State Ownership and Bank Competition: A Mixed Oligopoly Approach

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January 5, 2009

Abstract: Recent events have led many governments to buy equity in banks thereby engendering mixed oligopoly in banking markets. We model two such cases where a state-owned bank competes with a private bank in collecting deposits. In the first case, the government is a welfare maximizer, but also profit oriented leading to partial privatization of the state owned bank. Managerial incentives of the private bank induce even greater privatization and transfer of profit from the public to the private bank with unchanged social welfare. In the second case, the government is purely a welfare maximizer and the private bank does not offer managerial incentives. Both banks face risks in the loan market. We show that if the risk of default is sufficiently high and there is limited liability, then the state owned bank tries to mitigate depositors’ losses by mobilising less deposits leading to contraction of aggregate deposit. This contradicts the certainty case and contradicts the standard mixed oligopoly result from the literature.

Keywords: Banking, mixed duopoly, managerial incentive, default risk

JEL classification: G21, L13, L33.

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1 Introduction

In the recent financial crises policy makers around the world have turned their attention to state ownership of banks. Ironically as recently as a decade ago there was an apparent consensus in favour of bank privatization, and governments in developing countries were compelled to divest a significant proportion of the equity of the state-owned banks. This was seen as a cure for non-performing assets, a chronic problem that was associated with public sector banks. But in a dramatic reversal of fortune, it is now the private sector banks that are at the centerstage of recent meltdown and have even been blamed in the media for their pursuit of profit. While economists are divided in their prescription for solution, policy makers have rushed to bail out distressed private banks, and even (significantly) nationalized many of them. One immediate benefit of nationalization is recapitalization. But how will it affect bank competition in the deposit market and in the loan market? The banking literature has studied financial contagion (Allen and Gale, 2000), bank runs and liquidity crisis (Smith, 1991; Diamond and Rajan, 2005), and risk-shifting (Boyd and De Nicolo, 2005); but it has mainly considered interactions among private banks, and therefore may not be adequately helpful in understanding the outcome of emerging competition between public and private banks.

There is a separate body of work in industrial organization called mixed oligopoly that exclusively studies competition between public and private firms (De Fraja and Delbono, 1989; Matsumura, 1998). The underlying view of mixed competition is that state ownership provides an indirect means of regulating the conduct of private firms using market instruments rather than using administrative-judicial interventions like the US Anti-trust system. A general result of this literature is that the publicly owned firm will act more aggressively and have greater market share than the private firms; but social welfare may not always rise, though in most cases will especially if the public firm’s degree of state ownership is optimally cho-
sen. Our aim is to adapt the mixed oligopoly approach to bank competition and see how state ownership affects bank interactions. There is a significant difference between banks and ordinary firms. In the industrial organization literature firms mainly interact in the output market and have no or little interactions in the input market. Banks, on the other hand, have to interact in both the deposit (input) market, and the loan (output) market; moreover by lending to each other they add another dimension to their relationship. Above all, unlike ordinary firms banks have to deal with loan default risk and its knock-on effects on different aspects of banking.

Apart from the above-mentioned arguments, there are good empirical reasons why banking should be modelled using the mixed oligopoly approach, especially for emerging economies. Despite large scale divestments many state-owned banks in developing and transition economies, entry of private and foreign banks has still remained a subject of state control. In fact, state presence is a common feature in banking systems all over the world. See Barth et al (2001) for evidence on EU countries, Sherif et al (2003) for transition economies and Shirai (2002) for China and India. It is, therefore, reasonable to expect that the banking industry in many economies (especially emerging) resembles mixed oligopoly with strategic interactions among banks occurring at many dimensions. There have been only a few articles that have considered mixed ownership or non-profit maximizing behavior of banks. Purroy and Salas (2000) studied competition between a private bank and a savings institution, and Saha and Sensarma (2004) between a private bank and a public bank. Both articles have demonstrated that the mixed duopoly approach can be useful; but they share a common limitation of focussing only on deposit competition and not default risk, which we aim to overcome in this paper.

In modelling public banks, our first hurdle is to define the objective of a public bank. Following the mixed oligopoly literature we assume that it maximizes social welfare, which is the sum of the payoffs earned by all participants in the banking industry, which principally are banks’ shareholders, borrowers (which are
essentially entrepreneurs seeking funds for risky projects) and depositors. But by no means this is the only possibility. One can add other objectives, specifically linked to loan or deposit market, that might better describe the circumstances and motivation for state intervention in a particular country.

Throughout our analysis we will consider only a duopoly setup. We begin by considering only deposit competition under the assumption of ‘no uncertainty’. This is just to see how a standard mixed oligopoly model can capture deposit competition. Here we allow a the interactions to be fairly general. The government can be to some extent profit-oriented. The private bank can offer managerial incentives to counter the generally aggressive behaviour of the public bank. Optimal privatization and optimal managerial incentives are the key questions that we seek to answer with this simple model.

