Comparative Analysis of Commodity & Stock Indices Daily Movement using Techniques of Nonlinear Dynamics

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[Analysis of the time path of financial series in terms its underlying dynamic characteristic features has been the recent focus of scientists and practitioners studying the financial market. Understanding the dynamics of a financial series, as a stock market index or a commodity market index is still however, a complex task having its specific requirements. Efforts have been made to analyse the different time series and try to unearth the dynamic similarities, if any. In this paper we have tried to analyse the daily movement of stock index vis-à-vis the same of commodity index. For this purpose we have considered commodity(MCX -COMDEX )and S & P CNX NIFTY (NIFTY) indices of Multi Commodity Exchange and National Stock Exchange, India, respectively, for the study along with Dow Jones Industrial Average (DJI) and Dow Jones-AIG Commodity Index (DJ-AIGCI)indices for stock and commodities, USA from June 2005 to August 2008. To analyse the dynamics of the time movement of the indices we have used the techniques from nonlinear dynamics like Recurrence Plot analysis, Power Spectrum analyses, Delay based cross correlation function. Our studies show that the dynamics of the time path of daily movement of Indian stock and commodity exchanges are much similar in nature while those of the US market are quite different.]

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1 Introduction

In the contemporary global scenario, the working of the complex and multidimensional factors on the time behaviour of financial series necessitates an investigation of the underlying characteristic features of the same. In addition to the state of the art econometric approaches, analysis in terms of state space dynamics considering the financial time series as deterministic chaos has emerged to be of utmost importance. Understanding the dynamics of a financial series, as a stock market index or a commodity market index is still however, a complex task having its specific requirements. Market trend, in case of both commodity and securities is reflected by the average trading price movement. Thus the time movement of the indices in the respective markets are, therefore, an indication of the similarities/dissimilarities in the dynamics of the two markets. The purpose of this paper is to analyse the time movement of the indices of commodity and stock markets.

We examine the characteristics the time movements of the two indices to find out whether there exists any similarity in time evolution of prices in the two segments, the degree of correlation, the underlying patterns and randomness. We have used some new techniques developed from studies in nonlinearity and chaos in understanding the dynamics of financial time series. Our study also tries to address the question: "Is there a similarity in the basic trading dynamics of different markets?" To analyse this we use the commodities indices and NIFTY indices of Multi Commodity Exchange and National Stock Exchange, India for the study respectively along with DOW JONES indices for stock and commodities, USA. The reason for choosing these indices is to compare the time series characteristics of the stock and commodity indices of the two different markets. So we basically compare the daily movement of the commodity and stock prices in the two countries separately. The aspects under study are the basic trend of the time series followed by analysis of the state space evolution treating the time series as deterministic chaos.

The structure of the paper is as follows. In section 2, we briefly describe the source and nature of data. In section 3 we discuss all the tests performed, giving the background theory of our analysis before commenting upon the results. In section 4 we perform the comparative analysis of the results of our tests. Finally we summarise our conclusions in section 5.

2 Data & Methodology

Our analysis is based on S & P CNX NIFTY (NIFTY) and MCX-COMDEX (commodity) Index of India as well as Dow Jones Industrial Average (DJI) stock Index and Dow Jones-AIG Commodity Index (DJ-AIGCI) of USA. From the respective Exchange web sites, data has been collected for the period June 2005 - August 2008 (both month inclusive). S & P CNX Nifty is a well diversified 50 stock index (Traded in National Stock Exchange, India) accounting for 25 sectors of the Indian economy. MCX-COMDEX is a composite Futures index comprising of commodity futures of diversified sectors traded in MCX India. DJI is a composite index computed from stock prices of 30 largest and most widely held public companies in the USA. DJ-AIGCI is a composite index composed of future contracts on physical commodities traded on US exchanges.

Intra-day trading data, if analysed, would have certainly helped us in doing a scaling analysis, which would have refined our study. However, due to non-availability of such data for free, we have restricted our study to inter-day or daily closing data. This constraint,
however, does not pose any significant impediment to our endeavour to seek answer to
the main question on underlying index behaviour similarity between stock and commodity
markets of two different countries.

