Inefficient Shocks and Optimal Monetary Policy*

Sargam Gupta†
ISI Delhi

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Abstract

The importance of trade-offs between inflation and output gap stabilization for monetary policy evaluation is well known. Real disturbances in the economy which lead to such trade-offs, however have not been studied much in the context of monetary policy setting in emerging market and developing economies (EMDEs). We identify market price support present in the agriculture sectors of the EMDEs as a real disturbance leading to such trade-offs. Using a three-sector NK-DSGE model built in Ghate et al. (2018), featuring food procurement policy in the Indian economy, we derive welfare loss function and characterize optimal monetary policy under discretion and commitment. We show that under both discretionary and commitment policy, *trade-offs* exist between core-inflation and output gap stabilization, and between headline inflation and output gap stabilization. This result departs from the existing popular view that *strict* core-inflation targeting is the optimal monetary policy for developing countries susceptible to sectoral relative-price changes. We also compare the response of the economy to a positive procurement shock and a negative productivity shock under different monetary policy rules. It is observed that an optimal simple rule with sectoral terms of trade/ relative price gaps improves welfare outcomes significantly.

Keywords : Optimal Monetary Policy, Multi-sector New Keynesian DSGE Models, Inefficient shocks, Terms of trade, Agricultural Procurement

JEL Codes : E31; E43; E52; E58; Q18

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†Corresponding Author: Economics and Planning Unit, Indian Statistical Institute, New Delhi – 110016, India. Tel: 91-11-4149-3942. E-mail: sargamgupta.6@gmail.com.

1 Introduction

Monetary policy making in emerging markets and developing economies (EMDEs) is a challenging task as these economies are often characterized by inefficiencies such as incomplete financial markets, distorted agriculture sectors and large informal sectors that affect monetary policy effectiveness (see Hammond et al. (2009), Ghate and Kletzer (2016)). Most of the existing literature in monetary policy design for EMDEs focusses on determining the optimal inflation index that a central bank should target to reach the flexible price equilibrium.[1] Recently, Anand et al. (2015) showed that in EMDEs headline inflation targeting improves welfare outcomes by adding incomplete financial markets to the standard multi-sector small scale NK-DSGE model. This is different from Aoki (2001), who shows that strict core inflation targeting is an optimal monetary policy to reach the flexible price equilibrium in developing countries which are susceptible to sectoral relative price movements (or terms of trade shocks).[2] One common aspect in these papers is the assumption that variations in the flexible price equilibrium are efficient.[3] However, there could be possibilities when variations in the flexible price equilibrium are not efficient and thus strict inflation targeting will not be an optimal monetary policy, as there exists a trade-off between inflation and output stabilization (see Woodford (2003), Chapter 6). In other words, any attempt to stabilize inflation and bring output to the flexible price level will make output deviate further from its efficient allocation. Even having a multi-sector Aoki type model with sectoral terms of trade shocks/ relative price shocks does not show any tension between core-inflation and output stabilization.

Generally, inefficient variations in the flexible price equilibrium are modelled as inefficient supply shocks, such as price/ wage mark-up shock (see Justiniano, Primiceri, and Tambalotti (2013) , Gilchrist, Ortiz, and Zakrasjek (2009), Gali, Gertler, and Lopez-Salido (2007) , and Bhattarai et al. (2014)).[4] Inefficient shocks do have a practical importance in monetary policy making but the sources of such shocks have not been studied much (Woodford (2003), p. 454).[5] This paper addresses this gap in the literature and shows how real disturbances present in a developing economy could be a source of *inefficient* shocks. To be precise, in

[1]In this paper flexible price equilibrium is defined as the equilibrium level prevailing under complete price flexibility.

[2]Also see Huang and Liu (2005) , Benigno (2004) and Erceg and Levin (2006) .

[3]In this paper the efficient equilibrium is defined as the equilibrium level prevailing under perfect competition.

[4]For the estimates of inflation/ output trade-offs in US see Fuhrer, J. C. (1997). Gilchrist, Ortiz, and Zakrasjek (2009) shows trade-offs in presence of financial frictions.

[5]The term *real disturbance* refers to the existence of structural disturbances in the economy which can lead to trade-offs mentioned here. Generally in New-Keynesian literature the trade-off are generated with *exogenous* price/ wage mark-up shocks. But what leads to such shocks is not studied much.

this paper we identify market price support present in the agriculture sectors of the EMDEs as an inefficient distortion and show its implications for optimal monetary policy design.

Market price support estimates are over 2.2 trillion US dollars, between 2011-2015, across the world (OECD (2016*a*)).[6] This comprises 55% of the total producer support estimates (PSE) which are over 4 trillion US dollars during the same period.[7] Market price supports primarily take two forms, i) border protection measures such as, tariffs, import quotas and export subsidies as in Canada, Colombia, European Union, Iceland, Israel, Kazakhstan, Korea, Mexico, Norway, Russia, Turkey, United States and Vietnam; and ii) target pricing of a commodity both with and without government purchases such as in China, India, Indonesia, Japan, Norway and Vietnam.[8] There is an extensive literature studying the effects of the agricultural price supports on output, consumption and trade (see Bale and Lutz (1981), Anderson and Hayami (1986), Acemoglu and Robinson (2001), Timmer (1989), Dewbre, J., Anton, J., and Thompson W. (2001), Benjamin N. and Talab, I. (2011)). Figure 1a below shows the share of market price support as a percentage of GDP for EMDEs and advanced economies (AEs). As can be seen, between 2011-2015, the share for EMDEs is 0.78%, which is almost double the share in AEs (which is 0.40%).[9] What accentuates the effect of market price support in EMDEs is there large agriculture sectors. Figure 1b below shows the share of agriculture sector as a percentage of GDP between 2011-2015 for EMDEs and AEs. The share is 13.4% and 1.8% for EMDEs and AEs respectively.[10]

[INSERT FIGURE 1a & 1b]

[6]The Organization for Economic Cooperation and Development (OECD) agriculture statistics database has the agriculture support data for only 50 countries. The Market Price Support (MPS) is defined by OECD as an indicator of the annual monetary value of gross transfers from consumers and taxpayers to agricultural producers arising from policy measures creating a gap between domestic market prices and border prices of a specific agricultural commodity measured at the farm-gate level.

[7]The Producer Support Estimate (PSE) is defined by OECD as an indicator of the annual monetary value of gross transfers from consumers and taxpayers to support agricultural producers, measured at farm gate level, arising from policy measures, regardless of their nature, objectives or impacts on farm production or income.

[8]Refer OECD(2016*b*) for each country (except India) to get more detailed analysis. For India refer OECD(2009). Under target pricing, Indonesia and India have target/ support prices with government purchases and China, Japan, Norway and Vietnam have target/ support prices without government procurement.

[9]The author uses OECD agriculture statistics database (doi: dx.doi.org/10.1787/agr-pcse-data-en (accessed on 16 June, 2017). According to the availability of data, advanced economies (AE) constitutes United States, European Union (28 countries), Australia, Canada, Iceland, Israel, Japan, Korea, New Zealand, Norway, Switzerland and emerging markets and developing economies (EMDEs) constitutes, Brazil, Chile, China, Colombia, Indonesia, Kazakhstan, Mexico, Russia, South Africa, Turkey, Ukraine, Vietnam.

[10]The figures are calculated by author using macro indicators data available on Food and Agriculture Organization of the United Nations (FAO) (http://www.fao.org/faostat/en/#data/MK accessed in June, 2017). The percentage figures 13.4% and 1.8% are share of value of agriculture, fishing and forestry in GDP on average for EMDEs (152 countries) and AEs (38 countries) respectively, between 2011-2015. The author uses International Monetary Fund's (IMF) categorization of AE and EMDEs (WEO, October 2016).

Recently, Ghate et al. (2018) have shown how such market price supports in agriculture sector of India leads to sectoral and aggregate inflation, output gap and resource reallocation using multi-sector NK-DSGE model. In India, the target pricing of certain agricultural commodities (such as wheat and rice) is accompanied by government purchases of the commodity. This policy in known as food grain procurement policy.[11] Ghate et al. (2018) introduce procurement inefficiency in food grain sector as a shock and discuss the transmission of such shock to the aggregate economy. They also show that these shocks weaken monetary policy transmission.

In this paper, using the NK-DSGE model built in Ghate et al (2018), we derive a welfare loss function for a central bank of an economy characterized by market price support. Although we build on a NK-DSGE model specific to the Indian economy, the results can also be generalized to other EMDEs featuring similar inefficiencies. To derive the welfare loss function we use micro-founded utility based approach following Rotemberg and Woodford (1997, 1999) and Woodford (1999, 2003). The model has three sectors: grain, vegetable and manufacturing sector. The grain and vegetable sectors are part of the flexible price agriculture sector. The manufacturing sector is a sticky price sector. The model features a procurement inefficiency in the flexible price sector namely, grain sector. Using a welfare loss function, we characterize optimal monetary policy under discretion and commitment and study how the trade-off between inflation and output gap stabilization gets affected in the presence of procurement inefficiency. We then compare and rank optimal monetary policy rules with some implementable rules.

