

Hedging performance of Nifty index futures[☆]

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ABSTRACT

The primary objective of futures market is to provide a facility for hedging against market risk. The L.C. Gupta committee on Indian derivative markets clearly supported the introduction of exchange traded futures to aid risk management strategies. This paper attempts to investigate whether the introduction of index futures trading in the National Stock Exchange (NSE) of India has been an effective risk management instrument for the spot market of Nifty portfolio. The daily return distributions are modelled through a rigorous exploratory data analysis and risk management in futures market is quantified by employing four alternative time series models with the objective to bring out the model that provides the best fit to the returns series and the highest hedge performance. All the models prove to be useful in providing significant reduction in the variance (more than 90%) of the hedged portfolio. However, GARCH model outperforms others as it provides the highest hedge effectiveness and the best fit to the data generating process of the two return series. Over all the results indicate that hedging with Nifty futures is effective (97%) for managing risk in the spot (Nifty) market.

Key words index futures; hedge ratio; hedge effectiveness; volatility clustering; excess kurtosis

JEL classification GI

Introduction

There has been a heightened interest in futures trading in India since financial derivatives were introduced in Indian market in 2000, following the recommendations of L. C. Gupta Committee (1998). According to the committee, financial derivatives provide “*the facility for hedging in the most cost-efficient way against market risk*”. A market wide survey conducted by the committee reported, hedging to be the primary purpose for participants to engage in derivative trading. Here, stock index futures were found to be the most popular and preferred type of financial derivatives. The Committee cited several reasons for the wide acceptance and strong preference of stock index futures.¹ Accordingly, index futures were the first to be introduced in India’s derivatives markets.

Literature on hedging offers a wide variety of alternative models that can be used to model and quantify the hedge and hedge effectiveness of derivatives products. However, the results on the performance of these models have been mixed. This is a non-trivial problem, since it has been found that different models yield different hedge measures for the same derivative product. Therefore, practical application of hedging with futures requires choosing among these alternatives for best fit and hedge performance.

It has been more than nine years, since index futures were introduced in India and hence it would be of great interest to analyze the performance of index futures contract in terms of the level of hedge effectiveness that these contracts have offered to investors. This paper attempts to investigate whether the introduction of index futures trading in the National Stock Exchange (NSE) of India has been an effective risk management instrument for the spot market of Nifty portfolio.

[☆] I thank Dr. Mandira Sarma for her useful comments and suggestions.

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¹ For details, see L. C. Gupta Committee (1998).

In this paper we have employed a framework of four alternative time series models, where a rigorous exploratory data analysis on the daily return distributions precedes the estimation and comparison of hedge ratio and hedge effectiveness. Our results indicate that all the models are able to provide significant reduction in the variance (more than 90%) of the hedged portfolio. However, GARCH model outperforms others as it provides the highest hedge effectiveness as well as the best fit to the data generating process of the two return series.

The paper is organized as follows. Section II will discuss the trading mechanism of stock index futures contract. Section III describes the concept and alternative models which are used in dealing with the statistical properties of financial return series and quantifying the hedge and hedge effectiveness. In section IV we present an exploratory data analysis. Section V presents the empirical work along with the major findings, and section VI presents the conclusion.

II. Stock Index Futures

Stock index futures are *contracts* that are based on a stock market index. That is, these are derivatives that derive their value from an underlying index². By trading in Index futures, participants bet on the movement of the entire stock market.

While trading on stock index futures, the participant takes a view on the way the market will move and then accordingly take *long* or *short position*. On the *settlement date* or the *expiration date*, if the closing index value is higher than the value at which the index futures was initially bought, then the participant makes profit. Nevertheless, if the closing index value is lower than the level at which it was bought, the participant makes a loss. However, in this case, the participant will make profit if she had anticipated a downswing in the market and had sold earlier. It is similar to buying low and selling high or conversely selling and then buying back when the market goes down.

Index futures do not trade in shares rather they are traded in terms of number of contracts. Each contract has a standard *lot size* set up by the exchange. Futures trading involve the payment of initial margins that is an exchange prescribed percentage of the entire amount of the contract, to be paid to the exchange via the broker. This way the participant only pays the margin amount and not the entire amount of the contract³.

Further the participant's position in the futures market is '*marked to the market*' every day. This is done by the exchange to assess the value of a participant's position on a daily basis. At the end of each trading day, the margin amount is adjusted to the tune of the participant's gain or loss. This simply implies that if the market value of a participant's position shows a loss, then the difference is debited from her margin payment. Similarly, if her position has gained in value, the margin account is credited by profit amount⁴.

² The index is a financial asset whose price or level is a weighted average of stocks constituting an index.

³ If Mrs. X bought 100 units of NIFTY January expiry contract @ Rs 1400/- and if the daily margin is 5%, then the margin payment is just Rs 7000/- (i.e. 5% of 1400 * 100) and not the entire amount of the contract which is 140000/- (i.e. 1450 * 100).

⁴ Say, Mrs. X bought 100 units of NIFTY December expiry contract @ Rs 1450/- on 3rd October and suppose, that by the end of the day the futures price increases to 1460/-. In such a case the participant makes a mark to market profit of Rs 1000/- (i.e. (1460 - 1450) * 100) on 3rd October.

The exchange also allows participants to close the contract before expiration date. In such a case, the participant is required to enter into a transaction which will square up her position i.e. if the participant initially bought an index futures and want to close it now, then she can sell an index futures of same quantity and thus square off (or nullify) her futures position in the market. Such a transaction will ‘*close out*’ the existing buy or sell position of the participant and the difference between the participants buy or sell prices will be her gain or loss.

A stock index futures contract gets *cash settled* on the expiration day. This means that there is no physical delivery of securities, but only the difference between the contracted value and the closing index value is settled on the expiry day, depending upon whether the participant made a profit or a loss.

III. a. Hedging with futures

Hedging implies minimizing the risk of an investment by taking an offsetting position. The primary purpose of futures market is to provide an efficient and effective mechanism for hedging. Participants buy or sell futures contract which establishes a price level now for the asset to be delivered later, in order to insure themselves against the adverse changes in the price of the asset being traded. This is termed as *hedging with futures*. This way a futures hedge reduces the price risk by making the outcome more certain.

Hedger’s are participants who enter the futures market to offset the risk in an underlying risky investment made in the spot market. In a long hedge, participants buy futures to offset a short position in the spot market. In a short hedge, participants sell futures to offset a long position in the spot market. Thus, hedging with futures theoretically works on a simple rule: ‘Long a cash security, sell futures’ which means that any loss /profit in the spot market should be compensated by a profit /loss in futures.

The above concept of hedging with futures, as a risk management strategy undertaken to manage the risk associated with the investments made in the spot market has been widely accepted in the finance profession. While the concept is simple and effective, measuring or quantifying the hedge is not a simple one. There are several alternative econometric techniques that can be used to measure the hedge and capture the characteristics of the data on financial time series that is used in the analysis.

The fundamental quantitative tool used for measuring hedge is the “hedge-ratio”. Researchers have used several methodologies for measuring hedge ratios to quantify hedging with futures contract. Among these the following four methodologies are most commonly used 1) the Ordinary least square (OLS), 2) the Vector auto regression (VAR), 3) the Vector error correction Model (VECM) and 4) Multivariate GARCH. Empirical studies have extensively explored these four models and their conclusions report that OLS, VAR and VECM estimate static hedge ratio as these models ignore the time varying component of the variables and that the hedge ratio will vary over time as the conditional distribution between spot and futures prices changes. Hence, in order to take account of the time varying dynamic financial time series these studies have estimated hedge ratio using GARCH techniques.

III. b. Quantifying Hedge ratio and Hedge effectiveness

The *hedge ratio* (h) is defined as the number of futures contract to hold for a given position in the spot market. Given the overall spot position of a participant, the hedge ratio is defined as the ratio of the size of the position taken in the futures market to the size of the position in the spot market (Hull 2000).

$$h = \frac{\text{futures position}}{\text{spot market position}}$$

A perfect hedge would exactly offset all the risk in an underlying risky investment. The simple strategy to achieve a perfect hedge, involves entering into a futures position that is equal in magnitude but opposite in sign to the spot market position i.e. long (short) spot market position, short (long) futures market position in same magnitude. This is known as the one to one risk minimization approach, which makes the hedge ratio equal to -1.

The execution of a perfect hedge completely eliminates the market risk and is important as the portfolio gets immunized, but the approach has the following drawbacks: first it is hard to find a perfect hedge. A perfect match i.e. choosing a futures contract on an underlying asset that is as similar as possible to the asset to be hedged are sometimes not possible for financial futures, and cross-hedges⁵ are more common. Secondly, an important feature of perfect hedge is that the price of the spot portfolio must be perfectly correlated with the movement of the asset underlying the futures contract. However in practice, futures and spot price movements are not perfectly correlated, generating a large basis risk. A basis risk is the residual risk (futures price – spot price) implying that the price of the asset being hedged and the futures contract price, will not move together perfectly over time. Closer the price movements of the asset and the futures instrument, less the basis risk.

In the backdrop of the above limitations, for a hedger whose objective is to minimize risk, the hedge ratio of 1 may not necessarily be optimal and so we need an approach that takes into account the less than perfect relationship between spot and futures prices.

The literature dates back to Johnson (1960) and Stein (1961, 1964) who were the first to point out the imperfect correlation between spot and futures market prices. Their investigations opposed the naïve one to one fixed hedge ratio and advocated the use of portfolio approach to estimate minimum variance hedge ratio (MVHR) known as the '*Optimal hedge ratio*' (h). Their model applied mean variance approach of Markowitz (1956) to define optimal hedge ratio as a ratio that provides the minimum portfolio variance because hedgers can maximize their utility only by minimizing the conditional variance of the hedged portfolio.

In the portfolio approach hedging with futures is considered as a portfolio selection problem, where futures can be used as one of the assets in portfolio to minimize risk of the overall position in the spot market (Johnson 1960, Grant 1982). Therefore, the objective is to choose a hedge ratio that will minimize the risk of the spot portfolio and the futures position.

⁵ In actual hedging application, the spot and the futures position may differ in terms of either time span covered or other characteristics of the instruments. In such cases a cross hedge can be executed in which the characteristics of spot and the futures position do not match perfectly.

