First Workshop in Financial Econometric

Wavelets: a new tool in the Spectral Analysis

José G. Clavel
Dpto. de Métodos Cuantitativos para la Economía
Universidad de Murcia (Spain)
jjgarvel@um.es

Preface for a revised lecture...

- *Here you have a revised version of my 5th January presentation. It is **just** the 1.2 version... I'm sure I can improve it: any comments are very welcome*
- *I'll be working at IGIDR, (and living in India) God willing, till September 2005.*
- *You can reach me at **jjgarvel@um.es**, now and beyond September 2005: it is my former university e-mail address*

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State of Murcia (Spain)

- Surface:
  - 11.314 km2
- Coast:
  - 274km
- Temperature:
  - 19º C average
  - 3000 sun hours/year
- Population:
  - State of Murcia:
    - 1.250.000 (2002)
  - City of Murcia
    - 350.000 (3.5lacks)

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Murcia (Spain)

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Murcia (Spain)

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Summary

- Part I: Introduction
  - Spectral analysis: the frequency domain
  - Tools in the frequency domain
    - the fast fourier transformation
    - the periodogram
    - the filters
  - Drawbacks of the frequency domain
- Part II: The wavelets

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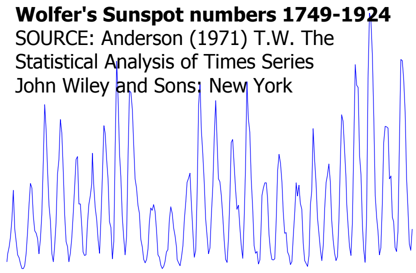
Summary

- Part I: Introduction
- Part II: The wavelets
  - Some applications...
  - Basic Elements of the wavelets world
    - the mother wavelet
    - the scale/frequency concept
  - The continuos wavelet transformation
    - some applications
  - The filters revisited: the DWT
    - some aplication

Spectral analysis: the frequency domain

- The spectral analysis gives us a way of studying stationary time series which **complements** those used in time domains
- The spectral representation is rooted in the basic notion of **Fourier Analysis** J.B. Fourier (1768-1830)

A typical example: Wolfer's Sunspot data: 1700-1987



Fourier Analysis, I

- Functions can be approximated over a finite interval, to any degree of accuracy, by a **weighted combinations of sine and cosine functions**, whose harmonically rising frequences are integral multiples of a fundamental frequency

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

Fourier Analysis, II

- The **periodogram** is simply a device for determining how much of the variance of f(t) is attributable to any given harmonic component
- Its value at ω_n

$$I(\omega_n) = \frac{N}{2}\{a_n^2 + b_n^2\}$$

Fourier Analysis, III

- In 1965, Cooley and Tuckey published an algorithm known as the **Fast Fourier Transformation** (FFT) that produces the same coefficients but with fewer arithmetic operations

Fourier Analysis, IV

- The relationship between the **spectral density function** and the sequence of **autocovariances,** (Norbert Wiener- A. Khintchine theorem, 1930s) provides a **link between** the TD and the FD analysis

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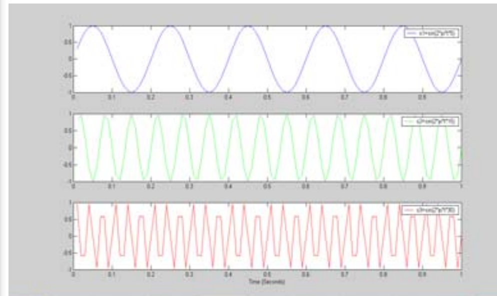
Fourier Example (1)



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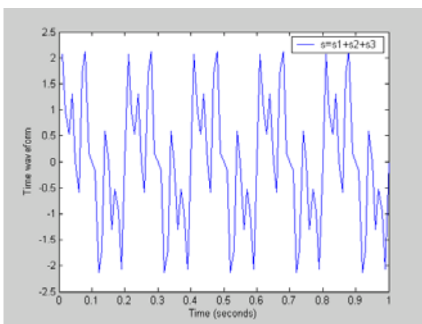
TD: The serie s=s1+s2+s3