The results of the paper contribute both theoretically as well into policymaking. Theoretically, we contribute by identifying a real disturbance in the form of market price support in agriculture sector as a source of inefficient shocks to an economy. In particular, we identify government induced procurement policy as a source of inefficient shocks for the Indian economy and derive welfare loss function of the central bank. For a policymaker the contributions

[11] Under this policy, government announces the target price known as minimum support prices (MSP) for a variety of food grains before the cropping season starts. Once the harvest is done, the food grain producers sell their output to the government at a set MSP. The procured food grain is then stored in the Food Corporation of India (FCI) warehouses. A part of the procured food grain is then distributed among the poor section of the society at subsidized prices through the public distribution system (PDS) and the rest remains in the warehouses as buffer stock. The effects of government induced procurement policy on the macroeconomy of India are non-negligible. Ramaswamy et al. (2014) have shown that the accumulated welfare losses of procurement policy to the Indian economy between 1998 and 2011 are 1.5 billion US dollars. In the recent years, rising minimum support prices has fueled food inflation in India (see Anand et al. (2016), Basu (2011), Dev and Rao (2015), Ramaswamy et al. (2014), Ghate et al. (2018)). High food inflation is a cause for concern, specially in a developing country like India where food expenditure shares are very high. For instance, share of food in consumer expenditure is 52.9% and 42.6% in rural and urban India, respectively (NSS (2013)). Also the food subsidy bill rose by 300% between 2006-07 and 2011-12 (see Sharma and Alagh (2013)).

of the paper are twofold. Firstly, we analyze optimal monetary policy under discretion and commitment with procurement inefficiency, and study the trade-offs encountered by the policymaker while setting monetary policy especially in EMDEs. Secondly, we compare some implementable instrument (here interest rate) rules with optimal rules to find out which interest rate rule should a central bank in EMDEs follow given that the agriculture sectors of these economies are characterized by inefficiencies such as procurement distortion.

1.1 Main Results

We find that the inefficiency due to procurement in the agriculture sector affects the economy through two distinct channels. First, it raises prices in the grain sector by affecting price mark-ups. Second, by reducing aggregate consumption directly, it deprives households of a part of the output. These channels lead to variations in the flexible-price equilibrium which are not efficient. The derived welfare loss function is a function of squares of core-inflation, consumption gap, and the terms of trade gap, where gaps are not natural gaps (from the flexible-price equilibrium) but from a welfare relevant level.[12] The welfare relevant level is defined as the flexible-price level with no mark-up effect of the procurement inefficiency i.e. without the first channel mentioned above.

The optimal monetary policy under discretion and commitment show that a central bank cannot stabilize core-inflation, output gap and terms of trade gap together, as there exists a trade-off between core-inflation and output gap stabilization and between terms of trade gap and output stabilization. Due to this the minimum losses are not zero. This happens due to the presence of procurement inefficiency which makes the flexible price equilibrium deviate from the efficient allocation and any attempt to bring core-inflation to zero (and output to the flexible price counterpart), makes output deviate further from its efficient allocation. This result departs from Aoki (2001), who shows that strict core-inflation targeting is an optimal monetary policy for developing countries featuring sectoral relative price movements (or terms of trade shocks). This implies that central banks in developing countries need more caution while setting monetary policy, as the inefficiencies in the real sector of their economy can modify standard results and alter the policy response.

We also compare the response of the economy under different optimal and implementable rules when the economy is hit by a positive procurement shock and a negative productivity shock. A comparative analysis among different monetary policy rules shows that the commitment rule leads to the least welfare losses and is thus best among all the considered monetary policy rules. Within the class of implementable monetary policy rules, a simple

[12] Note that the welfare relevant level is not same as the efficient level. An efficient allocation coincides with the flexible-price equilibrium when there is no procurement inefficiency.

Taylor rule with target variables as inflation and output gap performs the worst. The welfare losses reduce significantly when terms of trade gaps are added to the simple Taylor rule. We thus find the optimal coefficients on the simple Taylor rule with terms of trade gaps to get an optimal simple rule for the economy. It is observed that an optimal simple rule with sectoral terms of trade/ relative price gaps improves welfare outcomes significantly. We find that welfare losses reduce by 21% and 62% with optimal simple rules for a positive procurement shock and a negative productivity shock, respectively.

2 The Model

The basic structure of the model is adapted from Ghate et al. (2018). It is a closed economy three-sector NK-DSGE model, a variant to Aoki (2001) and Gali and Monacelli (2005). Ghate et al. (2018) differs from Aoki (2001), as it adds a procurement distortion in the flexible price sector (grain sector). This added distortion in the flexible price sector is the source of inefficiency, which we exploit in the present paper to study optimal monetary policy. The model consists of three entities, namely, households, firms and a central bank. The government's only role in the model is to procure a certain proportion of grain produce.[13] The procurement of grain is financed using the revenue collected from households in the form of lump-sum taxes.[14] The procured good, does not add any utility to the consumers and simply goes waste.

2.1 Households

The economy consists of a continuum of infinitely lived households. A representative household i consumes differentiated goods of all three sectors, namely, open market grain (OG), vegetables (V), and manufacturing (M) and maximizes the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(i)) - v(N_t(i)) \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $u(.)$ is the utility from aggregate consumption bundle, $C_t(i)$, $v(.)$ is the disutility from labour supply, $N_t(i)$, and the index $i \in [0,1]$. We assume a standard increasing and concave function, $u(C_t(i)) = \frac{C_t(i)^{1-\sigma}}{1-\sigma}$ where, σ, is the

[13] We do not focus on how the level of procurement set here but rather use an estimated AR(1) shock process on procurement as discussed in detail in Ghate et al. (2018).

[14] The government also provides an employment subsidy to do away with the inefficiency due to market power, as will be discussed later.

inverse of the inter-temporal elasticity of substitution and an increasing and convex function, $v(N_t(i)) = \frac{N_t(i)^{1+\psi}}{1+\psi}$ where, ψ, is the inverse of the Frisch elasticity of labor supply.

Aggregate consumption, $C_t(i)$, is a composite Cobb-Douglas index of consumption of manufacturing, $C_{M,t}(i)$, and agriculture sector goods, $C_{A,t}(i)$, and is defined as:

$$C_t(i) \equiv \frac{(C_{A,t}(i))^{\delta}(C_{M,t}(i))^{1-\delta}}{\delta^{\delta}(1-\delta)^{(1-\delta)}}, \tag{2}$$

where

$$C_{A,t}(i) \equiv \frac{(C_{V,t}(i))^{\mu}(C_{OG,t}(i))^{1-\mu}}{\mu^{\mu}(1-\mu)^{(1-\mu)}}, \tag{3}$$

is a composite Cobb-Douglas index of consumption of grain bought by consumers in the open market, $C_{OG,t}(i)$, and vegetables, $C_{V,t}(i)$. Further,

$$C_{s,t}(i) \equiv \left(\int_0^1 C_{s,t}(i,j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \; s = OG,\; V,\; M. \tag{4}$$

$\delta \in (0,1)$ is the share of total consumption expenditure allocated to agriculture sector goods and $\mu \in (0,1)$ is the share of total food expenditure allocated to vegetable sector goods. The elasticity of substitution between the varieties within each sector, θ, is greater than one and is assumed to be the same in all sectors. The index $j \in [0,1]$ refers to the j^{th} variety of differentiated good in each sector s. The optimal sectoral consumption demand functions are,

$$C_{OG,t} = (1-\mu)\left(\frac{P_{OG,t}}{P_{A,t}}\right)^{-1} C_{A,t}, \tag{5}$$

$$C_{V,t} = \mu\left(\frac{P_{V,t}}{P_{A,t}}\right)^{-1} C_{A,t}, \tag{6}$$

and

$$C_{M,t} = (1-\delta)\left(\frac{P_{M,t}}{P_t}\right)^{-1} C_t, \tag{7}$$

where $C_{A,t} = \delta\left(\frac{P_{A,t}}{P_t}\right)^{-1} C_t$.[15] Here the aggregate price index for the economy, or equivalently the consumer price index (CPI), is $P_t \equiv (P_{A,t})^{\delta}(P_{M,t})^{1-\delta}$ where $P_{A,t}$ and $P_{M,t}$ are prices of the composite agricultural and manufacturing goods, respectively. Also the price of agricultural goods is given by, $P_{A,t} \equiv (P_{OG,t})^{1-\mu}(P_{V,t})^{\mu}$ where $P_{OG,t}$ and $P_{V,t}$ are the prices

[15] The index i is supressed as the consumption decisions are identical across all households.

of open market grain and vegetables, respectively.[16] To get the optimal consumption plan, households maximize (1) subject to the following inter-temporal budget constraint,

$$P_t C_t + E_t\{Q_{t,t+1} B_{t+1}\} \leqslant B_t + W_t N_t - T_t + Div_t \tag{8}$$

where B_{t+1} is the nominal pay-off in period $t+1$ of the bond held at the end of period t. $Q_{t,t+1}$ is the stochastic discount factor. The transversality condition, $\lim_{T\to\infty} E_t\{B_t\} \geq 0$ $\forall\, t$, is assumed to be satisfied. W_t is the economy wide nominal wage rate.[17] T_t are lump-sum taxes to the government, and Div_t are the dividends or profits distributed to the households by monopolistically competitive firms. Money is excluded from both the budget constraint and utility function as the demand for money is endogenized. Also note that financial markets are complete here, such that the households have no credit constraints and thus each household faces a single intertemporal budget constraint.[18] The maximization yields the following optimal consumption-savings choice,

$$E_t\left[\beta R_t \left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{1-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] = 1 \tag{9}$$

and optimal consumption-leisure choice,

$$\frac{(N_t)^{\psi}}{(\Gamma_t)^{1-\sigma}(C_t)^{-\sigma}} = \frac{W_t}{P_t}. \tag{10}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross nominal return on the riskless one-period bond.