We then introduce a loan market and allow uncertainty in the form of default risk. The government’s objective in this model is entirely social welfare maximization and the private bank does not offer managerial incentives; it is a profit-maximizer. Both banks have equity capital along with deposit liabilities. We first consider the case where the banks bear the entire risk; they pay depositors the due amount, even if the borrowers have defaulted. The results of this section is similar to the certainty case. The public bank acts aggressively, and this leads to greater deposit mobilisation. Then we introduce limited liability, which essentially puts the depositors at risk; if borrowers default, depositors lose money, as banks (like their borrowers) are protected by limited liability. We show that the public bank in this case takes into account the depositors’ loss, and if the default risk is significant it will act conservatively and collect less deposits. Though this will benefit the private bank in their deposit mobilisation, aggregate deposits can fall below the pure duopoly level (which corresponds to the case of competition between two private banks). This result is quite opposite to what standard mixed duopoly models predict.
Standard mixed duopoly models, which usually focus on output competition, predict more aggressive behaviour of the public firm largely because it puts a positive weight on social welfare, and in the usual way greater output leads to greater welfare. This argument largely holds for bank competition as well, if depositors do not face any risk (due to either lack of limited liability or complete deposit insurance). But as has been seen in recent crises, limited liability on the part of the entrepreneur-borrowers and banks put depositors at risk. A social welfare oriented public bank takes into account the potential loss that depositors might face when default occurs. If this risk is not small, the public bank will try to restrict its deposit mobilisation below a profit-maximizing level. The overall outcome may be a contraction of total deposits.

Our result can be related to a well established literature that has studied the effect of competition on risk taking. Both the theoretical and empirical results are mixed, though policy makers tend to believe that competition has destabilising effect on risk-taking by the bankers. Boyd and De Nicolo (2005) provide an overview of this literature, and argues that the negative view of competition is largely based on the theories that consider mainly single market interactions, viz. the deposit market (see also Hellman et al, 2000). If a loan market is introduced, they argue, bank competition will reduce the interest rate on loans and thus reduce the risk of default. In our paper, the state owned bank has a similar effect on aggregate deposit as competition does in a setup of private banks. We show that if the probability of default is below a critical level, then the Boyd and De Nicolo (2005) type of positive effect emerges, but if the probability of default is above this critical level, then the negative effects may dominate.

The paper is organized as follows. Section 2 presents the first model of deposit competition with no uncertainty. Section 3 introduces the loan market and default risk, wherein we first consider the benchmark case of no limited liability, which largely retains the certainty case results. Then we introduce limited liability, which shifts the risk onto depositors. Here we derive our main results on the effects of
2 Profit oriented public banks and deposit competition

We assume that government is a social welfare maximizer, but is also somewhat profit oriented. This profit orientation forces the government to privatize the public bank to some extent, even if there is no other bank in the market. The presence of another bank makes the government internalize some of the strategic effects that state ownership, howsoever partial, might have on the other bank’s profit. On its part, the private bank can counter the competitiveness of the public bank by offering revenue-linked incentives to its manager. The combination of managerial incentives and profit orientation will cause even greater privatization of the public bank. As a consequence, a particular type of mixed duopoly emerges in which privatization is always partial and the private bank always departs from (pure) profit maximization. More interestingly, in this mixed duopoly the government’s profit orientation determines a certain level of industry profit, and the private bank’s managerial incentives determine the distribution of profits leaving the social welfare unchanged. This pure redistributive role of managerial incentives is possible only in a mixed duopoly, and this has not been identified earlier in the literature.

In the context of banking Purroy and Salas (2000) studied competition between a profit maximizing private bank and a savings institution. Their savings institution exhibits ‘expense preference behavior’, i.e. utility maximization where utility is a weighted average of profit and workers’ wage-bill. They show that the private bank can partly restore the asymmetry created by the savings institution’s utility function by offering managerial incentives. However their results cannot be generalized to public banks and the question of privatization cannot be addressed. Our model tries to fill this gap.
There are many papers that have studied mixed duopoly (starting with the work of De Fraja and Delbono, 1989), but partial privatization concerns only a few (such as Fershtman, 1990; Matsumura, 1998). But there is a large literature on managerial incentives following the seminal work of Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987). We integrate these two literatures with the objective of simultaneously determining optimal privatization and managerial incentives. There is also evidence that firms in emerging economies offer managerial incentives.\footnote{See e.g. Eriksson (2005) and Kato and Long (2006) for empirical evidence on pay-performance relationships in the Czech Republic and China.}

### 2.1 Model I

We consider a two-stage game between a partially public and a fully private bank. In the first stage, the government decides on the share of public ownership in the partially public bank, while the private bank decides on managerial incentives. In the second stage the two banks engage in deposit competition.\footnote{When interest rate is regulated, as in many developing economies, banks tend to compete more in deposit.} Profits are subsequently realized.