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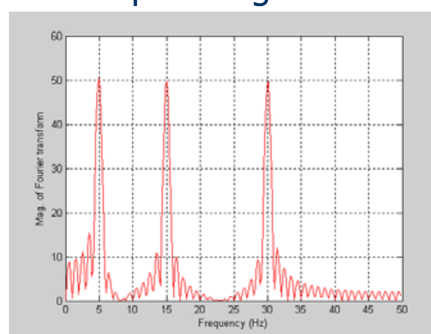
FD: The periodogram of s



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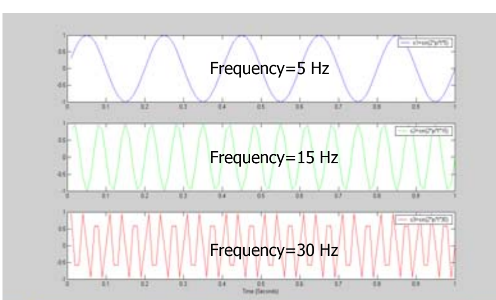
Fourier Example (II)



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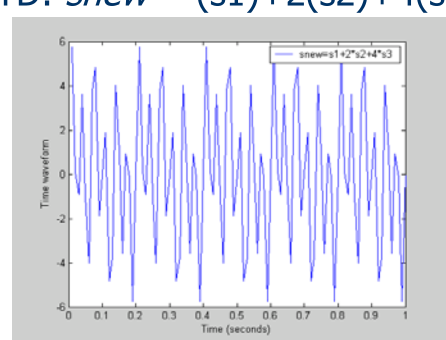
TD: *snew* = (s1)+2(s2)+4(s3)



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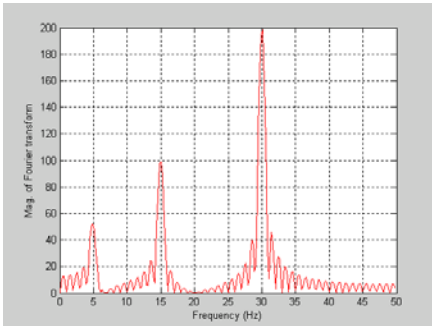
FD: The periodogram of *snew*



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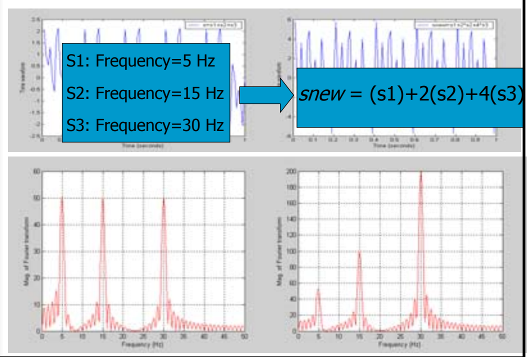
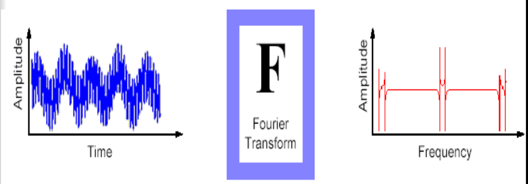
TD and FD: *s* and *snew*



Fourier Analysis in a nutshell...



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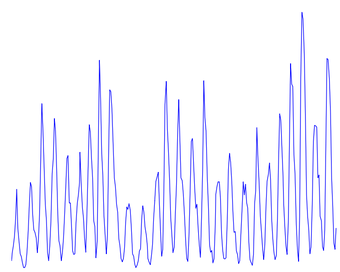
Wolfer's Sunspot data revisited



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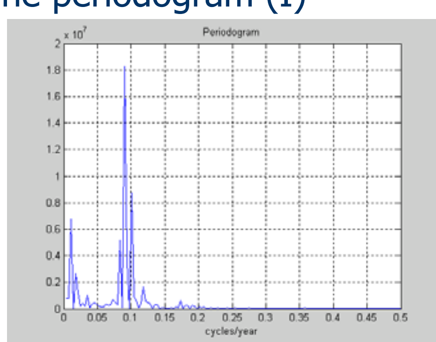
The periodogram (I)



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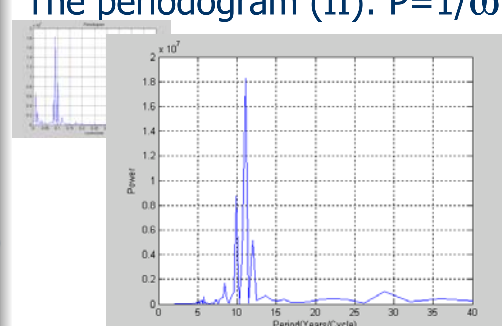
The periodogram (II): P=1/ω