2.2 Firms

All the firms in all three sectors (grain, vegetable and manufacturing sector) have a linear production function $Y_{s,t}(j) = A_{s,t} N_{s,t}(j)$, where $s = G, V$ and M, is the sector containing a continuum of firms indexed by $j \in [0,1]$ and $A_{s,t}$ is the sector specific productivity shock.[19] The flexible price agriculture sector (both vegetable and grain sector) firms optimize there

[16] Also note that the optimal consumption demand for the j^{th} variety in the s^{th} sector is given by $C_{s,t}(j) = \left(\frac{P_{s,t}(j)}{P_{s,t}}\right)^{-\theta} C_{s,t}$, where $P_{s,t} \equiv \left(\int_0^1 P_{s,t}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$ is the sector s specific price index.

[17] Labor is assumed to be completely flexible across all sectors. This assumption assures equal nominal wages across all sectors.

[18] Anand et al. (2015) in their paper relax this assumption and assume that a fraction of households are credit constraint and thus they have two budget constraints for two different type of households.

[19] Note that grain sector produce, $Y_{G,t}(i)$, comprises of two parts, one part is procured by the government, $Y_{PG,t}(j)$, which is not consumed and another goes to the open market, $Y_{OG,t}(i)$, to be consumed by households. Each firm j in a sector produces a variety j of the sectoral good.

profit function each period, to set prices,

$$P_{OG,t}(j) = \frac{\theta}{(\theta-1) - \frac{Y_{PG,t}}{Y_{OG,t}(j)}} MC_{G,t} \tag{11}$$

in grain sector, and

$$P_{V,t}(j) = \frac{\theta}{\theta-1} MC_{V,t} \tag{12}$$

in the vegetable sector.[20] In the above price setting equation (11) of the grain sector, notice that the price mark-up, $\frac{\theta}{(\theta-1)-\frac{Y_{PG,t}}{Y_{OG,t}(j)}}$, is time-varying and increasing in $Y_{PG,t}$, where $Y_{PG,t}$ follows an AR(1) process described later. Given the level of procurement in a particular year, $Y_{PG,t}$, grain producers set a price $P_{OG,t}$. The government and the households buy the grain at the newly set prices, $P_{OG,t}$. Procurement thus acts as an *inefficient* supply shock in the grain sector. Following Calvo (1983), we assume that only a fraction, $(1-\alpha_M) \in (0,1)$, of firms in the sticky price manufacturing sector adjust prices while the rest, α_M, of firms do not. Price of the $(1-\alpha_M)$ fraction of firms depends on current and discounted expected future marginal costs and is given by,

$$P^*_{M,t}(j) = \frac{\theta}{\theta-1} \frac{E_t \sum_{k=0}^{\infty} \alpha_M^k Q_{t,t+k} Y_{M,t+k}(j) MC_{M,t+k}}{E_t \sum_{k=0}^{\infty} \alpha_M^k Q_{t,t+k} Y_{M,t+k}(j)}, \tag{13}$$

where $Q_{t,t+k}$ is the stochastic discount factor. The rest α_M fraction keep their prices fixed to last year prices, $P_{M,t-1}$. The aggregate price in the manufacturing sector is $P_{M,t} = \left[\alpha_M (P_{M,t-1})^{1-\theta} + (1-\alpha_M)(P^*_{M,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}$. The government provides a fixed employment subsidy, τ, to firms such that the nominal marginal cost, $MC_{s,t} = \frac{(1-\tau)W_t}{MPN_{s,t}} = \frac{(1-\tau)W_t}{A_{s,t}}$, where $MPN_{s,t}$ is the marginal product of labor in sector $s = G, V$ and M.[21] The purpose of adding an employment subsidy is to get rid of the inefficiency in the model due to monopolistic competition.

2.3 Goods market clearing

In the goods market, aggregate consumption demand for vegetable and manufacturing goods equals output produced i.e. $C_{V,t} = Y_{V,t}$ and $C_{M,t} = Y_{M,t}$ respectively. In the grain sector,

[20] Firms in the grain sector maximize profits, $\pi_{OG,t}(j) = P_{OG,t}(j)[Y_{OG,t}(j) + Y_{PG,t}(j)] - MC_{G,t}[Y_{OG,t}(j) + Y_{PG,t}(j)]$, subject to the demand constraint $Y_{OG,t}(j) = \left(\frac{P_{OG,t}(j)}{P_{OG,t}}\right)^{-\theta} Y_{OG,t}$. The procurement of grain by government from each firm, $Y_{PG,t}(j)$, is assumed to be same across all firms in the grain sector, such that, $Y_{PG,t}(j) = Y_{PG,t}\ \forall j \in [0,1]$.

[21] Here τ is the rate at which the cost of employment is subsidized by the government, such that $(1-\tau) = \frac{\theta-1}{\theta}$. Note that this subsidy is not provided in Ghate et al. (2018).

demand for grain is derived from two sources, one is the government demand of grain produce for procurement, $Y_{PG,t}$ and the second is from households who consume grain sold in the open market. Thus the total consumption demand equals total output left in the open market after procurement, i.e. $C_{OG,t} = Y_{OG,t}$ and aggregate demand for the grain good equals the total output produced in grain sector i.e. $Y_{OG,t} + Y_{PG,t} = Y_{G,t}$. These equilibrium conditions hold true at the firm level j also, i.e. $C_{M,t}(j) = Y_{M,t}(j)$, $C_{OG,t}(j) + Y_{PG,t} = Y_{G,t}(j)$ and $C_{V,t}(j) = Y_{V,t}(j)$. We can now write aggregate output of the economy in consumption units as,

$$Y_t = C_t + \frac{P_{OG,t}}{P_t} Y_{PG,t}. \tag{14}$$

In the last section we saw that procurement shocks translate into mark-up shocks as in equation (11), but as shown above, procurement also has a more direct effect on output by acting as a demand shock. For a given level of aggregate output level, procurement competes with aggregate consumption, C_t in two ways; (i) directly by higher demand effect, $Y_{PG,t}$; (ii) by increasing the relative price of open market grain to the aggregate price level, $\frac{P_{OG,t}}{P_t}$.

2.4 Log-linearized model

In this section, we will log-linearize the model using Taylor's first order approximation around the steady state with constant prices.[22]

2.4.1 Terms of trade and inflation rates

Aggregate or headline inflation rate is defined as $\pi_t = \ln P_t - \ln P_{t-1}$ and similarly, $\pi_{s,t} = \ln P_{s,t} - \ln P_{s,t-1}$ for $s = A, M, OG, V$ denotes the inflation rates in agriculture, manufacturing, grain and vegetable sector respectively. Manufacturing or sticky price sector inflation is referred in the literature to as core-inflation rate. We define the inter-sectoral terms of trade (ToT) as,

$$T_{AM,t} \equiv \frac{P_{A,t}}{P_{M,t}}, \tag{15}$$

which are terms of trade between the flexible price sector (agriculture sector) and the sticky price sector (manufacturing sector), and the intra-sectoral ToT as

$$T_{OGV,t} \equiv \frac{P_{OG,t}}{P_{V,t}} \tag{16}$$

[22]For a variable X_t with a steady state, X, $\widehat{X}_t = \ln(X_t) - \ln(X)$, is the log-deviation from its steady state. A variable $\widehat{X}_t^n$ is its natural level or flexible price level and $\widetilde{X}_t = \widehat{X}_t - \widehat{X}_t^n$ is the gap from its natural level or natural gap.

which are terms of trade within the flexible price sectors, i.e., between the grain sector and the vegetable sector. Log-linearizing the above equations give, $\widehat{T}_{AM,t} = \widehat{P}_{A,t} - \widehat{P}_{M,t}$ and $\widehat{T}_{OGV,t} = \widehat{P}_{OG,t} - \widehat{P}_{V,t}$ respectively. Headline inflation rate can be written in terms of the flexible price sector inflation rate and sticky price sector inflation rates as, $\pi_t = \delta\pi_{A,t} + (1-\delta)\pi_{M,t}$. It can further be re-written in terms of core-inflation rate and changes in terms of trade as,

$$\pi_t = \pi_{M,t} + \delta\Delta\widehat{T}_{AM,t}, \tag{17}$$

where, $\Delta\widehat{T}_{AM,t} = \widehat{T}_{AM,t} - \widehat{T}_{AM,t-1}$. The last equation shows that changes in headline inflation is affected by the core-inflation rate, which is more persistent in nature, and by changes in the relative price movements of flexible price sector (agriculture) price, which are more transitory in nature.