Figures 1(a) , 1 (b) 2(a) & 2(b) depict the time series paths of S & P CNX NIFTY,
MCX-COMDEX of India, Dow Jones-AIG Commodity Index and Dow Jones stock Index
of USA respectively. We compare the series based on techniques from nonlinear dynamics
viz., Recurrence Plot analysis, Recurrence Histogram, Power Spectrum analysis and cross
correlation studies.

3 Background of Examinations

3.1 Recurrence Plot

A recurrence plot (RP) is a graph that shows all those times at which a state of the dynamical
system recurs. In other words, the RP reveals all the times when the phase space trajectory
visits roughly the same area in the phase space. 1

Natural processes can have a distinct recurrent behaviour, e.g. periodicities (as seasonal
or Milankovich cycles), but also irregular cyclicities (as El Niño Southern Oscillation).
Moreover, the recurrence of states is a fundamental property of deterministic dynamical
systems and is typical for nonlinear or chaotic systems. The recurrence of states in nature
has been known for a long time and has also been discussed in earlier publications (e.g.
recurrence phenomena in cosmic-ray intensity, Monk, 1939).

Eckmann et al. (1987)[1] have introduced a tool which can visualize the recurrence
of states xi in a phase space. Usually, a phase space does not have a dimension (two
or three) which allows it to be pictured. Higher dimensional phase spaces can only be
visualized by projection into the two or three dimensional sub-spaces. However, Eckmann’s
tool enables us to investigate the m-dimensional phase space trajectory through a two-
dimensional representation of its recurrences. Such recurrence of a state at time i at a
different time j is marked within a two-dimensional squared matrix with ones and zeros
dots (black and white dots in the plot), where both axes are time axes. This representation
is called recurrence plot (RP). Such an RP can be mathematically expressed as

\[ R_{i,j} = \Theta(\epsilon_i - ||x_i - x_j||), \quad x_i \in \mathbb{R}^m, i, j = 1, \ldots, N \]

where \( R_{i,j} \) is the recurrence plot, N is the number of considered states \( x_i \), \( \epsilon_i \) is a threshold
distance, \( ||\cdot|| \) a norm and \( \Theta(\cdot) \) the Heaviside function. The Heaviside step function is
given by:

\[ \Theta(x) = 0 \text{ if } x < 0 \]
\[ \Theta(x) = 1 \text{ if } x \geq 0 \]

The threshold distance is called the delay factor and the number of considered state is
called the embedding dimension

3.1.1 Embedding Parameters

The most natural question pertains to how to choose an appropriate value for the time delay
d and the embedding dimension m. Several methods have been developed to best guess m

---

1A phase space, introduced by Willard Gibbs in 1901, is a space in which all possible states of a system
are represented, with each possible state of the system corresponding to one unique point in the phase
space. In a phase space, every degree of freedom or parameter of the system is represented as an axis of a
multidimensional space. A phase space may contain very many dimensions.
and d. The most often used methods are the Average Mutual Information Function (AMI) for the time delay, as introduced by Fraser and Swinney in 1986 [2] and the False Nearest Neighbors (FNN) method for the embedding dimension developed by Kennel et al. [3].

3.1.2 Time Delay

First of all, the time delay has to be estimated, since most logically the method to find the embedding dimension needs an estimation of d. There are two main methods. In the first one, the value for which the autocorrelation function first passes through zero is searched, which gives d. The function is given by,

\[
C(d) = \frac{1}{N-d} \sum_{i=1}^{N-d} (X_i - \bar{X})(X_{i+d} - \bar{X})
\]

Where N is the length of the sample and \(X_i\) are the random sample.

In the second, one chooses the first minimum location of the average mutual information function, where the mutual information function is defined as follows. Let us start with partitioning the real numbers. Let \(p_i\) be the probability to find a time series value in the \(i-th\) interval of the partition, let \(p_{ij}(d)\) be the joint probability to find a time series value in the \(i-th\) interval and a time series value in the \(j-th\) interval after a time d, i.e. the probability of transition in d time from the \(i-th\) to the \(j-th\) interval. The average mutual information function is

\[
S(d) = -\sum_{ij} p_{ij}(d) \ln \frac{p_{ij}(d)}{p_i p_j}
\]

The value d that firstly minimizes the quantity \(S(d)\) is the method choice for finding a reasonable time delay.

The difference between these two methods resides in the fact that while the first looks for linear independence, the second measures a general dependence of two variables. For this reason the second method seems to be preferred in non-linear time series analysis.