Johnson (1960) defines risk as the variance of return on two - asset hedged position. Hence, the hedge question here is what trade in futures will produce minimum variance in the value of the spot portfolio. To illustrate the above idea, suppose that if an investor is long one unit of the asset in spot market, she needs to be short h units of the futures to hedge her price risk (Hull 2001, 2003).

We write the value of the initial position as: $R(t) = S(t) - h F(t, T)$

Where $S(t)$ = spot price at time t

$F(t, T)$ = futures price at time t of a contract that expires at time T

The value of the position at τ will be: $R(\tau) = S(\tau) - h F(\tau, T)$

Where $S(\tau)$ = spot price at time τ

$F(\tau, T)$ = futures price at time τ expiring at time T

Here both $S(\tau)$ and $F(\tau, T)$ are unknown at time t which makes $R(\tau)$ a random variable. The change in the value of the positions will be: $\Delta R = \Delta S - h\Delta F$

Variance of $\Delta R = \sigma_{\Delta S}^2 + h^2\sigma_{\Delta F}^2 - 2h\rho_{\Delta S\Delta F}\sigma_{\Delta S}\sigma_{\Delta F}$

For the variance to be minimized, the f.o.c is $\frac{\partial \sigma_{\Delta R}^2}{\partial h} = 0$

Which gives $h = \rho_{\Delta S\Delta F} \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$

where $\rho_{\Delta S\Delta F}$ = coefficient of correlation between ΔS & ΔF

$\frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$ = standard deviation of ΔS to standard deviation of ΔF .

The above can also be written as $h = \frac{\text{cov}(\Delta S\Delta F)}{\text{var}(\Delta F)}$

The optimal hedge ratio (h) is calculated as the ratio of the covariance between changes in spot and futures prices to the variance of the change in futures price. Here, h performs two functions: a) it estimates the correct number of futures contracts that will minimize the risk from spot market fluctuations b) it shows how the variance of the hedger's position depend on the hedge ratio chosen .i.e. if $\rho_{\Delta S\Delta F} = 1$ and $\sigma_{\Delta F} = \sigma_{\Delta S}$, the $h = 1$, indicating that change in futures price will be similar to change in spot price. However if $\rho_{\Delta S\Delta F} = 1$ and $\sigma_{\Delta F} = 2\sigma_{\Delta S}$, the $h = 0.5$, and indicates that the change in futures price will always be twice as much as the change in spot price (Baillie and Myers 1991).

Further, along with the hedge ratio it is also important to examine the hedge effectiveness. While the hedge ratio quantifies the appropriate hedge, the hedge effectiveness determines how effective the hedge will be or is likely to be. Estimation of hedge effectiveness is important in the sense, that it measures the usefulness and success of the futures contract in risk reduction (Silber 1985, Pennings & Meulenberg 1997). Johnson (1960) defined the measure of the *Hedge effectiveness* (E) of the hedged position in

terms of the reduction in variance of the hedged position ($\text{var}(H)$) over the variance of unhedged position ($\text{var}(U)$). The return on an unhedged and a hedged portfolio between time point t and $t+1$ can be written as:

$$R_u = S_{t+1} - S_t \quad \text{and} \quad R_H = (S_{t+1} - S_t) - h(F_{t+1} - F_t).$$

Where F_t and S_t are the futures and spot prices at time t , and h is the hedge ratio. R_H is the return generated when going long on one unit of spot and short on h units of futures at time t . R_u is the return on the unhedged position. Hedge effectiveness, then is defined by the proportion of reduction in variance of the position due to hedging relative to the variance of the unhedged position.

Variance of an Unhedged and Hedged Portfolio are:

$$\text{Var}(U) = \sigma_S^2 \quad \text{and} \quad \text{Var}(H) = \sigma_S^2 + h^2\sigma_F^2 - 2h\sigma_{S,F}$$

Where, S_t and F_t are natural logarithm of spot and futures prices, h is the hedge ratio, σ_S and σ_F are standard deviation of the spot and futures return and $\sigma_{S,F}$ is the covariance.

$$\text{We write hedge effectiveness as } E = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)} = 1 - \frac{\text{Var}(H)}{\text{Var}(U)}$$

Given the minimum variance hedge ratio (h), the hedge effectiveness (E) gives the maximum possible variance reduction of the overall unhedged spot position due to hedging. In other words, E can be defined as the degree to which the change in the value of the spot position is offset by changes in value of the futures position. Assuming that a hedged position will always have a lower variance than an unhedged position, the above formulation implies that the value of E will be bounded above by 1 i.e. higher the value of E (i.e., closer it is to 1), the better the hedge effectiveness.

In an important paper, Ederington (1979) extended the foundation work of Johnson (1960) and Stein (1961, 1964), by focusing on the empirical estimation of optimal hedge ratio within the portfolio framework. He adopted the OLS regression methodology to derive risk-minimizing hedge ratio. In this method changes in spot price, S_t is regressed on the changes in futures price, F_t . The optimal hedge ratio is estimated from the slope coefficient obtained by this OLS regression. The coefficient of determination (R^2) of the model indicates the hedging effectiveness. Since R^2 lies between 0 and 1, closer it is to 1, greater is the hedge effectiveness i.e. lowers the portfolio risk.

The OLS equation is given by:

$$R_{St} = \alpha + \beta R_{Ft} + \epsilon_t \quad \dots \dots \dots (1)$$

Where R_{St} = spot price returns

R_{Ft} = futures price return

β = slope coefficient of OLS regression

ϵ = error term

The optimal hedge ratio (h) is calculated as the slope coefficient, $\beta = \frac{\sigma_{SF}}{\sigma_F^2} = h$, where σ_{SF} is the covariance between spot and futures price returns and σ_F^2 the variance of futures price returns. Later, several other authors examined the robustness of hedge ratio calculated through the conventional OLS approach for different futures markets. Among them were studies done by Hill and Schneeweis (1981),

Anderson and Danthine (1981), Figlewski (1984), Witt et al. (1987), Myers and Thompson (1989), Benet (1992) and Lien (1990).

Although the OLS approach long dominated the literature on hedging in futures markets it was subjected to a number of theoretical and empirical criticisms. With the advancement of financial econometric techniques, researchers like Engle and Granger (1987), Myers (1991), Herbst et al. (1993), Lien (1997), Engle (1982) and Bollerslev (1990) argued that, though OLS technique is easy to implement, it is applied under the following assumptions:

1. No serial correlation between the residuals i.e. errors/residuals are independent.
2. Spot and futures prices changes are not co-integrated implying that both prices follow random walk and exhibit no co-movement in long run.
3. Residuals have constant variance, implying absence of heteroscedasticity.

In order to justify that the use of OLS technique is valid and efficient in estimation of optimal hedge ratio, we will perform (1) Jarque - Bera test for normality of the residuals, (2) Breusch-Godfrey Lagrange multiplier test for no serial correlation between error terms i.e. absence of autocorrelation in the residuals and (3) White's test statistic for presence of homoscedasticity. However, any inability to accept the null of these tests would suggest that standard error and t-statistic values of OLS method are non-informative and that OLS ignores conditional information such that the hedge ratio estimated will be static. This would also imply that OLS does not consider futures returns as endogenous variable and ignores the serially correlated disturbances.

To deal with the first assumption of OLS model i.e. the presence of serial correlation in residuals, we follow Herbst et al (1992, 1993, 1994). They employed bivariate VAR (m) Model, emphasizing that the model is an improvement over the conventional OLS estimation because it takes into account the presence of autocorrelation between errors and treat futures prices as endogenous variable. In the bivariate VAR (m) method there are two variables in each regression equation and the system allows the current values of a variable to depend on the past values of both the variables.

The VAR model is represented as:

$$R_{St} = \mu_S + \sum_{i=1}^m \beta_{Si} R_{St-i} + \sum_{i=1}^m \gamma_{Fi} R_{Ft-i} + \epsilon_{St} \dots \dots \dots (2)$$

$$R_{Ft} = \mu_F + \sum_{i=1}^m \alpha_{Si} R_{St-i} + \sum_{i=1}^m \lambda_{Fi} R_{Ft-i} + \epsilon_{Ft} \dots \dots \dots (3)$$

Where μ is the intercept term, $\beta, \gamma, \lambda, \alpha$, are parameters in VAR for $i=1, 2, 3, \dots, m$, R_{St-i} and R_{Ft-i} are lagged spot and futures returns, ϵ_{St} and ϵ_{Ft} are error terms that are i.i.d random vectors. The optimal lag m is selected through minimum Akaike's and Schwarz's Bayesian information criteria and maximum likelihood ratio test (AIC/SBC/LR). The minimum variance hedge ratio is calculated as: $h = \frac{\sigma_{SF}}{\sigma_F^2}$, where $\sigma_{SF} = \text{cov}(\epsilon_{St}, \epsilon_{Ft})$ and $\sigma_F^2 = \sigma^2(\epsilon_F)$ the variance of futures price returns.

Further, relaxing the second assumption of OLS methodology, we employ the VECM (m, r) developed by Engle and Granger (1987) to account for co-integration between spot and futures prices. According to

Lien (1997) “a hedger who omits the co-integration relationship will adopt a smaller than optimal futures position, which results in a relatively poor hedging performance”. Empirical investigations done by Ghosh (1993) on US S&P 500 index futures, Chou et al. (1996) for Nikkei Stock Index futures and Kenourgios and Samitas (2001) on currency futures markets, also supported the advantages of VECM over the traditional methods. The Vector Error correction methodology (VECM) is thus estimated as it incorporates the following factors:

- Takes into account that both the spot and futures prices are influenced by past and present values of each other.
- Understands that VAR (m) regression technique model’s relationship between non-integrated time series. To address this gap VECM includes an error correction term that takes care of both the short-run dynamics as well as the long-run co-integration.

