OPTIMUM LEVEL OF INTERNATIONAL RESERVES FOR INDIA IN THE PRESENCE OF SOVEREIGN RISK

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ABSTRACT

This paper empirically determines the optimal level of international reserves for India by explicitly incorporating the country’s sovereign risk associated with default of external debt due to financial crisis. The optimum level of reserves is determined by minimizing the central bank’s cost function consists of cost due to high reserve holdings and cost due to reserve depletion. The simulated optimum reserves for the period 1994-2008 indicate that actual reserves are higher than optimum across the sample period except during 1997-98.

Keywords: International Reserves, Sovereign Risk, Optimization, ARCH, Cointegration.

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1. INTRODUCTION

Recent years have been marked by massive accumulation of international reserves with the central banks of developing countries around the world, especially in the aftermath of the East-Asian crisis of 1997-98. This increase in reserves can be attributed to the extensive trade and financial integration of developing countries and associated risks. Therefore, the share of developing countries in the global reserves has risen from 45 percent in 1997-98 to 72 percent in 2007-08 and Asian economies account for most of this increase. High reserve holdings with the central banks provide safe-guard against an impending financial crisis, improve the external position of a country and help to maintain international credit worthiness. At the same time, maintaining a high level of reserves is costly due to the opportunity cost associated with alternative uses. Hence managing international reserves is a challenging problem for the central banks in high reserve holding economies. The present study takes an attempt to determine optimum international reserves for India by considering recent massive accumulation of international reserves with the central bank of India, the Reserve Bank of India (RBI).

The RBI held more than 270 billion US dollar as international reserves as of end March 2008 and this level of reserves accounts for more than 25 percent of India’s Gross Domestic Product (see Figure.1). Table 1 shows India’s international reserves have surpassed standard adequacy based on imports and broad money\(^1\) during 2007-08.

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\(^{1}\) Import based reserve adequacy criteria suggests that 30 percent or 4 months of import covering reserves can be considered as a minimum benchmark for reserve adequacy (Triffin (1947; IMF, 1958). Similarly, Wijnholds and Kapteyn (2001) proposed that countries on managed float or on fixed exchange rate regime could maintain reserves to cover around 10 and 20 percent of broad money.
Likewise, the ratio of total external debt to reserves and short-term debt to reserves has also declined considerably during the same period.

**Figure 1: Trends in International Reserve Holdings in India**

![Graph showing trends in international reserve holdings in India from 1990-91 to 2007-08.](image)

**Table 1. Reserves to Import, Broad Money and External Debt**

<table>
<thead>
<tr>
<th>Year</th>
<th>Import cover in Months</th>
<th>Reserves as percentage of M3</th>
<th>Total External Debt To Reserves</th>
<th>Total Short-term External Debt to Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-91</td>
<td>3.17</td>
<td>4.57</td>
<td>1427.82</td>
<td>146.94</td>
</tr>
<tr>
<td>2007-08</td>
<td>15.00</td>
<td>30.81</td>
<td>72.50</td>
<td>14.30</td>
</tr>
</tbody>
</table>

Source: Handbook of Statistics on Indian Economy, RBI, (various years)

Table 2 shows a comparison between return from foreign exchange reserves and the domestic interest rate. It can be seen that the return from foreign currency assets and gold (after accounting for depreciation) has declined from 3.1 percent in 2002 to 2.1 percent in
2003-04 and increased there after\(^2\). A noteworthy observation is that domestic interest rate, which is often used as a proxy for opportunity cost of holding reserves, is higher than returns from reserves, which points to substantial cost of holding huge reserves during this period. Given the excess reserve holdings and high opportunity cost associated with it, the present paper tries to determine optimum level of reserves for India, considering the cost and benefit of holding reserves.

### Table 2 Return on Reserves and Domestic Interest Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>Return (Percentage)</th>
<th>91 days T-bill rate (Average and Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-03</td>
<td>3.1</td>
<td>5.8 (5.2-7.0)</td>
</tr>
<tr>
<td>2003-04</td>
<td>2.1</td>
<td>4.5 (4.2-5.4)</td>
</tr>
<tr>
<td>2004-05</td>
<td>3.1</td>
<td>5.1 (4.3-5.6)</td>
</tr>
<tr>
<td>2005-06</td>
<td>3.9</td>
<td>5.3 (5.1-6.6)</td>
</tr>
<tr>
<td>2006-07</td>
<td>4.6</td>
<td>6.6 (5.7-8.0)</td>
</tr>
<tr>
<td>2007-08</td>
<td>4.8</td>
<td>7.1 (4.4-7.9)</td>
</tr>
</tbody>
</table>

Source: Report on Foreign Exchange Reserves, RBI (various years).