The public bank is indexed 0, and the private bank indexed 1. Depositors earn interest rate \( r \) by the following rule

\[
r = b(D_0 + D_1), \quad b > 0.
\]

On the revenue side, both banks face a constant rate of return \( R \) on each unit of investment made out of these deposits.\footnote{This is a simplification. Results do not change if \( R \) varies inversely with \( D \). We can also allow for a statutory reserve ratio, in which case \( R \) is to be taken as an average rate of return, averaged over the reserve deposit and investible deposit.} Fixed \( R \) can be justified by assuming that money markets and loan markets are competitive, where banks are price takers, though they can have market power in raising deposits. The public bank
is jointly owned by the government and a private partner, and the choice of the volume of deposit is made by the bank’s board of management consisting of a government representative and the private partner. It maximizes the following objective function:

\[ Z = \theta SW + (1 - \theta)\pi_0. \]  

(1)

where \( \theta (\theta \in [0, 1]) \) is the degree of public ownership.\(^4\) The resultant deposit choice gives rise to the public bank’s reaction function which is

\[ D_0 = \frac{(R - bD_1)}{b(2 - \theta)}. \]  

(2)

There is an alternative approach to public firm suggested by Fershtman (1990), in which the same weights (\( \theta \) and \( 1 - \theta \)) can be applied to a fully public bank’s and a fully private bank’s reaction function to arrive at the partially public firm’s reaction function. Saha and Sensarma (2004) followed this approach. Both approaches offer qualitatively similar conclusions. In fact, Kumar and Saha (2008) have shown that both approaches are formally equivalent.

While the public bank’s deposit choice is specified in the above manner, it may be preceded by a decision of how much to divest or privatize, and this decision lies at a higher level of government, whose concern is to maximize social welfare and possibly at the same time ensure solvency of the public bank. Thus, the government chooses \( \theta \) to maximize a modified social welfare function which places a higher weight on profit. The modified social welfare function is denoted as

\[ V = DS + \beta(\pi_0 + \pi_1) \]  

which can be rewritten as \( V = SW + (\beta - 1)(\pi_0 + \pi_1) \). With \( \beta > 1 \), the government demonstrates its profit orientation by placing an additional positive weight on profit in its social welfare objective. Note that the government’s objective function differs from the objective of the government representative in the public bank’s management board. However, this difference is only in terms of

\(^4\)While the ownership exceeds 50 percent is very important, it cannot be denied that any change in \( \theta \) will have some effect on the bank behavior.
the profit orientation.

The private bank, though technologically identical to its public counterpart, may hire a manager and offer her incentives to boost its profit. Following the strategic delegation literature (see Vickers, 1985; Fershtman and Judd, 1987; Skilvas, 1987), we assume a linear incentive scheme which may reward (or penalize) the manager for generating revenue beyond the standard profit maximizing level. Formally, the manager is instructed to choose $D_1$ to maximize

$$M = (1 - \rho)\pi_1 + \rho RD_1.$$ 

Depending on the owner’s preference, $\rho$ can take a wide range of values. The standard case of profit maximization is given by $\rho = 0$. But if $\rho > 0$, the manager is encouraged to pursue sales more than profit. Conversely, $\rho < 0$ implies that the manager will be encouraged to pursue profit more than sales.

The manager maximizes the above objective function and her choice of $D_1$ gives the private bank’s deposit reaction function

$$D_1 = \frac{R - (1 - \rho)bD_0}{2b(1 - \rho)}. \tag{3}$$

The reaction curves of the two banks as given by (2) and (3) are shown in figure 1. Two thick curves, denoted as $RF_0$ and $RF_1$ are drawn with the assumption that $\theta \in (0, 1)$ and $\rho \in (0, \frac{1}{2})$. The downward slopes indicate that the deposits are strategic substitutes. If the private bank chooses zero deposit, the public bank will choose its monopoly deposit as $\frac{R}{b(2 - \theta)}$, and similarly, if the public bank chooses zero deposit the private bank’s manager will choose $D_1 = \frac{R}{2b(1 - \rho)}$. Conversely, if the private bank chooses $D_1 = \frac{R}{2}$, the public bank will simply close down, and similarly, the public bank’s choice of $D_0 = \frac{R}{b(1 - \rho)}$ will force the private bank to close down. Thus, the monopoly and entry-deterring levels of deposits of each bank can be defined in the usual way as quantity setting firms’ outputs are defined. The equilibrium deposits are given by point $M$ comprising of $D_0^*$ and $D_1^*$, which we
obtain as

\[ D_0 = \frac{R(1 - 2\rho)}{b(3 - 2\theta)(1 - \rho)}, \]
\[ D_1 = \frac{R[(1 - \theta) + \rho]}{b(3 - 2\theta)(1 - \rho)}. \]

(4)

It is clear that we must have \( \rho < \frac{1}{2} \) for \( D_0 \) to be positive. If indeed it were the case that \( \rho \geq \frac{1}{2} \), the public firm would be forced to close down, and the private bank would mobilize \( D_1 = \frac{R}{b} \), however its profit will fall to zero, which also suggests that the private bank will never set \( \rho > \frac{1}{2} \). This extreme situation is described by the two dashed reaction curves. On the other extreme, if \( \theta = 0 \) and both banks were profit-maximizers (i.e. \( \rho = 0 \)) we would have a pure (or equivalently private) duopoly. Both reaction curves would then shift inward and we have the Cournot Deposits as \( D_0 = D_1 = \frac{R}{3b} \). This is given by point \( N \) at the intersection of two thinner reaction curves. Since point \( M \) lies North-East of point \( N \), it is clear that the mixed duopoly generates much greater individual and aggregate deposits than the private duopoly.

As can be seen from (4), managerial incentive of the private bank and privatization of the public bank will both favor the private bank, and hurt the public bank in terms of their deposit choice. Formally, \( \frac{\partial D_0}{\partial \rho} < 0, \frac{\partial D_0}{\partial \theta} > 0, \frac{\partial D_1}{\partial \rho} > 0, \frac{\partial D_1}{\partial \theta} < 0 \), if \( \rho < \frac{1}{2} \).