3.1.3 Embedding Dimension

The method used to find the embedding dimension is based on the concept of false neighbor. A false neighbor is a point in the data set that looks like a neighbor to another because the orbit is seen in a too small embedding space. For example two points on a circle can appear close to each other, even though they are not, if e.g. the circle is seen sideways (as a projection), thus is appearing like a line segment, whence increasing by one the dimension \(m\) of the reconstructed space often permits to differentiate between the points of the orbit, i.e those which are true neighbors and those which are not.

Let \(y(i)\) be a point of the reconstructed space. Note as \(y(i)^r\) the \(r-th\) nearest neighbour and compute the Euclidean distance \(L_2\) between them

\[
R_m^2(y(i), y(i)^r) = \sum_{k=1}^{m-1} [y(i + kd) - y^r(i + kd)]^2
\]

Next increase \(m\) to \(m + 1\) and compute the new distance, i.e. \(R^2\)

\[
R_{m+1}^2(y(i), y(i)^r) = R_m^2(y(i), y(i)^r) + [y(i + kd) - y^r(i + kd)]^2
\]
The point \( y'(i) \) is said a false nearest neighbor if

\[
\left[ \frac{R_{m+1}^2(y(i), y(i)^r) - R_m^2(y(i), y(i)^r)}{R_m^2(y(i), y(i)^r)} \right] > R_{tol}
\]

(3.5)

where \( R_{tol} \) is a predetermined threshold. Note that the number of false nearest neighbors depends on \( R_{tol} \). The sensitivity of the criterion to \( R_{tol} \) is not discussed here. Kennel et al. found that for \( R_{tol} = 10 \) the false nearest neighbor is clearly identified and we stick by this value below, but for a more profound discussion we suggest to read [3].

In practice, the percentage of false nearest neighbors (FNN) is computed for each \( m \) of a set of values; the embedding dimension is said to be found for the first percentage of FNN dropping to zero. Notice that when the signal is noisy this percentage never reaches a true zero value. For computation of the embedding parameters we used the software VRA 4.2 freely available on net.

### 3.1.4 Structures in Recurrence Plots

The initial purpose of RPs is the visual inspection of higher dimensional phase space trajectories. The view on RPs gives hints about the time evolution of these trajectories. The advantage of RPs is that they can also be applied to rather short non-stationary data.

The RPs exhibit characteristic large scale and small scale patterns. The first patterns were denoted by Eckmann et al. (1987) as typology and the latter as texture. The typology offers a global impression which can be characterized as homogeneous, periodic, drift and disrupted.

Homogeneous RPs are typical of stationary and autonomous systems in which relaxation times are short in comparison with the time spanned by the RP. An example of such an RP is that of a random time series.

Oscillating systems have RPs with diagonal oriented, periodic recurrent structures (diagonal lines, checkerboard structures). For quasi-periodic systems, the distances between the diagonal lines are different. However, even for those oscillating systems whose oscillations are not easily recognizable, the RPs can be used in order to find their oscillations.

The drift is caused by systems with slowly varying parameters. Such slow (adiabatic) change brightens the RP’s upper-left and lower-right corners.

Abrupt changes in the dynamics as well as extreme events cause white areas or bands in the RP. RPs offer an easy possibility to find and to assess extreme and rare events by using the frequency of their recurrences.

The closer inspection of the RPs reveals small scale structures (the texture) which are single dots, diagonal lines as well as vertical and horizontal lines (the combination of vertical and horizontal lines obviously forms rectangular clusters of recurrence points).

Single, isolated recurrence points can occur if states are rare, if they do not persist for any time or if they fluctuate heavily. However, they are not a unique sign of chance or noise (for example in maps).
A diagonal line $R_{i+k,j+k} = 1$ (for $k=1...l$, where $l$ is the length of the diagonal line), $R_{i,j}$ is the usual notation for RP as defined in section 3.1, occurs when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments. The direction of these diagonal structures can differ. Diagonal lines parallel to the Line of identity (LOI) (angle $\pi$) represent the parallel running of trajectories for the same time evolution. The diagonal structures perpendicular to the LOI represent the parallel running with contrary times (mirrored segments; this is often a hint for an inappropriate embedding). Since the definition of the Lyapunov exponent uses the time of the parallel running of trajectories, the relationship between the diagonal lines and the Lyapunov exponent is obvious.