2.4.2 Sectoral and aggregate demand functions

Using the optimal sectoral demand functions as described in section 2.1 and the goods market clearing conditions in section 2.3, we get the following sectoral demand functions,

$$\widehat{Y}_{M,t} = \widehat{C}_t + \delta\widehat{T}_{AM,t} \tag{18a}$$

$$\widehat{Y}_{V,t} = \widehat{C}_t - (1-\delta)\,\widehat{T}_{AM,t} + (1-\mu)\,\widehat{T}_{OGV,t} \tag{18b}$$

$$\widehat{Y}_{OG,t} = \widehat{C}_t - (1-\delta)\,\widehat{T}_{AM,t} - \mu\widehat{T}_{OGV,t} \tag{18c}$$

$$\widehat{Y}_{G,t} = (1-c_p)\,\widehat{Y}_{OG,t} + c_p\widehat{Y}_{PG,t} \tag{18d}$$

where, $c_p = \frac{Y_{PG}}{Y_G}$, is the steady state share of procured grain in total grain output. The parameter, c_p, is the distortionary and takes value between $\left[0, \frac{\theta-1}{\theta}\right]$. A positive value of c_p means that long run values of the procurement level is positive, which leads to distorted steady states in the model.[23] The above equations $(18a - 18c)$ imply that the demand for sectoral output not only depends on aggregate consumption (income effect) but also on the terms of trade between sectors (inter-good substitution effect). The aggregate demand equation (14), can be log-linearized as,

$$\widehat{Y}_t = (1-\lambda_c)\,\widehat{C}_t + \lambda_c[\widehat{Y}_{PG,t} + \mu\widehat{T}_{OGV,t} + (1-\delta)\widehat{T}_{AM,t}] \tag{19}$$

[23]Note that the procurement affects the equilibrium conditions of the model only when $c_p > 0$, which we calibrate later in the model. With $c_p = 0$, the model reduces to a standard multi-sector NK-DSGE model, similar to Aoki (2001).

where, λ_c is a combination of parameters in the model and takes value zero when $c_p = 0$, i.e. absence of inefficiency due to procurement.[24] It can be seen from equation (19) above, that procurement creates a wedge between aggregate output and aggregate consumption. With $c_p = 0$, the wedge goes away and equation (19) reduces to $\widehat{Y}_t = \widehat{C}_t$.

2.4.3 NKPC and DIS equation

Log-linearizing the price setting equation (13) of manufacturing sector firms with its aggregate prices, $P_{M,t}$, gives the following manufacturing sector NKPC,

$$\pi_{M,t} = \beta E_t\{\pi_{M,t+1}\} + \lambda_M \left(\psi\Theta_1 + \sigma\right) \widetilde{C}_t + \lambda_M \delta \widetilde{T}_{AM,t}. \tag{20}$$

where $\lambda_M = \frac{(1-\alpha_M)(1-\alpha_M\beta)}{\alpha_M}$, Θ_1 is a combination of parameters, equal to one and less than one for $c_p = 0$ and $c_p > 0$, respectively. Any shock to the agriculture sector thus changes, $\widetilde{T}_{AM,t}$ and shifts the NKPC, affecting core inflation. The aggregate NKPC can be written using the relation between headline and core-inflation in equation (17) and the following relation between $\widetilde{Y}_t$ and $\widetilde{C}_t$,[25]

$$\widetilde{Y}_t = (1-\lambda_c)\,\widetilde{C}_t + \lambda_c(1-\delta)\widetilde{T}_{AM,t}, \tag{21}$$

as,

$$\begin{aligned}\pi_t \;\; &= \;\; \beta E_t\{\pi_{t+1}\} + \lambda_M \frac{(\psi\Theta_1 + \sigma)}{(1-\lambda_c)}\widetilde{Y}_t + \lambda_M \left(\delta - \frac{\lambda_c\,(\psi\Theta_1+\sigma)\,(1-\delta)}{1-\lambda_c}\right)\widetilde{T}_{AM,t} \\ &\quad + \delta\Delta\widehat{T}_{AM,t} - \beta\delta E_t\{\Delta\widehat{T}_{AM,t+1}\}. \end{aligned} \tag{22}$$

The aggregate dynamic-IS equation for the model can be obtained by combining Euler's equation (9) and equation (21), as

$$\widetilde{Y}_t = E_t\{\widetilde{Y}_{t+1}\} - \frac{(1-\lambda_c)}{\sigma}[(\widehat{R}_t - E_t\{\pi_{t+1}\}) - \widehat{r}^n_t] - \lambda_c(1-\delta)E_t\left\{\Delta\widetilde{T}_{AM,t+1}\right\}, \tag{23}$$

where, $\widehat{r}^n_t = \sigma E_t\{\Delta\widehat{C}^n_{t+1}\}$, is the natural rate of interest. Ghate et al. (2018) show that the presence of procurement ($c_p > 0$) makes both NKPC and DIS curve steeper. This implies that the effect of any terms of trade shock or relative price shock gets amplified in the presence of an inefficiency such as procurement in developing countries. They also show that

[24] For details on the composite parameters refer to Ghate et al. (2018).

[25] Since, both grain and vegetable sectors are flexible price sectores, $\widehat{T}_{OGV,t} = \widehat{T}^n_{OGV,t}$ and $\widetilde{T}_{OGV,t} = 0$.

monetary policy transmission weakens in the presence of procurement since procurement creates a wedge between consumption and output and only a fraction, consumed out of output, gets affected by monetary policy.

2.4.4 Shock processes

In the model we have four structural shock processes namely, a procurement shock in the grain sector, $Y_{PG,t}$, and productivity shocks in the grain sector, $A_{G,t}$, vegetable sector $A_{V,t}$ and manufacturing sector, $A_{M,t}$, respectively. For the present paper we will only focus on shocks to the grain sector, i.e. $Y_{PG,t}$ and $A_{G,t}$. The shocks in log-linearized form are assumed to follow AR(1) processes as follows,

$$\Delta \ln A_{G,t} = \rho_{A_G} \, \Delta \ln A_{G,t-1} + \epsilon_{A_G,t} \ , \ \epsilon_{A_G,t} \sim i.i.d. \ (0, \sigma_{A_G}) \tag{24a}$$

$$\ln Y_{PG,t} - \ln Y_{PG} = \rho_{Y_{PG}} (\ln Y_{PG,t-1} - \ln Y_{PG}) + \epsilon_{Y_{PG},t} \ , \ \epsilon_{Y_{PG},t} \sim i.i.d. \ (0, \sigma_{Y_{PG}}) \tag{24b}$$

2.5 Welfare loss function

We now turn to study the implications of procurement inefficiency on optimal monetary policy. We will also compare some implementable instrument rules, to find out what rule should a central bank in developing country follow given that their economies are characterized by inefficiencies such as procurement distortion.

Our objective in this section is to derive a welfare loss function which central banks can use to evaluate the policy implications for the model economy described above. We use the seminal work of Rotemberg and Woodford (1997, 1999) and Woodford (1999, 2003) and take a second order approximation of the discounted sum of utility flows incurred by a representative consumer in a rational expectations equilibrium. A standard form of the welfare loss function depends on the squares of inflation and output gap. In case of a multi-sector model we get a welfare loss function depending on squares of the terms of trade gaps besides square of inflation and output gap.[26] The approximation to utility here is taken as its deviation from the efficient allocation and gaps are generally a deviation from this level. A standard one sector NK-DSGE model has two sources of inefficiencies namely, a sticky price sector (nominal rigidity) and monopolistically competitive firms with constant mark-ups (real rigidities).[27] In such a model, if the government provides an appropriate employment subsidy to firms to do away with the inefficiency due to monopolistic competition, the flexible price equilibrium coincides with the efficient allocation, such that natural gaps are same as

[26]See Aoki (2001), Huang and Liu (2005) and Benigno (2004).
[27]See Gali (2008, Chapter-3).

efficient gaps. Now, if the economy is characterized by price/wage markup time-varying shocks (generally referred to as *inefficient* supply shocks), the flexible price allocation does not coincide with the efficient one.[28] Here the natural gaps are not same as efficient gaps.[29] In the present paper there are three sources of inefficiencies namely, sticky prices in the manufacturing sector (nominal rigidities), monopolistic competition (real imperfection) and procurement distortion (real imperfection). We do away with the market power distortion completely in vegetable and manufacturing sector and partially in the grain sector by giving an appropriate employment subsidy as mentioned in section 2.2. The mark-up in the grain sector as shown in equation (11) is however scaled up by the presence of procurement in the model and a fixed employment subsidy, $(1-\tau)$, does not remove the market power completely. If we did not have procurement here (with an employment subsidy), then the flexible price equilibrium coincides with the efficient equilibrium, but we have a different scenario with procurement. As discussed in section 2.3 the procurement of grain by the government, impacts the economy by two channels. First, by raising prices in the grain sector by affecting mark-up. Second, by reducing the aggregate consumption level directly, as it deprives households of a part of the output produce. The monetary authorities are only concerned with the effect of such a procurement policy on prices (through the mark-up channel) and not the direct affect of procurement on consumption. Due to this, in the current model, welfare loss function depends on output and terms of trade gap from not from efficient level but from a flexible price level with no mark-up effect of procurement. We call this flexible price level with no mark-up effect as *welfare relevant* level and gaps from this level as welfare relevant gaps.[30] The welfare loss function takes the following form,[31]

$$W_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t}\right)^2 + \lambda_{\widetilde{C}} \left(\widetilde{C}_t^*\right)^2 + \lambda_{\widetilde{TAM}} \left(\widetilde{T}_{AM,t}^*\right)^2 \right] \tag{25}$$

where, $\lambda_{\pi M}$, $\lambda_{\widetilde{C}}$, $\lambda_{\widetilde{TAM}}$ are combinations of parameters of the model detailed in the technical appendix. $\widetilde{C}_t^*$ and $\widetilde{T}_{AM,t}^*$ are welfare relevant consumption and terms of trade gaps, respectively. The loss function depends on the squares of core-inflation/ manufacturing sector inflation, $\pi_{M,t}$, which is expected as the manufacturing sector is the only sticky price sector. This kind of loss differs from Aoki (2001) in two respects. First the gaps here are

[28] See Bhattarai et al. (2014) and Woodford (2003, Chapter-6).