The studies on optimum approach to international reserve holdings suggest that very little attention is given to calculate an optimum level of reserves for India. Available studies such as Ramachandran (2004), and Ramachandran and Srinivasan (2006) derived optimum reserves for India for the period 1999-2003 and 2001-2005 respectively, following Frenkel and Jovanovic reserve optimizing model (1981). Frenkel and Jovanovic model assumes country’s balance of payment disturbance is random and hence, Ben-Bassat and Gottlieb (1992) argued that this model may not be valid for most developing countries which are characterized by sustained current account deficit. This is also because these countries generally borrow from international market and hence BoP deficit is a common

\(^2\) Data on returns on foreign exchange is available only since 2002-03.
phenomenon. In such a case, they suggested that ‘sovereign risk’ of the country should be considered while estimating optimum reserves.

Considering sustained current account deficit in India over the last few years and high external borrowings in the form of External Commercial Borrowings (ECB) and Non-Resident Indian (NRI) deposits, the sovereign risk is an important factor while determining optimum reserves for India. Moreover, since one of the objective of the reserve management of India is to maintain high credit rating (Reddy, 2002), there is a dearth of studies determining an optimum level of reserves for India by measuring the sovereign risk of the country. Therefore, unlike other studies, the present study incorporates sovereign risk while determining optimum level of reserves for India. Apart from this, the present study also considers country specific factors such as the risk associated with the volatile nature of Foreign Institutional Investment and the fiscal deficit for determining sovereign risk.

The rest of the paper is organized as follows: section 2 and 3 deal with theoretical model and data sources. Econometric methodology and empirical results are given in section 4 and 5 respectively. Section 6 presents the conclusion.

2 THEORETICAL MODEL

In the framework of cost-benefit approach, a central bank considers the benefit from and cost associated with reserve holdings and the optimum level is attained when the marginal cost equals marginal benefit (Heller, 1966). The measurement of benefits and cost vary across studies. The major benefit of reserve holdings is the ability of a central bank to avoid economic loss due to fluctuations in BoP or exchange rate while the cost of reserve holdings corresponds to the return from forgone investment opportunities.
Based on the framework due to Ben-Bassat and Gottlieb (B-G, 1992), the optimum level of reserves for India is derived by minimizing RBI’s expected cost of reserve holdings. The cost of holding reserves consists of two components; a cost associated with reserve depletion or no reserves and another due to positive reserves or forgone earnings. The expected cost function is expressed as follows

Minimize

\[ E(\text{TC}) = \pi C_0 + (1 - \pi)C_1 \]

Where, \( TC \) is total cost, \( E \) is the expectation operation, \( C_0 \) refers to cost due to low reserves and \( C_1 \) denotes total opportunity cost or cost due to positive reserves. Likewise, \( r \) and \( R \) are the opportunity cost of reserve holdings and total reserve holdings respectively, and \( \pi \) and \( (1 - \pi) \) correspond to probability of reserve depletion and probability of reserves being positive respectively. Here, \( \pi \) is taken to be the probability of a country’s default of external debt due to financial or economic turmoil or crisis. Equation 1 implies that the central bank optimizes the level of reserves by trading-off between output loss and return loss. Reserve holdings and probability of default are negatively related, implying high reserve holding countries are less likely to default on their external debt. The probability of default can be described as

\[ \pi = f(R, Z) \quad \text{and} \quad \frac{\partial \pi}{\partial R} = \pi_r < 0 \]
where $Z$ is a set of economic variables that are likely to determine default risk of a country. $\pi_R$ is the first order derivative of $\pi$ with respect to $R$, which is expected to be negative since an increase in reserves would reduce the default risk.

The level of optimum reserves can be derived by minimizing the expected cost function. The first order differentiation of equation 1 with respect to $R$ is as follows

$$\frac{\partial E(TC)}{\partial R} = \pi_R (C_0 - rR) + (1 - \pi) r = 0$$

and optimum reserves $R^*$ can be written as

$$R^* = \frac{(1 - \pi)}{\pi_R} + \frac{C_0}{r}$$

where $R^*$ is the optimum reserves when total cost of reserve holdings is minimum. The steps for calculating optimum reserves are shown in box 1.
Box 1: Steps in Calculating Optimum Reserves

<table>
<thead>
<tr>
<th>E(TC) : Expected Total Cost</th>
<th>( \pi_R ) : First derivative of ( \pi ) w.r.t. ( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi ) : Probability of Default</td>
<td>( r ) : Opportunity Cost</td>
</tr>
</tbody>
</table>

Expected Cost of Holding Reserves:
\[
E(TC) = \pi C_0 + (1 - \pi)C_i
\]

Minimize w.r.t. \( R \):

First Order Condition
\[
\frac{\partial E(TC)}{\partial R} = \pi_R(C_0 - rR) + (1 - \pi)r = 0
\]

Estimate Parameters

Cost of Default (\( C_0 \)):
Difference between actual output and potential output

Probability of Default (\( \pi \)):
Function of macroeconomic variables, including reserve ratio.