A vertical (horizontal) line $R_{i+z,j+z} = 1$ (for $z=1...v$, where $v$ is the length of the vertical (horizontal) line) marks a time length in which a state does not change or changes very slowly. It seems, that the state is trapped for some time. This is a typical behaviour of laminar states (intermittency).

These small scale structures are the base of a quantitative analysis of the RPs. The visual interpretation of RPs requires some experience. The study of RPs from paradigmatic systems gives a good introduction into characteristic typology and texture. However, their quantification offers a more objective way for the investigation of the considered system. With this quantification, the RPs have become more and more popular within a growing group of scientists who use RPs and their quantification techniques for data analysis.

### 3.2 Quantification of Recurrence Plots (Recurrence Quantification Analysis)

**Definition:** The recurrence quantification analysis (RQA) is a method of nonlinear data analysis which quantifies the number and duration of recurrences of a dynamical system presented by its phase space trajectory.

A quantification of recurrence plots was developed by Zbilut and Webber Jr. (Zbilut and Webber Jr., 1992[4]; Webber Jr. and Zbilut, 1998[5]) and extended with new measures of complexity by Marwan [6]. Measures which base on diagonal structures are able to find chaos-order transitions (Trulla et al., 1996)[7], measures based on vertical (horizontal) structures are able to find chaos-chaos transitions (laminar phases, Marwan et al., 2002)[8].

These measures can be computed in windows along the main diagonal. This allows us to study their time dependence and can be used for the detection of transitions (Trulla et al., 1996)[7]. Another possibility is to define these measures for each diagonal parallel to the main diagonal separately (Marwan and Kurths, 2002)[9]. This approach enables the study of time delays, unstable periodic orbits (UPOs; Lathrop and Kostelich, 1989[10]; Gilmore, 1996[11]), and by applying to cross recurrence plots, the assessment of similarities between processes (Marwan and Kurths, 2002[9]). Some authors have already used these techniques to financial data[12,13,14,15,16,17,18].

Initially Zbilut and Webber introduced 5 variables to quantify changes. The 5 basic RQA variables are briefly defined as follows with a brief comment on each one:

- **% REC:** the percentage of recurrent points; recall that a point $(i, j)$ is recurrent if the distance between the vectors $y(i)$ and $y(j)$ is less than the threshold; in other words.
% REC is the ratio of the number of recurrent states measured with respect to all possible states. The formula for computation of % REC is given by:

\[
\% REC = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}
\]  

(3.6)

% DET: the percentage of recurrent points forming line segments parallel to the main diagonal. The presence of these lines reveals the existence of a deterministic structure. The formula for computation of % DET is:

\[
\% DET = \frac{\sum_{i,j=1}^{N} D_{i,j}}{\sum_{i,j=1}^{N} R_{i,j}}
\]

(3.7)

MAXLINE: the longest line segment measured parallel to the main diagonal. In Trulla et al. [16], it is claimed that this quantity is proportional to the inverse of the largest positive Lyapunov exponent. A periodic signal produces long line segments, while short lines indicate chaos.

\[
L_{\text{max}} = \max(l_i = 1, \ldots, N)
\]

(3.8)

TREND: the slope of line-of-best-fit through % REC as a function of the displacement from the main diagonal (excluding the last 10% range). This variable quantifies the drift and the non stationarity of the time series. For example, a flat slope indicates stationarity because in an homogeneous plot the quantity of recurrent points at the left and at the right of the central line is almost the same; instead when on the RHS this quantity is less than the quantity on the LHS the variable TREND assumes negative values; this means that the system is going far away from the state it had, i.e. it has a trend. Formula for computation:

\[
TREND = \frac{\sum_{i=1}^{N}(i - \bar{N}/2)(RR_i - \bar{RR}_i)}{\sum_{i=1}^{N}(i - \bar{N}/2)^2}
\]

(3.9)

ENT: the Shannon entropy of the distribution of the length of line segments parallel to the main diagonal. The entropy gives a measure of how much information one needs in order to recover the system. A low entropy value indicates that few information are needed to identify the system, in contrast, a high entropy indicates that much information are required. The entropy is small when the length of the longest segment parallel to the diagonal is short and does not vary much. This has to be associated with information on determinism. A high entropy is typical of periodic behavior while low entropy indicates chaotic behavior. Formula for computation:

\[
ENT = - \sum_{l=1}^{N_l} p(l) \ln p(l)
\]

