Impact of Income Inequality and Spatial Distribution on Consumer Welfare

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Abstract

This paper develops a model that integrates consumer's income distribution with spatial distribution, and looks at the consequence of presence of one income group on the welfare of the other. Idea is to emphasise the role of geographic and socio-economic distance in shaping the market outcome and hence the welfare of individuals. To this end we look at two income groups and compare their utilities in different scenarios, for instance under different spatial distributions like when they stay in mixed community instead of being economically segregated and under different specifications of good, like when quality is determined in equilibrium than being given exogenously. We find that for exogenous quality though at low income level poor are better-off staying in the mixed community, but they might actually be worse-off once their income is above by certain cut-off. Income gap is particularly relevant in case of endogenous quality choice. For the low levels of inequality poor benefit by staying in the mixed community, but as the income gap widens presence of rich makes poor significantly poor.

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1 Introduction

Virtually every society has a wide range of income groups, staying together. In certain regions income disparity between people is more profound than the others. The kind of neighborhood where a person stays determines not just the price and the quality of the good but also the ease of access. For example, ratio of hospital beds to population in rural areas is fifteen times lower than that for urban areas and the ratio of doctors to population in rural areas is almost six times lower than that in the urban population. Though the health care facilities are overwhelmingly concentrated in urban areas, the economic distance, which includes cost of health care prevents access for the urban poor [1]. So the poor people, at the same income level, might be more disadvantaged staying is one place than the other. Are poor are better-off staying in a segregated economy or in a mixed economy. Would poor like to stay in poor neighborhoods because living in affluent one cost too much? Or does living in a poor neighborhood make poor people significantly poorer, as they do not even have the access to the facility. These are the kind of questions that we are interested in exploring in this paper.

The underlying factor influencing these regional differences stems from the variation in average income of people. People with higher income generally have higher valuation for services like health; education etc and so have higher willingness to spend on them. Firms while making strategic decisions take this into account. This insight was first developed by Gabszewicz and Thisse [1979], and Shaked and Sutton [1982] in the vertical differentiation models introduced by them. In our models we allow consumers to differ both with respect to their income and location. In the recent IO literature (Neven and Thisse [1990], Economidies [1989] and [1993], Tabuchi[1994], Hans Degryse [1996], look at the product specification in two characteristics though they did not allow for exclusion. Atkinson [1993] looks at non-consumption, arising out of income gap. But people even at the same income level might not consume because of the higher distance, and this is especially relevant in case of health and education.

In our models, we assume that consumer can either be rich or poor depending on his income level. More affluent consumers are assumed to have higher marginal willingness to pay, for the same marginal increase in quality level. It is also assumed that consumption, or improvement in quality of goods/ service, relevant for our model like food, health, education etc increases consumers welfare from consumption of all goods. As distance is an important factor influencing consumption decision of individual, so circular city model widely discussed in the I.O literature is convenient framework to discuss our problem. Consumer are assumed to be uniformly distributed across the circular city, and their location is assumed to be fixed.

[1] Socio-economic inequality and its effect on healthcare delivery in India: Inequality and healthcare

Hence a consumer in our models is characterized by income, which can be either Y_R or Y_P and location. A firm is described by fixed cost and marginal cost which is assumed to be invariant with respect to output level. Given this basic structure we play around with assumptions on quality choice. In section 3, to highlight role of income of income variation, we start with a basic model where quality is not a choice and show that how different income distribution, even with perfect competition affects the price. We show that when the income level of poor is very low, they are better-off staying with rich. This is because when income is low then it is not viable for the firm to just serve them, but for little higher income when firm is still a local monopolist, they are better-off being alone. This is because owing to low income level price charged to poor will be lower when they are alone than when they are residing with the rich and because of this all of them will be served. But once the income is above certain cut-off then it does not matter whether they stay in the mixed or segregated economy.

In the fourth section we look at the model where rich and poor stay side by side and quality is also a choice. Structure of equilibrium depends on the three threshold level of income of the poor. If the poor are too poor then none of them is served, but if they are relatively rich then all of them are served and between these two extremes is the case when some of them are served, and some of them are left out. These threshold levels of income of poor are increasing in the income of the rich, implying that as the income gap between rich and poor increases, poor become worse-off. We also show in mixed community poor can attain the welfare maximising level of quality when income gap between them and rich is narrow, but as the income gap rises they end up loosing. We conclude in the last section.

2 Importance of Income difference

Our analysis crucially relies on the importance of income differential between people and across regions. There are numerous studies across various disciplines which have highlighted that inimical impact of income inequality on health outcomes. This becomes especially relevant for the developed countries like United States where it is not the absolute deprivation but the relative poverty which is prominent. [2] One does not much need much empirical evidence to emphasize the significance of income gap, just the casual walk across your city streets, in India, will be lot revealing. For instance while moving across filthy streets one might come across many roadside vendors, selling tea, providing barber services, etc. These kinds of shops require minimal physical investment and offer very basic service / good. But as one moves to relatively posh area, one comes across more sophisticated counterparts of the same good. Road side shops are replaced by café-coffee days, beauty saloon, etc. Now similar goods become highly capital intensive and specialised in nature. Prices of these goods

[2] Commentary: Income inequality summarises the health burden of individual relative deprivation

move in tandem with quality. But more surprisingly, one might observe heterogeneous price even for the same product in different markets, depending on the extent of inequality. This is especially true for the food products. As reported food price in rich region like Gurgaon is much higher than in the poor region, reflecting higher paying capacity. Even more so certain facilities might be completely missing in certain areas as it is not be viable for the firm to operate, forcing individuals either to travel long distances or do completely without consumption. All this has a significant implication on the utility of the consumer. People with the same income level might be better-off staying in one place than the other as because of the presence of other income group. The idea can be illustrated by looking at the simple case described below.

3 Basic Model with given quality

Consider a simple setting: Circular city of length L units. There are n firms in the market. Each firm is defined by the location x_j and faces two types of cost, c, the constant marginal cost and F, fixed cost. We assume three-stage game for the firm, in the first stage, potential entrants chooses whether or not to enter. Equilibrium number of firms is determined from zero-profit condition. We assume that entrants do not choose their location, but rather are automatically located equidistant from one another on the circle. In the second stage, firms compete in prices, given locations. Each firm simultaneously chooses its price in every market so that its total profit is maximized. We assume that firm's do not price discriminate between consumers and charges them same price, irrespective of their location.

Firms problem:

$$\text{Max } \pi_j = [p_j - c]D_j - F$$

Each consumer is described by location z_i and income level. Utility of consumer of income Y, is given by:

$$U = \begin{cases} Y\theta - p_j - t \mid x_j - z_i \mid & \text{if he buys,} \\ Y & \text{if he does not buy.} \end{cases}$$

$Y\theta$ is the gross utility of the consumer, p_j is the charged and t is the per-unit travel cost. In this model when quality level is given, this implies that individual's welfare from consumption of all goods increase if he chooses to buy. For example a healthier individual is better able to enjoy consumption of all other goods and services. Reservation utility is given by Y. Consumer buys if his utility from consumption is higher than his reservation utility. As is clear from the above expression that the total price paid by the consumers (which includes the transportation cost) differs from the net market price received by the producer. Because of this difference in the mill and the delivered price there might be consumers even at the same income level, who are left out of the market.

