Optimal Monetary and Fiscal Policy in a Currency Union with Nontradables

OKANO, Eiji*

This Draft: Jan. 2008

Abstract

By constructing a form of DSGE model, this paper verifies the necessity for an optimal monetary and fiscal policy under a currency union with non-tradable goods. As indicated by canonical literature, a solitary optimal monetary policy can maximize social welfare through stabilizing the producer price inflation and output gap in each country simultaneously when all goods are tradable, in spite of there being two countries and a single central bank. However, a solitary optimal monetary policy cannot maximize social welfare because of the Balassa–Samuelson Theorem, which holds that stabilizing the producer price inflation and output gap in each country simultaneously is disrupted, resulting from a consumption disparity between two countries when non-tradable goods exist. In this case, an optimal monetary and fiscal policy maximizes social welfare. This

*Correspondence to: Center on Japanese Economy and Business, Columbia University, 321 Uris Hall, 3022 Broadway, New York, NY 10027, USA. Tel: +1-212-854-3981 (Direct); Fax: +1-212-678-6658; E-mail: eo2217@columbia.edu
result, which supports implications derived from the DGSE framework, underlines the necessity of a policy mix, and is affirmed by canonical literature.
1 Introduction

A currency union that had previously been pure academic speculation became a reality when the European Monetary Union (EMU) was established. Needless to say, the creation of the EMU has led to new challenges for policy makers. This paper provides a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union and studies its implications for the optimal design of such policies.

Discussions of optimal monetary policy in a currency union have become vigorous. Assuming that all goods are tradable, Benigno (2004) derives an interest policy implication that optimal monetary policy, a synonym for inflation targeting in a simple situation, can maximize social welfare in a currency union that consists of two countries, assuming that, in addition to perfect risk sharing at both domestic and international levels, the economies in the two countries are identical on the demand side. Thus, the solitary central bank in a currency union can achieve welfare maximization. This policy implication is meaningful and amazing from the viewpoint of canonical opinions by Grauwe (1992), who summed up implications derived by Mundel (1961), McKinnon (1963) and Kenen (1969), and mentioned that optimal currency area is realized when one of following conditions is satisfied: (i) flexible nominal wage; (ii) flexible labor mobility; and (iii) fiscal transfer by the centralized government. However, Benigno (2004) shows that none of these conditions is needed to realize optimal currency area if solitary optimal monetary policy is conducted.  

The policy implication derived by Benigno (2004) seems, at a glance, to be one of the New Keynesian’s greatest findings. However, usefulness of optimal monetary policy

\footnote{Benigno (2004) assumes price stickiness rather than wage stickiness, a work force who cannot cross borders, and that there are no fiscal transfers between the countries.}
under a currency area is not settled among New Keynesian authors.²

Gali and Monacelli (Forthcoming) insist on a monetary and fiscal policy mix using a currency union model that, rather than comprising two countries, consists of an infinite number of infinitesimal countries. Under this framework, the solitary central bank can maximize welfare at a union-wide level, whereas it needs some support brought about by fiscal authority to maximize welfare.

From this implication, it can be said that Gali and Monacelli (Forthcoming) show that there is room to discuss centralized government advocated by canonical authors. Because Gali and Monacelli (Forthcoming) assume a currency union that consists of an infinite number of infinitesimal countries, fiscal expenditures in each country can be ignored in a point of view of the whole currency union. If a two-country setting is allowed, as canonical authors and Benigno (2004) adopt, fiscal transfer by the centralized government, which is one of the requirements to make an optimal currency union, may have a great role to maximize social welfare. Although fiscal transfer by the centralized government has not been considered for a long time, it is important to investigate the role of fiscal transfer by the centralized government under the New Keynesian’s context.

To discuss the necessity of monetary and fiscal policy mix, and to discuss the necessity of the centralized government, which is advocated by canonical authors, we should pay attention to the presuppositions that may cause confusion on the policy implications between Benigno (2004) and Gali and Monacelli (Forthcoming). Although canonical studies consider the existence of nontrad-

²Ferrero (2005) also discusses monetary and fiscal policy rule under a currency union with distortionary taxation. Beetsmaa and Jensen (2005) study the implications of fiscal policies in a monetary union. These papers insist that some support brought about by fiscal authority is essential to maximize welfare.
able goods, these studies do not consider the existence of nontradable goods.\(^3\)
While the definition of nontradable goods is not simple, as was mentioned by McKinnon (1963), nontradables, in general, correspond to services towards goods in an actual economy. Following the definition that regards goods produced in the manufacturing industry, agriculture, forestry, fishery and mining as tradables and regards goods produced in other industries as nontradables, as used by Canzoneri, Cumby and Diba (1999), nontradables, in terms of current and purchaser’s price, accounted for 50.3% of the sum of nontradables and tradables in major euro area countries such as Belgium, Germany, France, Greece, Italy, the Netherlands, Portugal and Spain in 1999. It is obvious that the share of nontradables should not be ignored in analyzing monetary policy.

From the above discussions, there are two circumstances to consider when we discuss optimal policy rule under a currency union. One of the circumstances is the existence of nontradable goods and the other is the roles of fiscal transfers by the centralized government.

In consideration of these circumstances, this paper constructs a form of DSGE model that describes a currency union that consists of two countries with nontradable goods to analyze an optimal monetary policy, and an optimal monetary and fiscal policy mix. Needless to say, nontradable goods have a disregarded effect to an economy.\(^4\) Whereas a nominal exchange rate does not appear in our model, because the model is a closed system, the Balassa–Samuelson Theorem explains a disparity in the consumer price indices (CPIs)

---

\(^3\)Neither papers on monetary policy in a currency union nor papers on monetary policy in an open economy, such as Benigno (2004), Benigno and Benigno (forthcoming) and Gali and Monacelli (2005), consider the existence of nontradable goods, although these papers derive some important implications.

\(^4\)Analyzing exchange rate volatility rather than monetary policy, Stockman and Tesar (1995), Benigno and Thoenissen (2005) and Selaive and Tuesta (2006) focus on nontradable goods in the consumption–real exchange rate anomaly. These papers on the Balassa–Samuelson Theorem point out the relationship between the anomaly and the theorem.
between two countries composing a currency union. This circumstance is not considered by Benigno (2004) and is a notable feature in our paper.

Intentions and results in our paper are as follows. First, we show the difficulties of conducting a monetary policy and the necessity of a monetary and fiscal policy mix in a currency union with nontradables. Second, we allow implementation of fiscal policy by a centralized government, which is emphasized by Grauwe (1992) as one of the important conditions of the optimal currency area. We investigate the appropriateness of a centralized government, which is advocated by canonical authors from the viewpoint of welfare maximization, which is one of the important agenda items in recent DSGE literature. By calculating social welfare or loss under various share of nontradables, we show that policy mix is essential to maximize social welfare if nontrdables exist in a currency union. Also, we show that solitary monetary policy can maximize social welfare if all goods are tradable. This result support half of policy implications derived by Benigno (2004) who implies that solitary monetary policy can maximize social welfare in a currency union while denies half of one derived by him. Third, we show allocation brought about by centralized government can be replicated by self-oriented setting, namely, local government because particular attention could be paid to the optimal monetary and fiscal policy mix in a cooperative versus a noncooperative equilibrium. Our second and third result support half of policy implications derived by canonical authors such as McKinon (1963) and Grauwe (1992) who support centralized government while deny half of one derived by them.

The paper is organized as follows. Section 2 constructs the model. Section 3 log-linearizes the model. Section 4 defines and analyzes monetary policy quali-
tatively without a fiscal policy regime and a mixed optimal monetary and fiscal policy regime. Section 5 is a numerical analysis including a welfare analysis. Section 6 analyzes implementing a cooperative solution by self-oriented fiscal authorities. Section 7 concludes this paper. The technical details are derived in the appendix.\textsuperscript{5}

2 The Model

We construct a closed-system currency union model belonging to the class of DSGE models with nominal rigidities and imperfect competition, and refer to Obstfeld and Rogoff (2000), Gali and Monacelli (2005). Following Stockman and Tesar (1995), we allow imperfect substitution between tradables and non-tradables, while Obstfeld and Rogoff (2000) implicitly assume that these goods are perfect substitutes. The union-wide economy consists of two equally sized countries, countries $H$ and $F$. Country $H$ produces an array of differentiated goods indexed by the interval $h \in [0,1]$, while country $F$ produces an array of differentiated goods indexed by $f \in [1,2]$.