Finally, if the private bank had set \( \rho < 0 \), its output would fall against any given \( \theta \), because its reaction function would shift inward starting from the situation of profit maximization. Consequently, its deposit would fall below the pure duopoly level. However, such a scenario is never profitable for the private bank. Therefore, we will not consider \( \rho < 0 \). Henceforth, our attention will be restricted to \( \rho \in [0, \frac{1}{2}] \).

Insert Figure 1 here
2.2 Optimal privatization and managerial incentives

We now move to the first stage of the game and analyze the strategic interactions in terms of managerial incentives and privatization. For this we need to derive the private banks’s profit and the government’s modified social welfare from the second stage equilibrium. Utilizing (4) we get

\[ V = \frac{R^2[2 - \theta \rho][(2 - \theta - \rho) + \beta(1 - \theta)(1 - 2\rho)]}{b(1 - \rho)^2(3 - 2\theta)^2}, \]

\[ \pi_0 = \frac{R^2(1 - \theta)(1 - 2\rho)^2}{b(1 - \rho)^2(3 - 2\theta)^2}, \]

\[ \pi_1 = \frac{R^2(1 - \theta)(1 - 2\rho)[1 - \theta + \rho]}{b(1 - \rho)^2(3 - 2\theta)^2}. \]

A crucial point to note is that both for \( \pi_1 > 0 \) and \( \pi_1 > 0 \) it is necessary that \( \theta < 1 \). Without some privatization two banks cannot operate. Further, in the pure duopoly case, i.e. when \( \theta = \rho = 0 \), each bank earns \( \pi = \frac{R^2}{b}\).

The government and the private bank determine their respective choice variables, viz. \( \theta \) and \( \rho \), simultaneously. The private bank’s owner chooses \( \rho \) by maximizing (7) as follows

\[ \rho(\theta) = \frac{1}{5 - 2\theta}. \]

This is the private bank’s ‘incentive reaction function’, which is clearly upward sloping in \( \theta \) (see figure 2). Starting from a situation of complete nationalization, as the public bank increases divestment (i.e. reduces \( \theta \)), \( D_0 \) fall, \( D_1 \) rises (assuming \( \rho < \frac{1}{2} \)), the private bank then reduces its aggressiveness by cutting down on its sales incentives (or incentives to mobilize deposit). Alternatively stated, in a situation of pure duopoly, the private bank can enjoy its highest profit by setting \( \rho = 1/5 \). Now if the government gets some stake in the rival bank, the private bank will experience a loss in profit. To make up for the lost profit, it will then raise its revenue incentive above 1/5. Thus, greater the \( \theta \), greater is \( \rho \). Thus, managerial incentive is strategic
complement to nationalization, or strategic substitute to privatization (which is measured by \((1 - \theta)\)).

To solve for the public bank’s response, we maximize \(V\) with respect to \(\theta\). This gives us

\[
\theta = \frac{2 - \beta - \rho(1 + \beta)}{1 - 2\rho\beta}.
\]

Several points are noteworthy. First, if \(\beta = 1\), optimal \(\theta\) is 1 regardless of \(\rho\). Second, given \(\rho < 1\) and \(1 < \beta < 2\), government’s choice of privatization is partial. That is, \(\theta \in (0, 1)\). Third, when \(\rho = 0\), the resulting privatization is still partial, \(0 < \theta < 1\). This is because, the government is now concerned about profit, and driving depositor surplus to its maximum by setting \(\theta = 1\) involves inflicting loss on both banks. Given \(\beta > 1\) that cannot be optimal. Fourth, from the government’s point of view nationalization (privatization) and managerial incentives are strategic substitutes (complements), as \(\theta'(\rho) = -\frac{(\beta - 1)(2\beta - 1)}{(1 - 2\rho\beta)^2} < 0\). This is exactly opposite of the perspective the private bank has. This is where mixed duopoly is crucially different from pure duopoly. As the government is not only concerned about private bank’s profit, but also values the industry profit relatively more than depositor surplus, it internalizes some of the negative effects on profit that would follow from aggressive deposit mobilization by both banks. So when the private bank is expected to increase its deposit incentives (thus induce greater aggression by its manager in the second stage), the public bank divests its ownership to reduce the public bank’s aggression, in order to contain the overall level of deposit mobilization. Thus, the government accommodates the private bank’s aggression through privatization.

We derive the Nash equilibrium from the intersection of the two reaction functions, i.e. the nationalization reaction function of the public bank and the incentive reaction function of the private bank. Equilibrium \(\theta\) and \(\rho\) are given as follows
\[
\theta^* = 3 - 2\beta, \\
\rho^* = \frac{1}{4\beta - 1}.
\] (10)

The solution is graphically shown in figure 2. In order to ensure an interior solution we need to assume that vertical intercept of the nationalization reaction function is greater than that of the incentive reaction function. This gives an upward limit on \( \beta \), i.e. \( \beta < \frac{3}{2} \). Beyond this level of \( \beta \), the government becomes too profit oriented and hence would prefer to fully privatize the public bank. Therefore, for \( \beta \in (1, \frac{3}{2}) \), (10) gives the equilibrium solution of \( \theta \) and \( \rho \).