Opportunity Cost (\( C_i \)):
Return from domestic investment

Solve Simultaneously:

Optimal Reserves (\( R^* \))

Source: Ben-Bassat and Gottlieb (1992)
### 2.1 Cost of Default ($C_0$)

Most of the studies used adjustments cost as the cost associated with low level of reserves or cost of reserve depletion. The propensity to import (Heller, 1966) and variability in reserves (Frankel and Jovanovic, 1981) are used as proxies for adjustment cost due to temporary disequilibrium in BoP. Given that most developing economies are borrowing economies characterized by sustained BoP disequilibrium, the cost of reserve depletion can be viewed as cost of default on external debt (B-G, 1992). Insolvency and financial crisis may lead to economic slowdown and hence output may decline over several years. Thus, GDP foregone due to financial crisis can be a better proxy for measuring cost of default. The present study uses the difference between actual growth rate of GDP and potential GDP during the period of BoP crisis 1991-92 as a measure of cost of default. Following Ozyildirim and Yaman (2005), the present study assumes that the percentage of GDP forgone in the crisis period represents cost of reserve depletion throughout the sample period.

### 2.2 Probability of Default ($\pi$)

Unlike other studies, the B-G study takes into account sovereign risk of a country to estimate optimum reserves. They measure sovereign risk by estimating the probability of default of external debt ($\pi$). They also assume a logistic probability function for $\pi$, which is a function of soundness of macroeconomic variables that reflect external liquidity and solvency. The function for probability of default can be written as

$$\pi = \frac{e^f}{1 + e^f}$$  \hspace{1cm} (5)
where,

\[ f = f\left(\frac{\text{res}}{\text{im}}, \frac{\text{edt}}{\text{ex}}, \text{im}, \text{y}\right) \]

Where, \( f \) is a function of variables such as reserves to imports, external debt to exports, value of imports and output. B-G followed Feder and Just (1977) to define \( f \) which states that the odds of default \( \frac{\pi}{(1-\pi)} \) are equal to the discounted risk premium in a perfect capital market.

\[
\frac{\pi}{(1-\pi)} = \frac{i-i^*}{(1+i)}
\]

where, \( i \) is the interest rate offered by the risky borrower, \( i^* \) is the risk free rate. By substituting equation 5 in equation 6, we obtain

\[
\frac{\pi}{(1-\pi)} = e^f
\]

Similarly,

\[
\log\left(\frac{\pi}{1-\pi}\right) = \log(e^f) = f
\]

Therefore,

\[
f = \log\left(\frac{i-i^*}{1+i}\right)
\]

or \( f \) is equal to the log of discounted risk premium or spread. The equation of \( f \) can be estimated by regressing a discounted risk premium equation with macro economic variables. In this study \( f \) is specified as:

\[ f = f\left(f_{ii}, \text{sted/im}, \text{fd/gdp}\right) \]
where $fii$ stands for volatility of Foreign Institutional Investment in India, $std/res$ is the short-term external debt to reserves, and $fd/gdp$ is fiscal deficit to GDP. The discount premium equation to be estimated can be written as

$$
\ln\left(\frac{i-i^*}{1+i}\right)_t = a_0 + a_1 \ln fii + a_2 \ln\left(\frac{st ed}{res}\right)_t + a_3 \ln\left(\frac{fd}{gdp}\right)_t + \epsilon_t
$$

where, $i$ is an average of interest rate paid by India for its ECBs and interest rate offered to NRI deposits. $i^*$ is the risk free interest rate proxied by London Interbank Offer Rate (LIBOR), \(\ln\) indicates the logarithmic transformation. $a_1, a_2$ and $a_3$ are parameters to be estimated and $a_0$ and $\epsilon_t$ are intercept and error term respectively. The definition of variables in the model and their relationships are discussed below.

**Spread**: the dependent variable, $i - i^* \left[\frac{1}{1+i}\right]$, shows the spread between domestic interest rate and foreign interest rate and this can be taken to refer to risk premium. The domestic interest rate $i$ is measured by taking the average interest rate on India’s ECB and interest rate offered to NRI deposits. The interest rates of ECB and NRI are taken because they constitute the major debt components in India’s external debt. The share of ECB and NRI deposits to India’s external debt is 27.6 percent and 25.6 percent respectively (Ministry of Finance, 2007). Since these debt components constitute more or less the same share to total external debt (26-28 percent), a simple average of interest rates of these components is used. The high positive spread between domestic interest rate and foreign interest rate indicate high risk premium due to high sovereign risk.
Volatility of Foreign Institutional Investment (fii): Short-term volatile capital flow is characterized by sudden stop and reversal in response to market sentiments. Due to macroeconomic risk associated with volatile capital flows, a country may be charged high interest rate on its borrowings from international market. Hence fii is expected to have a positive relationship with spread. It is also found that risk associated with Foreign Institutional Investment in India induces RBI to hold reserves as a precautionary savings (Prabheesh et al, 2009).

Short-term External Debt to Reserves (sted/res): A high short-term external debt to international reserves may indicate the inability of a country to meet its external obligations and it may adversely affect the credit worthiness of the country leading to higher premium being charged for external borrowings. Therefore, a positive relationship is expected between short-term external debt to reserves and spread\(^3\).