2.1 Households

Preference of the representative household in country $H$ is given by:

$$\mathcal{U}_t \equiv E_t \sum_{t=0}^{\infty} \delta^t \left( \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi} \right)$$

where $E_t$ denotes the expectation, conditional on the information set at period $t$, $\delta \in (0,1)$ denotes the subjective discount factor, $C_t$ denotes consumption in country $H$, $N_t \equiv N_{H,t} + N_{N,t}$ denote hours of work in country $H$, $N_{H,t}$ and $N_{N,t}$ denote hours of work to produce tradable goods produced in country $H$.\footnote{An additional mathematical supplement with more complete analytical derivations and further details available on the web at http://www.cku.ac.jp/okano/papers_e.html}
and nontradable goods produced in country $H$, respectively, $\gamma$ denotes the share of tradables in the CPI, and $\varphi$ denotes the inverse of a labor supply elasticity. $U^*$, denoting preference of the representative household in country $F$, is defined analogously. We note that quantities and prices peculiar to country $F$ are denoted by asterisks while quantities and prices without asterisks are those in country $H$ or common to both countries.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[ \gamma^{\frac{1}{\eta}} C_{T,t}^{\eta-1} + (1 - \gamma)^{\frac{1}{\eta}} C_{N,t}^{\eta-1} \right]^{\frac{1}{\eta}}$$  \hspace{1cm} (2)

where $C_{T,t} = 2C_{H,t}^{\frac{1}{\eta}} C_{F,t}^{\frac{1}{\eta}}$ denotes the consumption index for tradables, $C_{H,t}$, $C_{F,t}$ and $C_{N,t}$ denote Dixit–Stiglitz-type indices of consumption across the tradables produced in country $H$ and produced in country $F$, and nontradables produced in country $H$, respectively, $\theta > 1$ denotes the elasticity of substitution across goods produced within a country, and $\eta > 0$ denotes the elasticity of substitution between tradable and nontradable goods.

Total consumption expenditures by households in country $H$ are given by $P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + P_{N,t} C_{N,t} = P_t C_t$, with $P_{H,t}$ and $P_{F,t}$ being Dixit–Stiglitz-type indices of the price of tradable goods produced in countries $H$ and $F$, respectively, and $P_{N,t}$ being Dixit–Stiglitz-type indices of the price of nontradable goods produced in country $H$. A sequence of budget constraints in country $H$ is given by:

$$B_t + W_t N_t + S_t \geq P_t C_t + E_t Q_{t,t+1} B_{t+1}$$ \hspace{1cm} (3)

where $Q_{t,t+1}$ denotes the stochastic discount factor, $B_t$ denotes the nominal payoff of the portfolio, $W_t$ denotes the nominal wage, $S_t$ denotes the lump-sum
taxes, and $P_t$ denotes the CPI defined by:

$$P_t \equiv \left[ \gamma P_{T,t}^{1-\eta} + (1 - \gamma) P_{N,t}^{1-\eta} \right]^{1/\eta}. $$ (4)

We also note that the producer price index (PPI) in country $H$ is defined by:

$$P_{P,t} \equiv \frac{P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t}}{Y_{H,t} + Y_{N,t}}, $$ (5)

where $Y_{H,t}$ and $Y_{N,t}$ denote a Dixit–Stiglitz-type index of the aggregate output of tradables produced in country $H$ and of nontradables, respectively.

The optimal allocation of any given expenditure within each category of goods implies the demand functions as follows:

$$C_{H,t} = \frac{1}{2} \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-1} C_{T,t}; \quad C_{F,t} = \frac{1}{2} \left( \frac{P_{F,t}}{P_{T,t}} \right)^{-1} C_{T,t} \quad C_{T,t} = \gamma \left( \frac{P_{T,t}}{P_{t}} \right)^{-\eta} C_t; \quad C_{N,t} = (1 - \gamma) \left( \frac{P_{N,t}}{P_{t}} \right)^{-\eta} C_t, $$ (6)

where $P_{T,t} \equiv \frac{P_{H,t}^t P_{F,t}^t}{2}$ denotes the tradables price index (TPI).

The representative household maximizes Eq.(1) subject to Eq.(3). Optimality conditions are given by:

$$\delta E_t \frac{C_{t+1} P_t}{C_{t}^{-1} P_{t+1}} = 1 \frac{R_t}{R_t} \quad C_t N_t^{\phi} = \frac{W_t}{P_t}, $$ (7)

where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = Q_{t.t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (for short, the gross nominal interest rate), and $r_t$ denotes the net nominal interest rate. Eq.(7) is an intertemporal optimality condition, namely the Euler equation, and Eq.(8) is an intratemporal optimality condition.$^6$ Combining and

$^6$Optimality conditions analogous to Eqs.(7) and (8) must hold in country $F$. 

7
iterating both Eq. (7) and its counterpart in country \( F \), we have an optimal risk-sharing condition as follows:

\[
C_t = \vartheta C_t^* Q_t
\]

with \( Q_t \equiv \frac{P^*_t}{P_t} \) denoting the CPI differential between the two countries and \( \vartheta \) denoting a constant depending on the initial value. Following Chari, Kehoe and McGrattan (2002), we assume that \( \vartheta = 1 \).

### 2.2 Firms

Each firm is classified into one of two groups: tradables producers and non-tradables producers. Each producer can use a linear technology to produce a differentiated good as follows:

\[
Y_{H,t}(h) = A_{H,t} N_{H,t}(h) \quad \text{;} \quad Y_{N,t}(h) = A_{N,t} N_{N,t}(h),
\]

(10)

where \( Y_{H,t}(h) \) and \( Y_{N,t}(h) \) denote the output of tradable goods \( h \) produced in country \( H \) and of nontradables \( h \), respectively, and \( A_{H,t} \) and \( A_{N,t} \) denote stochastic productivity shifters associated with tradables produced in country \( H \) and nontradables produced in country \( H \), respectively. Each producer in country \( F \) can use a technology similar to country \( H \).

Each firm of a single differentiated good prices its goods in response to the elasticity of substitution across goods produced within the SOE given the CPI. This is because each firm plays an active part in the monopolistically competitive market. We assume that Calvo–Yun-style price-setting behavior applies, and, therefore, that each firm resets its price with a probability of \( 1 - \alpha \) in each period, independently of the time elapsed since the last adjustment.

When setting a new price in period \( t \), firms seek to maximize the expected
discounted value of profits. The FONCs are as follows:

\[
E_t \left[ \sum_{k=0}^{\infty} (\alpha \delta)^{k} \Lambda_{t+k} \tilde{C}_{H,t+k} \left( \tilde{P}_{H,t} - \zeta P_{P,t+k} MC_{H,t+k} \right) \right] = 0,
\]

\[
E_t \left[ \sum_{k=0}^{\infty} (\alpha \delta)^{k} \Lambda_{t+k} \tilde{C}_{N,t+k} \left( \tilde{P}_{N,t} - \zeta P_{P,t+k} MC_{N,t+k} \right) \right] = 0 \quad (11)
\]

where \( \Lambda_t \) denotes the marginal utility of nominal income in country \( H \), \( MC_{H,t} \equiv \frac{W_t (1 - \tau)}{P_{P,t} A_{H,t}} \) and \( MC_{N,t} \equiv \frac{W_t (1 - \tau)}{P_{P,t} A_{N,t}} \) denote the marginal costs associated with tradables produced in country \( H \) and nontradables produced in country \( H \), respectively, \( \tilde{C}_{H,t+k} \) and \( \tilde{C}_{N,t+k} \) denote the total demands when the prices are changed of tradables produced in country \( H \) and nontradables produced in country \( H \), respectively, \( \tilde{P}_{H,t} \) and \( \tilde{P}_{N,t} \) denote the adjusted prices of tradables produced in country \( H \) and nontradables produced in country \( H \), respectively, and \( \zeta \equiv \frac{\theta}{1-\tau} \) is a constant markup and \( \tau \) denotes an employment subsidy. We take it as given that the law of one price always holds.

We also note that using Eq.(8), marginal cost can be rewritten as follows.

\[
MC_{H,t} = \frac{(1 - \tau) C_t N_t^2 P_t}{P_{P,t} A_{H,t}} \quad ; \quad MC_{N,t} = \frac{(1 - \tau) C_t N_t^2 P_t}{P_{P,t} A_{N,t}} \quad (12)
\]

Note that the subsidy \( \tau \) has an important role to derive the first best under the flexible price equilibrium through eliminating monopolistical competitive power.

To derive the first best, we set \( \tau = 1 - \zeta^{-1} \). Because of this, \( \tau \) does not appear in our log-linearized model.\(^7\)

We define country wide real marginal cost as \( MC_t \equiv \frac{MC_{H,t} V_{H,t} + MC_{N,t} V_{N,t}}{V_{H,t} + V_{N,t}} \).

\(^7\)Our setting \( \tau = 1 - \zeta^{-1} \) makes marginal utility of consumption equal to marginal disutility of labor which is derived by solving maximization problem of utility function subject to international risk sharing condition, technological constraint and market clearing condition under flexible price equilibrium. Gali and Monacelli (2005) detail the role of an employment subsidy.
2.3 Centralized Government

As mentioned above, we verify alternative policy regimes, i.e., an optimal monetary policy without a fiscal policy regime and an optimal monetary and fiscal policy mix regime. In the former case, in this union, no government absolutely defrays its fiscal deficit in the first place. This phenomenon is indicated by $G_t = G_t^* = 0$ for all $t$ in our model, where $G_t = \frac{P_{H,t}^G G_{H,t}}{P_{G,t}^G} + \frac{P_{N,t}^G G_{N,t}}{P_{G,t}^G}$ denotes government expenditure on goods produced in country $H$, $G_{H,t}$ and $G_{N,t}$ denote government expenditure on tradable goods produced in country $H$ and non-tradable goods produced in country $H$ and $P_{G,t}^G = \frac{P_{H,t}^G G_{H,t}}{G_{H,t}} + \frac{P_{N,t}^G G_{N,t}}{G_{N,t}}$ denotes average price of goods purchased by government in country $H$.