3.1 Mixed Community:

We assume that there are two income groups staying side by side, one has income level Y_R and other Y_P, where $Y_R > Y_P$. Individuals with income Y_R and Y_P, are assumed to be uniformly distributed, across the circular city with density δ_R and δ_P, respectively.

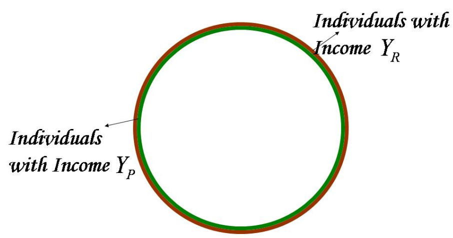

Figure 1: Circular City with Two Income Groups

Firm's compete with respect to rich: Initially we assume that Y_R is high enough to ensure that the marginal rich indifferent between the two adjacent firms is better-off buying, implying that adjacent firms compete for the marginal rich consumer and look at the different equilibrium possibilities depending on the income of poor.

3.1.1 Case 1. Marginal POOR indifferent between two adjacent firms is better-off consuming:

This would imply that the firms compete for the marginal poor as well. Demand here is derived as in the standard IO literature, by looking at the location of the marginal consumer, indifferent between the two adjacent firms. Let the marginal consumer with income Y indifferent between firm j and firm $j+1$ be located at z_i.

$$\text{Then } U(Y, x_j, p_j, z_i) = U(Y, x_{j+1}, p_{j+1}, z_i) \;\Rightarrow\; z_i(Y) = \frac{p_{j+1} - p_j + t(x_j, x_{j+1})}{2t}$$

Similarly one can work the position of the marginal consumer indifferent between firm j and $j-1$ and evaluate total demand corresponding to income group Y.

$$\Rightarrow D_j = \frac{\delta_R + \delta_P}{2t} \times [p_{j+1} + p_{j-1} - 2p_j + t(x_j, x_{j+1}) + t(x_j, x_{j-1})]$$

First order condition with respect to price implies:

$$D_j = \frac{\delta_R + \delta_P}{t} \times [p_j - c]$$

Income of poor is so high that it completely mitigates the affect of distance. Only the mass of the consumers determines the demand for the firm and hence the equilibrium values for n and p. Symmetric location together with **Zero-Profit** implies:

$$n = L\left[\frac{t(\delta_R + \delta_P)}{F}\right]^{\frac{1}{2}} \quad \text{and} \quad p_j = \left[\frac{tF}{\delta_R + \delta_P}\right]^{\frac{1}{2}} + c$$

Above case holds when the poor consumer who is indifferent between the two adjacent firms is better-off consuming.

$$\Rightarrow \ Y_P \ < \ Y_P\theta - p - \frac{tL}{2n}$$

Substitution of equilibrium values and simple rearrangement implies:

$$\frac{3}{2}\left[\frac{tF}{\delta_R + \delta_P}\right]^{\frac{1}{2}} < Y_P\theta - c < Y_R\theta - c \tag{1}$$

This is the case when the income of the poor is also high enough that the marginal poor is better-off consuming. This cut-off level of income of poor is the function of exogenous parameters. It warrants a mention that in this model without quality choice and with free entry once the income level Y_P is above certain cut-off then the income gap between poor and rich has no role. Equilibrium outcome for price, number of firms, is same, irrespective of high the income is, testifying the importance of competitive set-up. Price mark up above the marginal cost is just sufficient enough to cover the fixed cost.

3.1.2 case 2. Marginal Poor indifferent between the two adjacent firms is better-off not buying:

This implies that poor constitutes firms captive market. Demand from rich is evaluated in the same way as above and demand from the poor is determined by the distance of the marginal consumer indifferent in buying and not buying.

$$D_j = \frac{\delta_R}{2t} \times [p_{j+1} + p_{j-1} - 2p_j + t(x_j, x_{j+1})] + 2\delta_P \left[\frac{Y_P\theta - p_j}{t} \right]$$

Income of poor shows up in the aggregate demand, making demand more elastic to price as compared to previous case. This is because firms are monopolist with respect to poor.

Solving in exactly the same way as above yields:

$$p = \sqrt{\frac{tF}{\delta_R + 2\delta_P}} + c \quad \& \quad n = \frac{tL\delta_R}{\delta_R + 4\delta_P \left[\sqrt{\frac{tF}{\delta_R+2\delta_P}} - \frac{2\delta_P}{\delta_R+4\delta_P}[Y_P\theta - c] \right]} \tag{2}$$

This case holds when $Y\tilde{u} - p_j - \frac{tL}{2n} \; < \; Y_P\bar{u}$ *i.e.* when the marginal consumer gets less than his reservation utility and so is better-off not consuming. After substituting for the equilibrium values, implied cut-off of Y_P for which this holds is given by:

$$Y_P\theta - c < \frac{3\delta_R + 4\delta_P}{2[\delta_R + \delta_P]} \left[\frac{tF}{\delta_R + 2\delta_P} \right]^{\frac{1}{2}} \tag{3}$$

Here distance becomes relevant to the extent that it influences the consumption decision of individual. Even at the same income level there are some poor, relatively at the higher distance, who are better-off not consuming. This case highlights the importance of spatial dimension. For example in rural India, many poor people are unable to avail primary education as schools are not conveniently located.

As is clear from equation 2, that the number of firms is increasing in the income level of poor, though price is at the same level. As Y_P increases demand size of each firm goes up, implying that each firm is making more than zero-profit. To mop-up that extra profit number of the firm goes up. Once Y_P reaches the cut-off level implied by equation 3 every one is served.

For Y_P, such that, $$\frac{3\delta_R + 4\delta_P}{2[\delta_R + \delta_P]} \left[\frac{tF}{\delta_R + 2\delta_P} \right]^{\frac{1}{2}} \; < \; Y_P\theta - c \; < \; \frac{3}{2} \left[\frac{tF}{\delta_R + \delta_P} \right]^{\frac{1}{2}}$$

there is a kink equilibrium where the marginal poor who is indifferent between the two adjacent firms is also indifferent in buying and not buying.

Also it warrants attention that compared to the *case 1,* price here is lower, though the number of firms is also low. And within the parameter restriction as Y_P increases, price remains unchanged and the number of firms goes up. This has a direct implication on the utility of the rich , implying that as Y_P increases, welfare of rich improves, as the average distance traveled reduces.