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Summary Part I: Introduction

- Spectral analysis: the frequency domain
- Tools in the frequency domain
  - the fast fourier transformation (FFT)
  - the periodogram
  - the filters

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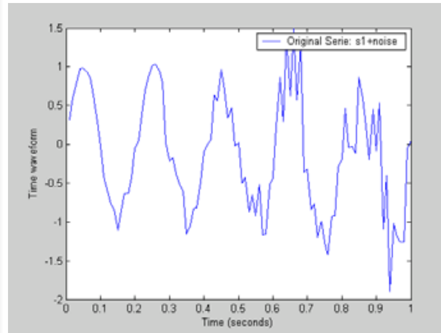
S1 + noise... (rem: freq s1=5)



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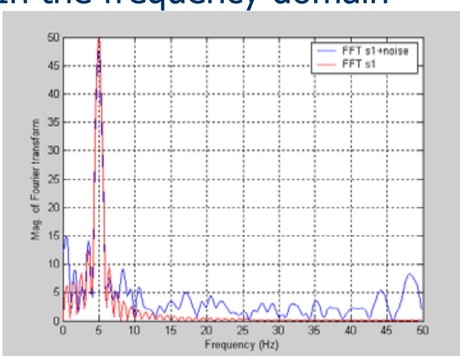
In the frequency domain



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Filters in time domain

- A linear filter simply converts a time series X(t) into another time series Y(t) by a linear transformation

X(t) → Filter → Y(t)

- Several kinds: linear vs nonlinear
  - Infinite Impulse Response filters (IIR)
  - Finite Impulse Response filters (FIR)
    - NonCausal Finite Impulse Response filters
    - Causal Finite Impulse Response filters

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X(t) → Filter → Y(t)

- The output Y(t) is the result of the **convolution** of the input X(t) with a coefficient vector ω(t)

$$y_t = \sum_{i=-\infty}^{\infty} \omega_i x_{t-i} \Rightarrow y_t = \sum_{i=0}^{\infty} \omega_i x_{t-i}$$

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Filters in time domain

- Infinite Impulse Response (IIR):

$$y_t = \sum_{i=1}^{L} a_i y_{t-i} + \sum_{i=0}^{M} \omega_i x_{t-i}$$

- Finite Impulse Response (FIR):
  - NonCausal FIR filters: $y_t = \sum_{i=-N}^{M} \omega_i x_{t-i}$
  - Causal FIR filters:

$$y_t = \sum_{i=0}^{M} \omega_i x_{t-i}$$

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A causal FIR filter...

$$y_t = \sum_{i=0}^{M} \omega_i x_{t-i}$$

- The M+1 period simple moving average

$$y_t = \sum_{i=0}^{M} \omega_i x_{t-i} = \frac{1}{M+1}(x_t + x_{t-1} + \ldots + x_{t-M})$$

- If M=1...

$$y_t = \sum_{i=0}^{1} \omega_i x_{t-i} = \frac{1}{2}(x_t + x_{t-1}) = 0.5x_t + 0.5x_{t-1}$$

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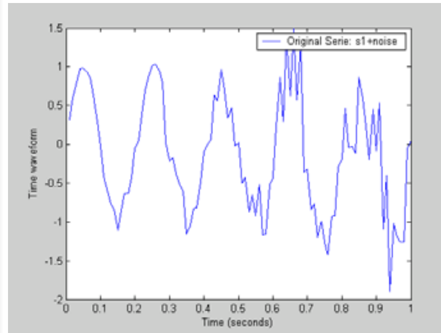
S1 (freq=5hz)+noise...



