's$ as a measure of persistence of volatility - i.e., the tendency of a volatile/nonvolatile state to persist once it gets started. For example, one may use $P = correlation(w_t, w_{t-1})$ as a measure of persistence of volatility. By definition, $P \in (-1,1)$ and a positive value of *P* means a tendency of large (small) observed value of *w* to follow a large (small) observed value and larger the value of *P*, the greater will be this inertia and hence persistence of volatility[2].

Given the observed values $(w_t, t = 1,T)$, the components *S, D* and *P* of observed volatility over the entire sample period may be estimated as follows: First, the empirical pdf of *w* is estimated using the non-parametric univariate kernel method of density estimation of Silverman (1986). Thus, for the given sample observations kernel estimate of the ordinate of the pdf for every observed value of *w* is obtained as $\hat{f}_T(w) = \frac{1}{T.h}\sum_{t=1}^{T} K[(w - w_t)/h]$, where $K[.]$ is the kernel function with the property $\int_{-\infty}^{\infty} K(u)du = 1$ and *h* denotes the bandwidth or smoothing parameter[3]. Once the empirical pdf of *w* is estimated this way, *S* and D are calculated according to the definition of these measures given above. Finally, *P* may be measured in terms of the sample autocorrelation of the observed *w* values[4].

The pattern of volatility of a variable may change over time. For example, if one has a time series of daily or more frequently recorded observations on a variable covering a reasonably long time period (say, a number of years), the pattern of volatility may change gently over time or may discretely change within the sample period. To bring out such *changing volatility* hidden in an observed time series data, one may consider a *rolling sample* estimation of the *S*, *D* and *P* measures of volatility explained above based on data for moving sample sub-periods and examine the over time variation of the individual components of volatility. For example, suppose one has a time series of daily observations on a variable covering a number of years. One may take a sample sub-period of 90 days, say, on a rolling sample basis, for every such sub-period estimate the three components of volatility and examine the time series of rolling sample estimates of each component to detect possible changes in volatility pattern over time. Needless to mention, such results should help a great deal in understanding the nature of volatility of the variable concerned.

III. AN ILLUSTRATIVE APPLICATION

For the purpose of illustration, we have applied the methodology that we have proposed above to a set of time series data of daily observations on three variables pertaining to India. The variables are the SENSEX stock price index of the Bombay Stock Exchange (BRET), net inflow of foreign institutional investment in equity (FII) and inter-bank call money rate (CMR). Using this data set we have compared the nature of volatility of these variables. This data set, compiled on the basis of information available in relevant websites, covers a sample period from January 1999 to May 2002 and consists of 840 daily observations.

As explained above, the method requires elimination of trend and other non-stationary elements, if any, from the given observed time series. To do so, we have first tested stationarity of the time series of individual variables using the Augmented Dickey-Fuller unit root test procedure. Summary statistics and results of unit root test are presented in Tables 1 and 2 respectively. As these results show, all the three time series are stationary.

TABLES 1 & 2 HERE

Next, to ascertain that the variables under consideration are indeed subject to volatile movements, we fitted GARCH models for each of these variables. In all the cases GARCH (1,1) turned out to be an adequate model specification. The GARCH (1,1) estimation results are presented in Table 3 below. It may be noted that the estimated parameters of the variance equation are all highly significant for all the variables and, more importantly, vary widely across variables.

TABLE 3 HERE

In the next step of analysis, we estimated the three components of volatility of the individual variables under the assumption that the pattern of volatility remained unchanged over the entire sample period. Estimated values of *S, D* and *P* measures are presented in Table 4. It may be noted that for individual components the estimated values for different variables are not widely different from each other. However, the strength of volatility (i.e., *S*) is highest for CMR and lowest for FIIN. Coming to the duration of volatility, CMR again has the largest value of *D* and hence highest proportion of volatile days in the entire sample period of 840 days seem to be in volatile state for this variable. The values of *D* for the other two variables are rather close. As regards the persistence of volatility as measured by the autocorrelation coefficients of *w*, it may be noted that the all the estimated autocorrelation coefficients of different orders are positive and range between 0.51 (1^{st} order autocorrelation coefficient for CMR) and 0.11 (3^{rd} order autocorrelation coefficient for BRET). The pattern of variation in the value of autocorrelation with the order of lag, however, is quite dissimilar across variables. Thus, while for BRET and CMR the strength of autocorrelation declines as the lag increases, for FIIN such a tendency is visibly absent.