[29] Note this is important here because the model equations like NKPC and the dynamic-IS curve are written in terms of natural gaps. If the welfare loss function is in terms of efficient gaps, then some modifications should be done to important equations mentioned above to do welfare analysis.

[30] For a variable X_t, $\widehat{X}_t^*$ is the welfare relevant level and $\widetilde{X}_t^*$ is the gap from welfare relevant level, $\widetilde{X}_t^* = \widehat{X}_t - \widehat{X}_t^*$. Any reference to gap will be welfare relevant gap by default from now in the text. Also note that, without the presence of procurement, welfare relevant gap coincides with efficient gap and natural gap.

[31] Refer to part A of the technical appendix for detailed derivations..

welfare relevant gaps as opposed to natural gaps in Aoki. Second, we have consumption gap here instead of an output gap, as the consumption gap and the output gap are not same, as described below. Both these differences appear here due to the presence of procurement inefficiency ($c_p > 0$).[32] When we remove the inefficiency due to procurement, i.e. $c_p = 0$, the welfare loss function reduces to a standard welfare loss function,

$$W_t = -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t}\right)^2 + (\sigma + \psi) \left(\widetilde{Y}_t\right)^2 + (\psi + 1)(1 - \delta)\,\delta \left(\widetilde{T}_{AM,t}\right)^2 \right]$$

Note that the coefficient on core-inflation is not affected by the presence of procurement and only the coefficient in front of real variables get affected by it. It is important to re-write some equations in the model in terms of welfare relevant gaps for further analysis.[33] The aggregate output gap equation (21), the manufacturing sector NKPC equation (22) and the DIS equation (23) can be written in terms of welfare relevant gap as,

$$\widetilde{Y}_t^* = (1-\lambda_c)\widetilde{C}_t^* + \lambda_c(1-\delta)\widetilde{T}_{AM,t}^* - (1-\lambda_c) z_{1,t}^* \tag{26}$$

$$\pi_{M,t} = \beta E_t \left\{\pi_{M,t+1}\right\} + \lambda_M \left(\sigma + \psi\Theta_1\right) \widetilde{C}_t^* + \lambda_M \delta \widetilde{T}_{AM,t}^* + z_{2,t}^* \tag{27}$$

$$\begin{aligned} \widetilde{Y}_t^* &= E_t\left\{\widetilde{Y}_{t+1}^*\right\} - \frac{(1-\lambda_c)}{\sigma}\left[\widehat{R}_t - E_t\left\{\pi_{M,t+1}\right\} - \widehat{r}_t^*\right] + \\ &\quad \left(\frac{(1-\lambda_c)\delta}{\sigma} - \lambda_c(1-\delta)\right) E_t\left\{\Delta \widetilde{T}_{AM,t+1}^*\right\} \end{aligned} \tag{28}$$

respectively, where

$$\begin{aligned} z_{1,t}^* &= \frac{1}{(1-\lambda_c)}\left(\widehat{Y}_t^* - \widehat{Y}_t^n\right) - \left(\widehat{C}_t^* - \widehat{C}_t^n\right) - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)}\left(\widehat{T}_{AM,t}^* - \widehat{T}_{AM,t}^n\right), \\ z_{2,t}^* &= \lambda_M\left(\sigma + \psi\Theta_1\right)\left(\widehat{C}_t^* - \widehat{C}_t^n\right) + \lambda_M \delta\left(\widehat{T}_{AM,t}^* - \widehat{T}_{AM,t}^n\right), \end{aligned}$$

[32] For the calibrated model, we find that that the welfare losses increases monotonically with increasing value of c_p both for procurement shock as well as productivity shock. Moreover, on impact the welfare losses increase by 11% for the productivity shock when procurement distortion is present.

[33] Refer to part B of technical appendix for detailed derivations.

and

$$\begin{aligned}\widehat{r}_t^* &= \widehat{r}_t^n + E_t\left\{\delta\Delta\widehat{T}_{AM,t+1}^*\right\} - \frac{\lambda_c\sigma(1-\delta)}{(1-\lambda_c)}E_t\left\{\Delta\widehat{T}_{AM,t+1}^* - \Delta\widehat{T}_{AM,t+1}^n\right\} \\ &\quad -\frac{\sigma}{(1-\lambda_c)}\left(\widehat{Y}_t^* - \widehat{Y}_t^n\right) + \frac{\sigma}{(1-\lambda_c)}E_t\left\{\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^n\right\}.\end{aligned}$$

Note that $z_{1,t}^*$, $z_{2,t}^*$ and $\widehat{r}_t^*$ are functions of exogenous shock processes.[34]

2.6 Monetary Policy Rule

For comparative analysis of monetary policy rules later in the paper, we will use a simple Taylor rule as described in Taylor (1993) with an added relative price/ terms of trade term, which takes following form,

$$R_t = (R_{t-1})^{\phi_R}(\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_{\widetilde{y}}}\left(\frac{T_{AM,t}}{T_{AM,t}^*}\right)^{\phi_{\widetilde{tam}}}$$

where ϕ_R is interest rate smoothing parameter, ϕ_π, $\phi_{\widetilde{y}}$ and $\phi_{\widetilde{tam}}$ are weights on headline inflation, output gap and terms of trade gap respectively.[35] We keep headline inflation as the measure of inflation rate here following Anand et al. (2015), where it is shown that headline inflation targeting improves welfare outcomes. The terms of trade term is added to the Taylor rule following Cuevas and Topak (2008), where the authors estimate such a Taylor rule for South Africa and some other countries. They show that countries with high inflation and inflation expectation show a more aggressive response to relative prices/ sectoral terms of trade. The log-linearized version of above rule Taylor-rule is:[36]

$$\widehat{R}_t = \phi_r\widehat{R}_{t-1} + \phi_\pi\pi_t + \phi_{\widetilde{y}}\widetilde{Y}_t^* + \phi_{\widetilde{tam}}\widetilde{T}_{AM,t}^* \tag{29}$$

When $\phi_{\widetilde{tam}} = 0$, the above rule reduces to a standard simple Taylor rule,

$$\widehat{R}_t = \phi_r\widehat{R}_{t-1} + \phi_\pi\pi_t + \phi_{\widetilde{y}}\widetilde{Y}_t^* \tag{30}$$

[34] In the absence of procurement inefficiency, all welfare relevant levels converge to the respective natural level such that $z_{1,t}^* = z_{2,t}^* = 0$ and $\widehat{r}_t^* = \widehat{r}_t^n$. Also, for any variable X_t, $\widetilde{X}_t^* = \widetilde{X}_t$.

[35] We assume that the inflation target is zero.

[36] Note that gaps are from the welfare relevant levels.

2.7 Description of parameters

Since we specifically model inefficiencies present in developing and emerging market economies in this paper, we limit our search for deep parameters to these countries. In particular, we pick most of the parameters estimated for Indian economy in literature as it features procurement policy which we model as an inefficient shock in this paper. We set the discount factor at $\beta = .9823$ as calibrated in Levine et al. (2012). We choose the value of the inverse of the Frisch elasticity of substitution, $\psi = 3$ as used in Anand et al. (2015). The values of the inter-temporal elasticity of substitution, σ, elasticity of substitution between varieties of the same sector goods, θ, and the measure of stickiness for the manufacturing sector, α_M, are set to 1.99, 7.02 and 0.75 respectively, as estimated in Levine et al. (2012) for the Indian economy.[37] We set the expenditure share on agriculture sector goods and vegetable sector goods to be, $\delta = 0.52$, $\mu = 0.44$, respectively, as calculated by Ghate et al. (2018) for the Indian economy. We choose shock parameters for productivity shocks in the grain sector namely persistence, ρ_{A_G}, and standard deviation, σ_{A_G}, as 0.25 and 0.03 respectively following Anand et al. (2015). The shock parameters for procurement shock , namely, persistence, $\rho_{Y_{PG}}$, and standard deviation, $\sigma_{Y_{PG}}$, are set to 0.4 and 0.66 respectively as estimated in Ghate et al. (2018). Besides this the steady state value of procured grain to total grain output, c_p, is also set to 0.08 using Ghate et al. (2018) . The Taylor parameters, namely, interest rate smoothing parameter, ϕ_R, weights on inflation, ϕ_π, and the output gap, $\phi_{\widetilde{y}}$ are set using Anand et al. (2015) to 0.7, 2 and 0.5 respectively. The weight on terms of trade gap, $\phi_{\widetilde{tam}}$, in the Taylor rule is set to 0.864 as estimated in Cuevas and Topak (2008) for the South African economy. Table 1 summarizes the description of the parameter values.[38]

[37]Levine et al. (2012) estimate a closed economy DSGE model for India using Bayesian estimation. They use data for real GDP, real investment, the GDP deflator, and the nominal interest rate for India from 1996:1 (i.e. first quarter)-2008:4 (i.e. last quarter). We use the estimated values for the 2-sector NK model from their paper.