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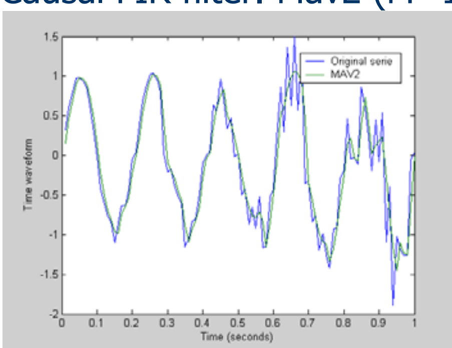
Causal FIR filter. Mav2 (M=1)



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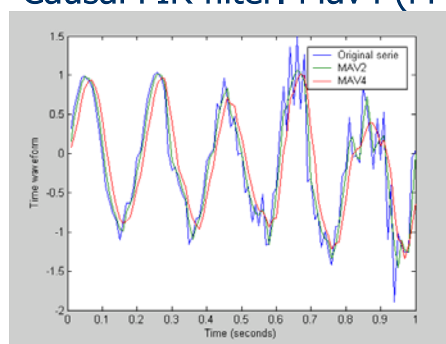
Causal FIR filter. Mav4 (M=3)



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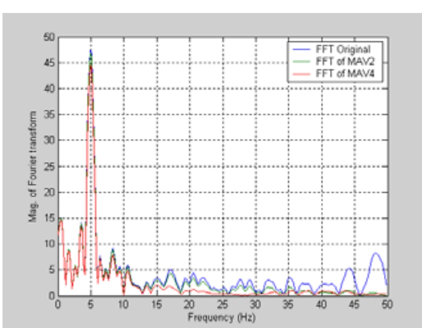
In the frequency domain...



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- Spectral analysis: the frequency domain
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- Drawbacks of the frequency domain

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Spectral analysis: the frequency domain

- The spectral analysis gives us a way of studying stationary time series which **complements** those used in time domains

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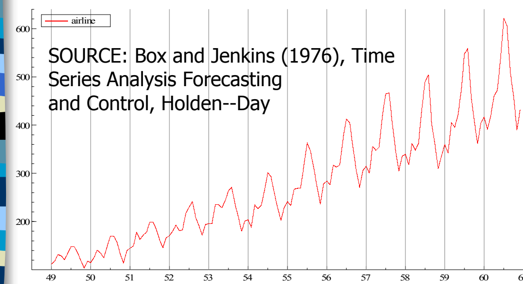
International airline passengers:
monthly totals; thousands of passengers;
January 1949 to December 1960



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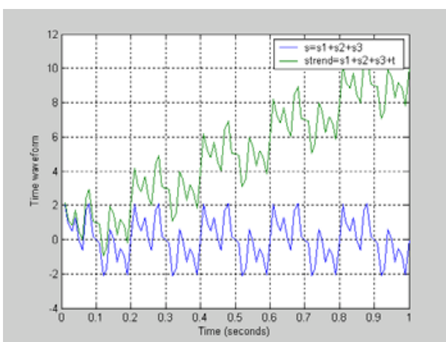
TD: *Strend*=s+time



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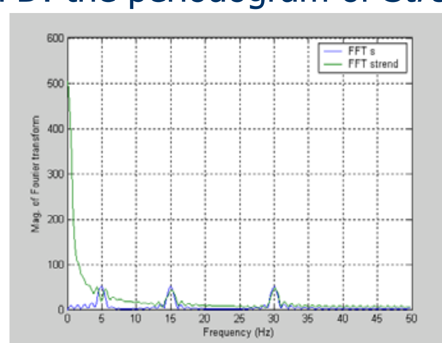
FD: the periodogram of *Strend*



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Drawbacks of the FD

- In transforming to the frequency domain, **time information is lost**.
- However, most interesting signals contain numerous nonstationary or transitory characteristics:
  - drift, trends, abrupt changes...
  - beginnings and ends of events...
  - structural changes...

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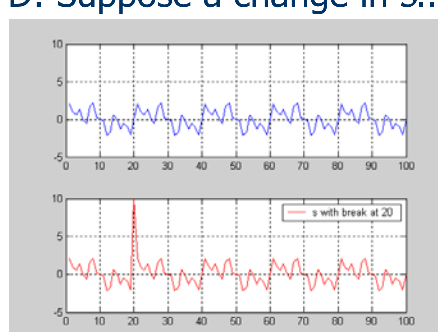
TD: Suppose a change in *s*...