(3.10)

3.3 Recurrence Histogram

Recurrence times are certainly not new. Poincare is perhaps the most famous for describing them in the context of dynamical systems as points which visit a small region of phase
space. The statistical literature also points out that recurrences are the most basic of mathematic relations. In this respect, it is important to reiterate the fact that calculation of recurrence times, unlike other methods such as Fourier, Wigner-Ville or wavelets, requires no transformation of the data, and can be used for both linear and non-linear systems. Because recurrences are simply tallies, they make no mathematical assumptions. The histogram corresponding to recurrence time shows the characteristic periodicity in the time series. It plots percentage of recurrence at different time lags.

3.4 Power Spectrum Analyses

One of the simplest and useful tool to investigate chaos is the power spectrum analysis. For a given data series, the power spectrum gives a plot of the portion of a signal’s power (energy per unit time) falling within given frequency bins. The most common and effective way of generating a power spectrum is by using a discrete Fourier transform, but there are some other techniques such as the maximum entropy method that can also be used.

The continuous Fourier transform \( \tilde{f}(\nu) \) of a function \( f(t) \) is defined as

\[
\tilde{f}(\nu) = F_1[f(t)](\nu) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \nu t} \, dt
\]  

(3.11)

For the discrete function \( f(t) \to f(t_k) \) we can modify our definition as

\[
F_n = \sum_{k=0}^{N-1} f_k e^{2\pi i k \nu N}
\]  

(3.12)

The corresponding inverse transform can be written as

\[
f_k = \sum_{k=0}^{N-1} F_n e^{2\pi i k \nu N}
\]  

(3.13)

where \( f(t_k) = f_k \) for \( k=0,1,2... N-1 \).

Very often, the most efficient way of estimating \( f_k \) from some data is via the Fourier transform, using the Convolution Theorem (the convolution of two infinite sequences is equal to the inverse Fourier transform of the product of the Fourier transforms of the individual sequences). The Power spectrum analysis is an effective tool to quantify chaos. Any data series which is in chaotic state shows random behavior and lots of peaks arises corresponding to the spectrum analysis. Also a data set in either steady or periodic state will converge to its corresponding number of peaks in Power spectrum diagram.

3.5 Cross Correlation function between two data series

For further investigation we have studied the nonlinear correlation analysis for the two sets. The autocorrelation is basically a mathematical tool for finding repeating patterns, such as the presence of a periodic signal which has been buried under noise, or identifying the missing fundamental frequency in a signal implied by its harmonic frequencies. It is used frequently in signal processing for analyzing functions or series of values, such as time domain signals. In other words it is a mathematical representation of the degree of similarity between a given time series and a lagged version of itself over successive time intervals. It is the same as calculating the correlation between two different time series, except that the
same time series is used twice - once in its original form and once lagged one or more time periods. The discrete version of the autocorrelation function can be written as

\[ R_{xx}(j) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x_n \bar{x}_{n-j} \]  

(3.14)

where \( N \) is the total sample and \( x_i, \bar{x}_i \) are the values from two data series. When computed, the resulting number can range from +1 to -1. An autocorrelation of +1 represents perfect positive correlation (i.e. an increase seen in one time series will lead to a proportionate increase in the other time series), while a value of -1 represents perfect negative correlation (i.e. an increase seen in one time series results in a proportionate decrease in the other time series).

The cross-correlation is a statistical measure timing the movements and proximity of alignment between two different information sets of a series of information. Cross correlation is generally used when measuring information between two different time series. The range of the data is -1 to 1 such that the closer the cross-correlation value is to 1, the more closely the information sets are.

In signal processing, cross-correlation is a measure of similarity of two waveforms as a function of a time-lag applied to one of them. This is also known as a sliding dot product or inner-product. It is commonly used to search a long duration signal for a shorter, known feature. It also has applications in pattern recognition and cryptanalysis.