In the latter case, a centralized government, which is advocated by canonical authors, and is imaginary, conducts fiscal policy as a policy authority as well as the central bank. We refer to this centralized government as merely “government”. In this case, the government’s budget constraint is relaxed, i.e., $G_t = -G_t^*$, which implies that government expenditure on goods in country $H$ is financed by levying taxes on goods in country $F$ and vice versa, whereas union-wide government expenditure is always zero. The government expenditure index is given by the Dixit–Stiglitz type. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods to minimize total cost. Thus, a set of government demand schedules is analogous to, and associated with, private consumption.

Similar to Gali and Monacelli (Forthcoming), we do not assume government budget constraint explicitly. However, our steady state is not distorted because

---

8To be exact, this phenomenon is indicated by $dG_t^* = \frac{dG_t}{Y} = 0$ in our log-linearized model, where $Y$ denotes a steady-state value of output.
of this. As mentioned, Benigno (2004) imply that solitary monetary policy maximize social welfare in a currency union while Ferreo (2007) and Beetsma and Jensesn (2005) insist necessity not only of optimal monetary policy but also of optimal fiscal policy to maximize social welfare. These three papers assume that all goods are tradable and a currency union which consists of two countries. Difference in policy implications between Benigno (2004) and Ferreo (2007) and Beetsma and Jensesn (2005) stem from setting of steady state. Benigno (2004) assumes non-distorted steady state while Ferreo (2007) and Beetsma and Jensesn (2005) assume distorted steady state. Our paper investigates the effects of existence of non-tradable goods which causes Balassa–Samuelson Theorem effect. If we assume not only existence of non-tradable goods but also distorted steady state, it becomes very hard to find the role of fiscal policy or income transfer. From this context, it can be said that not assuming government budget constraint explicitly is one of advantage in our paper. In other words, we can compare simply policy implications between Benigno (2004) who assume all goods are tradable and our paper which investigates effects on welfare under various share of nontaradables.

2.4 Market Clearing

The market in country $H$ for tradables clears when domestic demand equals domestic supply as follows:

$$Y_{H,t} (h) = C_{H,t} (h) + C_{H,t}^* (h) + G_{H,t} (h),$$  \hspace{1cm} (13)

where $C_{H,t}^* (h)$ denotes country $F$’s demand for generic tradables produced in country $H$. As for nontradables, equilibrium requires that:

$$Y_{N,t} (h) = C_{N,t} (h) + G_{N,t} (h).$$  \hspace{1cm} (14)
Let $Y_{H,t}$ denote a Dixit–Stiglitz-type index of the aggregate output of tradables produced in country $H$. Combining this definition and Eqs. (6) and (9), Eq. (13) can be rewritten as:

$$Y_{H,t} = \frac{\gamma}{2} \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-1} C_t \left[ \left( \frac{P_{T,t}}{P_t} \right)^{-\eta} + \left( \frac{P_{T,t}}{P^*_t} \right)^{-\eta} Q_t^{-1} \right] + G_{H,t}. \quad (15)$$

Eq. (15) and its counterpart in country $F$ imply that:

$$\frac{Y_{H,t} - G_t}{Y_{F,t} - G^*_t} = T_t,$$

where $Y_{F,t}$ denotes the aggregate output of tradables produced in country $F$ and $T_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT). Thus, the differential of output of tradables between country $H$ and country $F$ is equal to the TOT.

Let $Y_{N,t}$ denote a Dixit–Stiglitz-type index of the aggregate output of tradables produced in country $H$. Combining this definition and Eqs. (6) and (9), Eq. (14) can be rewritten as follows:

$$Y_{N,t} = (1 - \gamma) \left( \frac{P_{N,t}}{P_t} \right)^{-\eta} C_t + G_{N,t}. \quad (16)$$

Eq. (16) and its counterpart in country $F$ imply that:

$$\frac{Y_{N,t} - G_t}{Y_{N,F,t} - G^*_t} = N_t^\gamma Q_t^{-(\eta-1)}$$

where $N_t \equiv \frac{P_{N,t}}{P_{N,F,t}}$ denotes a nontradables price differential between countries $H$ and $F$ (NPD). Analogous to the differential of output of tradables, the differential of output of nontradables between the two countries is equal to the price differential of nontradables between them.

We define the aggregate domestic index of output as:

$$y_t = \frac{P_{H,t}}{P_{P,t}} Y_{H,t} + \frac{P_{N,t}}{P_{P,t}} Y_{N,t}, \quad (17)$$

analogous to that introduced for consumption index Eq. (2).
2.5 The Trade Balance

Following Gali and Monacelli (2005), net exports in country $H$ is defined as follows:

$$NX_t \equiv Y_t - \frac{P_t}{P_{P,t}}C_t - G_t,$$  \hspace{1cm} (18)

where $NX_t$ denotes net exports in country $H$.

3 Log-linearization of the Model

This section describes the stochastic equilibrium that arises from perturbations around the deterministic equilibrium. Lowercase letters denote percentage deviations of steady-state values for respective uppercase letters when there is no note to the contrary, i.e., $v_t \equiv \frac{dV_t}{V}$, where $V_t$ denotes the voluntary variable and $V$ denotes the steady-state value of $V_t$. Lowercase letters accompanied with $R$ as superscript mean logarithmic differential between the two countries for the respective uppercase letters, i.e., $v_t^R \equiv v_t - v_t^*$, while lowercase letters accompanied with $W$ as a superscript mean the logarithmic weighted sum of the two countries for the respective uppercase letters; i.e., $v_t^W \equiv \frac{1}{2} (v_t + v_t^*)$. Lastly, small letters accompanied with $\Delta$ mean changes in the large-letter variable, i.e., $\Delta v_t \equiv v_t - v_{t-1}$.

3.1 Aggregate Demand and Output

Log-linearizing Eqs. (7) and (9), we obtain the following:

$$c_t = E_t c_{t+1} - \hat{r}_t + E_t \pi_{t+1}$$

$$c_t^R = q_t,$$  \hspace{1cm} (19)

where $\hat{r}_t \equiv \ln R_t$ denotes the logarithmic union-wide gross nominal interest rate, $\pi_t$ denotes the CPI inflation rate in country $H$, and $q_t$ denotes the logarithmic
CPI differential between the two countries. Notice that the second equality in Eq.(19) implies that the logarithmic consumption differential between the two countries depends on the logarithmic CPI differential.

Log-linearizing and manipulating Eqs.(4) and (5), we obtain:

$$\pi_t = \gamma \pi_{T,t} + (1 - \gamma) \pi_{N,t}$$

(20)

$$\pi_{P,t} = \gamma \pi_{H,t} + (1 - \gamma) \pi_{N,t},$$

(21)

with $\pi_{T,t} = \frac{1}{2} \pi_{H,t} + \frac{1}{2} \pi_{F,t}$, where $\pi_{T,t}$ denotes the TPI inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denote the inflation rate of tradables produced in countries $H$ and $F$, respectively, $\pi_{N,t}$ denotes the inflation rate of nontradables produced in country $H$, and $\pi_{P,t}$ denotes the PPI inflation rate in country $H$.

Log-linearizing Eq.(17), we have:

$$y_t = \gamma y_{H,t} + (1 - \gamma) y_{N,t}.$$  

(22)

Log-linearizing Eqs.(15) and (16) and plugging these equalities into Eq.(22), we have:

$$y_t = c_t + \frac{\gamma}{2} t_t + \frac{\psi}{2} n_t + \hat{g}_t$$

(23)

with $\psi \equiv (1 - \gamma) \gamma (\eta - 1)$, where $t_t$ denotes logarithmic TOT, $n_t$ denotes logarithmic NPD, and $\hat{g}_t \equiv \frac{dG_t}{Y}$ denotes percentage deviations of government spending from steady-state output levels in country $H$.

Eq.(23) and its counterpart in country $F$ imply that:

$$y_t^R = \gamma t_t + (1 - \gamma) \omega n_t + \hat{g}_t^R$$

(24)

with $\omega \equiv 1 + (\eta - 1) \gamma$. Because of existing nontradables, the output differential between the two countries depends not only on the TOT, but also on the NPD.
When $\gamma = 1$, implying that there are no nontradables, this equality is reduced to $y_t^R = t_t + g_t^R$, which is familiar in many New Open Economy Macroeconomics (NOEM) studies. This equality shows that an increase in the price of domestic nontradables diminishes domestic output when we ignore the effect of $\eta$.