In the VECM(m, r) model, m is the number of selected lags determined by the minimum AIC/SBC and r is the rank of co-integrating vector determined through Johansen and Juselius (1990) to estimate the optimal hedge ratio (Appendix A) . VECM (m, r) for the two series is given as:

$$R_{St} = \mu_S + \sum_{i=1}^{m-1} \Phi_{1i} R_{St-i} + \sum_{i=1}^{m-1} \Phi_{2i} R_{Ft-i} + \beta_1 \alpha' (R_{Ft-1} - R_{St-1}) + \epsilon_{St} \dots\dots\dots (4)$$

$$R_{Ft} = \mu_F + \sum_{i=1}^{m-1} \Phi_{1i} R_{St-i} + \sum_{i=1}^{m-1} \Phi_{2i} R_{Ft-i} + \beta_2 \alpha' (R_{Ft-1} - R_{St-1}) + \epsilon_{Ft} \dots\dots\dots (5)$$

Where, R_{St} and R_{Ft} are logarithmic daily spot and futures returns, μ_F and μ_S are constants, R_{St-i} and R_{Ft-i} are lagged spot and futures returns, $\Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$ are the coefficients of the lagged spot and futures returns for $i= 1, \dots, m-1$, $\epsilon_{t-1} = \beta \alpha' (R_{Ft-1} - R_{St-1})$ is the error correction term for $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$ as the coefficients of the co-integrating vector, α' is the normalized co-integrating vector⁶ and ϵ_{St} and ϵ_{Ft} are serially-uncorrelated disturbances. The minimum variance hedge ratio and hedging effectiveness are estimated by following similar approach as in case of VAR model.

Empirical investigations have reported that the conventional OLS, VAR and VECM estimate a static minimum variance hedge ratio as they ignore the time varying nature of the variance. These models are built on the assumption that the covariance and variance of the return series are time invariant. However, such an assumption is empirically proved to be incorrect in practice, Bollerslev (1986), Kroner and Sultan (1991), Baillie and Myers (1991), Park and Switzer (1995), Brooks (2002) and others have shown that returns data exhibit time varying conditional heteroscedasticity. This observation paved the way to the development of several alternative estimation methods to model Dynamic hedge ratio. In such a case the expected returns and the variance of a portfolio (p) consisting of one unit of the underlying asset (S) and β units of the futures contract (F), at time t-1 will be given as: $E(R_{p,t}) = E(S_t - \beta_{t-1} E(F_t))$ and $E(\sigma_{p,t}) = \sigma_{S,t}^2 + \beta_{t-1}^2 \sigma_{F,t}^2 + 2\beta_{t-1} \sigma_{SF,t}^2$

⁶ r restrictions are obtained on α' by normalization and in the case when r=1 (one co-integrating vector), then r=1 is all that is required to normalize $\alpha' = \mathbf{1} - \mathbf{1}$ (Mills and Markellos 2008)

Where conditional (time varying) variance of spot and futures returns are denoted as $\sigma_{S,t}^2$ and $\sigma_{F,t}^2$, while the conditional covariance is given as $\sigma_{SF,t}^2$. The Dynamic hedge ratio that minimizes the variance of the spot and the futures portfolio returns is hence calculated as: $h = \beta_{t-1} = -\frac{\sigma_{SF,t}}{\sigma_{F,t}^2}$

The literature on financial time series suggests use of multivariate generalized autoregressive heteroscedasticity models (MGARCH) to estimate Dynamic hedge ratio. The MGARCH (p, q) model takes account of the empirical observation that assets and market volatilities appear to be correlated over time and that the co-variances vary through time as the returns are affected by some set of available information.

Bollerslev (1990) developed the constant correlation-MGARCH (CC-MGARCH) model to circumvent the complicated estimation, specification and inference procedure of conditional variance and covariance matrix in unrestricted VEC-GARCH model initially introduced by Bollerslev, Engle and Wooldrige (1988). The VEC-GARCH model was found to be practically infeasible and cumbersome to estimate for two or more than two variables and could not ensure the conditional variance-covariance matrix to be positive definite because of the large number of parameters contained in it. Consequently several authors proposed number of restrictions and specifications to reduce the number of parameters needed to be estimated.

Bollerslev (1990), CC-MGARCH model assumes that the conditional correlation between the observed residuals is constant through time. To illustrate, let's assume that an observed residual series has zero mean and conditional variance structure limited to lag one. For a vector of k residuals ε_t , the conditional variance of ε_t when allowed to vary over time can be written as $V(\varepsilon_t | I_{t-1}) = \Omega_t$, where I_{t-1} is the information set available at time t-1. This way the conditional variance $\sigma_{i,t}^2$ and conditional covariance σ_{ij} of Ω_t in the CC-GARCH (1, 1) model are given by:

$$\begin{aligned}\sigma_{i,t}^2 &= \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \text{ for } i = 1, 2, \dots, \dots, k \\ \sigma_{ij,t} &= \rho_{ij} \sigma_{it} \sigma_{jt} \quad 1 \leq i, j \leq k\end{aligned}$$

Where the ρ_{ij} are the constant correlations, $\omega_i, \alpha_i, \beta_i > 0$ and $\alpha_i + \beta_i < 1$ for all $i = 1, 2, \dots, k$ and the matrix of correlation is positive definite. Thus though the conditional correlation is constant, the model allowed for time variation in the conditional covariance (Mill and Markellos 2008).

We will employ constant correlation multivariate generalized autoregressive conditional heteroscedasticity (CC-MGARCH) model proposed by Bollerslev (1990) to estimate the dynamic hedge ratio. Since we are dealing with two variables (Nifty and Nifty futures) the execution of the bivariate CC-GARCH model requires estimating two set of equations, first a pair of conditional mean equations used to generate the residuals and second the conditional variance covariance equations used to model the behavior of the residuals generated from the mean equation. For which we will use VAR (m) to model the mean equation and CC-GARCH (p, q) to model the variance equation. Equation [2] and [3] presents the VAR (m) - conditional mean equation for Nifty Index and Nifty futures.

Following Bollerselv (1990), we assume that the conditional correlation is constant over time so that all the variations over time in the conditional covariance are caused by changes in each of the corresponding conditional variances. This implies that if $\varepsilon_t = (\varepsilon_{St}, \varepsilon_{Ft})'$, then $(\varepsilon_t | I_t) \sim N(0, \sigma_t)$, where I_t is generated out of all the available information up to time t-1 and the conditional covariance matrix σ_t is positive definite for all t: $\sigma_t = \begin{vmatrix} \sigma_{S,t} & \sigma_{SF,t} \\ \sigma_{SF,t} & \sigma_{F,t} \end{vmatrix}$

Thus the CC-GARCH (p, q) conditional variance covariance equation for residuals in VAR (m) Nifty and Nifty futures returns equation can be written as:

$$\sigma_{St}^2 = \omega_S + \sum_{i=1}^p \alpha_i \varepsilon_{St-i}^2 + \sum_{j=1}^q \beta_j \sigma_{St-j}^2 \dots\dots\dots (6)$$

$$\sigma_{Ft}^2 = \omega_F + \sum_{i=1}^p \alpha_i \varepsilon_{Ft-i}^2 + \sum_{j=1}^q \beta_j \sigma_{Ft-j}^2 \dots\dots\dots (7)$$

$$\sigma_{SF} = \rho_{SF} \sigma_{St} \sigma_{Ft} \dots\dots\dots (8)$$

Where $p \geq 0, q \geq 0$, the weights satisfy nonnegative condition ω_S and $\omega_F \geq 0, \alpha_i \geq 0, \beta_j \geq 0$ for $i = 1, 2, \dots, q$ and $j=1, 2, \dots, p$. The dynamic hedge ratio is written as: $h = \frac{\sigma_{SF,t}}{\sigma_{F,t}^2}$

Although GARCH class of models appears to be ideal models for estimating time-varying optimal hedge ratios for different types of financial price series. But, no existing study has provided compelling evidence that such time-varying hedge ratios are statistically different from a constant hedge ratio. Lien and Luo (1994) and Fackler and McNew’s (1994) studies on agricultural futures (corn) criticized the superiority of GARCH framework in estimation of hedging strategy. They pointed out that estimation of GARCH models requires the use of nonlinear optimization algorithms and the imposition of inequality restrictions on model parameters which makes it a less than ideal technique for estimating time-varying hedge ratios. Similarly, Holmes (1995) investigation on ex- ante hedging effectiveness of UK index futures contracts (FTSE 100) reported that hedge estimates of OLS method outperforms the error correction model and GARCH (1,1) approach. However, we have empirical investigations by Chou, Denis and Lee (1996) and Wilkinson, Rose and Young (1999) that advocated the supremacy of error correction method over traditional and advanced methods.

More recent investigations, such as those conducted by Lien, Tse and Tsui (2000), Yang (2001), Wang and Low (2002), Moosa (2003), Choudhry (2003), Rose, Zou, Wilson, and Pinfold (2005), Butterworth and Holmes (2005), Floros and Vougas’s (2006), Chaudhry et.al, (2008), Bhaduri and Durai (2008) and Emin AVCI and Murat ÇİNKO (2010) have extensively utilized different advanced and complex econometric techniques to estimate and compare the optimal hedge ratios obtained for wide variety of futures markets in different countries over varying sample periods.

These studies have made significant contribution towards the theoretical understanding and empirical application of different hedging tools and strategies, but there is no unanimous view among the academic community on practical application of these models and on the substantial improvement in hedging activities that these models offer. It is thus, evident from the above explanation that it is important to explore the statistical properties of the return distributions used in the analysis, prior to estimation and comparison of the results obtained from different models. Such an approach will aid in

highlighting the strengths and weaknesses of the alternative models in treatment of the data, measuring the hedge and comparing the hedge effectiveness.

IV Exploratory Analysis of Nifty Spot and Nifty Index Futures Returns

The objective of this section is to explore the properties of the financial time series on Nifty index and futures returns and to establish the best fitted data generating process for the two return series. We use the daily return series of NSE's most heavily traded, near month Nifty futures contract and the underlying Nifty index. The data has been retrieved from the website of National Stock Exchange of India Ltd. (NSE)⁷. The time period of the study is June 12, 2000 till September 30, 2009 which gives 2326 data points.

Graphical analysis

A preliminary graphical analysis of Nifty index and Nifty futures daily price series aims at formulating tentative inferences about the behavior and formation of the series over time using quantitative methods.