[38]We use MATLAB version 2013 and Dynare version 4.4.3 for calibration.

Parameter	Notation	Value	Source
Discount factor	β	.9823	Levine et al. (2012)
Inverse of Frisch elasticity of labor supply	ψ	3	Anand et al. (2015)
Inverse of inter-temporal elasticity of substitution	σ	1.99	Levine et al. (2012)
Share of total consumption expenditure allocated to agriculture sector goods	δ	0.52	Ghate et al. (2018)
Share of total food consumption expenditure allocated to vegetable sector goods	μ	0.44	Ghate et al. (2018)
Elasticity of substitution between the varieties of same sector goods	θ	7.02	Levine et al. (2012)
Measure of stickiness	α_M	0.75	Levine et al. (2012)
Shock processes			
Procurement in grain sector			
Persistence	$\rho_{Y_{PG}}$	0.4	Ghate et al. (2018)
Standard deviation	$\sigma_{Y_{PG}}$	0.66	Ghate et al. (2018)
Productivity shock in grain sector			
Persistence	ρ_{A_G}	0.25	Anand et al. (2015)
Standard deviation	σ_{A_G}	0.03	Anand et al. (2015)
Taylor rule parameters			
Interest rate smoothing	ϕ_R	0.7	Anand et al. (2015)
Weight on inflation gap	ϕ_π	2	Anand et al. (2015)
Weight on output gap	$\phi_{\widetilde{y}}$	0.5	Anand et al. (2015)
Weight on output gap	$\phi_{\widetilde{tam}}$	0.864	Cuevas and Topak (2008)

Table 1: Summary of parameter values

3 Optimal monetary policy

This section will discus monetary policy rules that can minimize the welfare loss function described in section 2.5. A monetary policy rule that minimizes the welfare loss function is termed as optimal monetary policy. We will characterize optimal monetary policy under discretion and commitment for the model described above using the welfare loss function in equation (25).

3.1 Optimal monetary policy under discretion

Optimal discretionary policy is a policy where the monetary authority optimizes on its decision in each period without committing itself to any future actions.[39] Formally the problem can be written as,

$$\min_{\{\pi_{M,t},\, \widetilde{C}_t^*,\, \widetilde{T}_{AM,t}^*,\}} \frac{1}{2}\left[\pi_{M,t}^2 + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}}\left(\widetilde{C}_t^*\right)^2 + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}}\left(\widetilde{T}_{AM,t}^*\right)^2\right]$$

subject to,

$$\pi_{M,t} = \lambda_M\left(\sigma + \psi\Theta_1\right)\widetilde{C}_t^* + \lambda_M\delta\widetilde{T}_{AM,t}^* + z_{2,t}^*$$

where the constraint is NKPC as described in equation (27) with given $E_t\{\pi_{M,t+1}\}$. Using first order conditions from the above optimization and the aggregate output gap equation (26), we get following 'targeting rules'[40],

$$\pi_{M,t} = -\frac{1}{X_1(1-\lambda_c)}\widetilde{Y}_t^* - \frac{1}{X_1}z_{1,t}^* \tag{31}$$

$$\pi_t = -\frac{X_2}{X_1(1-\lambda_c)}\widetilde{Y}_t^* - \frac{X_2}{X_2X_1}z_{1,t}^* + z_{3,t}^* - \frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\pi_{M,t-1} \tag{32}$$

$$\pi_{M,t} = -\frac{\lambda_{\widetilde{TAM}}}{\delta\lambda_M\lambda_{\pi M}}\widetilde{T}_{AM,t}^* \tag{33}$$

where X_1 and X_2 are combinations of parameters and where, $z_{3,t}^* = \Delta\widehat{T}_{AM,t}^*$. As can be seen from equation (31) above, a central bank cannot stabilize core-inflation and the output gap together. In other words, it is not possible to achieve, $\pi_{M,t} = 0$ and $\widetilde{Y}_t^* = 0$, simultaneously.[41] When $\pi_{M,t} = 0$, the output gap, $\widetilde{Y}_t^*$, would be $-(1-\lambda_c)z_{1,t}^*$ and when $\widetilde{Y}_t^* = 0$, $\pi_{M,t} = -\frac{1}{X_1}z_{1,t}^*$, thus there exists a trade-off in stabilizing core-inflation and the output gap.[42] At this point we depart with Aoki (2001), where it is shown that strict core inflation targeting is an optimal monetary policy for developing countries, given that these countries are susceptible to terms of trade shocks. The departure happens because the developing country like India are characterized by certain inefficiencies such as procurement intervention by the government. Due to this inefficiency, as shown earlier, the flexible price equilibrium differs from the efficient allocation and any attempt to bring core-inflation to zero (and output to

[39] Refer to part C of the technical appendix for detailed derivations.

[40] A targeting rule is the relation between target variables that the central bank seek to maintain at all times.

[41] It is also not possible to achieve $\widetilde{T}_{AM,t}^* = 0$ and $\widetilde{Y}_t^* = 0$, simultaneously.

[42] Trade-off is defined as the variability in inflation (for any measure of inflation under consideration) that needs to be forgone to stabilize variability in output gap.

the flexible price counterpart) makes output deviate further from its efficient allocation.[43] Trade-off also exists between stabilizing headline (or aggregate) inflation and output gap as shown in Aoki (2001) , but here the trade-offs will be higher as they get amplified by the presence of procurement. The trade-offs for optimal monetary policy under discretion are plotted in Figure 2.

[INSERT FIGURE 2]

Figure 2 shows the *efficient frontier* for a trade-off between core-inflation and output gap stabilization, and between headline inflation and output gap stabilization (in 1a and 1b, respectively).[44] An efficient frontier is a loci of all values of variance of inflation (core-inflation, $\pi^2_{M,t}$, or headline inflation, π^2_t) and variance of output gap $\left(\widetilde{Y}_t^{*^2}\right)$ that minimizes the welfare loss function for arbitrary values put on the weight given to output gap in the welfare loss function. Since we do not have the output gap in the welfare loss function, we vary the value of $\lambda_{\widetilde{C}}$, as it would be proportional to the weight given to the output gap, such that $\lambda_{\widetilde{C}} \in [0,\ 500]$.[45] A point A and P in Figure 1a and 1b respectively, correspond to the optimal policy results when $\lambda_{\widetilde{C}} = 0$, i.e. when there is no weight on output gap stabilization. As a result we see a large variance in the output gap. On the other extreme point C and R in Figure 2a and 2b respectively, correspond to the optimal policy results when $\lambda_{\widetilde{C}}$ is sufficiently large. For the present model, the optimal policy under discretion represent point B and Q in Figure 2a and 2b respectively. Figure 3 shows how the trade-off varies with the procurement level.

[INSERT FIGURE 3]

Figure 3 plots the efficient frontier for values of c_p namely, 0.06, 0.08, 0.10, 0.12. As the value of c_p, a parameter capturing level of procurement, rises, the efficient frontier pushes out such that minimum variances of inflation and output gap and thus minimum losses under discretion are strictly higher for higher values of c_p.[46] The efficient frontier does not exist for $c_p = 0$, i.e. no trade-off exists between core-inflation and output gap stabilization in the absence of procurement inefficiency. In other words, the minimum losses possible are not zero but positive in the presence of procurement inefficiency. This means that central

[43] Note that this trade-off is generated when $c_p > 0$. As shown earlier, with $c_p = 0$, $z^*_{1,t} = 0$, $z^*_{2,t} = 0$ (in NKPC) and we again converge to Aoki's result.

[44] We observe that a similar trade-off exists between stabilizing the terms of trade gap, $\widetilde{T}^*_{AM,t}$, and the output gap, $\widetilde{Y}^*$, but not between terms of trade gap, $\widetilde{T}^*_{AM,t}$, and core-inflation, $\pi_{M,t}$.

[45] We minimize the welfare loss function under discretion here. We keep the weights on $\pi_{M,t}$ and $\widetilde{T}^*_{AM,t}$ constant at, $\lambda_{\pi m}$ and $\lambda_{\widetilde{Tam}}$, respectively. For details see, Woodford (2003, Chapter-6) .