Fiscal Deficit to GDP (fd/gdp): A high fiscal deficit to GDP is an indication that the government is unable to cover current expenses including its debt servicing. A weak fiscal position also implies a higher likelihood that external shocks may generate a default. The fiscal balance of a country is the major determinant of creditworthiness and hence plays an important role in determining risk premium especially in the cause of emerging market economies (Baldacci et al, 2008). Hence a positive relationship between fiscal deficit to GDP and spread is expected.

\(^3\) Short-term external debt to reserves is chosen, rather than total debt to exports used by B-G, because the former captures the liquidity risk of a country especially when capital account is open.
2.3 Opportunity Cost (C1)

The total opportunity cost is calculated by multiplying reserves with return from domestic investment \((C_i = rR)\), where \(r\) is proxied by yield rate of 91 days Treasury bill in India.

3 DATA

In order to derive optimum international reserve for India, quarterly data from 1994:02 to 2007:04 has been used. Most of the data are collected from RBI publications such as Handbook of Statistics on Indian Economy, and the Ministry of Finance publication such as Status Report on India’s External Debt. All variables are measured in current prices. Quarterly estimates of GDP are available only from 1996 and hence, for the earlier period, estimates developed by Virmani and Kapoor (2003)\(^4\) have been used. Since these estimates are represented in constant prices with base 1993-94, the series is converted into current prices using GDP deflator\(^5\). Similarly, short-term external debt series is not available on a quarterly basis and hence the annual series is converted into quarterly series using extrapolation technique. The volatility of Foreign Institutional Investment is proxied by conditional variance derived from ARCH (2) model. Since data on Foreign Institutional Investment is given in monthly, the ARCH variances are also derived in monthly terms. Hence the derived variance series is converted into quarterly series by taking the average of three months corresponding to each quarter. The variables namely \(res\), \(fii\), \(sted/res\) and

\(^4\) The authors estimated back series of India’s GDP by adopting the methodology developed by Central Statistical Organization.

\(^5\) GDP deflator is the ratio of real GDP to Nominal GDP. To derive nominal quarterly series of GDP, the available annual ratio is assumed to be constant through the four quarters.
is expressed in crores of Rupees. Data on LIBOR is drawn from the website of British Banker’s Association (www.bba.org.uk).

4 ECONOMETRIC METHODOLOGY

The systems method of cointegration proposed by Johansen and Juselius (1992) is used to estimate the spread equation (10). The integration property of the variables is examined using unit root tests such as ADF test and P-P test. ARCH model has been used to derive the volatility series of Foreign Institutional Investment and the H-P filter method is used to measure potential GDP which measures output loss due to financial crisis. A brief description about these methods is given below.

4.1 ARCH model

To generate the volatility measure of Foreign Institutional Investment, \( fii \), we have applied the Autoregressive Heteroscedastic (ARCH) model developed by Engel (1982). The main advantages of ARCH model compared to traditional volatility estimation method such as rolling standard deviation etc are it helps to model the volatility clustering features of the data and incorporates heteroscedasticity into the estimation procedure. The ARCH(p) model specification can be written as

\[
\begin{align*}
fii_t &= \mu + \varepsilon_t \\
\varepsilon_t \sim N(0, h_t) \\
h_t &= \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2
\end{align*}
\]  

(11)
Equation (11) is the conditional mean equation, where \( \mu \) is the mean of \( f_{it} \). \( \varepsilon_t \) is the error term conditional on the information set \( \alpha_{t-i} \) and is normally distributed with zero mean and variance \( h_t \). Equation (12) is the variance equation which shows that the conditional variance \( h_t \) depends on mean \( \omega \) and the information about the volatility from previous periods \( \varepsilon^2_{t-i} \). The size and significance of \( \alpha_t \) indicates the presence of the ARCH process or volatility clustering in the series.

4.2 HP filter method

Hodrick- Prescott (1997) or HP filter method is employed to derive the output loss due to financial crisis or to calculate the cost of default. The HP filter is a smoothing method which obtains smooth estimates of the long-term trend component of a series. It has an advantage over simple de-trending procedure based on linear trend in that it is a time varying method and allows the trend to follow a stochastic process, whereas, the traditional method assumes that the trend series grows at constant rate. HP filter method computes the smoothed series of GDP, \( gdp^T \), by minimizing the variance of \( gdp \) around \( gdp^T \), subject to a penalty that constrains the second difference of \( gdp^T \). The HP filter chooses \( gdp^T \) to minimize

\[
\sum_{t=r}^n (gdp_t - gdp^T)^2 + \lambda \sum_{t=2}^{n-1} (\Delta gdp_{t+1}^T - \Delta gdp_t^T)^2
\]  

(13)
Where, $\lambda$ is the smoothing parameter and $n$ is the sample size. $\lambda$ takes value of 1,600 for quarterly series (Harvey and Jaeger 1993). The difference between the actual series ($gdp_t$) and the smoothed series ($gdp_t^\lambda$) is the output gap, or cost of default.