Using the definition of the TOT and Eqs.(20) and (21), we have:

$$\Delta t_t = -\frac{1}{\gamma} \pi_{P,t}^R - \frac{1 - \gamma}{\gamma} \Delta n_t.$$  \hspace{1cm} (25)

When $\gamma = 1$, Eq.(25) is reduced to $\Delta t_t = -\pi_{P,t}^R$, which implies that the TOT depreciation has an exact relationship with the PPI inflation differential. \footnote{Eq.(25) can be rewritten as $\pi_t - \frac{1}{\gamma} \Delta t_t = \pi_{P,t}$ or $\pi^*_t + \frac{1}{\gamma} \Delta t_t = \pi_{P,t}^*$, which implies that there is no difference between CPI and PPI when there are no tradables, i.e. $\gamma = 0$.}

Plugging Eqs.(23) and (24) into the first equality in Eq.(19), we have aggregate demand curves or so-called new Keynesian IS curves (NKIS) as follows:

$$y_t = E_t y_{t+1} - \hat{r}_t + E_t \pi_{P,t+1} - \frac{\psi}{2} \Delta E_t n_{t+1} - \Delta E_t \hat{g}_{t+1}.$$  \hspace{1cm} (26)

When all goods are tradable, i.e. $\gamma = 1$, Eq.(26) is reduced to:

$$y_t = E_t y_{t+1} - \hat{r}_t + E_t \pi_{P,t+1} - \Delta E_t \hat{g}_{t+1}.$$  

This shows no expected percentage change of the NPD in the NKIS and is a familiar expression when all goods are tradable.

Now, we refer to government expenditure constraints according to the Maastricht Treaty. At least, our currency union has the government expenditure constraint as follows:

$$\hat{g}_t^W = 0$$  \hspace{1cm} (27)

for all $t$. Eq.(27) implies that there are no union-wide government surpluses or deficits. If government expenditure is not allowed strictly in each country,
constraint is not only Eq.(27), but the following equality:

\[ \hat{y}_t^R = 0 \]  

(28)

which implies that the government expenditure differential is zero. Thus, when both Eqs.(27) and (28) are imposed, government expenditure is zero in each country. In a later section, we analyze monetary and fiscal policy under alternative constraints on government expenditures.

### 3.2 Aggregate Supply and Inflation

Log-linearizing Eq.(11) and rearranging, we can describe the dynamics of inflation in terms of marginal cost as follows:

\[
\begin{align*}
\pi_{H,t} &= \delta E_t \pi_{H,t+1} + \lambda (1 - \gamma) p_{N,t} - \lambda (1 - \gamma) p_{H,t} + \lambda m_{cH,t} \\
\pi_{N,t} &= \delta E_t \pi_{N,t+1} - \lambda \gamma p_{N,t} + \lambda \gamma p_{H,t} + \lambda m_{cN,t}
\end{align*}
\]

(29)

with \( \lambda \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha} \).

Plugging Eq.(29) into Eq.(21), we have a PPI-based inflation dynamics equation, namely, a New Keynesian Philips Curve (NKPC), as follows:

\[
\pi_{P,t} = \delta E_t \pi_{P,t+1} + \lambda m_{c,t}
\]

(30)

where \( m_{c,t} \) denotes logarithmic domestic marginal cost in country \( H \). Combining the second equality of Eq.(29) and its counterpart in country \( F \), the nontradables inflation differential is given by:

\[
\pi_{N,t}^R = \delta E_t \pi_{N,t+1}^R + \lambda \gamma n_t - \lambda \gamma t_t + \lambda m_{cN,t}^R
\]

(31)

which is a type of NKPC where, at first glance, Eq.(31) evolves into this version of a Balassa–Samuelson Theorem equality and can be called the New Keynesian Balassa–Samuelson Theorem equation (NKBS). Our model is a closed
system while a two-country economy is assumed; however, as with the Balassa–
Samuelson Theorem, Eq. (31) explains the CPI disparity between the two coun-
tries, although the Balassa–Samuelson Theorem addresses the problem of why
the nominal exchange rate deviates from purchasing power parity in the canonical
international money and finance literature. Details on Eq. (31) are mentioned
in a later section.

By log-linearizing the aggregated Eq. (10) and combining it with Eq. (22), we
have:

\[ y_t = \gamma a_{H,t} + (1 - \gamma) a_{N,t} + n_t. \]  

(32)

Combining log-linearized Eq. (8), Eqs. (23) and (32), we have:

\[ mc_{H,t} = (1 + \varphi) y_t - \frac{\psi}{2} n_t - \hat{g}_t - (1 + \varphi) a_{H,t} - (1 - \gamma) \varphi a_{N,t} \]

\[ mc_{N,t} = (1 + \varphi) y_t - \frac{\psi}{2} n_t - \hat{g}_t - \varphi a_{H,t} - [1 + (1 - \gamma) \varphi] a_{N,t}, \]  

(33)

which implies that marginal cost depends not only on domestic output, but also
on the NPD.

Using the definition of the logarithmic marginal cost, Eq. (33) can be rewrit-
ten as follows:

\[ mc_t = (1 + \varphi) y_t - \frac{\psi}{2} n_t - \hat{g}_t - (1 + \varphi) \gamma a_{H,t} - (1 + \varphi) (1 - \gamma) a_{N,t}, \]  

(34)

which implies that domestic marginal cost depends on the NPD. When \( \gamma = 1 \),
Eq. (34) reduces to:

\[ mc_t = (1 + \varphi) y_t - \hat{g}_t - (1 + \varphi) a_{H,t}, \]

because of \( \psi = 0 \) when \( \gamma = 1 \). This equality is a familiar expression in DSGE
applied to the NOEM literature.
Combining the second equality in Eq.(33) and its counterpart in country \( F \), the logarithmic marginal cost differential associated with nontradables is given by:

\[
mc_{N,t}^R = (1 + \varphi) y^R_t - \psi n_t - \tilde{g}^R_t - \varphi \gamma a_{H,t} + \varphi \gamma a_{F,t} - [1 + (1 - \gamma) \varphi] a_{N,t} + [1 + (1 - \gamma) \varphi] a^*_N. \tag{35}
\]

### 3.3 Dynamics of Relative Price and The Trade Balance

Log-linearizing Eq.(4) and rearranging yields:

\[
q_t = (1 - \gamma) n_t. \tag{36}
\]

It is clear by paying attention to Eqs.(19) and (36) that the logarithmic consumption differential depends on both the logarithmic CPI differential and the logarithmic NPD. When \( \gamma = 1 \), Eq.(36) is altered as \( q_t = 0 \), implying that the CPI between the two countries has an identity. In ordinary international finance literature, this means that purchasing power parity holds.

Combining Eqs.(19), (20) and (36), and rearranging, we have:

\[
\Delta E_t n_{t+1} = \frac{1}{1 - \gamma} \Delta E_t c^R_{t+1}. \tag{37}
\]

This equality implies that expected changes in the NPD are exactly related to expected changes in the logarithmic consumption differential between the two countries.

Using the definition of the NPD and the inflation rate of nontradables, expected changes in the NPD can be written as:

\[
\Delta E_t n_{t+1} = -E_t \pi^R_{N,t+1}. \tag{38}
\]

There is some relationship between the NPD and the trade balance. Combining the log-linearized Eqs.(18) and (23), we obtain:

\[
\tilde{a} \tilde{x}_t = \frac{\psi}{2} n_t \tag{39}
\]

18
where \( \hat{\Delta} x_t = \frac{dX_t}{dY} \) denotes net exports in terms of domestic output, expressed as a fraction of steady state output. Eq.(39) implies that a relative increase of nontradables price in country \( F \) brings about a trade balance surplus in country \( H \) and vice versa. When the nontradables price increases relative to that of tradables, demand for tradables increases while that for nontradables decreases. Hence, when the nontradables price in country \( F \) increases relative to that in country \( H \), demand for tradables in country \( F \), including tradables produced in country \( H \), increases, and the trade balance in country \( H \) goes into the black. When \( \gamma = 1 \), implying that all goods are tradable, Eq.(39) is reduced to \( \hat{\Delta} x_t = 0 \), implying balanced trade. In this model, the degree of relative risk aversion and the elasticity of substitution between tradables produced in countries \( H \) and \( F \) are implicitly assumed to be unity. These assumptions are adopted in Gali and Monacelli (2005): assuming that all goods are tradable, they showed that balanced trade is achieved under such parameter constraints. In our model, however, balanced trade is not assured because of nontradables.

There is another case that \( \hat{\Delta} x_t = 0 \) holds: when \( \eta = 1 \), implying the elasticity of substitution between tradables and nontradables is unity, namely, both tradables and nontradables are a perfect substitution.