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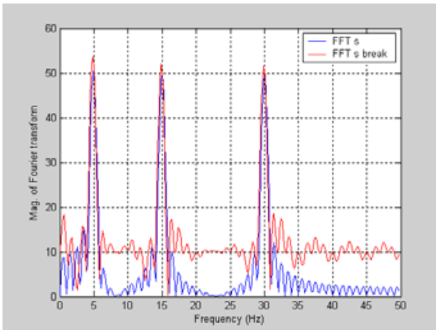
FD: ... no news



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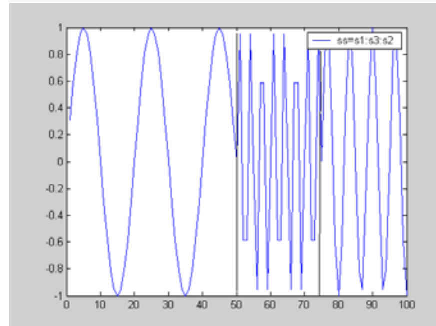
TD: Suppose a structural change...



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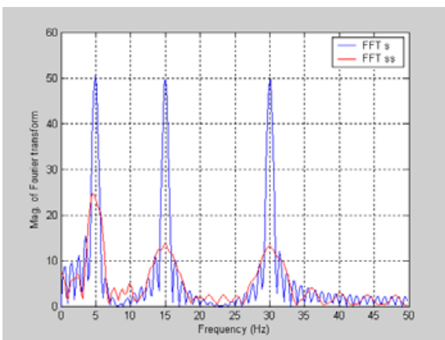
FD: ... no news



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Summary Part I: Introduction

- Spectral analysis: the frequency domain
- Tools in the frequency domain
  - the fast fourier transformation
  - the periodogram
  - the filters
- Drawbacks of the frequency domain
- The wavelets approach: Part II

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Summary Part II: The wavelets

- Some wavelets applications
- Basic Elements of the wavelets world
  - the mother wavelet
  - the scale/frequency concept
  - the output
- The continuos wavelet transformation
- The filters revisited: the DWT

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Some wavelets applications

- Analysis (detection of crashes, edges...)
- Compression (reduction of storage)
- Smoothing (attenuation of noise)
- Synthesis (reconstruction after comp.)
- Economics uses:
  - Seasonality Filtering
  - Denoising
  - Identification of structural breaks
  - Multiscale cross-correlation

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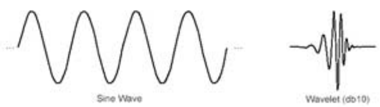
What is a Wavelet?

- A **wavelet** is a waveform of effectively limited duration that has an average value of zero.
- Comparing with sine waves...
  - sinusoids **do not have** limited duration and,
  - sinusoids **are smooth and predictable** whether wavelets tend to be irregular and asymmetric.
- See:



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Wavelet vs. Fourier analysis

- **Fourier analysis** consists of breaking up a signal into sine waves of various frequencies.
- **Wavelet analysis** is the breaking up of a signal into **shifted and scaled versions** of the original (or mother) wavelet.

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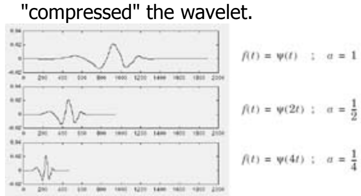
Scaling and shifting...

- **Scaling** a wavelet simply means stretching (or compressing) it.
  - The smaller the scale factor, the more "compressed" the wavelet.



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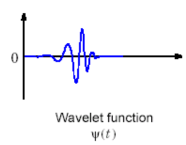
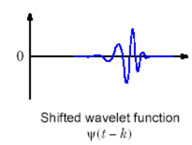
Scaling and shifting...

- **Shifting** a wavelet simply means delaying (or hastening) its onset.





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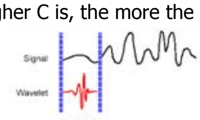
Continuos Wavelet Transform in 4 steps (CWT)

- 1. Take a wavelet,
  - and compare it to a section at the start of the original signal.
- 2. Calculate C,
  - C represents how closely correlated the wavelet is with this section of the signal.
  - The higher C is, the more the similarity.