### 4.3 Multivariate Cointegration

If the variables in the model are non-stationary and integrated with same order then the systems method of cointegration proposed by Johansen and Juselius (1992) helps to check for long-run relationship between variables. If the variables under consideration are cointegrated, i.e. the long-run relationship is established, then cointegrating vector is normalized with respect to the targeted variables, spread in this context, which then provides estimates of long-run relationship between spread and its determinants. Johansen’s cointegrating analysis involves estimating the following Vector Error Correction Model in reduced form

$$
\Delta Y_t = \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \Pi Y_{t-1} + \lambda D + \varepsilon_t
$$

(14)

Where, $Y_t$ is a vector of non-stationary variables, $\Gamma$, $\Pi$, and $\lambda$ are matrices of parameters to be estimated. The rank of $\Pi$ matrix determines the long-run relationship and can be decomposed as $\Pi = \alpha \beta'$, where $\alpha$ and $\beta$ contain adjustments and cointegrating vectors respectively. $D$ is a vector of deterministic variables that may include constant term, linear trend and dummy variables. $\Delta$ and $\varepsilon_t$ refer to change and error term respectively. Johansen has proposed two likelihood ratio statistics, the trace static and the maximum eigen value statistic, both of which determine the number of cointegrating vectors based on significant eigen values of $\Pi$. The trace statistic tests the null of $r$ cointegrating vectors against the
alternative of more than \( r \) cointegrating vectors, while the maximum eigen statistic tests the null of \( r \) against the alternative of exactly \( r + 1 \) cointegrating vectors. Once the number of cointegrating vectors is determined, hypotheses on both adjustment and cointegrating vectors can be tested.

5 EMPIRICAL RESULTS

5.1 Estimation of Cost of Default (\( C_0 \))

The cost of default is measured by taking the difference between actual and potential growth rate of GDP in India during the BoP crisis period in the early nineties. Table 3 shows the annual growth rates of actual GDP and potential GDP in both current and constant terms with growth gap. The annul growth rate is calculated by taking the average growth rates of quarterly GDP corresponding to each period\(^6\). The potential GDP is the trend values of the GDP derived by using H-P filter method. The table shows that during the crisis period, 1991-92, current and constant output contracted by 4.7 and 4.4 percentage respectively. Similarly, the cumulative loss of GDP growth due to crisis is also calculated to measure the overall impact of crisis on output. The cumulative GDP growth contraction due to crisis for the period 1991-92 to 1993-94 is found to be 7.5 and 5.2 percentage at current and constant price respectively. In this circumstance, the optimum level of reserves for India is calculated by considering a range of impact of crisis on output reduction which is a minimum of 4.8 percent of GDP and maximum of 7.5 percent of GDP.

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\(^6\) Annual, rather than quarterly, growth rate is shown to depict aggregate magnitude of impact of crisis on output growth.
Table 3  Actual and Potential Growth Rate of Nominal GDP

<table>
<thead>
<tr>
<th>Year</th>
<th>At Current Price</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Potential</td>
<td>Deviation</td>
<td>Actual</td>
<td>Potential</td>
<td>Deviation</td>
</tr>
<tr>
<td>1991-92</td>
<td>14.44</td>
<td>19.19</td>
<td>-4.75</td>
<td>1.29</td>
<td>5.73</td>
<td>-4.43</td>
</tr>
<tr>
<td>1992-93</td>
<td>15.68</td>
<td>17.45</td>
<td>-1.76</td>
<td>5.11</td>
<td>5.85</td>
<td>-0.73</td>
</tr>
<tr>
<td>1993-94</td>
<td>15.78</td>
<td>16.78</td>
<td>-1.00</td>
<td>5.90</td>
<td>5.98</td>
<td>-0.07</td>
</tr>
<tr>
<td>1994-95</td>
<td>17.48</td>
<td>16.43</td>
<td>1.04</td>
<td>7.25</td>
<td>6.09</td>
<td>1.16</td>
</tr>
<tr>
<td>1995-96</td>
<td>17.21</td>
<td>15.84</td>
<td>1.37</td>
<td>7.34</td>
<td>6.16</td>
<td>1.17</td>
</tr>
<tr>
<td>1996-97</td>
<td>17.66</td>
<td>14.81</td>
<td>2.85</td>
<td>7.83</td>
<td>6.18</td>
<td>1.65</td>
</tr>
</tbody>
</table>

5.2 Estimation of Probability of Default (\( \pi \))

5.2.1 ARCH variance of Foreign Institutional Investment

The volatility of Foreign Institutional Investment or variable \( fii \) is estimated through ARCH (2) model\(^7\). The results reported in the Table 4 show that the ARCH effect is significant in the conditional variance. The model diagnostics do not indicate serial correlation in the standardized squared residuals or ARCH effect on residuals.