TABLE 4 HERE

Finally, to examine how the pattern of volatility of the individual variables might have changed over the given sample period, we estimated the components of volatility on a *rolling sample* basis. For this purpose, two different window-widths, viz., 15 and 90 days, were used in turn. For each variable we thus have two different time series of estimated rolling sample values relating to 15- and 90-day window-width for each component of volatility. These two window-widths are supposed to show the pattern of movement of volatility over time in very short period and medium period, respectively. A graphical examination of the time series of rolling sample estimates of individual components of volatility would undoubtedly be revealing. For each component of volatility one should examine if the graphs showed systematic rising or falling tendency over the entire sample period. In the present exercise, no such trend rise or decline was observed in any of the cases presumably because the time period covered by the data set was a little less than three and a half years only. However, for every variable the time series graph of a component of volatility turned out to be flatter for the longer window-width[5].

A summary of the results of rolling sample estimation of volatility is presented in Table 5. For each variable, window-width and component of volatility, this Table gives the mean value of the rolling sample estimates and the corresponding coefficient of variation (measured as a proportion, rather than percentage), which is supposed to reflect the extent of variation of the estimated value of a component over the entire sample period. For the purpose of comparison, the corresponding estimate based on the entire sample is also presented in each case.

TABLE 5 HERE

The results in Table 5 may be summarized as follows: First, for each variable and each component of volatility except *S* for CMR, the mean value of component increases with the window-width, the value being largest for the estimate based on the entire sample. Secondly, In all the cases, the coefficient of variation of the values of a component of volatility is smaller for the larger window-width, which suggests that the intensity of volatility in very short period is somewhat stronger than that in medium period. Coming to specific components of volatility, for both window-widths, CMR has a greater variability of *S*, although the mean value of *S* for CMR is comparable with those for the other two variables. As regards *D*, the measure of duration of volatility, the mean values for CMR are a little higher than those for the other two variables. An opposite is true for the day to day variability of the estimates of this component as the coefficient of variation for CMR is smaller than those for the other two variables. Compared to *S*, the day to day fluctuation of the value of *D* is much less for all the variables for every choice of window-width. The persistence of volatility as reflected by the value of *P* is much greater for CMR together with a much smaller day to day variability.

TABLE 6 HERE

Finally, we tried to see how the volatility patterns for different variables might be correlated. To do so, we examined, separately for each of the three components of volatility, the contemporaneous correlation coefficient of the rolling sample estimates of the component for different pairs of variables[6]. These computed correlation coefficients are presented in Table 6. As these results show, except for the *S* component of volatility measured for the BRET-FIIN pair based on the 90-day window-width, all the other correlation coefficients turned out to be non-significant.

IV. CONCLUSION

Volatility of a variable is empirically examined either nonparametrically in terms of data-driven rolling sample estimates of the time-varying variance/standard deviation of the variable concerned or by using parametric models like GARCH (p,q) or some variant of it. In this paper we have suggested a unique decomposition of the volatility of a variable into three distinct components, viz., the strength, duration and persistence of volatility and suggested empirical measures of these components that can be estimated for a given univariate time series data set under the assumption of an unchanged volatility pattern for the entire sample period and also on a rolling sample basis under the assumption of a changing volatility pattern within the given sample period. We have made illustrative application of the proposed methodology on a time series data set of daily observations on three variables, viz., stock return, call money rate and foreign institutional investment pertaining to India.

The proposed decomposition of volatility into three components, being essentially descriptive in nature, is purely empirical. We have made no attempt to examine the stochastic properties of the proposed measures for the three components of volatility that we have suggested. Furthermore, unlike the parametric approach to volatility based on the GARCH methodology, the procedure suggested here cannot be used to generate prediction of future pattern of volatility. Our purpose here has essentially been to suggest a method of a comprehensive analysis of the pattern of volatility that remains implicit in a given body of observed time series data on a variable – a type of analysis that will help understand better the volatility of a variable and, more importantly, compare volatility patterns of a set of variables in a qualitative as well as quantitative manner.