As the income of the poor keeps falling more and more poor are left unserved and in the limit all poor are left out. This occurs when the poor who is even at the location of the firm is better-off without consumption. Relevant income level of poor for which this holds is determined from the following equality:

$$Y_P\theta - p \;=\; Y \;\;\Rightarrow\; Y_P\theta \;=\; \left[\frac{tF}{\delta_R + 2\delta_P}\right]^{\frac{1}{2}} + c \tag{4}$$

Observe that the two cut-off of Y_P implied by equation 1 , 3 and 4 are relaxed as θ increases. θ can also be interpreted as individuals valuation. So if valuation increases, then more poor will avail the relevant service. For example if people become more informed about the benefits of education or become more health conscious then individuals participation will go up. This emphasises on role of programs directed towards making people more aware.[3]

It is important to note, that unlike in the Betrand model, there are different prices for the same good/service depending on the income distribution, even with perfect competition. And interestingly, on controlling for distribution, as the density of population increases, price for the same good falls. This is possible as now individuals are also distributed with respect to their location, aswell.

3.1.3 case 3. All POOR are left unserved:

Below the income level implied by equation 4, poor even at the location of the firm is better-off not buying at all. This is true when income of poor is so low, that they can not even

[3]WaterAid-India's rural sanitation program was making slow progress in 1995-96. A lack of demand from households meant that partner NGOs had constructed only 460 out of 1,100 latrines planned for the 12-month period. WaterAid-India decided that it was time to reformulate its strategy and focus on marketing sanitation. As a result of this change in approach, by the first six months of 1997-98, partner NGOs had achieved a dramatic turnaround in demand and constructed 5,000 latrines

afford the mill price. Only rich constitutes the demand for the firm. Equilibrium p and n are given by:

$$p = \left[\frac{tF}{\delta_R}\right]^{\frac{1}{2}} + c \quad \text{and} \quad n = L\left[\frac{t\delta_R}{F}\right]^{\frac{1}{2}}$$

$$\text{Also} \quad Y_R\theta - p - \frac{tL}{2n} \ > \ Y_R \quad \Rightarrow \quad \frac{3}{2}\left[\frac{tF}{\delta_R}\right]^{\frac{1}{2}} + c \ < \ Y_R\theta \tag{5}$$

It warrants a mention that once the density is controlled for equilibrium values of n, p and the income level for which firms are competing for rich is the same as in *case 1,* when firms are competing with respect to both rich and poor. As of now we have considered the case when poor are staying in a mixed community. In order to make valid comparisons we need to look at the equilibrium when an individual stays in a segregated economy.

3.2 Segregated Economy:

Here we look at the equilibrium when poor are staying in the segregated community.

3.2.1 Monopoly equilibrium defining feasibility condition:

Potential monopoly market of the representative firm is given by: $\dfrac{2\delta_P[Y\theta - p]}{t}$

$$\text{For } \ Y\theta - c \ = \ \sqrt{\frac{2tF}{\delta_P}}, \quad n \ = \ L\sqrt{\frac{t\delta_P}{2F}} \quad \text{and} \quad p = \frac{Y\theta + c}{2} \tag{6}$$

It will be feasible for the firm to operate only if $\pi \geq 0 \Rightarrow Y\theta - c \geq \sqrt{\dfrac{2tF}{\delta_P}}$.

Implying that for Y_P below this level there will be no firm in the market and all consumers will be left unserved.

3.2.2 Competitive equilibrium:

Equilibrium here is same as *case 3,* where only one income type is being served.

$$\text{It holds for } \ Y\theta \geq \ \frac{3}{2}\sqrt{\frac{tF}{\delta_P}} \ + \ c \ \text{ and } \ p = \sqrt{\frac{tF}{2\delta_P}} + c \quad \text{and} \quad n = L\sqrt{\frac{t\delta_P}{F}}$$

There is *kink equilibrium* for $\sqrt{\dfrac{2tF}{\delta_P}} \ < \ Y\theta - c \ < \ \dfrac{3}{2}\sqrt{\dfrac{tF}{\delta_P}}$.

3.3 Comparisons

It is particularly intriguing that in the no quality model, we have different outcome in terms of price and number of firms, depending on the income gap. This has direct implication on the utility of the consumers. Exploiting this we are now in the position to make comparisons of the welfare under two set of conditions, one when individuals are staying in a mixed community and the other when they are in a segregated economy. To make any valid comparisons arising out of the income difference we need to control for the density, so we assume that $\delta_R + \delta_P = \delta$ in the mixed community also $\delta_P = \delta_R = \delta$ in the segregated economy.

1. As noted above for $Y_P\theta < \sqrt{\frac{2tF}{\delta_P}} + c$, there will be no firm in the region, as it is not operationally viable. So by living by themselves no poor will be served. Its the presence of rich which not just makes firms viable but also reduces the minimum cut-off for which any poor is served, (follows from *case 2*). So, poor are certainly better-off staying with rich for the following range of Y_P.

$$\sqrt{\frac{tF}{\delta_R + 2\delta_P}} + c < Y_P\theta < \sqrt{\frac{2tF}{\delta_P}} + c$$

   This is because income of rich by itself is high enough to make firms sustainable even in the absence of relatively economically backward, so once firm is there then the poor who are relatively closely located are better-off consuming instead of doing without it. Since at least few poor are getting more than their reservation utility so poor as the community gains.

2. On moving to the other extreme, where firms are competing for poor as well,

$$i.e \quad \text{for} \quad \frac{3}{2}\sqrt{\frac{tF}{\delta_R + \delta_P}} = \frac{3}{2}\sqrt{\frac{tF}{\delta}} < Y_P\theta - c \tag{7}$$

   equilibrium outcome is the same both in the segregated and the mixed community, so the individuals utility is the same. This is because as already discussed, in the no quality model, once the income level is above a certain cut-off level, it has no impact on the equilibrium outcome.

3. For the intermediate range of Y_P, *i.e.*

$$i.e. \quad \text{for} \quad \sqrt{\frac{2tF}{\delta_P}} + c \ \leq \ Y_P\theta \ \leq \ \frac{3\delta_R + 4\delta_P}{2[\delta_R + \delta_P]}\sqrt{\frac{tF}{\delta_R + 2\delta_P}} + c$$

comparison of the utility is not so straight forward. So we evaluate the welfare of the poor at these two extremes. We first look at the case where $Y_P\theta = \sqrt{\frac{2tF}{\delta_P}} + c$. In the *segregated community* as noted above all poor are consuming. So the welfare of the poor is given by:

$$2n\int_0^{\frac{L}{2n}} \{Y_P\theta - p - tx\}dx$$

Next consider the case where the consumers are staying in the *mixed* community, relevant case for this income level is the case 2, firms are competing only for the rich but is a monopolist with respect to poor. Some poor are left out of the market, so welfare of poor given by

$$2n\int_0^{\nu} \{Y_P\theta - p - tx\}dx + 2n\int_{\nu}^{\frac{L}{2n}} Y_P \, dx$$

where ν is determined such that marginal poor is indifferent in buying and not buying. It has been shown in the appendix that poor are better-off staying alone instead of mixing with the rich. This is because equilibrium price in the segregated economy given by $\sqrt{\frac{tF}{2\delta_P}} + c$ which equals $\sqrt{\frac{tF}{2[\delta_R+\delta_P]}} + c$ (on controlling for density) is strictly lower than the price in the mixed community given by $\sqrt{\frac{tF}{\delta_R+2\delta_P}} + c$, even though the number of firms is lower. It implies that the price affect dominates the distance affect, because of which in segregated economy all poor consume though some of them are left out in the mixed community, leading to higher welfare when they are staying alone.