Table 4  ARCH (2) results of Foreign Institutional Investment

<table>
<thead>
<tr>
<th>( fii_t )</th>
<th>122.48</th>
<th>+0.734 ( fii_{t-1} )</th>
<th>+ ( \epsilon_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1.90]***</td>
<td>[33.4]*</td>
<td></td>
</tr>
<tr>
<td>( h_t )</td>
<td>166923.3</td>
<td>+0.5.89 ( \epsilon_{t-1}^2 )</td>
<td>+1.12 ( \epsilon_{t-2}^2 )</td>
</tr>
<tr>
<td></td>
<td>[7.93]*</td>
<td>[3.91]*</td>
<td>[6.557]*</td>
</tr>
</tbody>
</table>

Log – Likelihood = -1398,  SR LB \( \chi^2 \) = 8.49 (0.58),
SSR LB \( \chi^2 \) = 7.70(0.65), ARCH = 0.82(0.60)

Note: SR = standardized residuals, SSR = standardized squared residual, LB = Ljung-Box statistics for serial correlation at 10 lags. ARCH = LM test for ARCH effects in the residuals. * and *** denotes significance at 1%, and 10% levels, respectively. Figures in square brackets and parenthesis show t-statistics and level of significance, respectively.

\(^7\) We also estimated the volatility series using GARCH model. However, the results are not stable.
5.2.2 Descriptive Statistics and Unit Root Tests

The summary statistics of the variables considered for risk premium equation are reported in the Table 5. Jarque-Bera test shows that the null hypothesis of normality is rejected in all variables indicating non-normality. In order to understand the property of integration of each variable, ADF and P-P unit root tests were performed and the results are reported in the Table 6. Results of ADF and P-P tests show that the null hypothesis of non-stationarity cannot be rejected in the case all variables in levels, implying these variables are non-stationary in levels. On the other hand, in first difference the null of non-stationarity is rejected in all cases implying that these variables are stationary in first difference. Overall, the unit root tests results indicate that the variables considered for estimation are integrated at order one, i.e., I (1). Sine all variables are found to be integrated at order one, the multivariate co-integration developed by Johansen and Juselius (1992) is performed to estimate the risk premium equation to measure the probability of default.

Table 5 Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>ln(i-i*)/(1+i)</th>
<th>ln fii</th>
<th>ln sted/res</th>
<th>ln fd/gdp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.588</td>
<td>6.930</td>
<td>-2.316</td>
<td>-3.028</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.988</td>
<td>0.650</td>
<td>0.582</td>
<td>0.608</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.120</td>
<td>0.653</td>
<td>0.225</td>
<td>-0.767</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.178</td>
<td>2.271</td>
<td>1.531</td>
<td>3.267</td>
</tr>
<tr>
<td>Jarque-Bera Probability</td>
<td>20.753</td>
<td>4.763</td>
<td>5.017</td>
<td>5.153</td>
</tr>
<tr>
<td>Probability</td>
<td>(0.000)</td>
<td>(0.075)</td>
<td>(0.003)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Note: The Jarque-Bera test is a test of the null hypothesis of normality in which skewness and kurtosis of the series is compared to the normal distribution.
Table 6  Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th>ADF Test Statistic</th>
<th>P-P Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>First Difference</td>
</tr>
<tr>
<td>$\ln(i - i^* / 1 + i)$</td>
<td>-0.63</td>
<td>-5.24*</td>
</tr>
<tr>
<td>$\ln(f_{ii})$</td>
<td>0.41</td>
<td>-6.89*</td>
</tr>
<tr>
<td>$\ln(sted / res)$</td>
<td>1.12</td>
<td>-6.01*</td>
</tr>
<tr>
<td>$\ln(fd / gdp)$</td>
<td>0.74</td>
<td>-11.53*</td>
</tr>
</tbody>
</table>

Note: * denotes rejection of unit root at 1 %.

5.2.3  Johansen’s approach of cointegration

Johansen’s approach of cointegration begins with the formulation of an unrestricted Vector Auto Regressive Model (VAR). Using AIC and SBC, the optimum number of lags for VAR is identified as one, where the residuals of the VAR are found to be uncorrelated and homoscedastic. Table 7 presents both trace and maximum eigen test statistics which provide evidence that the null of ‘None’ cointegrating vector can be rejected. However, both statistics could not reject the null of ‘At most 1’ cointegrating vector, implying that there exists one set of cointegrating relationship among the four variables considered. The normalized cointegrating coefficients with respect to $\ln(i - i^* / 1 + i)$ are given in Table 8.

Table 7  Johansen Cointegration Test

<table>
<thead>
<tr>
<th>Hypothesized no. of CV(s)</th>
<th>Trace Statistic</th>
<th>5 % Critical Value</th>
<th>Max-Eigen Statistic</th>
<th>5 % Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None*</td>
<td>55.60</td>
<td>54.07</td>
<td>30.53</td>
<td>28.58</td>
</tr>
<tr>
<td>At most 1</td>
<td>25.07</td>
<td>35.19</td>
<td>17.06</td>
<td>22.29</td>
</tr>
<tr>
<td>At most 2</td>
<td>8.00</td>
<td>20.26</td>
<td>5.48</td>
<td>15.89</td>
</tr>
<tr>
<td>At most 3</td>
<td>2.52</td>
<td>9.16</td>
<td>2.52</td>
<td>9.16</td>
</tr>
</tbody>
</table>