[46] The efficient frontier for the calibrated value of the model is with $c_p = 0.08$.

banks in developing countries need more caution while setting their monetary policy, as the inefficiencies in the real sector of their economy can modify standard results and alter the policy response, as shown above.

The rate of interest rule for policy under discretion can be obtained by putting optimal values of the inflation rate, output gap and the terms of trade gap in DIS equation as,

$$\widehat{R}_t^* = \widehat{r}_t^* + \frac{(1-X_4)}{X_3} E_t \left\{ \sum_{j=0}^{\infty} \left(\frac{\beta}{X_3} \right)^j z_{2,t+1+j}^* \right\} + \frac{X_4}{X_3} E_t \left\{ \sum_{j=0}^{\infty} \left(\frac{\beta}{X_3} \right)^j \Delta z_{2,t+j}^* \right\} - \sigma E_t \left\{ \Delta z_{1,t+1}^* \right\} \tag{34}$$

where X_3 and X_4 are combinations of parameters. Note that the above discretion policy rule is a function of current and future shock processes.

3.2 Optimal monetary policy under commitment

Optimal commitment policy is a policy where monetary authorities commit to a optimal policy plan at all possible dates and states of nature, current and future.[47] Formally, the problem can be written as,

$$\min_{\{\pi_{M,t},\, \widetilde{C}_t^*,\, \widetilde{T}_{AM,t}^*\}} -\frac{1}{2} \lambda_{\pi M} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{M,t}^2 + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}} \left(\widetilde{C}_t^* \right)^2 + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}} \left(\widetilde{T}_{AM,t}^* \right)^2 \right]$$

subject,

$$\pi_{M,t} = \beta E_t \left\{ \pi_{M,t+1} \right\} + \lambda_M \left(\sigma + \psi \Theta_1 \right) \widetilde{C}_t^* + \lambda_M \delta \widetilde{T}_{AM,t}^* + z_{2,t}^*$$

where the constraint is NKPC as described in equation (27). Using first order conditions from the above optimization and the aggregate output gap equation (26), we get the following 'targeting rules',

$$\widetilde{Y}_t^* = -\omega_1 \widehat{\widehat{P}}_{M,t} - (1-\lambda_c) z_{1,t}^* \tag{35}$$

$$\widetilde{T}_{AM,t}^* = -\omega_3 \widehat{\widehat{P}}_{M,t} \tag{36}$$

for $t = 0, 1, 2, ...$where $\omega_3 = \frac{\lambda_{\pi M} \lambda_M \delta}{\lambda_{\widetilde{TAM}}}$ and $\widehat{\widehat{P}}_{M,t} = \widehat{P}_{M,t} - \widehat{P}_{M,-1}$. $\widehat{P}_{M,-1}$ is the price level in the manufacturing sector that prevails one period before the central bank chooses its optimal plan. As can be seen, the targeting rule under discretion in equation (31) has inflation as its target, but in case of commitment we get a price level target as an optimal targeting rule. Trade-off between inflation and output stabilization also exists in optimal policy under

[47] Refer to part D of the technial appendix for detailed derivations.

commitment as plotted in Figure 4.

[INSERT FIGURE 4]

The Figure 4 shows the efficient frontier for the trade-off between core-inflation and output gap stabilization, and between headline inflation and output gap stabilization (in $4a$ and $4b$, respectively). The trade off exist between core-inflation and output gap stabilization, only for $c_p > 0$, as for the case under discretion as can be seen from equation (35). The difference between the two optimal policies i.e. under discretion and under commitment is that the trade-offs are higher under discretion than under commitment as can be seen from the figure. The efficient frontier for the discretionary policy has a higher slope than the efficient frontier for the commitment policy for all arbitrary values of $\lambda_{\widetilde{C}}$ except when $\lambda_{\widetilde{C}} = 0$, in which case the two policies coincide. A higher slope would mean higher trade-offs, as for any given level of variance in the output gap a higher variance is inflation is seen for policy under discretion then commitment. A commitment policy will thus give lower minimum losses than the discretionary policy. Note that minimum losses possible are not zero but positive in the presence of a procurement inefficiency both under the discretionary policy as well as commitment policy.

The rate of interest rule for policy under commitment can be obtained by putting the optimal values of the price level, the output gap and the terms of trade gap in the DIS equation as,

$$
\begin{aligned}
\widehat{R}_t^* &= \widehat{r}_t^* + \omega_5 \left(\varkappa_1 - 1\right) \widehat{\widehat{P}}_{M,t} + \frac{\omega_5}{\varkappa_2 \beta} E_t \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z_{2,t+1+k}^* \\
&\quad - \frac{\sigma}{(1-\lambda_c)} (1-\lambda_c) E_t \left\{ z_{1,t+1}^* - z_{1,t}^* \right\} \qquad (37) \\
\text{where, } \widehat{\widehat{P}}_{M,t} &= \frac{1}{\varkappa_2 \beta} \sum_{j=0}^{t} \varkappa_1^j \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z_{2,t+k-j}^*
\end{aligned}
$$

The nominal rate of interest is a function of past, current and future shocks.

Although the discretionary and commitment rule in equation (34) and (37) , respectively, are desirable, they have certain disadvantages. First, these rules do not guarantee a unique equilibrium, as these depend on specific parameter values. Second, they are not easy to implement as they depend on current and future path of shocks which are not known to the policymaker precisely. These imprecisions can lead to large welfare losses. At best these optimal rules can be used as a benchmark for normative analysis. We therefore discuss some simple rules which are easy to implement in the next section and do comparative analysis.

4 Comparative analysis

Taylor (1999) discusses advantages of a class of simple rules over a class of optimal rules. In this section we will calibrate the model and compare five monetary policy rules namely, a discretion rule, a commitment rule, a simple Taylor rule *without* terms of trade gaps, a simple Taylor rule *with* terms of trade gaps, as shown in equation (34) , (37) , (30) and (29) , respectively, and an optimal simple rule. Here the optimal simple rule is a rule like equation (29) with value of coefficients, ϕ_R, ϕ_π, $\phi_{\widetilde{y}}$ and $\phi_{\widetilde{tam}}$, such that the welfare loss function is minimized.[48] We do these comparisons for a positive procurement shock, $\widehat{Y}_{PG,t}$, and a negative productivity shock, $\widehat{A}_{G,t}$ to the grain sector.

4.1 Procurement shock

We analyze the response of the economy to a one period positive procurement shock in the grain sector (s.d. 0.66) when the central bank follows five different monetary policy rules as discussed above. Table 2 shows the welfare loss, values of coefficients, and standard deviation of nominal rate of interest of the shock with different rules.

Rule	Welfare losses	ϕ_R	ϕ_π	$\phi_{\widetilde{y}}$	$\phi_{\widetilde{tam}}$	s.d.(R)
Simple Taylor rule						
without ToT*	3.914×10^{-3}	0.7	2	0.5	0	0.0110
with ToT	3.565×10^{-3}	0.7	2	0.5	0.864	0.0117
Optimal Monetary policy						
Discretion	1.196×10^{-3}	n.a.	n.a.	n.a.	n.a.	0.0055
Commitment	9.280×10^{-4}	n.a.	n.a.	n.a.	n.a.	0.0127
Optimal simple rule	3.090×10^{-3}	0.576	2.029	0.741	0.601	0.0116

*ToT refers to terms of trade gap

Table 2: Monetary policy rules for positive procurement shock

A simple Taylor rule *without* terms of trade gap gives highest welfare losses. The losses reduce by 9% when terms of trade gap is added to the simple Taylor rule and by 21% with optimal simple rule.[49] The optimal weight in front of $\widetilde{T}_{AM,t}$ in the optimal simple rule is positive and takes a value of 0.601. This means that sectoral terms of trade/ relative price

[48] To get the optimal simple rule, we do the numerical optimization to minimize welfare loss function in Dynare. To do this we initialize the value of parameters with the calibrated values, i.e. $\phi_R = 0.7$, $\phi_\pi = 2$, $\phi_{\widetilde{y}} = 0.5$ and $\phi_{\widetilde{tam}} = 0.864$.

[49] Here optimal simple rule is the optimized simple Taylor rule with terms of trade gap which minimizes the welfare loss function.

gaps in the simple Taylor rule does improve welfare outcomes. Among the optimal monetary policy rules, the commitment rule gives lowest welfare losses, followed by discretionary policy and then the optimal simple rule. Since the optimal simple rule gives the lowest welfare losses, it is best among the class of implementable rules considered here.

4.1.1 IRFs for a positive procurement shock

Figure 5*a* compares the IRFs for optimal monetary policy rules namely, discretion, commitment and optimal simple rule for one period positive procurement shock.

[INSERT FIGURE 5a]

On impact response to output gap and consumption gap is smallest under commitment than discretion or optimal policy rule. Also under commitment the response of nominal rate of interest is negative on impact, which is in contrast to other two policy responses. Due to this, consumption falls less and aggregate output increases further up. The inflation seems to be less persistent under commitment as the price level (both core sector and aggregate) comes back to its initial level in the long run, and remain permanently high under discretion. The optimal simple rule performs very well for most of the nominal variables like inflation (both aggregate and headline), price levels and terms of trade. In fact the price level converge very close to its initial values in long run, similar to commitment policy. On impact this rule does contracts the economy more than the other two optimal rules, but in the long run it performs very close to commitment policy. Figure 5*b* compares IRFs for implementable simple rules namely, simple Taylor rule, simple Taylor rules with terms of trade gaps and optimal simple rule for one period positive procurement shock.