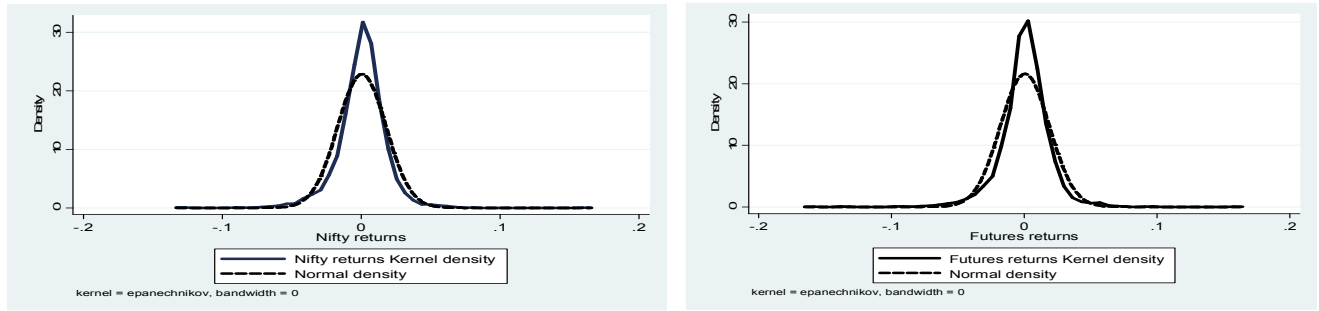
The literature on financial time series observes that returns on financial markets are not normally distributed. Mandelbrot (1963), empirically formulated four stylized facts of financial return series of: 1) fat tail and high peakedness or excess kurtosis, 2) volatility clustering, 3) asymmetry, and 4) leverage effects. Among these stylized facts, excess volatility and its associated clustering have been most widely studied due to their importance in theory and application of financial study. The logarithmic returns provides the most consistent basic measure on variation of price differences in the past and so will be employed to study these two characteristics. Returns on Nifty(S) and Nifty futures (F) have been calculated using the following: $R_S = \ln \left(\frac{P_{S,t}}{P_{S,t-1}} \right)$ and $R_F = \ln \left(\frac{P_{F,t}}{P_{F,t-1}} \right)$

Where $P_{S,t}$ and $P_{F,t}$ are current daily closing value of Nifty and Nifty Futures and $P_{S,t-1}$ and $P_{F,t-1}$ are previous day daily closing value for Nifty and Nifty futures.

The phenomenon of positive *excess kurtosis* is associated with the observation that financial return series cannot be modelled under the assumption of stationary normal distribution because it does not account for the extreme variations seen in the returns. The absolute magnitude of the variations in the returns is much larger due to which the probability distribution of the return series exhibit 'heavy tails and high peaks'. Such time series that exhibit fat tails and high peaks are called *leptokurtic*⁸.

⁷ www.nseindia.com

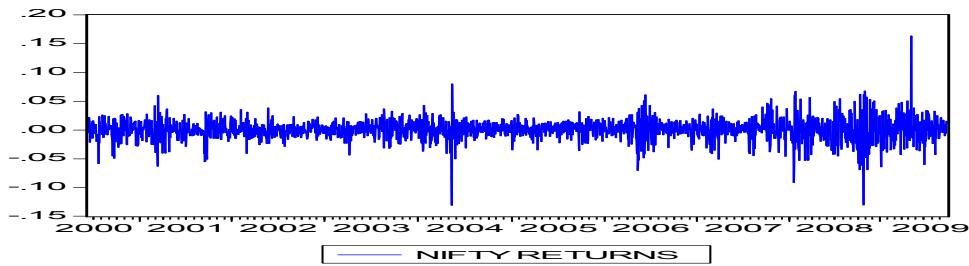
⁸ The measure of kurtosis, $K = \frac{\text{fourth moment about the mean}}{\text{square of the second moment}}$ when greater than 3 is called leptokurtic.



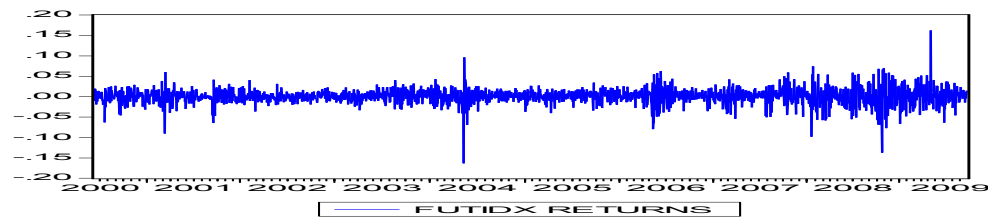
[Figure 1]

In [Figure 1] empirical densities shown are computed as a smoothed function of the histogram using normal kernel. Superimposed on the empirical density is the normal distribution for comparative analysis of the two densities. Both the series are highly peaked and exhibit fat tails as they put more mass on the tail than the standard normal distribution. The solid line in the figures illustrates excess kurtosis displaying a power declining tail rather than an exponential decline, as is the case with normal distribution depicted by the dashed line. This indicates that both the return series are non-normal.

Financial return series also exhibit *volatility clustering* or *persistence*. As described by Mandelbrot (1963), “*volatility clustering manifests itself as periods of tranquility interrupted by periods of turbulence. The change between these two extreme regimes is a slow process so that large returns slowly decline until a relatively tranquil state is reached*”. In simple words this means that the phenomenon of volatility clustering indicates patterns when large changes tend to follow large changes, and small changes tend to follow small changes. Figure [2 and 3] plots the daily log-returns for Nifty and Nifty futures. In both the figures, the phenomenon of volatility clustering can be observed as the presence of sustained periods of high turbulence with many high peaks clustering together and periods of calmness where volatility remains low or in the words of Mandelbrot (1963), “*Turbulent phases with large price changes follow relatively silent phases of small price changes*”.



[Figure2]



[Figure 3]

Descriptive statistics

Summary statistics and the results of the diagnostics checks on the distributional properties of Nifty index and Nifty future are presented in [Table 1]. Average returns of both the markets for the sample period are more or less equal. The variances in the two return series differ slightly with the futures returns showing relatively higher variance. The unconditional distributions of spot and futures returns are non-normal, as shown by high skewness, high kurtosis, and significant Jarque-Bera statistics. The futures and spot market returns are significantly positively correlated with the Nifty index and futures returns exhibiting a correlation as high as 98%. The high correlation between the index and the futures market suggests that both markets are related and exhibit strong positive co-movement. Jarque-Bera test statistics appear to be significantly high (p value = 0) implying that the returns are asymmetric and not normally distributed. Both the return series show excess kurtosis implying fatter tail than normal distribution and are skewed to the left. This is in line with the empirical observations of Fama (1965), Stevenson and Bear (1970) that flat tails are due to volatility clustering and asymmetry is due to asymmetric information and leverage effect.

	NIFTY_RETURNS	FUTIDX_RETURNS
Mean	0.000542	0.000538
Median	0.001462	0.00105
Maximum	0.163343	0.161947
Minimum	-0.130539	-0.162581
variance	0.000304398	0.000341289
Skewness	-0.311886	-0.47882
Kurtosis	10.71403	11.46364
Jarque-Bera	5802.369 (0.00)	7028.303 (0.00)
cross correlation matrix		
	NIFTY_RETURNS	FUTIDX_RETURNS
NIFTY_RETURNS	1	0.97985 (0.00)
FUTIDX_RETURNS		1
Observations	2325	

[Table 1] Figures in parenthesis denote p values

Time series specifications of Nifty index and Nifty futures returns

Stationary Return Series

We apply Unit root tests (ADF and PP) to test for stationarity of our data. Presence of unit root implies that the particular time series has a time varying mean or time varying variance or both, implying non-stationarity. According to Ganger and Newbold (1974), in such a case it becomes necessary to difference any nonstationary time series before further econometric analysis are conducted.

Unit root test

The Augmented Dicky-fuller (1979) and Phillip Perron (1988) tests are most widely used tests to examine the existence of unit roots, and to determine the degree of differencing necessary to make the series stationary. The results of ADF or PP determine the form in which the data should be used in any subsequent econometric analysis.

The unit root tests, ADF and PP test holds the following condition for stationary series: working with a simple equation like: $\Delta Y_t = \rho Y_{t-1} + \varepsilon_t$ where, ΔY_t is the first difference of the respective series (SPOT_t,FUT_t). If we are not able to reject the null that the coefficient (ρ) is different from zero then series has unit root showing the series to be non-stationary. To test ADF and PP, we estimate following three equations:

1. Regression equation with constant and trend

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

2. Regression equation with constant and no trend

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

3. Regression equation with no constant and no trend

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

Where $\Delta y_t = y_t - y_{t-1}$ and ε_t is a stochastic term that has mean equal to zero, a constant variance under the assumption of absence of autocorrelation. The ADF and PP test statistics are subject to the critical values of the Dickey-Fuller test and Phillip Perron test respectively. The optimal number of lags p of the dependent variable (y) is selected using two information criteria the Akaike information criterion (AIC) and the Schwartz's Bayesian information criterion (SBC). For sample size of ∞ the complete set of test statistics are as follows.

Summary of the Dickey-Fuller Tests

Model	Hypothesis	Test Statistics	Critical Values for 95% and 99% confidence Intervals
Model (1)	$\gamma = 0$	τ_τ	-3.45 and -4.40
Model (2)	$\gamma = 0$	τ_μ	-2.89 and -3.51
Model (3)	$\gamma = 0$	τ	-1.95 and -2.60

[Table 2] presents unit root tests on daily closing log price series of Nifty and Nifty futures rejects the null of $\gamma=0$ at the first difference as the computed values are greater than the critical values at 1% level. The results report that both the series are stationary at first difference or integrated of order one, [I (d) = I (1)] and significant. This is equivalent as saying that the return series of Nifty index and futures are stationary.