Insert Figure 2 here

We can contrast this solution with two situations. First, if there were no private banks at all, what would be the optimal privatization? This can be determined by considering that if \( D_1 = 0 \) optimal \( D_0 \) would be \( \frac{R}{\pi(2-\theta)} \) yielding \( V^M = \frac{R^2}{\pi^2(2-\theta)^2}[(1 + 2\beta(1 - \theta))] \) and maximizing \( V^M \) one obtains the optimal \( \theta \) as \( \frac{1}{3} \). This optimal privatization depends only on \( \beta \). If however, \( \beta = 1, \theta^* = 1 \), i.e. divestment under monopoly is optimal only if there is profit concern. Second, if the government had no additional concern for profit (i.e. if \( \beta = 1 \)) its reaction curve would be vertical at \( \theta = 1 \) and the only Nash equilibrium is then \( \rho = \frac{1}{3} \) and \( \theta = 1 \). This gives rise to a strange situation. Both firms make zero profit, and yet mobilize strictly positive deposits. At \( \rho = 1/3 \) we can see from figure 1 both \( D_0 > 0, D_1 > 0 \), though the private bank remains aggressive in the deposit competition, its aggressiveness hardly pays off. But this is also not a situation where the private bank’s best response is to exit. Thus, both banks will be stuck in a state of insolvency.

The mixed duopoly literature generally highlights the aggressiveness of public ownership and how it can force private firms to suffer loss and even exit; but what has not been considered at all is the potential response of the private firms. By
using sales-oriented managerial incentives the private firms can prevent their exit from the industry, albeit earning only zero profit. So the managerial incentive can be seen as a survival strategy of private firms in the face of complete nationalization. In this environment, therefore, the government can make the mixed duopoly solvent by having some additional concern for profit. Of course, an immediate implication of the profit concern is that the government will be forced to internalize some of the strategic effects of managerial incentives, and therefore will divest more. We now summarize our main result.

**Proposition 1**  If the government is profit oriented, (i.e. $\beta > 1$), the Nash equilibrium is characterized by the public bank being partially privatized and the private bank offering managerial incentives. Compared to the ‘no managerial incentives’ case, privatization will be greater. With an increase in $\beta$, equilibrium $\theta$ and $\rho$ fall.

While we have already discussed the equilibrium responses, here we consider the effect of an increase in $\beta$. A rise in the profit orientation of the government leads to greater privatization, and lower managerial incentives. Here the $\theta$ reaction curve shifts to the right, and the $\rho$ reaction curve remains unaffected. Hence equilibrium $\theta$ and $\rho$ decline. Thus, a higher profit orientation of the government induces both banks to move towards profit maximization behavior, and reduce their competitiveness.

Finally we check the social welfare implications of partial privatization and managerial incentives. Recall that $SW = (R - \frac{bD}{2})D$, where $D$ is the total deposit. It can be easily seen that $SW$ is an increasing function of $D$ as long as $R > bD$ (which holds in equilibrium). Hence higher $D$ would mean higher social welfare. Therefore comparing social welfares in two situations boils down to comparing total deposits. From the second stage equilibrium deposits as given in (5) we derive the total deposit as

$$D = \frac{R[2 - \theta - \rho]}{b(1 - \rho)(3 - 2\theta)}.$$  

(11)
Next we substitute the equilibrium values of $\theta$ and $\rho$ from (10) in (11) and obtain the equilibrium value of $D$ which is

$$D^* = \frac{R\beta}{b(2\beta - 1)}.$$  

We would like to compare $D^*$ with two situations: (i) pure duopoly without managerial incentives, and (ii) mixed duopoly without managerial incentives. In the first case, total deposit is $D = \frac{2R}{3b}$ which is obtained by substituting $\rho = \theta = 0$ in (11). Since $\beta > 1$, $D^* > \frac{2R}{3b}$. That is, social welfare is higher in the mixed duopoly as compared with the case of a private duopoly without managerial incentives.

In the second case, we find the equilibrium industry deposit without managerial incentive. This is obtained by substituting $\theta = 2 - \beta$ (which is the optimal value of $\theta$ when $\rho = 0$) in (9). This yields $D = \frac{R\beta}{b(2\beta - 1)} = D^*$. Therefore, social welfare in the mixed duopoly with managerial incentives is same as that in a mixed duopoly without managerial incentives. Since $D$ is same in both situations, depositor surplus is also same and therefore the industry profit is also unchanged. But we know from (7) that the public bank’s profit will fall with an increase in $\rho$. Then it must be the case that with managerial incentives the public bank’s profit has fallen and the private bank’s profit has risen, by exactly the same amount. Thus, the managerial incentive is playing a merely redistributive role with no effect on efficiency, which is entirely determined by the government’s profit orientation.