For continuous functions \( f \) and \( g \) the cross-correlation is defined as:

\[ (f \ast g)(t) = \int_{-\infty}^{\infty} f^*(\tau)g(t + \tau)d\tau \]  

(3.15)

The cross-correlation is similar in nature to the convolution of two functions. Whereas convolution involves reversing a signal, then shifting it and multiplying by another signal, correlation only involves shifting it and multiplying (no reversing). In an Autocorrelation, which is the cross-correlation of a signal with itself, there will always be a peak at a lag of zero. If \( X \) and \( Y \) are two independent random variables with probability distributions \( f \) and \( g \), respectively, then the probability distribution of the difference \( X - Y \) is given by the cross-correlation figure. In contrast, the convolution \( f \ast g \) gives the probability distribution of the sum \( X + Y \).

4 Discussion of Test results

4.1 Recurrence Plot and Recurrence Quantification Analysis

Fig 3a) and 3 b) represent the RP of Nifty and MCX time series, while Fig 4a) and 4b) represent that of DJ-AIGCI and DJI respectively. The recurrence Plot reveals interesting characteristics of the time series. While there is enough similarity in pattern formation incase of Nifty and MCX COMDEX index, there is a notable characteristic difference in Dow Jones and Dow Jones Commodity index time series evolution. While we see that both Nifty and Commodity have similar patterns in moving from randomness to trend, DJI shows pronounced chaotic behaviour(mush similar to the Indian data set) than DJ-AIGCI. A close look at the RQA results (Tables 1 & 2 and tables 3 & 4) also reinforce our findings above. The RQA parameter values also show a much greater similarity in case of NIFTY and MCX than in case of DJI and DJ-AIGCI.
4.2 Recurrence Histogram

Fig 5a) and 5 b) represent the Recurrence Histogram of Nifty and MCX time series, while Fig 6a) and 6b) represent that of DJ-AIGC and DJI respectively. A look at the corresponding histograms reveal the same story. The wide difference between DJI and DJ-AIGCI as compared to Nifty & MCX-COMDEX is quite noticeable.

4.3 Power Spectrum Analysis

Fig 7a) and 7b) show the power spectrum diagram of the Nifty and MCX-COMDEX data respectively. From the figure it is easy to predict that both the sets are chaotic in nature. Fig 8a) and 8b) show the power spectrum diagram of the DJ-AIGCI and DJI data respectively. From the figures we can find that the DJ-AIGCI data is much less chaotic than the DJI data set.

4.4 Cross Correlation

Fig 9 represents the cross correlation between Nifty and MCX-COMDEX data set. The plot shows that the two data sets have strong correlation. Fig 10 represents the cross correlation between DJI and DJ-AIGCI data sets. The plot shows that the two data sets have weak correlation.

5 Conclusion

The Recurrence Plots of NIFTY and MCX-COMDEX data sets show a similarity in terms of pattern change and nonlinearity. A close look at those of DJI and DJ-AIGCI data sets tells us that the two plots reveal widely different patterns. This indicates that the underlying nonlinear characteristics of the US Stock and Commodity data sets are quite different. The Recurrence Histograms of both NIFTY and MCX-COMDEX reveal a lack of periodicity. Though that of DJI data shows a similar characteristics, that of DJ-AIGCI data show presence of an underlying pattern. The Power Spectra of both Nifty and MCX data sets show chaotic behaviour. The same is not true for Dow Jones data set. Only the DJI data set shows chaotic behaviour while the DJ-AIGCI data set shows a less chaotic behaviour. The cross correlation diagrams reveal a greater correlation between the Indian stock and commodity indices data sets than the US ones. We can, therefore, reasonably conclude that based on the data sample, the underlying nonlinear dynamics of Indian stock and commodity markets are more closely related than in the US market. This result has important relevance towards formulation of policies relating to commodity and stock markets in macro perspective.

7 References:

## 6 Tables

### Table 1: RQA of NIFTY with a data shift of 90 days.

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<tr>
<th>Epoch</th>
<th>Start</th>
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<th>DET</th>
<th>ENT</th>
<th>MAXLINE</th>
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### Table 2: RQA of MCX-COMDEX with a data shift of 90 days.

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</tr>
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### Table 3: RQA of Dow Jones AIG Commodity index with a data shift of 90 days.

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<th>REC</th>
<th>DET</th>
<th>ENT</th>
<th>MAXLINE</th>
<th>TREND</th>
</tr>
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<td>14.198</td>
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### Table 4: RQA of Dow Jones Industrial Average with a data shift of 90 days.

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<th>ENT</th>
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