Unit Root test on Nifty index and futures returns				
Test Variables	H0: $\gamma = 0$ in Eq(1), τ_τ	H0: $\gamma = 0$ in Eq(2), τ_μ	H0: $\gamma = 0$ in Eq (3), τ	Unit Root
Augmented Dickey Fuller test				
$\Delta \ln \text{ spot}$	-34.77	-34.76	-34.76	no
$\Delta \ln \text{ fut}$	-47.01	-46.99	-46.96	no
Phillips Perron				
$\Delta \ln \text{ spot}$	-44.42	-44.42	-44.39	no
$\Delta \ln \text{ fut}$	-46.98	-46.97	-46.948	no
Test critical values:				
1% level	-3.961992	-3.432967	-2.565957	
5% level	-3.411741	-2.862582	-1.94096	
10% level	-3.127753	-2.56737	-1.616608	

[Table 2]

Mean dynamics of return series

Vector Autoregressive (VAR) framework (equation 2 and 3) is used to model the mean dynamics of return series. As given by the Box-Jenkins methodology, the buildup of the VAR (m) model for a time series requires three stages: (1) identification (2) estimation and (3) diagnostic checking. In the first step involves selection of the optimal lag m. The model is estimated in the second stage to determine the interrelationship between the two variables and to generate the innovations (residual) series to model the variance behavior of the residuals. Finally, diagnostic checks are done to discover any lack of fit.

VAR (m) Identification (Optimal lag selection):

The optimal lags are selected by employing minimum Akaike's and Schwarz's Bayesian information criteria (AIC/SBC) and maximum likelihood ratio (LR) test.

Optimal lag Selection			
Lag Selection	AIC	SBC	Log Likelihood
1 to 2	-13.82	-13.79526	16061.94
1 to 3	-13.8252	-13.79054	16065.06
1 to 4	-13.83400*	-13.78941	16072.36*
2 to 3	-13.6337	-13.60889	15838.67
2 to 4	-13.6363	-13.6016	15838.9
2 to 4	-13.6268	-13.60206	15823.94

The rule of minimum AIC/SBC and maximum LR as shown in the above table suggests, to use m=4 as the optimal lag in order to estimate the VAR equations (2) and (3).

Estimation of VAR (4):

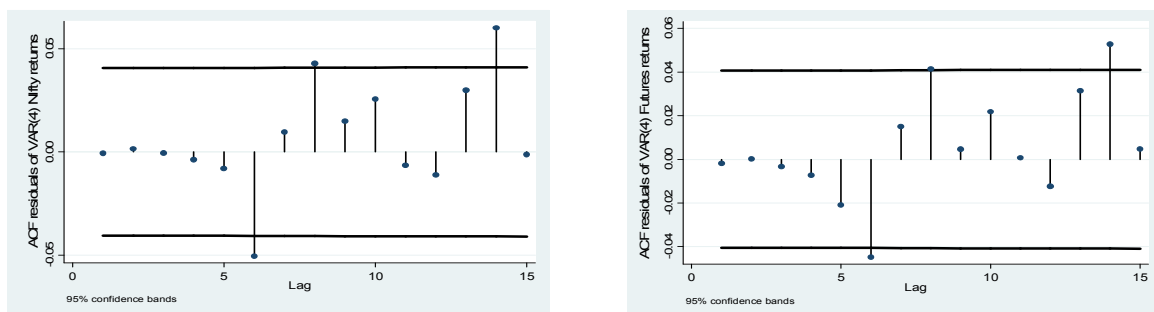
VAR equations (1) and (2) are estimated using the specification m= 4 and the residuals are extracted to examine the behavior of the residuals [table 3]. The parameters are found to be significant at 1% and 5% level.

VAR (4) model estimates			
	Nifty Index	Nifty Index Futures	
μ_S	0.0005	μ_S	0.000499*
β_{S1}	0.167648* [1.46423]	α_{S1}	-0.483757* [1.30338]
β_{S2}	0.027386* [0.22693]	α_{S2}	-0.314173* [-1.58557]
β_{S3}	-0.019973** [-0.16572]	α_{S3}	-0.093403* [-0.76455]
β_{S4}	0.101294 [0.91683]	α_{S4}	-0.483757** [-2.56468]
γ_{F1}	-0.07956* [-0.73560]	λ_{F1}	0.548607* -0.12118
γ_{F2}	-0.089204 [-0.77071]	λ_{F2}	0.262481** [2.05503]
γ_{F3}	0.029329* [0.25408]	λ_{F3}	0.113415 [0.88912]
γ_{F4}	-0.076975* [-0.72785]	λ_{F4}	0.200959* [1.71859]
R square	0.011412	R square	0.012454

[Table 3] [] denotes t-statistics, *, **denotes significant at 1% and 5%

Diagnostics tests:

The VAR (4) can now be tested for absence of serial correlation in the residuals. The correlogram for the respective residuals of both equation (1) and (2) are presented in [figure 4].



[Figure 4]

The low levels of autocorrelation immediately dropping at lag1 up to lag 6 or in other words the spikes lying within the 95% confidence limit, suggests that the residuals of VAR (4) are not serially correlated. Thus VAR (4) provides a good fit to the data generating process of both the return series.

Volatility dynamics of return series

In the section on graphical analysis we observed that the return series on Nifty and Nifty futures exhibit volatility clustering. Unlike other static (marginal or unconditional) statistical properties, volatility clustering describes a conditional dependence of volatility on past information about the dynamics of financial time series.

To illustrate, let us consider a first order autoregressive AR (1) process⁹ of a financial return series: $R_t = \beta_0 + \beta_1 R_{t-1} + \varepsilon_t$. Here, R_t is the return that is drawn from a conditional density function $f(R_t|R_{t-1})$ and ε_t is the white noise process that has zero mean and constant variance. This AR (1) process explains, that the forecast of today's value of return (R_t) are conditioned on past information of value of return (R_{t-1}) i.e. $E(R_t|R_{t-1}) = \beta_0 + \beta_1 R_{t-1}$. Thus, if returns were independently and individually distributed (i.i.d.)¹⁰ normal distributions, then volatility could have been simply measured as the sample standard deviation of the return distribution. But, as we have observed that Nifty index and futures return series exhibit non-normality and are uncorrelated but not independent, we need to estimate their volatility through other methodology which takes care of the unconditional non normal distribution of returns and the conditional variance of returns varying over time.

As an alternative measure to identify volatility flows, we will use daily absolute returns and squared returns as a proxy for volatility (Ederington and Gaun 2005). The daily absolute and squared returns are calculated as:

$$|\sigma_t| = |P_t - P_{t-1}| \text{ i.e. the absolute difference of daily price changes.}$$

$$\sigma_t^2 = (P_t - P_{t-1})^2 \text{ i.e. the squared difference of daily price changes.}$$

Accordingly, volatility clustering would quantitatively imply that, while returns (R_t) themselves are uncorrelated as they do not exhibit significant autocorrelation. The absolute returns $|R_t|$ or their squares R_t^2 exhibit a positive, significant and slowly decaying autocorrelation function: $\text{Corr}(|R_t|, |R_{t+1}|) > 0$ ¹¹. The time varying volatility model by Heston (1993) empirically showed that, although returns show little autocorrelation, they are usually not independent¹².

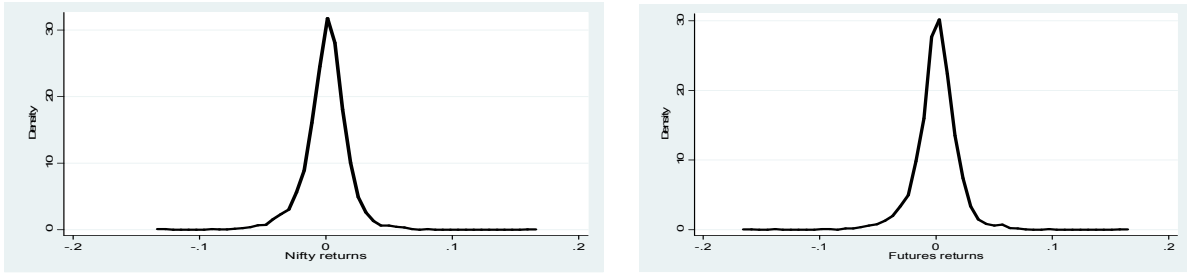
In order to set up our return series model of volatility clustering. We start by graphically examining and comparing the Kernel density graphs and the spicks of the correlogram (ACF and PACF values) of the Nifty and Nifty futures daily simple returns (R_t), absolute returns ($|R_t|$) and squared returns (R_t^2) with the objective to identify presence of time varying volatility in both the series.

⁹ An AR (p) process optimally incorporates the past information to forecast values of current period.

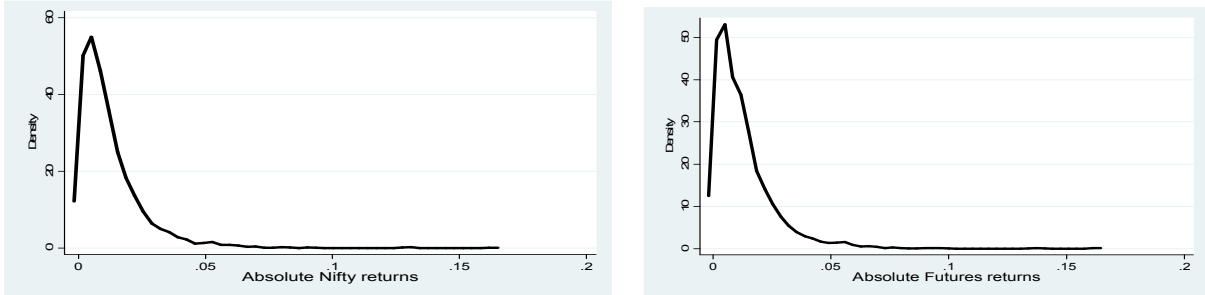
¹⁰ Independently and individually distributed means that the observations of a random variable (the returns of an asset over a period of time, for instance) are independent of each other and have the same distribution. Thus, they all have the same mean and variance.

¹¹ Mandelbrot (1963) defines independence as a situation when no investor can use his knowledge of past data to increase his expected profit. But he holds that the converse is not true. There exist processes, in which the expected profit vanishes, but dependence is of extremely long range, and knowledge of the past may be profitable to those investors whose utility function differs from the markets.

¹² Heston (1993) time varying volatility model refute the assumptions of uncorrelated and independent returns that underlie the constant volatility Blacksholes model (1973).