We believe, this is a new insight into managerial incentives. In a pure duopoly managerial incentives (offered by two private banks) lead to mutual over-production, which generates higher social welfare, but lowers industry profit. In a mixed duopoly with endogenous privatization, the government internalizes the strategic effects on the private bank and thus softens the intensity of deposit competition by privatizing appropriately. The extent of privatization will however depend on the extent of profit orientation. If, for instance, the government is not at all profit oriented ($\beta = 1$), industry profit will be zero; but with managerial incentives the
private firm will still be able to induce a redistribution of deposits with however no profit to redistribute. But if $\beta > 1$, the government’s concern for industry profit will be greater and its optimal privatization in the absence of managerial incentive will ensure an industry profit of $\pi = \frac{R^2 \beta(\beta-1)}{b(2\beta-1)^2}$. If now the private bank offers managerial incentives, the government will further divest, so that the industry profit and social welfare remain unchanged, but the private bank’s profit rises at the expense of the public bank’s and this redistribution of profit will take place via a redistribution of deposits. The following proposition summarizes our finding on social welfare.

**Proposition 2** Social welfare in a mixed duopoly with equilibrium privatization and managerial incentives is higher than that in a private duopoly without managerial incentives; but it is same as that in a mixed duopoly without managerial incentives. Thus in a mixed duopoly, managerial incentives become merely redistributive having no efficiency effect.

### 3 A model with a loan market

We now introduce a loan market where the interest rate is determined by the following loan demand curve $i = \bar{c} - cL$, $c > 0$, where $L$ is the total loans made. The banks give out loans from not only the deposits raised but also from equity ($E$) raised from the capital market. Total lending by a bank equals the sum of deposits and shareholder equity, i.e. $L = D + E$. Left to themselves the shareholders might not put in enough of their own funds into the bank leading to a highly leveraged bank (where the leverage ratio is $D/L$). Therefore, through capital regulation the regulator has to enforce a minimum level of shareholder equity (Milne, 2006). Let $\gamma$ be the capital to asset ratio imposed by the regulator (similar to risk weighted capital adequacy ratio e.g. 8% under Basel-I norms) such that $\gamma = E/L$. We can then rewrite the balance sheet equality as $L = kD$ where $k = \frac{1}{1-\gamma}$.
We also assume that the entrepreneur-borrowers face uncertainty in the realization of the value of their project. There are two states of nature: good and bad. The good state occurs with probability $p_1$ and bad state with probability $p_2$. In the good state the borrowers are able to pay back both principal and interest, but in the bad state they pay back nothing. However, the bank can liquidate the firm and recover the principal, but the interest is lost. Alternatively the borrowing can be assumed to be collateralized. In the bad state, as the bank loses interest earnings, its ability to repay the depositors is affected. the depositors get back the principal, but loses out on the interest payments. However, since the bank has equity ($k > 1$) it will be able to meet the interest obligations to the extent it has funds. Thus, the depositors stand to lose in the bad state. Note that if there is complete deposit insurance or if the banks are sufficiently capitalized ($k$ large) then the depositors do not lose. But we do not consider deposit insurance here, instead allow the depositors to bear the risk by allowing limited liability on the part of the banks and entrepreneur-borrowers.\footnote{Exogenously given schemes of deposit insurance can be introduced in our framework without much complications. But in principle the insurer should be allowed to monitor the behaviors of the banks, which is a non-trivial exercise and we do not pursue it here.} As a benchmark case we first consider deposits as debt obligations to be honored in all states of nature.

In this section and the remainder of the paper we will assume that government is a social welfare maximizer, and the private bank does not offer managerial incentive. In other words, in the government’s objective function $\beta$, the weight on bank profits, is 1 and in the private bank’s objective function $\rho = 0$.

### 3.1 No limited liability: A benchmark case

Bank $i$ lends $L_i(= kD_i)$ and earns at the end of the year

$$E\pi_i = p_1 [(1 + i)kD_i - (1 + r)D_i] + p_2 [kD_i - D_i(1 + r)],$$

$$= \left[k - \delta(D_0 + D_1)\right] D_i,$$

\[12\]
where $\tilde{k} = k - 1 + p_1k\bar{c}$ and $\tilde{\delta} = p_1k^2c + b$. Sum of the two banks’ expected profits is

$$E\pi = E\pi_0 + E\pi_1 = \left[\tilde{k} - \tilde{\delta}(D_0 + D_1)\right](D_0 + D_1). \quad (13)$$

The depositors expect to earn a surplus of

$$E[DS] = r(D_0 + D_1) - b\frac{(D_0 + D_1)^2}{2} = b\frac{(D_0 + D_1)^2}{2}. \quad (14)$$

This is same as the no-uncertainty case (which we analyzed previously).

Now we have another group - entrepreneur-borrowers - whose expected surplus is

$$E[BS] = p_1\left[Rk(D_0 + D_1) - c\delta^2\frac{(D_0 + D_1)^2}{2} - ik(D_0 + D_1)\right],$$

$$= p_1k^2c\frac{(D_0 + D_1)^2}{2}. \quad (15)$$

Expected social welfare of this economy is then defined as $SW = E\pi + E[BS] + E[DS]$. Adding equations (13), (14) and (15) we get

$$E[SW] = \left[\tilde{k} - \tilde{\delta}\frac{(D_0 + D_1)}{2}\right](D_0 + D_1). \quad (16)$$

The public bank as before pursues an objective function $W = \theta E[SW] + (1 - \theta)E\pi_0$, where $\theta$ is the degree of public ownership. Maximizing $W$ we derive the public bank’s deposit reaction function as

$$D_0 = \frac{\tilde{k}}{(2 - \theta)\bar{\delta}} - \frac{D_1}{2 - \bar{\theta}}.$$