### 3.4 Marginal Cost and Output Gap

In this section, we show that the linearized equilibrium dynamics have a representation in terms of an output gap. That representation has provided a basis for the analysis and evaluation of alternative policy regimes in much of the DSGE and NOEM literature. Following Gali and Monacelli (2005), we define the relation between output, its natural level and its gap as follows:

\[
y_t = \bar{y}_t + \hat{y}_t,
\]
where $\tilde{y}_t$ denotes the logarithmic output gap at its natural level, and $\tilde{y}_t^*$ denotes the logarithmic natural level output. Under flexible price, $\tilde{y}_t = \tilde{y}_t^* = 0$ must hold.

When fiscal authorities design their policies to dissolve distortions generated by monopolistically competitive markets, real marginal costs under flexible price equilibrium are unity, and their logarithm is given by:

$$mc_t = mc_t^* = 0.$$  

In addition, under flexible price equilibrium, all relative prices are unity. Thus, logarithmic NPD under flexible price equilibrium is given by:

$$n_t = 0.$$  

Combining these facts, Eq.(34) implies that:

$$\tilde{y}_t = \frac{1}{1 + \varphi} \hat{y}_t + \gamma a_{H,t} + (1 - \gamma) a_{N,t}, \quad (40)$$

Eq.(40) implies that the natural level of output consists of productivity, consumption disparity, and government spending.

Using Eq.(40), the log-linear approximated model can be rewritten in terms of the output gap. In addition, combining Eq.(37), Eq.(26) can be rewritten as:

$$\tilde{y}_t = E_t \hat{y}_{t+1} - 2\tilde{r}_t + E_t \pi_{P,t+1} + E_t \pi_{P,t+1}^* + \Delta E_t \hat{y}_{t+1}^* + \tilde{r}_t, \quad (41)$$

with $\nu \equiv \frac{\varphi}{1 + \varphi}$, where $\tilde{r}_t \equiv -\gamma a_{H,t} - \gamma a_{F,t} - (1 - \gamma) a_{N,t} - (1 - \gamma) a_{N,t}^*$ denotes a version of the real natural interest rate. Eq.(41) and its counterpart in country $F$ imply that under optimal risk sharing, NKISs in the two countries are homogeneous because Eq.(41) and its counterpart in country $F$ are identical.

NKPCs, in terms of the output gap, are given by:

$$\pi_{P,t} = \delta E_t \pi_{P,t+1} + \kappa \hat{y}_t - \frac{\psi \lambda}{2} n_t, \quad (42)$$
with $\kappa \equiv (1 + \varphi) \lambda$. These expressions become familiar when $\gamma = 1$. In this case, Eq.(42) can be rewritten as:

$$\pi_{P,t} = \delta E_t \pi_{P,t+1} + \kappa \tilde{y}_t,$$

which are derived by Gali and Monacelli (Forthcoming), who insist that inflation–output trade-offs can be dissolved simultaneously in a small open economy, under severe deep parameter restrictions, by inflation targeting. Indeed, when inflation targeting, such as $\pi_{P,t} = \pi^*_{P,t} = 0$ for all $t$, is introduced in our currency union with special restrictions, i.e. $\gamma = 1$, these equalities imply that $\tilde{y}_t = \tilde{y}_t^* = 0$ for all $t$ and that output gap is dissolved.

### 3.5 Canonical Balassa–Samuelson Theorem and NKBS

As mentioned in the former subsection, we now turn to the relation between the canonical Balassa–Samuelson Theorem and the NKBS. Using Eq.(40), NKBS Eq.(31) can be rewritten as:

$$\pi^R_{N,t} = \delta E_t \pi^R_{N,t+1} + \lambda \varphi \tilde{y}^R_t + \lambda \eta + \nu \lambda \tilde{g}^R_t - \lambda a_{N,t} + \lambda a^*_{N,t}. \quad (43)$$

Using Eq.(36), Eq.(43) can be rewritten as follows.

$$q_t = \frac{1 - \gamma}{\lambda} \pi^R_{N,t} - \frac{(1 - \gamma) \delta}{\lambda} E_t \pi^R_{N,t+1} - (1 - \gamma) \varphi \tilde{y}^R_t - (1 - \gamma) \nu \tilde{g}^R_t + (1 - \gamma) a_{N,t} - (1 - \gamma) a^*_{N,t},$$

which implies that the CPI disparity is dissolved between the two countries, namely, $q_t = 0$ holds when the currency union has no nontradables, i.e., as $\gamma = 1$. A problem with the CPI disparity is resolved, as each country has the same CPI. This implies that purchasing power parity holds under an ordinary open-economy model. Even the fact that $q_t = 0$ holds when $\gamma = 1$ can be depicted; however, a character of Eq.(43) as the canonical Balassa–Samuelson Theorem in international money and finance literature is obscure. This stems
from the fact that Eq.(43) is a dynamic equation, as in the New Keynesian literature, that has rightfully assumed nominal rigidities. To easily understand this character, we inspect Eq.(43) in a flexible price equilibrium. Under a flexible price equilibrium, Eq.(43) can be rewritten as:

$$q_t = -(1 - \gamma) \nu \tilde{y}_t^R + (1 - \gamma) a_{N,t} - (1 - \gamma) a_{N,t}^s,$$

because $\alpha = 0$ and $\tilde{y}_t = \tilde{y}_t^* = 0$ holds. By neglecting the first term on the RHS in this equality, the CPI disparity, which can be called the “real exchange rate” when a nominal exchange rate exists, is determined by productivity shifters in a currency union. In this equality, relatively increasing the productivity of tradables produced in country $H$, i.e., decreasing the productivity of nontradables produced in country $H$, causes a decrease in the CPI disparity $q_t$. As the canonical Balassa–Samuelson Theorem explains, a rise in productivity of the tradables sector in the home country causes a decreasing real exchange rate through an increase in nontradables prices in the home country, which stems from an increase in wages in both the tradables and the nontradables sector because of perfect labor mobility between each sector.\(^{10}\) This equality, the flexible price version of NKBS, can explain a decrease in the CPI differential stemming from an increase in the productivity of tradables produced in country $H$. Thus, Eq.(43) and similar equalities can be called an NKBS. Existing nontradables that cause disparities in the CPI and consumption is the principal friction taking rank with nominal rigidities in our currency-union model.

\(^{10}\)Labor mobility is not allowed between countries $H$ and $F$, whereas perfect labor mobility exists between the tradables and nontradables sectors in each country.
4 Monetary and Fiscal Policy

In this section, we analyze the macroeconomic implications of an alternative policy regime for the currency union: an optimal monetary policy without a fiscal policy regime and an optimal monetary and fiscal policy-mix regime. Under an optimal monetary policy without a fiscal policy regime, because Eqs.(27) and (28) are imposed as government expenditure, government expenditure in each country is given by:

\[ \hat{g}_t = \hat{g}_t^* = 0 \]  

(44)

which implies that government expenditure in each country is zero.

Under an optimal monetary and fiscal policy-mix regime, only Eq.(27) is available as a government expenditure constraint. The government expenditure constraint under this regime can be written as:

\[ \hat{g}_t = -\hat{g}_t^* \]  

(45)

Under this regime, government expenditure is allowed while the government keeps zero union-wide government expenditure.

4.1 Optimal Monetary Policy without Fiscal Policy

Under an optimal monetary policy without fiscal policy regime, only the central bank takes part as the authority because of Eq.(44). The central bank seeks to minimize the social loss function subject to our structural model.\(^{11}\) The period loss function is derived by second-order Taylor approximated Eq.(1), which is given by:

\[ \mathcal{U}^W_W = -\sum_{t=0}^{\infty} \delta^t L_t^W + \text{t.i.p.} + o \left( \| \xi \|^3 \right), \]

\(^{11}\)Our structural model consists of Eqs.(38),(41), (42), and (43) and their counterparts in country \( F \).
with:

\[ L^W_t = \frac{1}{4} \left[ \frac{\theta}{\lambda} \pi^2_{P,t} + (1 + \varphi) \tilde{y}^2_t + \frac{\theta}{\lambda} (\pi^*_{P,t})^2 + (1 + \varphi) (\tilde{y}^*_{t})^2 \right] \]

being period loss function, \( U^W = \frac{1}{2} (\mathcal{U} + \mathcal{U}^*) \) being union-wide utility, t.i.p. denoting the terms independent policy, and \( o \left( \| \xi \|^3 \right) \) denoting terms that are higher than third order. 12 13 Disregarding the terms independent policy and terms that are higher than third order, we have have discounted value of social losses as follows:

\[ L^W_t = \sum_{t=0}^{\infty} \delta^t L^W_t, \quad (46) \]

where \( L^W_t \) denotes discounted value of social losses.

Using the FONC of the Lagrangian, which consists of Eq.(46) and our structural model, we obtain the optimal monetary policy rule as follows:

\[ \hat{r}_t = \frac{1}{2} \bar{r}_t + \phi \pi^W_t, \quad (47) \]

with \( \phi \equiv \theta \) being the reaction coefficient to inflation. Gali and Monacelli (2005) analyzed the subject using a policy rule similar to Eq.(47), whereas they did not derive an interest rate policy rule from the optimization problem. As mentioned above, this policy rule implies a union-wide inflation targeting policy.