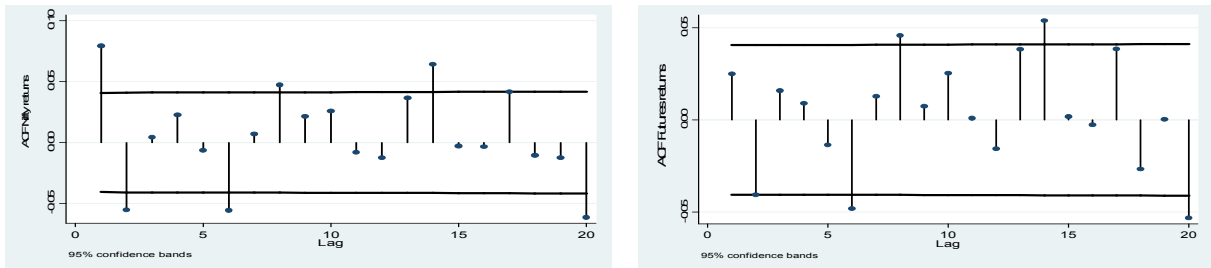


[Figure 5] Kernel density graph of Nifty and Nifty futures simple returns



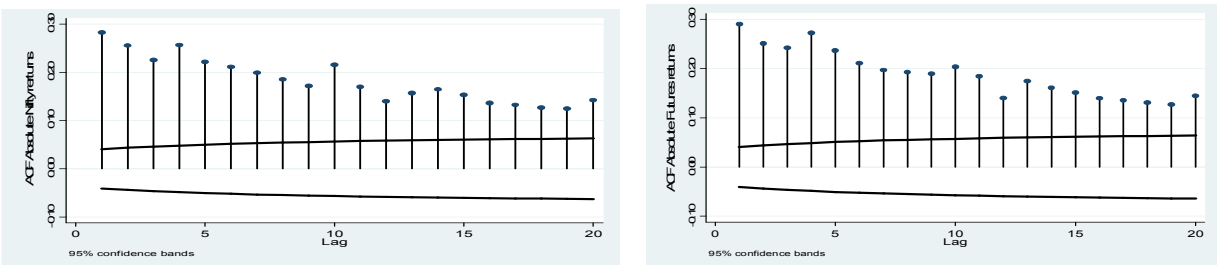
[Figure 6] Kernel density graph of Nifty and Nifty futures absolute returns

Kernel density graphs show normal distribution of simple returns [figure 5] in contrast to volatility graphs that show peakedness/ fat tail (excess kurtosis) and asymmetry (skewed distribution) [Figure6].

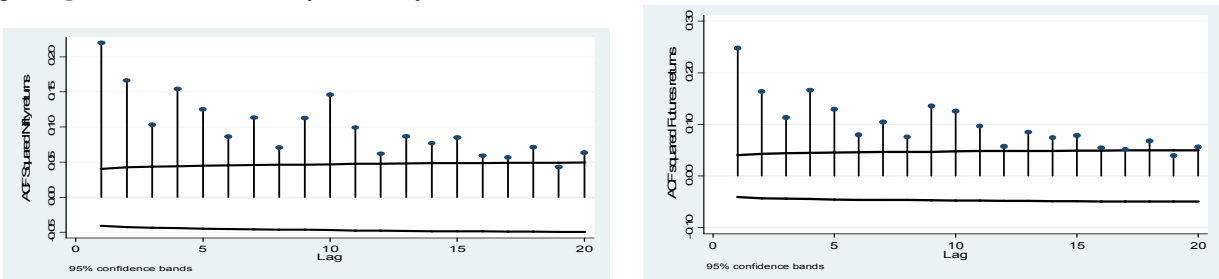


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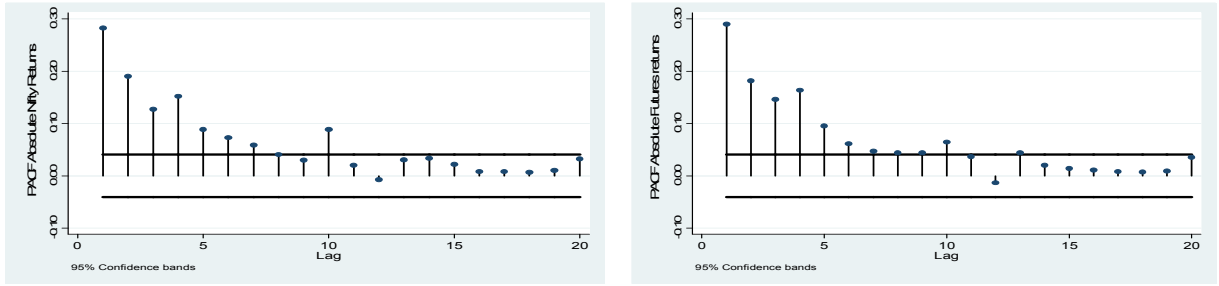
[Figure 7] ACF of Nifty and Nifty future simple returns



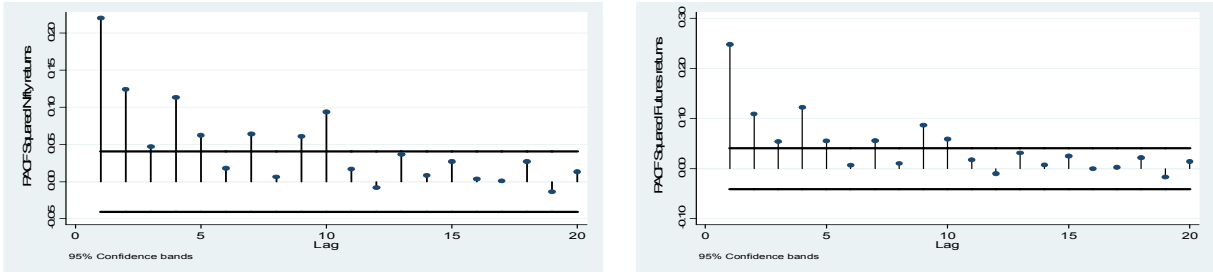
[Figure 8] ACF of absolute Nifty and Nifty futures returns



[Figure 9] ACF of squared Nifty and Nifty futures returns



[Figure 10] PACF of absolute Nifty and Nifty futures returns



[Figure 11] PACF of squared Nifty and Nifty futures return

Combining the plots (figures 7, 8, 9, 10,11) suggests that return series seem to be serially uncorrelated, but are indeed dependent. The correlogram of simple returns show few significant autocorrelation while the absolute and squared daily Nifty and futures returns show persistence in volatility as the ACF are longer and persists for larger lags (up till lag 20). The PACF of absolute and squared Nifty and Nifty futures returns show significant spicks up till 10 lags. The significant spicks of PACF suggest that the percentage changes are not serially independent highlighting the phenomenon of volatility clustering.

GARCH (p, q) methodology (equation 6and 7) is employed to model volatility clustering in the return series. Empirically, GARCH model have been found very useful in distinguishing between the conditional and unconditional variance of the innovations/residual process (ε_t) obtained from the conditional mean model (VAR (4)). The term conditional indicates dependence on past observations and unconditional implies no dependence on knowledge available in the past.

Bollerselv (1986) gave the long run stable conditional variance condition as:

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$$

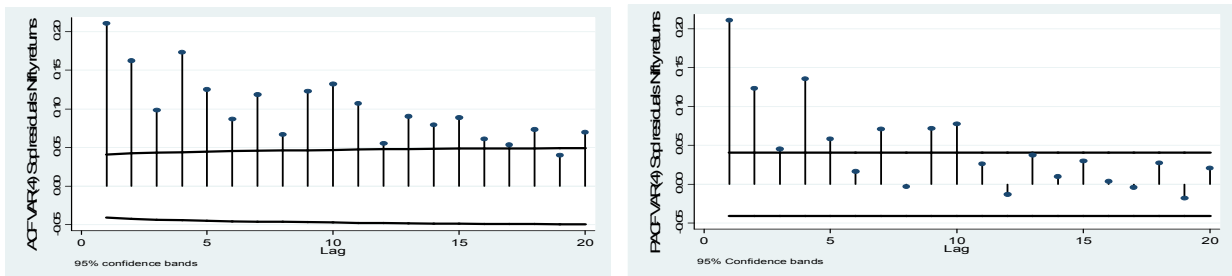
The above restriction implies that for the return series the variances are going to be sticky towards 1%¹³ in long run. Bollerselv (1986) defined $\alpha_i + \beta_j$ as the persistence of the return series. The greater the value more is the persistence of the return series and lesser the decay towards the long run variance. Thus $\alpha_i + \beta_j$ is a key parameter that controls the persistence from decay of the returns series. The closer $\alpha_i + \beta_j$ is to 1, the slower the decay of the autocorrelation of σ_t . Smaller the value of $\alpha_i + \beta_j$, greater is the decay which implies that the current variance will soon hit the long run variance.

¹³ The long run variance of the return series is taken as one percent.

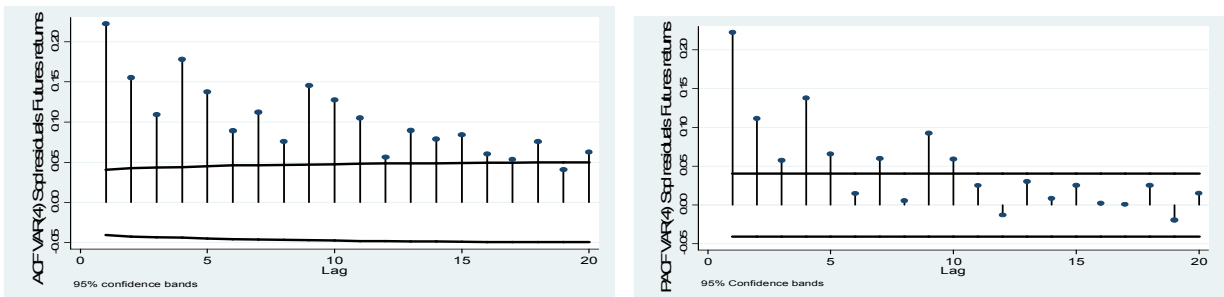
Therefore, significant positive α_1 & β_1 indicate that past volatility (positive or negative) has positive influence on the future volatility of the market and so we observe the clustered phenomenon in the return series. Bollerslev (1986) showed that the Box-Jenkins methodology can be applied to the squared residual series in order to identify, estimate and check the residual behavior of conditional variance equation of GARCH (p, q) form.