Similarly, the private bank’s reaction function is

$$D_1 = \frac{\tilde{k}}{2\bar{\delta}} - \frac{D_0}{2}. \quad (18)$$
From the above two equations we solve the Nash equilibrium deposits raised by the two banks:

\[
D_0 = \frac{\tilde{k}}{\delta(3 - 2\theta)}, \quad (17)
\]

\[
D_1 = \frac{\tilde{k}(1 - \theta)}{\delta(3 - 2\theta)}, \quad (18)
\]

The aggregate deposit in this model is

\[
D_o + D_1 = \frac{\tilde{k}(2 - \theta)}{\delta(3 - 2\theta)}.
\]

It can be readily checked that both the public bank’s deposit and aggregate deposit are increasing in the degree of nationalization. On the other hand, the private bank’s deposit is decreasing in the degree of nationalization. In fact, when the public bank is fully public, \( \theta = 1 \), the private bank’s deposit is reduced to zero. This is the case where the public bank crowds out the private bank, a phenomenon commonly occurring mixed oligopoly models.\(^6\)

In this case, when debt obligations are honored in all states of nature the standard results of mixed duopoly models continue to hold, in which the public bank acts more aggressively (in raising deposits) and the private bank acts more defensively. In other words, the standard profit-shifting motive of public ownership continues to hold even if there are some states of nature forcing defaults. It can also be checked that if the government was to choose optimal privatization, it would choose \( \theta = 1 \). Total deposit under optimal (zero) privatization would be \( D_0 = \frac{\tilde{k}}{3\delta} \) and the expected profit of the public bank will be zero. That is, its expected loss in the bad state would be equal to its expected profit in the good state. The private bank is driven out of business.

\(^6\)However, in standard mixed oligopoly models for this to happen constant marginal cost is required. In our banking model, neither the marginal return curve nor the marginal cost curve is constant. But on both dimensions they depend on aggregate deposits. This is a crucial feature of banks, as opposed to standard firm.
### 3.2 Limited liability

Now we assume that the banks are insufficiently capitalized, so that in the bad state it cannot meet its full obligations. This will be the case if \( k = 1 \) or \( k \) is slightly greater than 1, or the bank is not able to recover the full amount of the principal from the entrepreneurs. For simplicity we will assume that the full amount of the principal is recovered, but \( k \) is not large enough to cover the interest charges payable to the depositors in the equilibrium. Due to limited liability, depositors get back what is available with the bank, \( kD \), when \( kD < (1 + r)D \). Since \( r \) increases with the total deposit \( D \), it is quite possible that the depositors may not lose at all if the total deposit is not large relative to \( k \). That is, if \( k > (1 + r) \) then even in the bad state the depositors are safe. For our story to hold, we need to focus on the case where \( k < (1 + r) \).

With limited liability banks do not money in the bad state. Their expected profit is now given only by the good state.

\[
E\pi_i = p_1 \left[ (1 + i)kD_i - (1 + r)D_i \right],
\]

\[
= p_1 \left[ k^* - \delta(D_0 + D_1) \right] D_i,
\]

where \( k^* = k - 1 + k\bar{c} \) and \( \delta = k^2c + b \).

From the above one gets the aggregate expected profits as

\[
E\pi = E\pi_0 + E\pi_1 = p_1 \left[ k^* - \delta(D_0 + D_1) \right] (D_0 + D_1).
\]

The depositors now stand to lose in the bad state. Their surplus is

\[
E[DS] = p_1(1 + r)(D_0 + D_1) + p_2k(D_0 + D_1) - D - b\frac{(D_0 + D_1)^2}{2},
\]

\[
= \left[ \frac{1}{2}p_1b(D_0 + D_1) + p_2(k - 1) - p_2b\frac{(D_0 + D_1)}{2} \right] (D_0 + D_1).
\]

Note that with the restriction \( kD < (1 + bD)D \) or \( k - 1 < b(D_0 + D_1) \), (so that
limited liability applies) $E[DS]$ is now strictly less than $b(D_0 + D_1)^2$, which was the expected depositor surplus under the ‘no limited liability’ case.

The expected borrower surplus remains unchanged from equation (15). Adding the expressions in (15), (20), and (21) we derive the expected social welfare as

$$E[SW] = p_1 \left[ k^* - \frac{\delta}{2}(D_0 + D_1) \right] (D_0 + D_1) + p_1 b \frac{(D_0 + D_1)^2}{2} + p_2 (D_0 + D_1)(k - 1) - b \frac{(D_0 + D_1)^2}{2}. \quad (22)$$

Following the same procedure as before we get the following two reaction functions:

$$D_0 = \frac{p_1 k^* + \theta p_2 (k - 1)}{p_1 \delta (2 - \theta) + \theta p_2 b} - \frac{[p_1 \delta + \theta p_2 b]}{p_1 \delta (2 - \theta) + \theta p_2 b} D_1, \quad (23)$$

$$D_1 = \frac{k^* - D_0}{2\delta} \quad (24)$$

Solving the above system of equations we get the following Cournot-Nash deposits:

$$D_0 = \frac{p_1 k^* \delta + \theta p_2 \{(k - 1)2\delta - bk^*\}}{\delta \left[p_1 \delta (3 - 2\theta) + \theta p_2 b\right]}, \quad (25)$$

$$D_1 = \frac{p_1 k^* \delta (1 - \theta) + \theta p_2 \{bk^* - (k - 1)\delta\}}{\delta \left[p_1 \delta (3 - 2\theta) + \theta p_2 b\right]}, \quad (26)$$

resulting in the aggregate deposit

$$D_0 + D_1 = \frac{p_1 k^* (2 - \theta) + \theta p_2 (k - 1)}{p_1 \delta (3 - 2\theta) + \theta p_2 b}.$$