4. Moving to the other end, *i.e* when $Y_P\theta - c = \frac{3\delta_R + 4\delta_P}{2(\delta_R + \delta_P)}\left[\frac{tF}{\delta_R + 2\delta_P}\right]^{\frac{1}{2}}$. In the *segregated economy* for this level of Y_P, relevant equilibrium is the kinked one.

$$\text{as} \quad \frac{3\delta_R + 4\delta_P}{2(\delta_R + \delta_P)}\sqrt{\frac{tF}{\delta_R + 2\delta_P}} \ < \ \frac{3}{2}\sqrt{\frac{tF}{\delta_R + \delta_P}} \ = \ \frac{3}{2}\sqrt{\frac{tF}{\delta}}$$

So the price is set such that, the marginal consumer indifferent between the two adjacent firms is also indifferent in buying and not buying. In the mixed community is also, this level of Y_P defines the cut-off over which all poor is served, so the expression for total welfare is the same as above. Also this income level marks the start of the kink. Again the price determined in the same way as above. It has been shown in the appendix that the welfare of the consumer is same in both cases.

Above analysis implies that for the very low level of income, poor are benefit from the rich presence of rich, then there exists a range when they are better-off staying in segregated economy as because of rich price is relatively higher, but this difference in the welfare diminishes as Y_P increases and completely fades out once all poor are served. This leads to the following proposition:

Proposition 1. Poor are better-off staying with rich for income the income level $\sqrt{\frac{tF}{\delta_R+2\delta_P}}+c < Y_P\theta < \sqrt{2tF}+c$ for $\sqrt{\frac{2tF}{\delta_P}}+c \leq Y_P\theta \leq \frac{3\delta_R+4\delta_P}{2[\delta_R+\delta_P]}\sqrt{\frac{tF}{\delta_R+2\delta_P}}+c$ poor are worse-off staying with the rich, but rich benefit from the presence of poor, but for $\frac{3\delta_R+4\delta_P}{2[\delta_R+\delta_P]}\sqrt{\frac{tF}{\delta_R+2\delta_P}}+c < Y_P\theta$ it makes no difference whether poor are staying in the mixed or segregated economy.

Firms are monopolist with respect to rich: Through-out the above analysis we assumed that rich are rich enough, but in the initial stages of development income difference between poor and rich might not be substantial. We can not allow income of the rich to be too low for the technical reasons, so for the simplicity assume that the income level of rich is low such that the marginal rich who is indifferent between the two adjacent firms is also indifferent in buying and not buying. This implies that for this level of Y_R and for $Y_R > Y_P$ some poor will be left out for sure, but the cut-off level of Y_P for which no poor is served is lower here than above.

Proposition 2. *Cut-off level of Y_P for which no poor is served in the mixed community is lower when rich are relatively poor than the case when the firms have are competing with respect to them, implying that for low levels of Y_P welfare of poor improves in case income gap between poor and rich is not substantial.*

Proof of the above proposition follows by comparing the two cut-off levels of Y_P in the two scenarios in the mixed community. In the case where firms are competing with respect to rich the relevant cut-off level, as shown above is $\sqrt{\frac{tF}{\delta_R+2\delta_P}}+c$ where as for the case when firms are monopolist with respect to rich it is $\sqrt{\frac{tF}{2[\delta_R+\delta_P]}}+c$. This cut-off level of Y_P is evaluated in the appendix.

4 When quality choice is endogenous

As development makes inroads one observes qualitative different in the lives of people. But there are many who are not touched by income rise and are left in the wraps of poverty. Individuals are not segregated with respect to their incomes and what one generally observes is the juxtaposition of people of disparate circumstances in limited space. Firms responds to this income diversity while taking decisions. What role does this income difference plays, is especially important when quality is endogenous. Propelled by the income growth, malls culture takes over. Does this transformation aggravates situation of poor or even they are able to benefit from others fortunes? Is the gain of one group loss for the other? Does the market outcome might be such that it actually leads to exacerbation of poverty because of the increased market prices?

Like the previous model we continue to assume that there are two income groups, with income Y_R and income Y_P uniformly distributed across the circular city with density δ_R and δ_P respectively, on the circular city of length normalised to one. The value to consumer (z, Y) of one unit of product of quality θ_j sold at price p_j is:

$$U(z, Y, x_j, \theta_j, p_j) = Y\theta_j - p_j - t(z - x_j) \tag{8}$$

The utility function is continuous in both p_j and θ_j. Note that the particular form of the utility function implies that U everywhere satisfies "single-crossing" condition (SCI):

$$\delta\left(\frac{\delta U/\delta\theta}{\delta U/\delta Y}\right)/\delta Y \;>\; 0$$

Hence any indifference curve in (θ, Y)- plane of a higher income household cuts a indifference curve of lower-income household from below. So the marginal rate of substitution between price and quality, $MRS_{p_j\theta_j}$, is higher for the consumers with income Y_R. So for the same increase in the quality people with higher income are willing to pay more. This is a reasonable, as the richer individual is more likely to be informed about the benefits of better quality, hence would value it more. We assume three stage game for the firm, where in the first stage firm decides whether to enter the market or not. In the second stage firm decides about its location and in the third stage firms simultaneously decide on the price and quality. We restrict ourselves to symmetric equilibrium. Profit of the firm is given by:

$$\pi_j(p, \theta_j, x, n) = [p - c(\theta_j)]D_j(p, \theta_j, x, n) - F(\theta_j) \tag{9}$$

where $p = (p_1, ...p_n)$ $\theta = (\theta_1, ...\theta_n)$ $x = (x_1, ...x_n)$ are the n-tuples of strategies of prices , quality specifications and locations of firms respectively. We assume that the marginal cost is invariant with respect to output level (D_j) but varies costively with quality. Fixed cost is assumed to be convex in quality.

$$\begin{aligned} c_{\theta_j}(\theta_j) > 0, \qquad c_{\theta_j\theta_j}(\theta_j) \geq 0, \\ F_{\theta_j}(\theta_j) > 0, \qquad F_{\theta_j\theta_j}(\theta_j) \geq 0, \end{aligned} \tag{10}$$

As stressed earlier, our focus is on goods and services which are required at the frequent intervals, like barber service, education, health care etc, and hence the distance becomes the crucial factor in making the choice. Marginal cost of such services depends on the quality of resources. Superior quality means higher outlay, for instance better qualified professionals, better computers etc. So one would expect marginal cost to increase with the increase in the quality. Also the level of physical investment also depends on the quality. For example as an economy experiences huge increases in income one can see transition from small health centers to big hospitals equipped with all the modern day apparatus. Similarly coming to the education, one can see evolution of dilapidated schools in rural areas to dominating structures in the urban area.