Notes: CV denotes cointegrating vector and * denotes rejection of the hypothesis at 5 percent significance level
Table 8  Normalized Cointegrating Coefficients

<table>
<thead>
<tr>
<th>Normalized $\beta$</th>
<th>$\ln (i - i^<em>)/(1 + i^</em>)$</th>
<th>$\ln fii$</th>
<th>$\ln sted/res$</th>
<th>$\ln fd/gdp$</th>
<th>$\text{constant}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-3.46</td>
<td>-1.58</td>
<td>-20.791</td>
<td>-51.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.60)*</td>
<td>(-2.75)*</td>
<td>(-3.14)*</td>
<td>(-2.71)*</td>
</tr>
</tbody>
</table>

Note: Figures in parenthesis indicate t statistics and * denotes statistical significant at 5 percent level.

The long-run cointegrating equation for spread can be written as

$$\ln\left(\frac{i - i^*}{1 + i}\right) = 51.70 + 3.46\ln fii + 1.58\ln\left(\frac{sted}{res}\right) + 20.79\ln\left(\frac{fd}{gdp}\right)$$

(15)

The normalized cointegrating equation exhibits theoretically expected signs and the ‘t’ statistic in the parentheses of the Table 8 indicate that the explanatory variables are statistically significant at 5 percent level. This implies that all explanatory variables considered in the spread equation significantly explain risk perception of foreign lenders on Indian economy. The relationship between volatility of Foreign Institutional Investment and spread is positive and highly significant. This shows the short-term capital flows characterized by sudden stop and reversal reflects the risks in the economy. The impact of short-term capital flows on risk premium is found in the case of Turkey for the period 1988-2002 by Ozyildirim and Yaman (2005).

Similarly, short-term external debt to reserves is also found to be statistically significant in explaining spread. This indicates that the country’s ability to meet short-term obligations is an important determinant of risk premium of India. Similar result is found in the context of Chile during the period 1999-2002 by Rojas and Jaque (2003). Similarly, the impact of total external debt to reserves on spread is evidenced in the case of 19 emerging markets by Ades et al. (2000).
It is also interesting to see that the coefficient of fiscal deficit to GDP on spread is 20.79, which is higher than other explanatory variables and is also statistically significant. This is a clear indication that high and sustained fiscal deficit in India is perhaps a major concern among lenders when India approaches them for external borrowing. The significance of fiscal balance of the Government on spread is empirically supported in the case of Argentina by Nogués and Grandes (2001) during the period of 1994-1998. Ades et al. (2000) found a significant effect of fiscal balance on spread in 19 emerging markets for the period 1996-2000. Similarly, Baldacci et al. (2008) found that fiscal balance is a major determinant of spread in the case of 30 emerging market countries including India during the period of 1997-2007.

The cointegrating graph obtained from the vector error correction model is shown in Figure 2 which demonstrates that the relationship among the variables is stable across the sample period.

**Figure 2  Cointegrating Graph**
Using the estimated spread equation (15), it is possible to derive the probability of default ($\pi$) by estimating $\ln(i - \frac{i^*}{1+i})$ which is then plugged in the following equation.

$$\pi = e^f / (1 + e^f) \quad \text{where} \quad f = \log(i - \frac{i^*}{1+i})$$

$$\ln\left(\frac{\pi}{1-\pi}\right) = 51.70 + 3.46 \ln \left( \frac{students}{res} \right) + 1.58 \ln \left( \frac{fd}{gdp} \right) + 20.79 \ln \left( \frac{fd}{gdp} \right)$$

where, $$\frac{\pi}{1-\pi} = \frac{i - i^*}{1+i}$$

The estimated average probability of default $\pi$ is found to be 0.39 and maximum and minimum values are 0.73 and 0.01 respectively. This time varying probability of default better captures the sovereign risk of the country than the traditional approach of assuming a value, 0.5 for $\pi$.

From the spread equation (16), it is also possible to derive $\pi_R$ (or equivalent to $\pi_R$) by differentiating it with respect to reserves ($res$).

$$\frac{\partial^2 \pi}{\partial res} = \pi_R = -\pi(1-\pi) \frac{1.58}{res} < 0$$

This shows that the change in probability of default due to a small increase in reserves is negative, $\pi_R < 0$, indicating the probability of default diminishes as reserves increases.