At this point we can make the following observations. Neither $D_0$ and $D_1$ are unconditionally monotonic in $\theta$. In particular, it is striking that even if $\theta = 1$, the private bank’s deposit does not become zero. It remains strictly positive. More importantly, the aggregate deposit under the mixed duopoly (with the public bank being fully public) may not necessarily be greater than that under pure duopoly.
If both banks were privately owned (i.e. \( \theta = 0 \)), aggregate deposit is

\[
D(\theta = 0) = \frac{2k^*}{3\delta}.
\]

With full public ownership in the public bank, aggregate deposit is

\[
D(\theta = 1) = \frac{p_1k^* + p_2(k - 1)}{p_1\delta + p_2\delta}.
\]

By comparing these expressions and individual deposits of the banks we arrive at the following result.

**Proposition 3** If \( p_2 > \frac{\delta k^*}{k(3\delta c - 2k^* c)} \), aggregate deposit under full public ownership of the public bank will be less than the aggregate deposit under pure duopoly. That is, \( D(\theta = 1) < D(\theta = 0) \); moreover, \( D_0(\theta = 1) < D_0(\theta = 0) = D_1(\theta = 0) < D_1(\theta = 1) \). On the other hand, if \( p_2 < \frac{\delta k^*}{k(3\delta c - 2k^* c)} \), we have \( D(\theta = 0) < D(\theta = 1) \), and \( D_1(\theta = 1) < D_0(\theta = 0) = D_1(\theta = 0) < D_0(\theta = 1) \).

One implication of this proposition is that when the likelihood of the bad state is above a critical level, the depositors’ loss matters a lot and the publicly owned bank collects deposit very conservatively. However, this defensive act encourages the private bank to increase its deposit, because the two banks’ deposits are strategic substitutes. However, the aggregate deposit falls short of the fully private or pure duopoly level and thus the depositors’ loss is restricted to an optimal level. On the other hand, if the bad state is less likely then the government social welfare maximization objective encourages its bank to mobilize greater deposits and the market is expanded well beyond the pure duopoly level. In the latter case, we have a similar outcome as if there was no uncertainty. But in the former case, public ownership leads to a contraction in the market, quite in contrast to standard mixed duopoly models.

We may relate this to the finding of Boyd and De Nicolo (2005) where greater competition reduces default risk by lowering the cost of borrowing (as the loan
rate falls) and also by making more low-risk-low-profit projects viable. But our model suggests that this finding may not hold unconditionally. Though we considered only exogenous risk, it can be argued that in the presence of a public bank, results can go either way. If the default risk is significant, then the public bank’s conservative approach will lead to a contraction in total loans causing interest rate to rise. In the view of Boyd and De Nicolo then entrepreneurs will shift to more risky projects. If we allow endogenous default probability, we will be able to see which the public bank reacts and how the government’s choice of public ownership is affected.

4 Conclusion

This paper explores optimal partial privatization and managerial incentives in the framework of a ‘mixed oligopoly’ involving a partly divested public bank. We develop two models. In the first model, only deposit competition is considered and there is no risk of default. In this model we show that if the government is profit oriented, then the public bank has to be partially privatized, and in response the private bank will offer revenue-linked incentives to its manager. But privatization will be greater than in the case where the private bank does not offer managerial incentives. Thus, managerial incentives and partial privatization appear to be integral features of mixed duopoly. Moreover, profit orientation and managerial incentives appear to be playing two distinct roles. Profit orientation determines the industry profit and managerial incentives determine its distribution between the two banks. This merely redistributive role of managerial incentive seems to be possible only in a mixed duopoly.

In the second model we introduce a loan market and an exogenous risk of default. Here, if there is no limited liability or if there is complete deposit insurance, then in the event of loan default bank loses money, but depositors remain safe. But bank’s loss is equal to depositors’ gain. Therefore, in the social welfare this
loss does not matter. The public bank still acts aggressively and mobilises greater deposits leading to an expansion of aggregate deposits, though in the process the private bank’s operation is squeezed. This is very much in line with the predictions of standard mixed duopoly models. But if the banks are protected by limited liability and deposits are only partially insured, then depositors are exposed to risk, and they cannot pass on this risk to anybody else. Therefore, in the social welfare calculations the expected loss of the depositors matters. The public bank takes into account this loss, and changes its behaviour in the following way. If the default risk is significant, it will mobilise less deposits resulting in a contraction of aggregate deposits, though the private bank will be able to profit from this defensive act. Essentially, the public bank will try to borrow less and lend less. This is a reversal of the standard mixed duopoly result. However, if the risk of default is small, the public bank returns to aggressive competition.
References


Figure 1: Deposit reaction functions of public and private banks
Figure 2: Equilibrium privatization and managerial incentives