Again as in the previous section, we assume that income of rich is high enough that firms are competing with respect to them, and focus on equilibrium structure depending on the income level of poor. With respect to poor one can identify three cut-off levels of Y_P and demand from poor is determined by where does the actual income of poor lie.

4.1 Marginal poor consumer who is indifferent between the two adjacent firms is better-off buying

With the quality choice marginal consumer indifferent between firm j and firm $j+1$ is determined by equating, $U(Y, \theta_j, x_j, p_j, z_i) = U(Y, \theta_{j+1}, x_{j+1}, p_{j+1}, z_i)$

Working in the similar way as in no quality choice model implies equilibrium:

$$n = L\sqrt{\frac{t(\delta_R + \delta_P)}{F(\theta)}} \quad \text{and} \quad p = \sqrt{\frac{tF(\theta)}{\delta_R + \delta_P}} + c(\theta)$$

Where, θ is determined from:

$$\frac{Y_R\delta_R + Y_P\delta_P}{\delta_R + \delta_P} = F^{'}(\theta)\left[\frac{t}{F(\theta)(\delta_R + \delta_P)}\right]^{\frac{1}{2}} + c^{'}(\theta) \tag{11}$$

Y_P enters in the same as Y_R in determination of equilibrium outcome. Both n and p depend on θ implying that unlike the no quality choice model income has role to play even when firms compete for both poor and rich. Price is increasing in quality where as number of firms falls pointing towards the kind of economic development where the mall replaces the small retail shops. With the increase in quality level price goes up. Fixed cost also increases, implying that the total number of firms should fall to let each firm break-even.

This case holds when the marginal poor consumer, indifferent between the two adjacent firms is better-off consuming.

$$\Rightarrow \frac{Y_P(\theta_j - \theta_{j+1}) - (p_j - p_{j+1}) + t(x_j - x_{j+1})}{2t} = \frac{L}{2n} < \frac{Y_P(\theta_j - 1) - p_j}{t}$$

Substituting for the equilibrium implies that above case will hold when:

$$\frac{3}{2}\sqrt{\frac{tF(\theta)}{\delta_R + \delta_P}} < Y_P(\theta - 1) - c(\theta) \tag{12}$$

Observe that unlike in the basic model, cut-off level of Y_P depends on quality level which depends on the income level of rich as well. So the level of welfare of poor clearly depends on the income of rich and hence on the income gap. So it is not just the absolute level of income but relative income is important in determining the welfare of poor. Aggregate welfare of all poor is given by:

$$2n\int_0^{\frac{L}{2n}} \{Y_P\theta - p - tx\}dx$$

Simple substitution and assuming $L = 1$ implies that total consumer welfare equals

$$Y_P\theta - \frac{5}{4}\sqrt{\frac{tF(\theta)}{\delta_R + \delta_P}} - c(\theta) \tag{13}$$

From the above equation it follows that for a given level of Y_P there exists a level of θ say θ_s at which aggregate welfare of poor as a community is maximised. The level of θ which satisfies the following equation.

$$Y_P = \frac{5}{8}F^{'}(\theta)\left[\frac{t}{F(\theta)(\delta_R + \delta_P)}\right]^{\frac{1}{2}} + c^{'}(\theta) \tag{14}$$

In the mixed community level of θ is determined from:

$$\frac{Y_R\delta_R + Y_P\delta_P}{\delta_R + \delta_P} = F^{'}(\theta)\left[\frac{t}{F(\theta)(\delta_R + \delta_P)}\right]^{\frac{1}{2}} + c^{'}(\theta) \tag{15}$$

where as in the segregated poor community quality level is determined from:

$$Y_P = F^{'}(\theta)\left[\frac{t}{F(\theta)(\delta_R + \delta_P)}\right]^{\frac{1}{2}} + c^{'}(\theta) \tag{16}$$

It clearly follows that when the income gap between poor and rich is not substantial then poor are infact better-off staying in the mixed community. This follows clearly from the picture below.

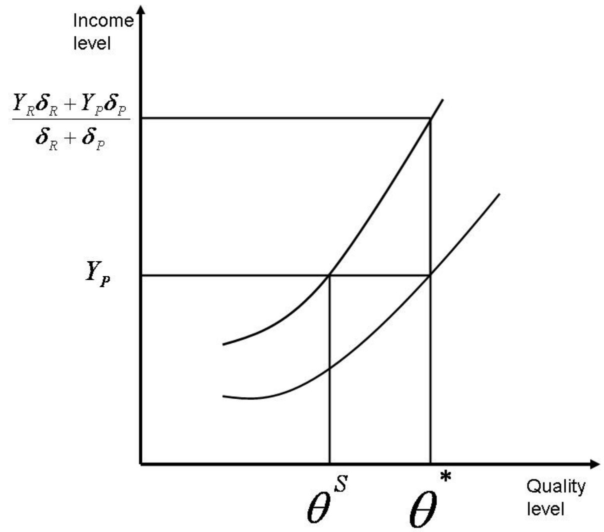

Figure 2: Relation between aggregate welfare of poor and Y_R

From the above equations it is also clear that aggregate welfare curve is an inverted U-shaped curve which attains maximum at θ_P^*. So, in the mixed community when the gap between poor and rich is not much then the welfare of poor initially increases and reaches it maximum, but as the income gap widens then increase in utility is not enough to offset the corresponding rising cost, so the welfare falls. This is shown in the figure below.

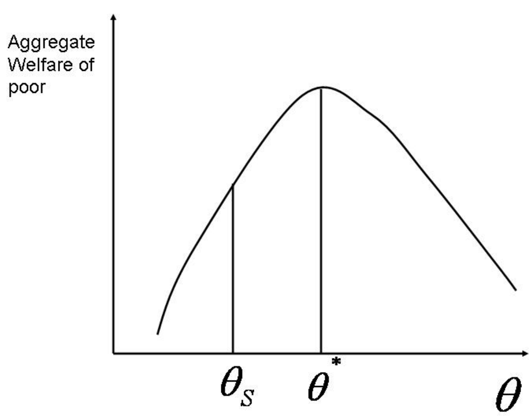

Figure 3: Relation between aggregate welfare of poor and income level

This leads us to *proposition3*, which follows from the above figures.