Identification: Order Determination of GARCH (p, q)

Following Wooldrige and Bollerslev (1992), ACF and PACF of ε_{st}^2 and ε_{ft}^2 extracted from the VAR-mean equation are examined to determine the order of GARCH (p, q) model [Figure 12 and 13]. Also, since GARCH model can be treated as ARIMA model for squared residuals, the traditional model selection criteria such as AIC and SBC can be used for selecting the model [Table 4]. For a pure ARCH (p) model it is empirically been found that large values of p are selected by AIC / SBC and for GARCH (p, q) models, those with p, q < 2 are typically selected by AIC and SBC. Empirical observation suggests that low order GARCH (p, q) are generally preferred to high order ARCH (p) model for the reason of parsimony and better estimation and that it is hard to beat the simple GARCH (1, 1) model.



[Figure 12]ACF and PACF of squared residuals of VAR (4) Nifty returns



[Figure 13] ACF and PACF of squared residuals of VAR (4) Nifty futures returns

In the above figures the plot of PACF for ε_{st}^2 and ε_{ft}^2 suggests that GARCH can be modeled using the following specifications i.e. p, q= 1, 2, 3, 4. Further, we check for the minimum values of AIC/SBC for the above specification.

p, q	Squared returns	AIC	SBC
(1,0)	Nifty	-213377	-193587
	Nifty futures	-293345	-274563
(2,0)	Nifty	-235658	-209689
	Nifty futures	-273322	-273486
(3,0)	Nifty	-244637	-209589
	Nifty futures	-278845	-272132
(4,0)	Nifty	-203372	-205387
	Nifty futures	-271023	-270125
(1,1)	Nifty	-202930	-202537
	Nifty futures	-281034	-280023
(1,2)	Nifty	-202930	-202549
	Nifty futures	-281236	-280203
(2,2)	Nifty	-202932	-202647
	Nifty futures	-281045	-280824
(2,1)	Nifty	-202937	-202548
	Nifty futures	-272652	-280123

[Table 4]

[Table 4] gives the model criteria for different GARCH (p, q) fitted to daily returns of Nifty and Nifty futures. For a pure ARCH (p) the majority rule of minimum AIC and SBC selects ARCH (4) for both the series and for GARCH (p, q), AIC selects GARCH (1, 2) and SBC selects GARCH (1, 1). However we chose the minimum among them, this rule suggests that we should estimate GARCH (1, 1) to model volatility.

Estimation of GARCH (1,1)

On estimation, GARCH (1, 1) model was found to give more significant values of the parameters (significant p- values) as compared to GARCH (1, 2). [Table 5] gives the results of GARCH (1, 1) model.

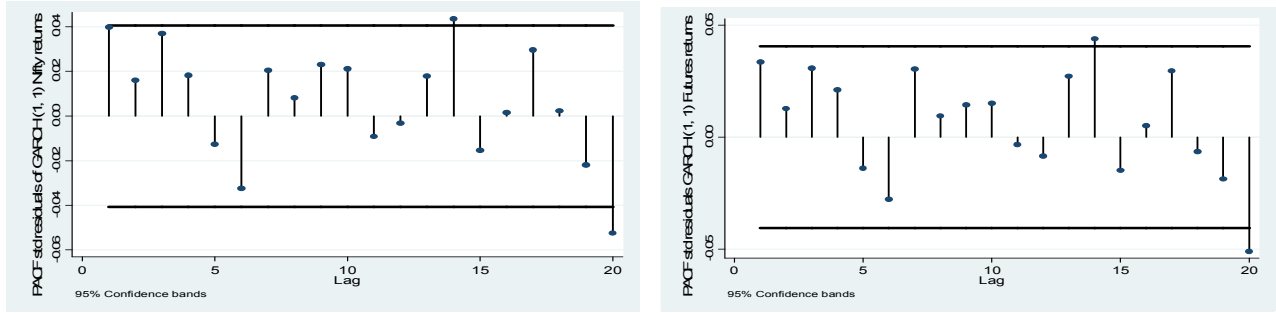
Daily returns Nifty Index				
	α_0	α_1	β_2	$\alpha_1 + \beta_2$
GARCH (1, 1)	9.01E-06	0.168485 [0.013292]	0.809581 [0.013272]	0.978066
Daily returns Nifty Futures				
GARCH (1, 1)	9.61E-06	0.166203 [0.011945]	0.812612 [0.012063]	0.978815

[Table 5][] denote t values

Table 5 reports that α_1 & β_2 coefficients are significantly positive, which indicates that the returns are not serially independent and exhibit ARCH effect i.e. the variance of the current innovations/ residuals are a function of the previous period innovations. The $\alpha_1 + \beta_2$ close to 1 (approximately 98%) for both the return series suggests that past fluctuations negative or positive have positive influence on the future fluctuation of the each market, hence implying to the presence of volatility clustering (persistence) in the returns. Also $\alpha_1 + \beta_2 < 1$ shows that the conditional variance sequence of returns is stable.

Check of GARCH(1,1) fit

The adequate fit of the GACH model can be tested by examining the PACF of standardized residuals and squared residuals of the GARCH (1, 1) for no serial correlation, absence of hetroscedasticity and non normality.



[Figure 14] PACF of standardized residuals of GARCH (1, 1) of Nifty and Nifty futures return series

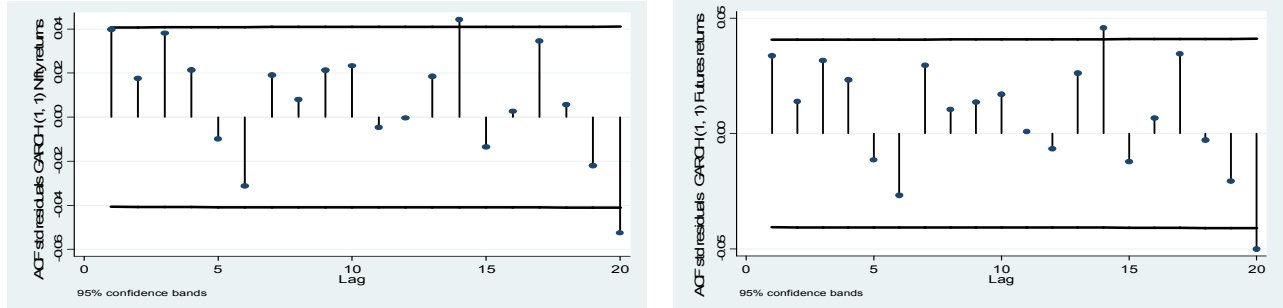


Figure 15] ACF of standardized residuals of GARCH (1, 1) of Nifty and Nifty futures returns

[Figure 14 and 15] show graphical diagnostics from the fitted GARCH (1, 1) for both series these include PACF and ACF of standardized residuals of GARCH (1, 1) for Nifty and Nifty futures return series. The plot of PACF and ACF of standardized residuals shows no significant spikes at any lag, suggesting that the mean and GARCH (1, 1) is adequate in describing the volatility clustering phenomenon observed in the Nifty and Nifty futures return series.

V. Empirical estimation of hedge ratios and hedge effectiveness

This section presents the estimates of hedge ratio and hedge effectiveness obtained from the alternative models described in Section III.b. Thereafter, their empirical performances are compared and conclusions made on the best performing model based the percentage of variance reduction they offer.

OLS Estimates

[Table 6] presents the estimated coefficients of the OLS regression obtained by running equation (1) along with the corresponding standard error (.) and the t-statistics [.]

OLS Regression Estimates	
α	0.0000447
Hedge Ratio ($\beta = h$)	0.925377**
Std. Error	(0.003914)
t-Statistic	[236.4383]
p-values	0.00
Hedge Effectiveness ($R^2=E$)	0.960104

[Table 6] “**” denotes significant at 5%

OLS estimation of hedge ratio provides 92% variance reduction and 96% hedge effectiveness. Thus suggests that the Nifty futures contracts are effective hedge instruments in case of Indian stock markets. Further in order to check for the validity of these results, the diagnostic tests are given in [Table 7].

Diagnostic Tests	Test Statistics	p-values	Decision
Jarque-Bera H₀: Residuals are normally distributed	13162.96	0	reject
Breusch GodfreySerial Correlation LM Test H₀: No serial correlation between residuals	123.6439	0	reject
White Heteroscedasticity Test: H₀: residuals are homoscedactic	60.16497	0	reject

[Table 7]

The analysis of the diagnostic tests reports our inability to accept the null of the three statistics, implying that values of standard error and t-statistic of the OLS regression can be non-informative and inappropriate. It also suggests that further in the investigation we must consider the presence of non normality, serial correlation and heteroscedasticity in the returns data.

VAR (m) estimates

VAR (m) equations (2) and (3) are solved and the residuals estimated to obtain the figures of hedge ratio and hedge effectiveness of Nifty futures contracts. The hedge ratio is estimated using covariance of ϵ_S, ϵ_F and the variance of ϵ_F , while the hedge effectiveness is calculated using the variance of estimated unhedged and hedged return series.

Following the VAR (4) estimates presented in section IV, there we observed that the correlogram for the respective residuals of both VAR (4) equations show low levels of autocorrelation immediately dropping after lag 1 up to lag 6 or in other words the spikes lying within the 95% confidence limit. This implies that VAR (4) has adequately taken into account the serial correlation previously detected in the OLS estimation. Hence, we now proceed to estimate the hedge ratio and hedge effectiveness using VAR (4) model [table 8].

VAR (4) estimation of Hedge Ratio and Hedge Effectiveness	
Cov(ϵ_S, ϵ_F)	0.000313437
var(ϵ_F)	0.000337282
Hedge Ratio(h)	0.9293018
variance(H)	74.02122747
variance(U)	3187.576901
Hedge Effectiveness(E)	0.976778214

[Table 8] $h = \frac{\sigma_{sf}}{\sigma_f}$ and $E = 1 - \frac{\text{var}(H)}{\text{var}(U)}$

The hedge ratio estimated through VAR (4) provides approximately 93% variance reduction and the hedge effectiveness provided is 97%. VAR estimates of both hedge ratio and hedge effectiveness perform better than those obtained through conventional OLS model ($h_{ols} = 92\%$ and $E_{ols} = 96\%$).