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Table 1. *Summary Descriptive Statistics*

	BRET	**CMR**	**FIIN**
Mean	-0.00004	8.38	34.07
Median	0.00092	8.03	23.10
Maximum	0.09	22.50	983.20
Minimum	-0.07	0.50	-509.50
Std. Dev.	0.02	2.10	120.04
Skewness	-0.07	2.58	0.76
Kurtosis	5.33	13.99	9.66
Jarque-Bera	190.15	5160.66	1632.13
Sample size	840	840	840

Table 2. *Results of Unit Root Test*

	BRET	**CMR**	**FIIN**
ADF-statistic	-16.48	-7.73	-9.11
5% Critical Value	-1.94	-3.97	-2.87
Model Selected	No trend or intercept	Trend and intercept	Intercept
lag order	2	3	4

Table 3: *Results of GARCH (1,1) estimation*

item	BRET		CMR		FIIN	
	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
mean equation						
intercept	0.000723	0.000592	7.990946	0.025773	24.63928	2.921969
variance equation						
intercept	4.56E-05	9.87E-06	0.196689	0.012257	274.2189	61.64922
ARCH(1)	0.161038	0.036078	1.08646	0.059423	0.124378	0.013505
GARCH(1)	0.713714	0.052307	0.144736	0.019215	0.863543	0.014828
Adjusted R^2	-0.005257		-0.03849		-0.00979	

Table 4. *Variable-specific estimates of Components of Volatility based on the entire sample data*

	BRET	**CMR**	**FIIN**
Amplitude of Fluctuation			
Average Amplitude of Normal Phase (w_m)	0.295	0.133	0.256
Average Amplitude of Volatile Phase ($\overline{w}_m$)	0.987	0.847	0.929
Strength of Volatility (*S*)	0.692	0.714	0.673
Duration of Volatility			
Proportion of Volatile days (*D*)	0.773	0.807	0.769
Persistence of Volatility (P)			
1st-order Autocorrelation of *w*	0.25	0.51	0.22

2nd-order Autocorrelation of *w*	0.18	0.36	0.20
3rd-order Autocorrelation of *w*	0.11	0.25	0.20

Table 5. *A Summary of Rolling Sample Estimation Results*

Volatility Component	Window-width	Mean/CV	Variable		
			BRET	**CMR**	**FIIN**
S	15-day	mean	0.66	0.67	0.63
		cv	0.51	1.12	0.51
	90-day	mean	0.68	0.75	0.65
		cv	0.22	0.54	0.33
	Entire sample		**0.692**	**0.714**	**0.673**
D	15-day	mean	0.60	0.72	0.63
		cv	0.23	0.12	0.17
	90-day	mean	0.70	0.79	0.71
		cv	0.09	0.05	0.06
	Entire sample		**0.773**	**0.807**	**0.769**
P	15-day	mean	-0.01	0.24	-0.01
		cv	-47.31	0.91	-15.78
	90-day	mean	0.20	0.45	0.08

		cv	0.67	0.33	1.54
	Entire sample		**0.25**	**0.51**	**0.22**

Table 6: *Correlation between day to day variations of estimated volatility components for different pairs of variables*

Volatility component	Window-width	Correlation for the variable-pair		
		BRET-CMR	**BRET-FIIN**	**CMR-FIIN**
S	15-day	-0.02	0.23	0.06
	90-day	0.43	0.51*	0.25
D	15-day	-0.34	0.05	0.06
	90-day	-0.38	0.23	0.19
P	15-day	-0.12	0.07	0.05
	90-day	-0.23	-0.16	0.42

Endnotes

[1] As volatility is typically measured in terms of variance (or equivalently in terms of standard deviation) of the variable concerned, comparability of volatility of variables measured in different units calls for this standardisation.

[2] One may examine the autocorrelation function of *w* for the purpose of comparison of persistence of volatility of two or more variables or of the same variable in two or more states.

[3] For the illustrative results reported later in this paper, this estimation has been done using SHAZAM. The default setting for the bandwidth parameter viz. h is $h = \{4/3T\}^{1/5}\hat{\sigma}_w$, where $\hat{\sigma}_w$ is the sample standard deviation of w and the Gaussian kernel function have been used.

[4] For the illustrative results reported later in this paper, we have used the autocorrelation up to 3 lags as measures of *P*.

[5] This is only to be expected. Because, the difference between the estimated value of a measure for two consecutive windows is only due to the difference in the first and last values of these two windows and as the

window-width increases, more values for two consecutive windows become common.

[6] Needless to mention, one may examine presence of lead-lag relationship in day to day variations of the volatility components of a set of variables and discover volatility spillovers as well.