Proposition 3. *There exists a level of Y_R which corresponds to the socially optimal level of quality for poor in the segregated community, implying that poor as a community are better-off staying in the mixed community when relative income gap is not much.*

This is intuitive and also quite apparent from the data. As the income of rich increases given the level of Y_P level of quality goes up hence the gross utility of individuals goes up, but with increase in θ price and the distance traveled by the marginal consumer also rise, leading to fall in utility. Ultimately what happens to consumers welfare depends on which affect dominates. For the low income divergence even poor gain as they have access to better quality but when income disparity is too stark then they loose out owing to increases cost. For example preliminary investigation of NSSO data reveals that health expenditure of person from the poorest quintile of urban population initially increases as the person moves from very poor urban region to poor urban region but it falls as he moves from very poor urban region to urban rich region, implying that there is U-shaped relation between the average health expenditure of person from the poorest quintile of urban population and state per capita GDP of that region.

Above is what happens at the aggregate, but what happens to the marginal consumer depends on the it can be shown that, $\bar{Y_P}$ and $\bar{\theta}$, the cut-off level of Y_P and θ, at which poor is just indifferent in buying and not buying increases as Y_R increases, implying that as quality goes up in response to exogenous factors, *marginal* poor consumer is worse-off as increase in gross utility is not enough to outweigh increased cost. So to bring back consumer to the same utility level $\bar{Y_P}$ increases. This leads to the following proposition.

Proposition 3. *$\bar{Y_P}$ and $\bar{\theta}$ increases as Y_R increases if $c^{"}(\theta) \geq 0 \quad$ and $\; F^{"}(\theta) > \frac{[F^{'}(\theta)]^2}{2F(\theta)}$, implying that the marginal poor consumer is worse-off if income gap between poor and rich widens.*

Above proposition has been proved analytically in the appendix. Following figure illustrates the idea.

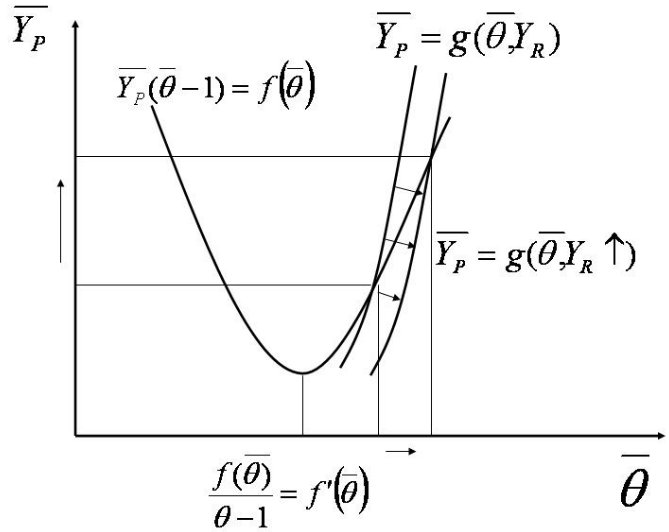

Figure 4: Impact of increase in Y_R on $\bar{Y_P}$ and $\bar{\theta}$

This implies that with convex cost structure as quality level increases without increase in Y_P , poor as community gains for certain income gap but the marginal consumer is always worse-off, as the benefit of quality increase is not enough to outweigh the loss due to higher price and distance.

4.2 Marginal consumer indifferent between the two adjacent firms is better-off not buying

As in the previous section this case implies that there are some poor who are left out of the market by the virtue of higher distance. Again it can be shown, that the cut-off level below which some poor are left unserved is increasing in the income of rich. This highlights the impact of rising income disparity. Demand from rich is evaluated in the same way as above and demand from the poor is determined by the distance of the marginal consumer indifferent in buying and not buying.

$$D_j = \frac{\delta_R}{2t} \times [Y_R(2\theta_j - \theta_{j+1} - \theta_{j-1}) - (2p_j - p_{j+1} - p_{j-1}) + t(x_j, x_{j+1}) + t(x_j, x_{j-1})] + 2\delta_P \left[\frac{Y_P(\theta_j - 1) - p_j}{t}\right]$$

As in the basic model demand is more elastic as poor constitutes firms captive market, and hence the gain to firm from fall in price is more from poor than from rich. So firms give more weightage to poor as compared to rich. This is reflected in the expressions for n and p below:

$$p = \sqrt{\frac{tF(\theta)}{\delta_R + 2\delta_P}} + c(\theta) \quad \text{and} \quad n = \frac{tL\delta_R}{\delta_R + 4\delta_P\left[\sqrt{\frac{tF(\theta)}{\delta_R+2\delta_P}} - \frac{2\delta_P}{\delta_R+4\delta_P}[Y_P(\theta-1) - c(\theta)]\right]}$$

θ in this case is determined from the following equation:

$$\frac{Y_R\delta_R + 2Y_P\delta_P}{\delta_R + 2\delta_P} = F^{'}(\theta) \times \left[\frac{t}{F(\theta)[\delta_R + 2\delta_P]}\right]^{\frac{1}{2}} + c^{'}(\theta) \tag{17}$$

Cut-off level of Y_P for which this is true is implied by the following equation:

$$Y_P(\theta - 1) - c(\theta) < \frac{3\delta_R + 4\delta_P}{2[\delta_R + \delta_P]}\left[\frac{tF(\theta)}{\delta_R + 2\delta_P}\right]^{\frac{1}{2}}$$

Let $\tilde{Y_P}$ be the cut-off level of Y_P, at which this equation holds with equality. For $Y_P < \tilde{Y_P}$ distance bites, in the sense some individuals even at the same income level are left unserved because of the increased cost.

5 Remarks

- $\bar{Y_P} > \tilde{Y_P}$, as $\bar{Y_P}$ is the level of Y_P above which the marginal poor consumer indifferent between the two adjacent firms is better-off buying, while $\tilde{Y_P}$ is the level of Y_P below which marginal poor consumer indifferent between two adjacent firms gets less than his reservation in case he buys, so $\tilde{Y_P} > \bar{Y_P}$, is never a possibility.

- In the above paper we have restricted ourselves only to the symmetric equilibrium, where there is just one quality, though generally depending on the income dispersion there are many qualities, but what the above analysis portrays is the average picture in some sense.

- In case minimum quality restriction is binding then poor as a community might be worse-off if $\theta > \theta^*$.

6 Conclusion

In this paper we tried to explore the relationship between the income dispersion and economic well-being of poor. The notion is compelling and thought provoking as it shows that how sheer presence of another income group can influence individuals choices and quality of life. For example, in India it has been revealed by the census data that the literacy level of poor, (individuals at the lowest decile of income level) is higher in the urban region as compared to the rural region. This points towards the lack of availability of good quality schools in the rural regions because of the low income level of poor. As is evident, the major theme that runs through the paper is that whether poor are better-off staying in the segregated community or in a mixed community. While in the scenario when there is no quality choice, poor gain when their income level is very low, as it is not sustainable for the firm to open up in the poor region, but this gain erodes once the income gap is narrow. In the model where there exists endogenous quality choice, poor gain from better quality when income gap is not substantial, but when this divergence rises, then the quality level which is determined by the weighted average of the income level does not reflects preference of poor. Even though the paper has limitation, as it restricts itself to the symmetric equilibrium but it raises an important issue that needs to further addressed in future research.