We can investigate the features of optimal monetary policy without fiscal policy by observing a structural model. Paying attention to the real natural interest rate \( \bar{r}_t \), which is common to the two countries, it is clear that all shifters

12Following Woodford (2001) and Gali and Monacelli (2005) and defining that the logarithm of each generic good PPI is the weighted sum of the logarithmic price of tradables and nontradables, we can obtain the second-order Taylor approximated utility function.

13If linear terms appear when a second order approximated utility function is obtained, second order approximation of the full model is required, as indicated by Benigno and Woodford (2005). We expand the model of Gali and Monacelli (2005) and terms of relative price appear in our model. However, linear terms can be eliminated entirely when a second order approximated utility function is obtained in our model. Thus, we do not require second order approximation of the full model. See Appendix E on the web for details.
affect the output gap in the same direction. Plugging Eq.(47) into Eq.(41), NKISs are altered as:

\[ \tilde{y}_t = E_t \tilde{yt}_{t+1} - \phi \pi_{P,t} - \phi \pi_{P,t}^* + E_t \pi_{P,t+1} + E_t \pi_{P,t+1}^* + \Delta E_t \tilde{y}_{t+1}^*, \]

implying that optimal monetary policy insulates the output gap from any shifters because, when the central bank’s interest rate rule is Eq.(47), the central bank seeks to make the nominal interest rate identical to the real natural rate.

Next, we inspect the supply side. Combining Eq.(42) and its counterpart in country \( F \), we have:

\[ \pi^W_t = \delta E_t \pi^W_{P,t+1} + \kappa \tilde{y}^W_t. \]

This equality implies that the union-wide output gap is always zero when the central bank conducts monetary policy, following an optimal policy rule such as Eq.(47) because the central bank seeks to stabilize perfectly the union-wide inflation rate. For instance, we suppose that \( \pi^W_t = 0 \) for all \( t \) is realized under the optimal policy rule. Paying attention to this equality, this means that \( \tilde{y}^W_t = 0 \) for all \( t \). The same finding is reported by Gali and Monacelli (2005) and Benigno (2004). The fact that inflation–output trade-offs can be resolved simultaneously holds not only at the union-wide level but also at the country level when there are no nontradables. Under Eq.(47), the PPI inflation rate in each country is also stabilized similarly to the union-wide inflation rate. This implies that the economy does not have the disparity of the PPI inflation rate, i.e., \( \pi^R_t = 0 \) for all \( t \). Subtracting the counterpart of Eq.(42) in country \( F \) from Eq.(42), we have:

\[ \pi^R_{P,t} = \delta E_t \pi^R_{P,t+1} + \kappa \tilde{y}^R_t, \]

where we assume that there are no nontradables, i.e., \( \gamma = 1 \). When \( \pi^R_{P,t} = 0 \)
for all \( t \), that \( \tilde{y}_t^R = 0 \) for all \( t \) is guessed by this equality. Not only \( \pi_t^W = 0 \) but also \( \pi_t^R = 0 \) for all \( t \) implies that \( \pi_{P,t} = \pi_{P,t}^* = 0 \) for all \( t \). Hence, under optimal monetary policy, the PPI inflation rates in countries \( H \) and \( F \) are fully stabilized, while the output gaps in countries \( H \) and \( F \) are fully stabilized, i.e., \( \tilde{y}_t = \tilde{y}_t^* = 0 \). The same is stated by Benigno (2004).

However, when nontradables exist in the economy, the state of affairs brought about by optimal monetary policy is altered. While union-wide NKPC is not affected by the share of tradables, NKPC in terms of the PPI inflation rate differential between the two countries is affected by the share of tradables. Without a restriction \( \gamma = 1 \) implying that there are no nontradable goods, NKPC in terms of PPI inflation rate differential is given by:

\[
\pi_{P,t}^R = \delta E_t \pi_{P,t+1}^R + \kappa \tilde{y}_t^R - \psi \lambda n_t.
\]

In this case, although optimal monetary policy is adopted, trade-offs between the inflation rate and the output gap in countries \( H \) and \( F \) arise. When does the NPD fluctuate? Rearranging Eq.(43), we obtain:

\[
n_t = \frac{1}{\lambda} \pi_{N,t}^R - \delta E_t \pi_{N,t+1}^R - \varphi \tilde{y}_t^R - \nu \tilde{y}_t^R + a_{N,t} - a_{N,t}^*.
\]

implying that when any shifters associated with nontradables result, the NPD changes. This change affects NKPCs unless \( \gamma = 1 \), which coincides with \( \psi = 0 \). Thus, inflation–output trade-offs cannot be dissolved unless \( \gamma = 1 \) under optimal monetary policy. This can be explained by fluctuation of the CPI disparity. Using Eq.(36), NKPC can be rewritten as:

\[
\pi_{P,t}^R = \delta E_t \pi_{P,t+1}^R + \kappa \tilde{y}_t^R - \gamma (\eta - 1) \lambda q_t.
\]

Unless \( \gamma = 1 \), \( q_t = 0 \) does not hold. When the union-wide economy produces some nontradables, changes in productivity result in a CPI disparity between
the two countries. The CPI disparity expands the disparity of the output gap between the two countries. Although union-wide inflation, output gap and the PPI inflation in each country are stabilized by the optimal monetary policy, the output gap in each country cannot be stabilized by the policy when there are nontradables.

Gali and Monacelli (Forthcoming) reach a similar conclusion to ours with regard to optimal monetary policy in a currency union that consists of inumerable small open economies. In such a currency union, optimal monetary policy can stabilize union-wide inflation and the output gap, whereas the PPI inflation and the output gap are not stabilized in each country because the small open economy has a peculiar CPI, different from the union-wide CPI. This difference stems from the fact that the scale of the small open economy is infinitesimal. Our model, however, does not assume small open economies. Instead, it assumes two countries’ economies and allows nontradables. Existing nontradables necessarily result in a disparity of CPIs between the two countries. On that point, we can double-check an implication derived by Gali and Monacelli (Forthcoming) using a two-country economic model with nontradables.

We refer to another case in which changes in NPD do not affect NKPCs, namely, optimal monetary policy can dissolve inflation–output trade-offs. When $\eta = 1$ implying perfect substitution between tradables and nontradables, $\psi = 0$ holds. In this case, changes in any shifters without union-wide preference shifters do not affect NKPCs through NKBS although nontradables exist. Thus, it can be said that elasticity of substitution between tradable and nontradables $\eta$ is related to the Balassa–Samuelson Theorem in the same way as $\gamma$. 

27
4.2 Optimal Monetary and Fiscal Policy Mix

Under a currency union with nontradables, monetary policy alone cannot stabilize both the PPI inflation and the output gap at the individual-country level. Now, we abandon Eq.(28) as a constraint but allow country-level government expenditure under Eq.(27). In this optimal monetary and fiscal policy-mix regime, both the central bank and the central government seek to minimize Eq.(46) subject to the structural model. The optimal fiscal policy rule derived by the FONCs of the Lagrangian is given by:

\[
\hat{g}_t^R = \bar{g}_t^R + (1 + \varphi) \theta \pi^R_{P,t} + \frac{1}{\nu \lambda} \pi^R_{N,t} - \frac{\lambda + \delta}{\nu \lambda} n_t, \tag{49}
\]

where \( \bar{g}_t^R \equiv \frac{1}{\nu} a_{N,t} - \frac{1}{\nu} a^*_N \) addresses real natural public expenditure disparity or real natural transfers of income. In contradistinction to the optimal monetary policy rule, Eq.(47), Eq.(49) consists of relative variables between the two countries. Plugging Eq.(49) into NKBS Eq.(43) yields:

\[
n_t = E_t \pi^R_{N,t+1} + \frac{\lambda \phi}{\delta} \hat{g}_t^R + \frac{\lambda \phi \theta}{\delta} \pi^R_{P,t}.
\]

By contradistinction between this equality and Eq.(48), it is clarified that no productivity shifter can affect the NPD as long as the central government conducts optimal fiscal policy. Moreover, optimal monetary policy insulates NKISs from any productivity and preference shifters, and the optimal monetary and fiscal policy mix insulates NKPCs, not only at the union-wide level, but also at the individual-country level, from any exogenous productivity and preference shifter, regardless of the share of nontradables.
5 Numerical Analysis