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8 In the equation 2 $\pi_R$ is defined as the first order differentiation of $\pi$ with respect to reserves (R). Though in the present context reserves is by denoted as res, $\pi_R$ is also refers to $\pi$ with respect res.
5.3 Optimum Reserves (\(R^*\))

After estimating \(\pi, \pi_p,\) and \(C_0,\) the next step is to compute optimum reserves by substituting these values together with opportunity cost \((r)\) in equation 4. The simulated optimum reserves for India for the period 1995:Q1-2007:Q4, by considering the impact of 4.8-7.5 percent of GDP contraction, are shown in the Figure 3 along with actual reserves. It can be observed that the optimum reserves show an increasing trend along with actual reserves. Importantly, the actual reserves tended to optimum level based on 4.8 percent GDP contraction during the period 1997-98, which can be attributed to the risk associated with the contagious effect of East-Asian crisis experienced during this period. However, the actual reserves are less than optimum level during the same period when maximum impact of 7.5 percent of GDP contraction is considered. The increased reserve accumulation after 2001 can be a reason for the divergence of actual reserves from optimum reserves in the subsequent periods. Overall, actual reserves are higher than optimum reserves except during the period of East-Asian crisis, 1997-98. After 2001, actual reserves are much higher than optimum reserves in India.
Additionally, the adequate levels of reserves are computed based on rule of thumb measures for purpose of comparison. Figure 4 shows the comparison between optimum reserves and adequate level of reserves based on three months imports cover along with actual reserves. The figure indicates that optimum reserves based on 7.5 percent of GDP contraction is higher than traditional adequacy criterion except during 1995-96 perhaps due to high import bill in that year. Thus the optimum reserves, even after considering sovereign risk is able to cover more than three months imports bill. Similarly, Table 9 shows that optimum level of reserves can cover 13.3 percent of broad money by the year 2007-08 and short-term debt constitutes around 35 percent of optimum reserves in the same year.

\[\text{9} \] Hike in short-term debt to reserves can be attributed to the change in the definition of short-term debt by the RBI since 2004-25 (Report on Foreign Exchange Reserves, RBI, 2008)
Figure 4  Comparison between Actual, Adequate and Optimum Level of Reserves

Table 9  Optimum Reserves and Money Supply and External Debt (in percentage)

<table>
<thead>
<tr>
<th>Year</th>
<th>Optimum Reserves to M3</th>
<th>Actual Reserves to M3</th>
<th>Short-term Debt to Optimum Reserves</th>
<th>Short-term Debt to Actual Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994-95</td>
<td>15.87</td>
<td>16.68</td>
<td>41.02</td>
<td>16.85</td>
</tr>
<tr>
<td>1995-96</td>
<td>5.07</td>
<td>13.45</td>
<td>62.33</td>
<td>22.36</td>
</tr>
<tr>
<td>1996-97</td>
<td>11.43</td>
<td>14.77</td>
<td>26.52</td>
<td>25.44</td>
</tr>
<tr>
<td>1997-98</td>
<td>23.84</td>
<td>15.41</td>
<td>15.26</td>
<td>17.19</td>
</tr>
<tr>
<td>1999-00</td>
<td>8.13</td>
<td>15.71</td>
<td>15.98</td>
<td>10.33</td>
</tr>
<tr>
<td>2000-01</td>
<td>9.91</td>
<td>16.11</td>
<td>16.41</td>
<td>8.57</td>
</tr>
<tr>
<td>2001-02</td>
<td>6.16</td>
<td>18.59</td>
<td>14.70</td>
<td>5.07</td>
</tr>
<tr>
<td>2003-04</td>
<td>7.97</td>
<td>26.33</td>
<td>12.70</td>
<td>7.92</td>
</tr>
<tr>
<td>2004-05</td>
<td>8.19</td>
<td>29.07</td>
<td>37.08</td>
<td>12.52</td>
</tr>
<tr>
<td>2005-06</td>
<td>14.63</td>
<td>27.36</td>
<td>24.33</td>
<td>12.88</td>
</tr>
<tr>
<td>2007-08</td>
<td>13.34</td>
<td>30.81</td>
<td>35.06</td>
<td>15.18</td>
</tr>
</tbody>
</table>

Note: Optimum reserves are based on 7.5% of GDP loss.
6 CONCLUSIONS

This paper has made an attempt to derive optimum level of international reserves for India for the period 1994-2008 using quarterly data. The study employed the reserve optimizing approach developed by Ben-Basset and Gottlieb (1992) for developing countries by taking into account the sovereign risk in the estimation. Optimum reserves are derived by minimizing a cost function of the central bank which consists of cost due to high reserve holdings as well as cost due to reserve depletion and their associated probabilities. The cost due to high reserve holding is measured by opportunity cost proxied by domestic interest rate and the cost due to reserve depletion is measured by taking the percentage contraction in output during the period of BoP crisis in India. The study has used a range of output contraction to derive the optimum reserves. The probability of reserve depletion is derived from estimating a risk premium equation which captures the sovereign risk associated with external borrowings. The empirical result shows that the volatility of Foreign Institutional Investment, short-term debt to reserves and the fiscal deficit to GDP significantly explain variation in risk premium.

The simulated optimum reserves indicate that actual reserves are higher than optimum reserves across the sample except for period during 1997-98. The East-Asian crisis and consequent risk associated with the contagious effect may be the reason for optimum reserves exceeding actual reserves in this period. Therefore, the present study concludes that international reserves in India are higher than the estimated optimum level of reserves even after considering sovereign risk associated with financial crisis.
Therefore, prudent reserve management policies are required to channel excess reserves into productive sectors of the economy.

REFERENCES