7 APPENDIX

Proof of proposition 1.

1. *Comparison of welfare of poor at* $Y_P\theta = \sqrt{\frac{2tF}{\delta_P}} + c$ *in a mixed community versus a segregated economy.*

   As noted welfare of poor in the *segregated community* is given by:

$$2n \int_0^{\frac{L}{2n}} \{Y_P\theta - p - tx\}dx$$

Simple substitutions and normalising $L = 1$ implies welfare equals:

$$Y_P + \frac{1}{4}\left[\frac{2tF}{\delta_R + \delta_P}\right]^{\frac{1}{2}}$$

In the *mixed* community, welfare of poor given by:

$$2n \int_0^{\nu} \{Y_P\theta - p - tx\}dx + 2n \int_{\nu}^{\frac{L}{2n}} Y_P \, dx$$

where $\nu = \frac{1}{t}[Y_P\theta - p]$. Simple substitution and normalising $L = 1$ implies, welfare equals:

$$nt\nu^2 + Y_P$$

On substituting for the equilibrium values above expression reduces to:

$$\frac{\delta_R\left[\left[\frac{2tF}{\delta_R+\delta_P}\right]^{\frac{1}{2}} - \left[\frac{tF}{\delta_R+2\delta_P}\right]^{\frac{1}{2}}\right]^2}{(\delta_R + 4\delta_P)\left[\left(\frac{tF}{\delta_R+2\delta_P}\right)^{\frac{1}{2}} - \frac{2\delta_P}{\delta_R+4\delta_P}\left[\frac{2tF}{\delta_R+\delta_P}\right]^{\frac{1}{2}}\right]} + Y_P$$

Poor will be better staying in the mixed community instead of segregated only if:

$$\frac{1}{4}\left[\frac{2tF}{\delta_R + \delta_P}\right]^{\frac{1}{2}} > \frac{\delta_R\left[\left[\frac{2tF}{\delta_R+\delta_P}\right]^{\frac{1}{2}} - \left[\frac{tF}{\delta_R+2\delta_P}\right]^{\frac{1}{2}}\right]^2}{(\delta_R + 4\delta_P)\left[\left(\frac{tF}{\delta_R+2\delta_P}\right)^{\frac{1}{2}} - \frac{2\delta_P}{\delta_R+4\delta_P}\left[\frac{2tF}{\delta_R+\delta_P}\right]^{\frac{1}{2}}\right]}$$

To simplify things we work with the case $\delta_R = \delta_P = \delta$. After substitution it turns out that poor are better-off staying in the segregated economy instead of mixing with rich.

2. *Evaluating welfare of poor, for the other extreme, i.e when* $Y_P\theta - c = \frac{3\delta_R + 4\delta_P}{2(\delta_R + \delta_P)}\left[\frac{tF}{\delta_R + 2\delta_P}\right]^{\frac{1}{2}}$

Segregated economy: Price is set such that, the marginal consumer indifferent between the two adjacent firms is also indifferent in buying and not buying $\Rightarrow p = Y_P\theta - \frac{tL}{2n}$

$$\pi_j = [p_j - c]D_j - F = [Y_P\theta - \frac{tL}{2n} - c]\frac{\delta L}{n} - F$$

$$\frac{L}{n} \text{ is determined such that } \pi_j = 0 \Rightarrow \frac{t\delta}{2}\left[\frac{L}{n}\right]^2 - [Y_P(\tilde{u} - \bar{u}) - c]\frac{\delta L}{n} + F = 0$$

Since all poor are consuming so welfare of the consumers is given by:

$$2n\int_0^{\frac{L}{2n}} \{Y_P\theta - p - tx\}dx$$

In case of *mixed community* also this income level marks the start of the kink. Price is again determined so as to make the marginal consumer indifferent in buying and not buying. And the relevant equation which determines the level of $\frac{L}{n}$ is also the same. These observations imply welfare of poor is same in both the cases.

Proof of proposition 2.

Demand when consumers of both types are just indifferent in buying and not buying:

$$\Rightarrow D_R = \frac{2\delta_R[Y_R\theta - p]}{t} \text{ and } D_P = \frac{2\delta_P[Y_P\theta - p]}{t}$$

$$\pi = 2[p - c]\left[\frac{(Y_R\delta_R + Y_P\delta_P)\theta - p(\delta_R + \delta_P)}{t}\right] - F$$

$$\text{FOC wrt Price: } \Rightarrow p = \frac{(Y_R\delta_R + Y_P\delta_P)\theta}{2(\delta_R + \delta_P)} + \frac{c}{2}$$

$$\text{Zero-profit condition implies: } p = \frac{(Y_R\delta_R + Y_P\delta_P)\theta}{(\delta_R + \delta_P)} - \left[\frac{tF}{2(\delta_R + \delta_P)}\right]^{\frac{1}{2}}$$

$$\text{On equating the two prices we have: } \frac{(Y_R\delta_R + Y_P\delta_P)\theta}{(\delta_R + \delta_P)} - c = \left[\frac{2tF}{(\delta_R + \delta_P)}\right]^{\frac{1}{2}}$$

Given that some poor are consuming would imply: $D_P = \frac{2\delta_P[Y_P\theta - p]}{t} > 0$

After substitutions this is equivalent to: $Y_P\theta - \left\{ \left[\frac{2tF}{(\delta_R+\delta_P)}\right]^{\frac{1}{2}} + c - \left[\frac{tF}{2(\delta_R+\delta_P)}\right]^{\frac{1}{2}} \right\} > 0$

$$\Rightarrow \text{ Some poor will be served only if: } Y_P\theta - c > \left[\frac{tF}{2(\delta_R+\delta_P)}\right]^{\frac{1}{2}}$$

Proof of proposition 3.

Equation 16 implies:

$$\frac{d\theta}{dY_P} = \frac{\delta_P}{\left[\frac{t(\delta_R+\delta_P)}{F(\theta)}\right]^{\frac{1}{2}}\left[F^{"}(\theta) - \frac{[F^{'}(\theta)]^2}{2F(\theta)}\right] + c^{"}(\theta)(\delta_R+\delta_P)}$$

$$\Rightarrow \quad \frac{d\theta}{dY_P} > 0 \text{ if } c^{"}(\theta) \geq 0, \quad \text{and} \quad F^{"}(\theta) > \frac{[F^{'}(\theta)]^2}{2F(\theta)} \quad \text{which we assume to hold.}$$

$$\text{Let } f(\bar{\theta}) = \frac{3}{2}\left[\frac{tF(\theta)}{\delta_R+\delta_P}\right]^{\frac{1}{2}} + c(\theta) \Rightarrow \bar{Y_P}(\bar{\theta}-1) - f(\bar{\theta}) = 0 \tag{18}$$

Given that $F^{"}(\theta) > \frac{[F^{'}(\theta)]^2}{2F(\theta)}$ sufficient condition for $\sqrt{F(\theta)}$ to be convex in quality is satisfied, implying that $f(\bar{\theta})$ is convex in quality.