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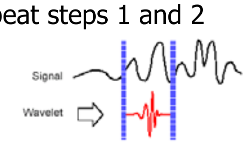
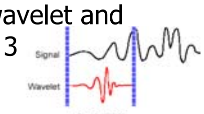
CWT

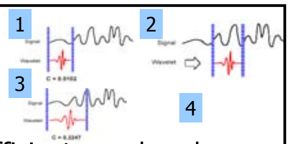
- 3. Shift the wavelet to the right and repeat steps 1 and 2



- 4. Scale (stretch) the wavelet and repeat steps 1 through 3



CWT output



- We have the coefficients produced
  - at different scales or levels
  - by different sections of the signal.
- A typical representation is a plot:
  - on which the x-axis represents position along the signal (time),
  - the y-axis represents scale,
  - and the color at each x-y point represents the magnitude of the wavelet coefficient C.
    - The brighter the color, the bigger the coefficient

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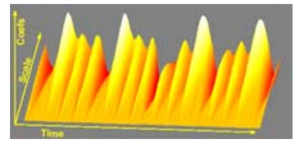
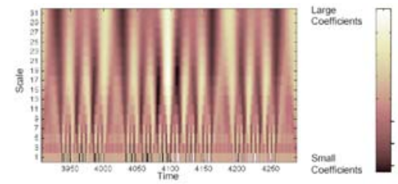
CWT output





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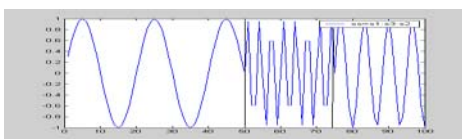
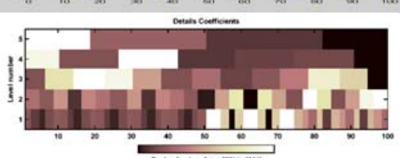
CWT of *ss*=[s1:s3:s2]





REM: The smaller the scale factor, the more "compressed" the wavelet.
The brighter the color, the bigger the coefficient

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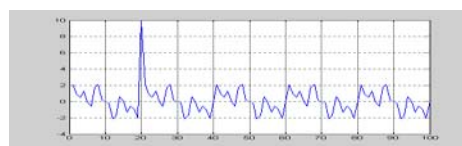
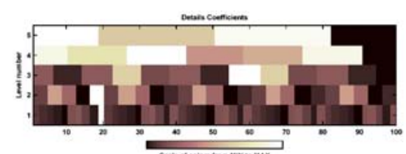
CWT of *sbreak:* s1+break





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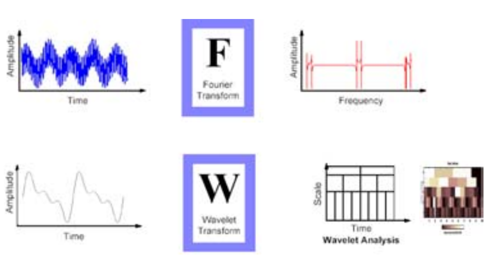
Wavelet vs. Fourier analysis



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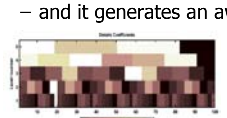
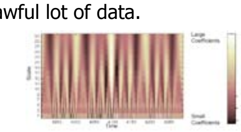
Continuos vs Discrete W.T.

- Continuos?
  - the CWT can operate at every scale
  - the CWT is also continuous in terms of shifting
- But:
  - Calculating wavelet coefficients at every possible scale is a fair amount of work,
  - and it generates an awful lot of data.





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Summary Part II: The wavelets

- Some applications...
- Basic Elements of the wavelets world
  - the mother wavelet
  - the scale/frequency concept
  - the output
- The continuos wavelet transformation
  - some applications
  - what is 'continuos' in the CWT?
- The filters revisited: the DWT

Continuos vs Discrete W.T.

- Discrete Wavelet Transform (DWT)
  - DWT choose only a subset of scales and positions
  - scales and positions are based on powers of two
- An efficient way to implement this scheme was developed in Mallat (1988)
  - a filter algorithm:
    - a convolution between wavelet and signal
  - a filter into which a signal passes,
  - and out of which wavelet coefficients emerge.
- It is fast wavelet transform

Remembering a CFIR filter...

- The M+1 period simple moving average

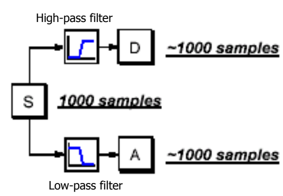
$$y_t = \sum_{i=0}^{M} \omega_i x_{t-i} = \frac{1}{M+1}(x_t + x_{t-1} + \ldots x_{t-M})$$

- If M=1...