VECM (m, r) Estimation

In the Appendix A, we have shown through Johansen and Juselius (1990) λ_{\max} and λ_{trace} statistics that Nifty and Nifty futures return are co-integrated in long run in rank, $r=1$. VECM (4, 1) equations (4) and (5) are solved and the estimated coefficients are presented in [Table 9].

VECM(4,1,1) Model Estimates			
Nifty Index		Nifty Index Futures	
μ_S	-8.56E-06	μ_F	-1.05E-05
Φ_{11}	1.610658** [-4.65619]	Φ_{21}	1.542292*** [4.28607]
Φ_{12}	-1.250682* [-4.43765]	Φ_{22}	1.060931* [3.63052]
Φ_{13}	-0.939952* [-4.56317]	Φ_{23}	0.803647* [3.77971]
Φ_{14}	0.530495** [-4.41811]	Φ_{24}	0.453928* [3.72304]
Φ_{21}	0.885376* [2.58732]	Φ_{11}	-2.306443** [-6.34074]
Φ_{22}	0.64606* [2.32479]	Φ_{12}	-1.690206* [-5.70316]
Φ_{23}	0.539221* [2.66679]	Φ_{13}	-1.214145* [-5.60533]
Φ_{24}	0.339269* [2.92607]	Φ_{14}	-0.650493* [-5.15191]
β_1	0.1093701** [2.75284]	β_2	0.3189006** [7.63321]
R square	0.360056	R square	0.403863

[Table 9]*, **, *** denotes significant at 1%, 5% and 10%. [] shows t-statistics

The coefficients of the error correction terms, β_1 and β_2 in VECM (4, 1) reported to be significant at 5% level implying that the long run co-integrating relationship between the spot and futures returns has been appropriately considered in VECM equations (4) and (5). Further, the residuals series of the system are extracted to estimate hedge ratio and hedge effectiveness, the results are given in [Table 10].

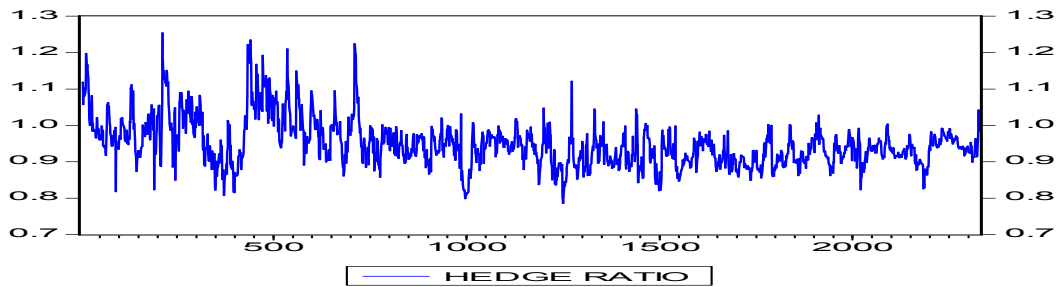
VECM (4, 1) estimation of Hedge Ratio and Hedge Effectiveness	
Cov(ϵ_S, ϵ_F)	0.000372473
var(ϵ_F)	0.000397162
Hedge Ratio(h)	0.937835967
variance(H)	74.25432624
variance(U)	3187.576901
Hedge Effectiveness(E)	0.976705087

$$[\text{Table 10}] \quad h = \frac{\sigma_{sf}}{\sigma_f} \text{ and } E = 1 - \frac{\text{var}(H)}{\text{var}(U)}$$

The hedge ratio and hedge effectiveness obtained through VECM (4, 1) shows approximately 94% variance reduction and 97% hedge effectiveness. Although VECM model does not consider the conditional covariance structure of spot and futures returns, it is the best model for calculating the constant hedge ratio and hedging effectiveness because it takes into account the long run co-integration between spot and futures price. VECM certainly performs better than the OLS and the VAR model.

CC-GARCH (p, q) estimation

The CC-GARCH model is employed with the objective to estimate the time varying hedge ratio. Following the estimation of GARCH model from section IV, [Table 5] reports the coefficients in GARCH (1, 1) to be statistically significant, implying that the forecast of conditional variance at different time periods depends on the information available in current period. Further, to estimate the time varying hedge ratio we generated the two conditional variance series (equation 6 and 7) in GARCH (1, 1) and estimated the constant correlation between the observed residual series ($\rho = 0.989048$) to obtain the conditional covariance series (equation 8). The resulting dynamic hedge ratio is presented in [figure 16] and its summary statistics are given in [Table 11].



[Figure 16] Dynamic hedge ratio – Nifty Futures

Descriptive statistics of Dynamic hedge ratio	
	Nifty futures
Mean	0.949964
Standard deviation	0.011024
Min	0.79112
Max	0.935219
Jarque- Bera	905.3562

[Table 11]

[Figure 16] plots the dynamic hedge ratio over the analysis period starting from 12 June 2000 till September 2009. Since the dynamics hedge ratio are less stable and exhibit several ups and down or fluctuations, this suggests that the hedgers of Nifty futures market have to adjust their futures positions more often. [Table 11] reports the descriptive statistics of dynamic hedge ratio estimated from the time-varying conditional variance and covariance between spot and futures return. The dynamic hedge ratio ranges from a minimum 0.79112 to a maximum of 0.935219. Also a high Jarque- Bera suggests that the distribution of the hedge ratios is not normal. The average dynamic hedge ratio estimated is approximately 95% variance reduction which is highest among other models. Based on this time varying hedge ratio we estimated the variances of hedged and unhedged portfolio to calculate the hedge effectiveness. The estimation reports that hedging with the dynamic hedge ratio of Nifty futures is 97% effective.

As the last step of our analysis, the static hedge ratio obtained from OLS, VAR, VECM and average of time varying hedge ratio obtained from VAR-CC-GARCH model with their respective hedge effectiveness are compared in [Table 12].

	Hedge ratio	Hedge Effectiveness
OLS	0.925377	0.960104
VAR (4)	0.9293018	0.9767782
VECM (4, 1)	0.9378359	0.9767050
CC- GARCH (1,1)	0.949964	0.9710321

[Table 12]

Comparison of hedge ratios estimated for the four different models reveals that CC-GARCH model performs best as it offer highest variance reduction among the other three models. While static hedge ratios estimated through VAR and VECM are clearly an improvement over OLS estimated hedge ratio they are out performed by the dynamic hedge ratio of CC- GARCH.

VI. Conclusion

In this paper, we investigate whether Nifty futures present itself as an effective hedging tool for risk management in spot Nifty index market. Using data for a period of 9 years, we employed four different methods to estimate the hedge performance of Nifty futures. The main objective of using four different estimation techniques is to incorporate the peculiar statistical properties of the financial return series. We started with the estimation of simple OLS model, which is found to be inappropriate in the sense that it ignored non-normality, existence of serial correlation and presence of hetroscedasticity. Thus to incorporate these features we further employ VAR, VECM and CC-GARCH in our analysis. The OLS, VAR and VECM models estimated constant hedge ratio whereas time varying optimal hedge ratios are calculated using bivariate CC-GARCH model. All the models are able to offer a significant reduction in the variance of hedged portfolio relative to the unhedged portfolio. Optimal hedge ratio and hedge effectiveness estimated through these models suggest that CC-GARCH offers the highest variance reduction (95%) as compared to the other models (approximately 92-93%). Over all the results indicate that hedging with Nifty futures is effective (97%) for managing risk in the spot (Nifty) market.

Appendix A

Johansen's co-integration test

The co-integration test on the variables deals with examining the linear combinations of integrated variables that may be stationary. If such a property holds for Nifty index and futures returns then the two can be said as co-integrated. Johansen (1988, 1989) and Johansen and Juselius (1990) suggested two statistic tests to determine the number of co-integrating vectors (r), the first one is the trace test (λ trace) and the second test is the maximum eigenvalue test (λ max).

For the first trace test (λ trace) the null hypothesis holds that the number of distinct co-integrating vector is less than or equal to r against the alternative of k co-integrating relations, where k is the number of endogenous variables, for $r = 0, 1, 2$. The test calculated as follows: $\lambda_{trace}(r) = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i)$, where T is the number of usable observations, and the $\hat{\lambda}_i$ is the estimated eigenvalue. The second statistical test is the maximum eigenvalue test (λ max). The null hypothesis holds that there is r co-integrating vectors against the alternative that $r + 1$ co-integrating vector and is calculated as: $\lambda_{max}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1})$.

Johansen's Co-integration Test (1988)				
Null Hypothesis	Alternative hypothesis	Eigen	Log Likelihood	0.05%
(H ₀)	(H ₁)	Values	Ratio Test	Critical Value
Trace test statistics (λ trace)				
r=0	r \geq 1	0.045	108.63	20.26
r \leq 1	r \geq 2	8.74E-04	2.028	9.16
Maximum Eigen Value (λ max)				
r=0	r=1	0.045	106.60	15.89
r=1	r=2	8.74E-04	2.028	9.164

Trace test statistics (λ trace)

We calculate λ trace (0) statistic to test the first hypothesis that the variables are not co-integrated ($r=0$) against the alternative of one or more co-integrating vectors ($r > 0$). Since 108.63 exceed the 5% critical value of 20.26 we reject the null of no co-integrating vector and accept the alternative null of one or more co-integrating vectors. In the second hypothesis we calculate λ trace (1) statistic to test for the null of $r \leq 1$ against the alternative null of two co-integrating vectors. We observe that the λ trace (1) value of 2.028 is less than the critical value of 9.163 and so we cannot reject the null of no more than one co-integrating vectors at 5% level of significance.

Maximum Eigen Value (λ max)

The Maximum Eigen Value (λ max) statistics further confirms the results obtained from λ trace statistics. Clearly the null of no co-integrating vectors ($r=0$) is rejected against the alternative of $r=1$ as the calculated value of λ max (0) = 106.60 is greater than the critical value of 15.89 at 5% level of significance. Also the calculated value of λ max (1) = 2.028 is less than the 5% critical value of 9.164 and hence the null of $r=1$ cannot be rejected.

The results obtained from the above two test statistics suggest that the data generating process contains only one co-integrating vector and implies that there exists a well defined long run relationship between Nifty futures and Nifty returns in India.

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