In this section, we illustrate the equilibrium behavior of the currency union under the alternative policy regime described above. We resort to a series of dynamic simulations and adopt the following benchmark parameterization. We assume an inverse of the labor supply elasticity $\varphi$, the elasticity of substitution across goods $\theta$, price stickiness consistent with an inverse of an average period of one year between price adjustments $\alpha$, the share of nontradables in the CPI $\gamma$, the elasticity of substitution between tradables and nontradables $\eta$, and the subjective discount factor $\delta$ set equal to 3, 7.88, 0.66, 0.5, 0.44 and 0.99, respectively, as if the timing of the model were quarterly. Except for $\gamma$ and $\eta$, these parameterizations are frequently used in DSGE literature, including Benigno (2004), Gali and Monacelli (Forthcoming) and (2005) and Rotemberg and Woodford (1997). As mentioned in the introduction, nontradables account for 50.3% of the major euro area, thus we set $\gamma = 0.5$. Following Stockman and Tesar (1995), we set $\eta = 0.44$.\(^{14}\) We notice that setting $\alpha = 0.66$ and $\delta = 0.99$ implies that the slope of the NKPC $\lambda$ is identical to 0.1786, $\theta = 7.88$ implies that the Taylor Principle $\phi$ is identical with 7.88, while the constant markup $\zeta$ is approximately 1.1453, and $\delta = 0.99$ implies that a riskless annual return is equal to about 4.04%. We also assume that the productivity and preference shifters are described according to the following AR (1) processes:

\[
\begin{align*}
    a_{H,t} &= \rho a_{H,t-1} + \xi_{H,t} ; \\
    a_{F,t} &= \rho a_{F,t-1} + \xi_{F,t} \\
    a_{N,t} &= \rho a_{N,t-1} + \xi_{N,t} ; \\
    a^*_{N,t} &= \rho a^*_{N,t-1} + \xi^*_{N,t}.
\end{align*}
\]

\(^{14}\)Setting $\eta = 0.44$ is used widely, including by Benigno and Thoenissen (2005) and Selaive and Tuesta (2006).
where $\xi_{H,t}$, $\xi_{F,t}$, $\xi_{N,t}$ and $\xi_{N,t}^*$ denote the i.i.d. shocks, and $\rho$ denotes the coefficient associated with AR (1) processes. Following Batini, Harrison and Millard (2001), we set $\rho$ equal to 0.7.\footnote{Batini, Harrison and Millard (2001) estimate AR (1) processes of productivity of tradables at 0.705 and of nontradables at 0.784. For simplicity, we adopt 0.7.}

### 5.1 Optimal Monetary Policy without Fiscal Policy

In accordance with the above finding, we inspect the benchmark parameterization case under optimal monetary policy without fiscal policy. Figure 1 displays the impulse responses to shocks under optimal monetary policy without fiscal policy. Neither $\gamma = 1$ nor $\eta = 1$ holds in the benchmark case; inflation–output trade-offs cannot be resolved by optimal monetary policy without fiscal policy. This can be confirmed by inspecting the top two panels in Figure 1. When shocks change the productivity shifter, the output gap results in a CPI disparity between the two countries, which is shown as the sixth panel in Figure 1. For instance, we consider the occurrence of changes in the productivity shifter of nontradables produced in country $H$. The nominal interest rate is lowered to maintain zero PPI inflation when this change occurs. When $\psi = 0$, this implies that $\gamma$ or $\eta$ is unity, thus insulating the reduction of the output gap from this shock. In the benchmark case, however, $\psi$ does not equal zero, so the NKPCs are affected by changes in the NPD, namely the CPI differential. As shown in the sixth panel in Figure 1, changes in the productivity shifter of nontradables produced in country $H$ cause changes in the CPI differential through NKBS. As mentioned in the former section, increasing productivity of nontradables produced in country $H$ increases the CPI differential through the Balassa–Samuelson Theorem. An increase in wages in the nontradables sector stemming from increases in the productivity of nontradables produced in coun-
try $H$ causes an increase in the price of tradables produced in both countries. This is the cause of a relative increase in the CPI in country $F$, because the price of nontradables produced in country $H$ somewhat decreases. As shown in Eq.(39), an increase in the CPI differential stemming from an increase in the price of tradables produced in both countries causes a trade balance surplus in country $H$, because of a rising demand for nontradables produced in country $H$. Thus, the output gap in country $H$ decreases, while that in country $F$ eventually increases. While monetary policy is optimal and can stabilize PPI inflation in both countries, NKBS affects NKPCs in the benchmark case, implying that half of the goods are nontradable. Thus, optimal monetary policy without fiscal policy is not adequate to resolve the inflation–output trade-offs when nontradables exist in the currency union.

Table 1 depicts macroeconomic volatility under optimal monetary policy without fiscal policy. Whereas the PPI inflation rate is fully stabilized to any changes in productivity shifters, the output gap in neither country is stabilized. It can be noted in Table 1 that any changes in productivity shifters associated with nontradables cause output gap fluctuations through changes in the CPI differential and the trade balance.

5.2 Optimal Monetary and Fiscal Policy Mix

As mentioned in the former section, an optimal monetary and fiscal policy mix regime can stabilize both the PPI inflation rate and the output gap. Figure 2 shows impulse responses to shocks under an optimal monetary and fiscal policy mix with benchmark parameterization. That output gap and PPI inflation are stabilized simultaneously when the productivity shifter changes can be confirmed by the first to the fourth panels in Figure 2. An increase in the pro-
ductivity shifter associated with nontradables produced in country $H$ pressures the output gap in country $H$ to decrease through NKBS. This is because the increase in prices associated with tradables produced in both countries stemming from an increase in productivity associated with nontradables produced in country $H$ shifts demand for goods from tradables produced in both countries to nontradables produced in country $H$. Under an optimal monetary and fiscal policy-mix regime, however, an increase in the NPD caused by an increase in prices associated with tradables is prevented by an increase in government expenditure in country $H$; namely, the fiscal deficit in country $H$. An increase in government expenditure in country $H$ controls the decrease in the PPI in country $H$, including tradables prices. Thus, the CPI differential is unchanged while a decrease in pressure associated with nontradables prices in country $H$ results. Finally, the output gap in country $H$ is unchanged because the trade balance is fully stabilized. In country $F$, adverse changes occur in response to a shock to the productivity shifter associated with nontradables produced in country $H$. This is shown in the last panel in Figure 2. An increase in a productivity shifter associated with nontradables produced in country $H$ increases prices in country $F$ relative to those in country $H$. This relative increase in prices in country $F$, namely, an increase in the NPD, places decreasing pressure on the output gap in country $F$. Under this regime, however, government expenditure in country $F$ becomes smaller, which, in turn, increases the CPI in country $F$. \textsuperscript{16}

As shown in Table 1, an optimal monetary policy regime can fully stabilize both the PPI inflation and the output gap, whereas optimal monetary policy without a fiscal policy regime stabilizes only the PPI inflation rate. Benigno

\textsuperscript{16}Paying taxes in kind in country $F$, consumption goods relatively decrease and the CPI is under pressure to increase.
(2004) asserts that the optimal monetary policy, namely simple union-wide inflation targeting, can stabilize both the inflation rate and the output gap simultaneously. When a currency union consists of homogeneous economies, a solitary instrument, namely the interest rate of the consolidated currency, can resolve inflation–output trade-offs. When a currency union consists of heterogeneous economies, however, the interest rate cannot correct a disparity across economies, although it can stabilize the union-wide economy. Similarly to our paper, Gali and Monacelli (Forthcoming) insist on the importance of fiscal policy in currency union. Gali and Monacelli (Forthcoming) do not assume the existence of nontradables, but rather a currency union that consists of infinitesimal countries. Because a currency union that consists of infinitesimal countries is much the same as our settings for heterogeneity, whereas we assume a two-country economy that is adopted by Benigno (2004) who assumes that all goods are tradable, the issue associated with an optimal monetary and fiscal policy mix should concern policy administration.

5.3 The Role of Share of Nontradable Goods

In this section, we investigate to what extent the welfare-based ranking of the regimes discussed above may be sensitive to the calibration of the central parameter characterizing the currency union: the share of nontradables $1 - \gamma$. Before this sensitivity analysis, we define the expected welfare loss criterion. Taking unconditional expectations on the second-order Taylor approximated Eq.(1), and letting $\delta \to 1$, the expected welfare loss function is given by:

$$\mathcal{L}^W = \frac{1}{4} \left[ \frac{\theta}{\lambda} \text{var} (\pi_{P,t}) + (1 + \varphi) \text{var} (\tilde{y}_t) + \frac{\theta}{\lambda} \text{var} (\pi_{P,t}^*) + (1 + \varphi) \text{var} (\tilde{y}_t^*) \right].$$
This shows that welfare losses can be evaluated by the variance of PPI inflation and output gap.

Figure 3 displays the effect on welfare of varying shares of nontradables. Although PPI inflation in both countries is zero, which applies in any share of nontradables, the volatility of the output gap depends on the share of nontradables. Reflecting the relationship between volatilities and share of nontradables, welfare loss results under optimal monetary policy without a fiscal policy regime while welfare loss does not result under an optimal monetary and fiscal policy-mix regime. From the viewpoint of dissolving inflation–output trade-offs and of eliminating a well-defined welfare loss, it can be said that an optimal monetary and fiscal policy mix is an important policy issue in an actual currency union, namely, the euro area where the half of goods are nontradable.