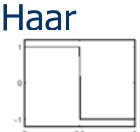
$$y_t = \sum_{i=0}^{1} \omega_i x_{t-i} = \frac{1}{2}(x_t + x_{t-1}) = 0.5x_t + 0.5x_{t-1}$$

The Discrete Wavelet Transform (DWT)

- The DWT algorithm (I):



Filters derived from Haar (1909) wavelets



- The High-pass filter:

$$y_t = \sum_{i=0}^{1} \omega_i x_{t-i} = \frac{1}{\sqrt{2}}(x_t - x_{t-1}) = 0.7071x_t - 0.7071x_{t-1}$$

- The Low-pass filter:

$$y_t = \sum_{i=0}^{1} \omega_i x_{t-i} = \frac{1}{\sqrt{2}}(x_t + x_{t-1}) = 0.7071x_t + 0.7071x_{t-1}$$

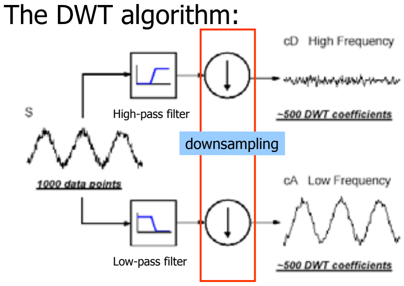
Filters derived from Haar (1909) wavelets

- Example s=[4,6,10,12,8,6,5,5]; N=8
- The High-pass filter:
  - s*HPf= sqrt(2)*[2 1 2 1 -2 -1 -0.5 0]
- The Low-pass filter:
  - s*LPf= sqrt(2)*[2 5 8 11 10 7 5.5 5]

$$y_t = \sum_{i=0}^{1} \omega_i x_{t-i} = \frac{1}{\sqrt{2}}(x_t + x_{t-1}) = 0.7071x_t + 0.7071x_{t-1}$$

The Discrete Wavelet Transform (DWT)

- The DWT algorithm:



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Filters derived from Haar (1909) wavelets

- Example s=[4,6,10,12,8,6,5,5]; N=8

- The High-pass filter + downsamplig:
  - s*HPF= sqrt(2)*[2 1 2 1 -2 -1 -0.5 0]
  - s*HPf*Downl=sqrt(2)*[1 1 -1 0]==> cD

- The Low-pass filter + downsampling:
  - s*LPf= sqrt(2)*[2 5 8 11 10 7 5.5 5]
  - s*LPf*Downl= sqrt(2)*[5   11 7 5]==>cA

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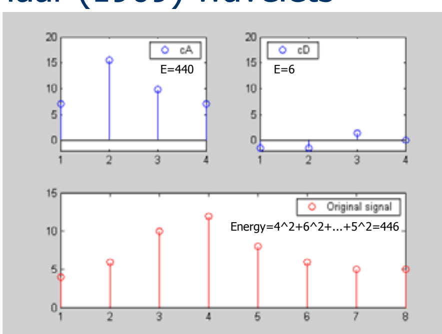
Haar (1909) wavelets



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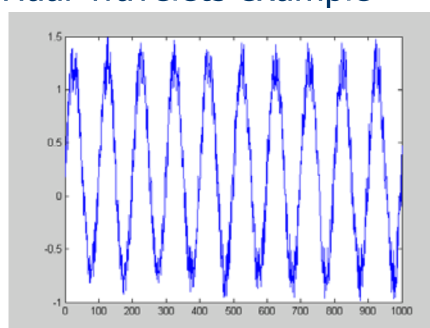
Haar wavelets example



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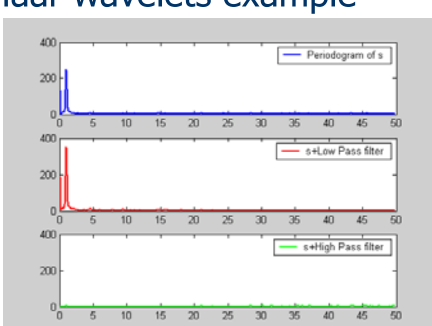
Haar wavelets example



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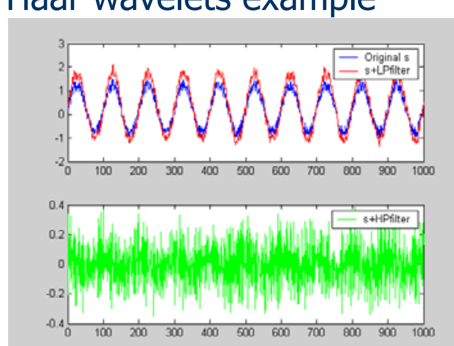
Haar wavelets example