Where $\bar{\theta}$ is evaluated from equation 16 , which can be rewritten as:

$$\bar{Y_P} = \frac{-Y_R\delta_R}{\delta_P} + \frac{\delta_R+\delta_P}{\delta_P}\left\{F^{'}(\bar{\theta})\left[\frac{t}{F(\bar{\theta})(\delta_R+\delta_P)}\right]^{\frac{1}{2}} + c^{'}(\bar{\theta})\right\}$$

This can be written as:

$$\bar{Y_P} - g(\bar{\theta}, Y_R) = 0 \tag{19}$$

On totally differentiating the two equations, 18 and 19 can be expressed as:

$$(\bar{\theta}-1)d\bar{Y_P} + [\bar{Y_P} - f^{'}(\bar{\theta})]d\bar{\theta} = 0 \tag{20}$$

$$\text{and} \quad d\bar{Y_P} - \frac{\delta g}{\delta \theta} d\bar{\theta} = \frac{\delta g}{\delta Y_R} dY_R \tag{21}$$

$$\text{After substitution equation 20 implies, } (\bar{\theta} - 1)d\bar{Y_P} \; + \; \left[\frac{f(\bar{\theta})}{\bar{\theta} - 1} - f^{'}(\bar{\theta})\right] d\bar{\theta} \; = \; 0$$

$$i.e. \; \frac{d\bar{Y_P}}{d\bar{\theta}} \; = \; -\frac{\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})}{\bar{\theta} - 1}$$

implying that it is U-shaped curve and it at its minimum at the level of θ where difference between marginal and the average equals zero, *i.e.* there exists a level of $\bar{\theta}$ at which $\frac{f(\bar{\theta})}{\bar{\theta}-1} = f^{'}(\bar{\theta})$ which equals Y_P from equation 18. Level of θ relevant for this case is determined from equation 16.

$$\frac{Y_R \delta_R + Y_P \delta_P}{\delta_R + \delta_P} = F^{'}(\theta)\left[\frac{t}{F(\theta)(\delta_R + \delta_P)}\right]^{\frac{1}{2}} + c^{'}(\theta)$$

Which implies a positive relation between Y_R and θ, so the relevant portion of the curve implied by the equation 18 is the upward sloping part.

$$\text{And equation 21 implies, } \frac{d\bar{Y_P}}{d\bar{\theta}} \; = \; \frac{\delta g}{\delta \theta} \; > 0$$

In matrix notation above equations can be written as:

$$\begin{pmatrix} \bar{\theta} - 1 & \bar{Y_P} - f^{'}(\bar{\theta}) \\ 1 & -\frac{\delta g}{\delta \theta} \end{pmatrix} \begin{pmatrix} d\bar{Y_P} \\ d\bar{(\theta)} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\delta g}{\delta Y_R} dY_R \end{pmatrix}$$

Following Cramer's rule:

$$d\bar{\theta} = \frac{(\bar{\theta} - 1)\frac{\delta g}{\delta Y_R} dY_R}{-(\bar{\theta} - 1)\frac{\delta g}{\delta \theta} - [\bar{Y_P} - f^{'}(\bar{\theta})]}$$

$$\Rightarrow \frac{d\bar{\theta}}{dY_R} = \frac{-\frac{\delta g}{\delta Y_R}}{\frac{\delta g}{\delta \theta} + \frac{\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})}{\bar{\theta}-1}}$$

It is clear that numerator is positive, as $\frac{\delta g}{\delta Y_R} \; < \; 0$ implying that $\frac{d\bar{\theta}}{dY_R} \; > \; 0$ if the denominator is positive. Also $\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta}) \; < \; 0$, when the curve slopes upward implying that the denominator will be positive if $\frac{\delta g}{\delta \theta} \; > \; -\frac{1}{\bar{\theta}-1}\left[\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})\right]$, *i.e.* if the slope of equation 19 is greater than $\frac{1}{\bar{\theta}-1} \times$ slope of equation 18.

This condition can be written as:

$$\frac{\delta g}{\delta \theta} \;>\; -\frac{1}{\bar{\theta}-1}\left[\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})\right]$$

$$\text{Where } \frac{\delta g}{\delta \theta} = \frac{\delta_R + \delta_P}{\delta_P}\left\{\sqrt{\frac{tF(\theta)}{\delta_R + \delta_P}} \times \left\{F^{"}(\theta) - \frac{1}{2}\frac{[F^{'}(\theta)]^2}{F(\theta)}\right\} + c^{"}(\theta)\right\}$$

For the relevant θ it has been argued, $f^{'}(\bar{\theta}) \;>\; \frac{f(\bar{\theta})}{\bar{\theta}-1} \;\Rightarrow\; f^{"}(\bar{\theta}) \;>\; -\frac{1}{\bar{\theta}-1}\left[\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})\right]$

It can be easily shown that $\frac{\delta g}{\delta \theta} \;>\; f^{"}(\bar{\theta})$ implying that denominator is positive.

As Y_R increases quality level goes up. But what does this imply for $\bar{Y_P}$ for this we need to check sign of $\frac{d\bar{Y_P}}{dY_R}$.

$$\frac{d\bar{Y_P}}{dY_R} = \frac{-\frac{\delta g}{\delta Y_R}[\bar{Y_P} - f^{'}(\bar{\theta})]}{-(\bar{\theta}-1)\frac{\delta g}{\delta \theta} - [\bar{Y_P} - f^{'}(\bar{\theta})]}$$

Substituting for $\bar{Y_P}$:

$$\Rightarrow \frac{d\bar{Y_P}}{dY_R} = \frac{\frac{\delta g}{\delta Y_R}[\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})]}{(\bar{\theta}-1)\frac{\delta g}{\delta \theta} + [\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta})]}$$

Again the numerator is positive as $\frac{\delta g}{\delta Y_R} \;<\; 0$ and $\frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta}) \;<\; 0$ and denominator is positive if above condition on slopes of two equations is satisfied.

$$\frac{d\bar{Y_P}}{dY_R} \;>\; 0 \text{ if } (\bar{\theta}-1)\frac{\delta g}{\delta \theta} + \frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta}) \;>\; 0 \text{ given } \frac{f(\bar{\theta})}{\bar{\theta}-1} - f^{'}(\bar{\theta}) \;<\; 0.$$

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