Note that in our model, $\gamma = 1$ corresponds to the case of Bebigno (2004). As shown in Figure 3, welfare loss is zero not only under optimal monetary policy without a fiscal policy regime but also under an optimal monetary and fiscal policy-mix regime if share of nontradables is zero. This implies that solitary monetary policy can maximize social welfare and additional fiscal support is not needed to enhance social welfare. This policy implication corresponds to one derived by Benigno (2004). However, half of goods are nontradable in the euro area. This is the reason why we accept half of the implication derived by him while we deny half of one derived by him.

6 Implementing a Cooperative Solution by Self-oriented Fiscal Authorities

Some works, such as Benigno (2002), Obstfeld and Rogoff (2002) and Benigno and Benigno (2006), have shown that self-oriented monetary authorities can
replicate the cooperative outcome in a decentralized framework so that there is no need for international monetary policy coordination. Following their context, we investigate whether it is possible that fiscal policies taken in a noncooperative environment can implement the optimal cooperative solution in this section.

While the central bank commits to minimizing union-wide social loss $L^W$, we assume that each fiscal authority commits to minimizing its respective losses as follows:

$$L^{NC} = E_0 \sum_{t=0}^{\infty} \delta^t L^N_{t}^{NC} ; \quad L^{NC*} = E_0 \sum_{t=0}^{\infty} \delta^t L^N_{t}^{NC*},$$

subject to structural model with:

$$L^N_{t}^{NC} = \frac{1}{2} \left[ \frac{\theta}{\kappa} \pi^2_{P,t} + (1 + \varphi) \tilde{y}^2_{t} \right] ; \quad L^N_{t}^{NC*} = \frac{1}{2} \left[ \frac{\theta}{\kappa} (\pi^*_{P,t})^2 + (1 + \varphi) (\tilde{y}^*_{t})^2 \right],$$

where $L^N_{t}^{NC}$ denotes social losses assigned to fiscal authority to replicate cooperative outcome in country $H$.\(^{17}\)

Next, we seek to find expected union wide social losses, $L^W$ and $L^{NCW} = \frac{1}{2} (L^{NC} + L^{NC*})$ being the union-wide social loss brought about by self-oriented fiscal authorities in both countries. It can be said that $L^{NCW}$ is the union-wide social loss in the Nash equilibrium under optimal monetary policy. When the noncooperative solution brings about the social loss that is brought about by cooperative solution, $L^W = L^{NCW}$ is applied. Let us assume that there are cost push shocks that prevent the central bank from being able to stabilize inflation and the output gap simultaneously. For simplicity, we assume the cost shock has i.i.d. and constant variance $\sigma^2_{c}$ and $(\sigma^*_{c})^2$ which are variance of costpush shocks in country $H$ and country $F$, respectively. This assumption helps us to

\(^{17}\)Following Beetsma and Jensen (2005), we split the per period union-wide social loss function Eq. (17) as follows: $L^W_{t} = \frac{1}{2} \left( L_{t} + L^*_t \right)$ with $L_t \equiv \frac{1}{2} \left[ \frac{\theta}{\kappa} \pi^2_{P,t} + (1 + \varphi) \tilde{y}^2_{t} \right]$ and $L^*_t \equiv \frac{1}{2} \left[ \frac{\theta}{\kappa} (\pi^*_{P,t})^2 + (1 + \varphi) (\tilde{y}^*_{t})^2 \right]$.\(^{35}\)
calculate the social loss analytically.\textsuperscript{18} With tedious calculations, we have $L^W$ and $L^{NCW}$ as follows:

$$L^W = \frac{1}{4} \left\{ \left( \frac{1}{(1+\kappa \theta)^2} \right)^2 \left[ \left( 1 + \frac{1}{1+\kappa \theta} \right)^2 + \left( 1 - \frac{1}{1+\kappa \theta} \right)^2 \right] \left[ \frac{\theta}{\kappa} + (1+\varphi) \theta^2 \right] \left[ \sigma^2 + (\sigma^2)^2 \right] \right\}$$

$$= L^{NCW}$$

This implies that social loses brought about by centralized setting is equal to one brought about by self-oriented setting. In the other word, self-oriented fiscal authority can achieve the cooperative allocation in the Nash equilibrium without any forged work. As mentioned in introduction, we support half of policy implications derived by canonical authors, McKinon (1963). Fiscal policy is essential to enhance social welfare in a currency union with non-tradable goods. However, we deny the rest of policy implications that centralized government is essential to enhance social welfare in a currency union with non-tradable goods. Self-oriented fiscal authority can maximize social welfare if domestic welfare based loss function is imposed.

Our result partially differernt from result of Benigno and Benigno (Forthcoming), Benigno and Benigno (2006) and Beetsma and Jensen (2005) who analyzes necessity of policy coordination under two-country version DSGE model investigate how self-oriented setting bring about the cooperative allocation in the Nash equilibrium. They show that allocation derived by centralized setting can be replicated by self-oriented setting. However, they do not necessarily deny gains from cooperative setting because of TOT externality. Thus, to replicate allocation under cooperative setting, they propose somewhat forged individual or respective loss function whose sum do not become equal to union-wide loss

\textsuperscript{18}Following Walsh (2003) and Monacelli (2004), we calculate period losses.
Our result implies that forged respective loss function is not needed. There are contradictory policy implications between our paper and those papers, at glance. However, Benigno and Benigno (Forthcoming) and Benigno and Benigno (2006) show that there are no need to cooperate internationally under special circumstance while some gains from policy cooperation is proven.\textsuperscript{19} Our utility function does not include any exogenous shifter. Because of this, the square terms of TOT does not appear in the second-order approximated utility function which implies that there is no TOT externality. On the contrary, second-order approximated utility function derived by Benigno and Benigno (Forthcoming), Benigno and Benigno (2006) and Beetsma and Jensen (2005) imply TOT externality which disappear under only special circumstances. Our case corresponds to special cases of these papers. Thus, our policy implication is not necessarily inconsistent with these papers.

\section{Conclusion}

We investigate an optimal monetary and fiscal policy mix, which has been forgotten for a long time. Some implications can be derived from our paper, the greatest contribution being that it indicates the importance of a monetary and fiscal policy mix in a currency union with nontradables because of the Balassa–Samuelson Theorem. Whereas perfect risk sharing in assets markets is formed in a union-wide economy, the existence of nontradable goods makes optimal monetary policy insufficient to eliminate welfare loss. Both monetary policy and fiscal policy conducted by the centralized government play parts in achieving

\textsuperscript{19}Benigno and Benigno (2006) show that there are no gains from cooperation when there are no mark-up shocks and government purchases. Benigno and Benigno (Forthcoming) that sum of respective loss functions to replicate cooperative allocation become strictly equal to union-wide loss function when both degree of relative risk aversion and the elasticity of substitution between goods produced in both countries are unity.
zero welfare loss in a currency union with nontradables. In connection with the necessity of a policy mix, our paper agrees with the proposal made by Gali and Monacelli (Forthcoming). From another viewpoint, this paper justifies approximately a canonical argument associated with policy mix in a currency union. While we agree with the canonical argument, the necessity of a centralized fiscal policy is denied.
References


Benigno, P. and Benigno, B. (Forthcoming) Implementing International Monetary Cooperation through Inflation Targeting, *Macroeconomics Dynamics*.


Central Bank of Chile Working Papers, No. 359.


Table 1: Macroeconomic Volatility under Alternative Regimes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regime</th>
<th>$a_{H,t}$</th>
<th>$a_{N,t}$</th>
<th>$a_{F,t}$</th>
<th>$a_{N,F,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{y}_t$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\tilde{y}_t^*$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0348</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\pi_{P,t}$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\pi_{P,t}^*$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\hat{r}_t$</td>
<td>MP</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.0467</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.0467</td>
</tr>
<tr>
<td>$n_t$</td>
<td>MP</td>
<td>0.0000</td>
<td>1.1808</td>
<td>0.0000</td>
<td>1.1808</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$q_t$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.5904</td>
<td>0.0000</td>
<td>0.5904</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\hat{g}_t$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.1245</td>
<td>0.0000</td>
<td>0.1245</td>
</tr>
<tr>
<td>$\hat{g}_t^*$</td>
<td>MP</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Mix</td>
<td>0.0000</td>
<td>0.1245</td>
<td>0.0000</td>
<td>0.1245</td>
</tr>
</tbody>
</table>

MP: Monetary Policy without Fiscal Policy
Mix: Optimal Monetary and Fiscal Policy Mix
Figure 1: Impulse Responses to Shocks under Optimal Monetary Policy without Fiscal Policy

- Output Gap in Country H
- Output Gap in Country F
- PPI Inflation Rate in Country H
- PPI Inflation Rate in Country F
- Nominal Interest Rate
- CPI Disparity
- Government Expenditure in Country H
- Government expenditure in Country F

Legend:
- Productivity Shock of Tradable Goods in Country H
- Productivity Shock of Nontradable Goods in Country H
Figure 2: Impulse Responses to Shocks under Optimal Monetary and Fiscal Policy Mix
Figure 3: Effect on Welfare of Varying Share of Nontradable Goods