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DWT applications

- Reconstruction
- Multiresolution Analysis
- Compression of audio signals
- Removing noise

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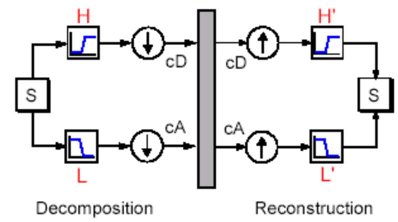
Wavelets reconstruction



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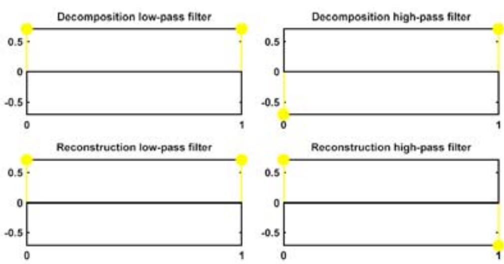
Haar (1909) wavelets: filters for reconstruction



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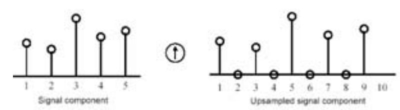
Haar wavelets: reconstruction of the approximation signal

- Upsampling:



- cA=[5r2,11r2,7r2,5r2]; N=4
- cA+Upsamp=[5r2,0,11r2,0,7r2,0,5r2,0]
- cA+Ups+HPRf=[5,5,11,11,7,7,5,5]=ApS

Original signal=[4,6,10,12,8,6,5,5]; N=8

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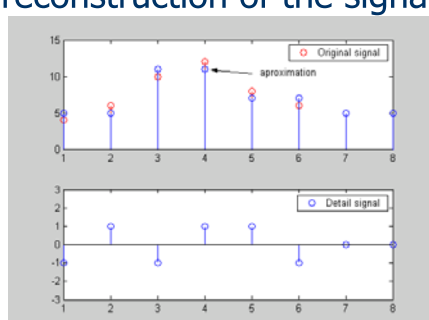
Haar (1909) wavelets: reconstruction of the signal



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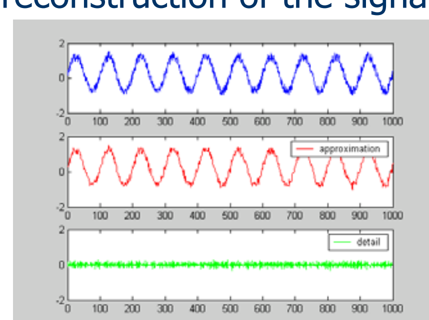
Haar (1909) wavelets: reconstruction of the signal



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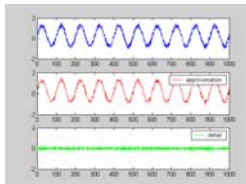
DWT applications

- Reconstruction
- Multiresolution Analysis
- Compression of audio signals
- Removing noise



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Summary Part II: The wavelets

- Some wavelets applications
- Basic Elements of the wavelets world
  - the mother wavelet
  - the scale/frequency concept
  - the output
- The continuos wavelet transformation
- The filters revisited: the DWT
  - some DWT applications
- References

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References

– Books:
  – Gençay, R.; Selçuk, F. and Whitcher, B. (2002): *An introduction to wavelets and other filtering methods in Finance and Economics.* Academic Press.
  – Walker, J. (1999): *Wavelets and their Scientific Applications.* Chapmand & Hall.
  – Nievergelt, Y. (1999): *Wavelets made easy.* Birkhäuser.
  – Pollock, S (1999): *A handbook of Time-Series Analysis, signal Processing and Dynamics.* Academic Press
– Http://www.wavelets.org
– Programs:
  • matlab (wavelet toolbox)
    – http://www.mathworks.com/
  • fawav
    – http://www.crcpress.com/e_products/downloads/download.asp?cat_no=8276

About Murcia, in English: http://www.carm.es/ctyc/murciaturistica/Portal/iya.menu.menu?idi=2
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First Workshop in Financial Econometric

Wavelets: a new tool in the Spectral Analysis

José G. Clavel
Dpto. de Métodos Cuantitativos para la Economía
Universidad de Murcia (Spain)
jjgarvel@um.es