[INSERT FIGURE 5b]

On impact, the simple Taylor rule response to the shock is insufficient to stabilize nominal variables like inflation (both core and headline inflation), terms of trade and price levels (which remain permanent high). On the other hand, the response of a simple Taylor rule with terms of trade gap is too aggressive, which deflates the economy such that the economy converges to a price level lower than its initial level. The optimal simple rule performs the best among all three implementable rules considered here in stabilizing inflation (both core and output) and price level as discussed earlier. Since the trade-off exists between inflation and output stabilization, we see that real variables response the least for simple Taylor rule. But as summarized in Table 2, the losses are 21% less in optimal simple rule as compared with simple Taylor and hence it is the best rule among considered implementable rules.

4.2 Productivity shock

We now analyze the response of the economy to a one period negative productivity shock in the grain sector (s.d. 0.03) when the central bank follows five different monetary policy rules as discussed above. We do this in two parts. In the first part we do away with the procurement inefficiency by putting $c_p = 0$; in the second part we analyze the policies in the presence of procurement with $c_p = 0.08$.

4.2.1 Without procurement inefficiency

We put $c_p = 0$ in this section, so that the results can be compared to any standard multi-sector model with a negative productivity shock. Table 3 shows the welfare loss, values of coefficients and standard deviation of the nominal rate of interest of the shock with different rules.[50]

Rule	Welfare losses	ϕ_R	ϕ_π	$\phi_{\widetilde{y}}$	$\phi_{\widetilde{tam}}$	s.d.(R)
Simple Taylor rule						
without ToT*	1.146×10^{-4}	0.7	2	0.5	0	0.0120
with ToT	7.630×10^{-5}	0.7	2	0.5	0.864	0.0109
Optimal Monetary policy						
Discretion	0	n.a.	n.a.	n.a.	n.a.	0.0040
Commitment	0	n.a.	n.a.	n.a.	n.a.	0.0040
Optimal simple rule	3.873×10^{-5}	1.240	1.792	0.568	1.005	0.0085

*ToT refers to terms of trade gap

Table 3: Monetary policy rules for negative productivity shock with no procurement

A simple Taylor rule without a terms of trade gap gives highest welfare losses. The losses reduce by 33% when a terms of trade gap is added to the simple Taylor rule and by 66% with the optimal simple rule. The optimal weight in front of $\widetilde{T}_{AM,t}$ in the optimal simple rule is positive and takes a value of 1.005, which is higher than our calibrated value of 0.864. This means that sectoral terms of trade/ relative price gaps in the simple Taylor rule improves welfare outcome. Among the optimal monetary policy rules, the discretion and commitment policy are the same as both policies completely stabilize the core-inflation, output gap and the terms of trade gap, i.e. $\pi_{M,t} = \widetilde{Y}_t = \widetilde{C}_t = \widetilde{T}_{AM,t} = 0$. Note that there are no trade-offs between stabilizing core-inflation and output gap when, $c_p = 0$. Optimal simple rule although

[50] Any values of losses less than 10^{-30} are put a zero.

performs worst among the optimal rules but is best among the considered implementable rules.

4.2.2 IRFs for a negative productivity shock without procurement inefficiency

Figure 6a compare the IRFs for optimal monetary policy rules namely, discretion, commitment and the optimal simple rule for one period negative productivity shock.

[INSERT FIGURE 6a]

The IRFs show that the discretion and commitment policies give the same response for all the variables in the economy, as explained above. The core-inflation, output gap and terms of trade gap are all zero under discretion and commitment policy rules, as there is no trade-off. The price level return to its original levels in these two policies. The optimal simple rule on the other hand performs well for the aggregate inflation and the aggregate price level, but poorly for core sector inflation rate and the price level. On impact an optimal simple rule also contracts the economy more than the other two optimal rules.

Figure 6b compare the IRFs for implementable simple rules namely, simple Taylor rule, a simple Taylor rules with terms of trade gaps and an optimal simple rule for one period negative productivity shock.

[INSERT FIGURE 6b]

The response of most variables seem similar for all three rules on impact except, core-inflation and price levels (both aggregate as well as core-sector) where optimal simple rule performs better. Under an optimal simple rule core inflation is strictly less for all periods and prices deviate less from the steady state in the long run. Between the second and fourth quarter, the output gap, consumption gap and terms of trade gaps are more stable. Overall the optimal simple rule performs the best by reducing losses upto 66% as compared to a simple Taylor rule.

4.2.3 With procurement inefficiency

We put $c_p = 0.08$, as calibrated for Indian economy, in this section. Table 3 shows the welfare loss, values of coefficients and standard deviation of nominal rate of interest of the shock with different rules.

Rule	Welfare losses	ϕ_R	ϕ_π	$\phi_{\widetilde{y}}$	$\phi_{\widetilde{tam}}$	s.d.(R)
Simple Taylor rule						
without ToT*	1.278×10^{-4}	0.7	2	0.5	0	0.0125
with ToT	8.142×10^{-5}	0.7	2	0.5	0.864	0.0114
Optimal Monetary policy						
Discretion	1.640×10^{-5}	n.a.	n.a.	n.a.	n.a.	0.0042
Commitment	1.292×10^{-6}	n.a.	n.a.	n.a.	n.a.	0.0036
Optimal simple rule	4.879×10^{-5}	1.153	1.8	0.548	1.001	0.0092

*ToT refers to terms of trade gap

Table 4: Monetary policy rules for negative productivity shock with procurement

As expected the welfare losses under *all* five rules are higher in the presence of procurement inefficiency as compared to Table 3. A simple Taylor rule without terms of trade gap gives the highest welfare losses here too. The losses reduce by 36% when a terms of trade gap is added to the simple Taylor rule and by 62% with the optimal simple rule. The optimal weight in front of $\widetilde{T}_{AM,t}$ in the optimal simple is positive and takes a value of 1.001, which is higher than the calibrated value of 0.864. This means that the sectoral terms of trade/ relative price gaps in the simple Taylor rule improves welfare outcome. With procurement, the welfare losses are positive under discretion and commitment, as a trade-off now exists and minimum values of $\pi_{M,t}, \widetilde{Y}_t^*, \widetilde{C}_t^*, \widetilde{T}_{AM,t}^*$ are not zero. Among the optimal monetary policy rules, the commitment rule gives lowest welfare losses, followed by discretionary policy and then the optimal simple rule.

4.2.4 IRFs for a negative productivity shock with procurement inefficiency

Figure 7*b* compare the IRFs for optimal monetary policy rules namely, discretion, commitment and optimal simple rule for one period negative productivity shock.

[INSERT FIGURE 7a]

The IRFs show that with the procurement inefficiency, discretion and commitment policies do not give same response for all the variables in the economy, specially the price levels (both aggregate and core-sector). Moreover, the core-inflation, output gap and terms of trade gap are not zero under discretion and commitment policy rules, as there is a trade-off. Between second to fourth quarter they become more stable for commitment policy. The optimal simple rule on the other hand performs well for the aggregate inflation and the aggregate price level, but poorly for core sector inflation rate and the price level. On impact, the optimal simple rule also contracts the economy more than the other two optimal rules.

Figure 7*b* compares the IRFs for implementable simple rules namely, a simple Taylor rule, simple Taylor rules with terms of trade gaps and an optimal simple rule for one period negative productivity shock.

[INSERT FIGURE 7b]

The graphs in Figure 7*b* are not qualitatively different from graphs in Figure 6*b*, although the presence of procurement affects the values of the variables. The output gap and consumption gap are higher in the presence of procurement for all time periods.[51] The response of most of the variables seems similar on impact except, core-inflation and price levels (both aggregate as well as core-sector) where optimal simple rule performs better. Under the optimal simple rule core inflation is strictly less for all periods and prices deviate less from the steady state values in the long run. Between the second and fourth quarter, the output gap, consumption gap and terms of trade gaps are more stable. Overall optimal simple rule performs the best among the rules considered by reducing losses upto 62% as compared to a simple Taylor rule.

5 Conclusion

Our paper contributes to a growing literature on monetary policy for India and other EMDEs. Most of the literature in monetary policy setting for developing countries focusses on the optimal inflation index that should be targeted to bring the economy close to the flexible-price equilibrium. The real disturbances which can be a source of inefficient shocks to these economies, and possibly bring trade-offs between inflation and output gap stabilization for central banks, have not been studied much. In this paper, we identify market price support present in the agriculture sectors of the EMDEs as a real disturbance leading to such trade-offs. In particular, we study market price support in the form of a government induced procurement policy in Indian economy, as a source of inefficient shocks. We derive the welfare loss function of central banks and characterize optimal monetary policy under discretion and commitment. We show that the presence of procurement induces trade-offs between core-inflation and output gap stabilization, and between headline inflation and output gap stabilization under both discretion and commitment rule. This result is a departure from the existing popular view point that strict core-inflation targeting is the optimal monetary policy for developing countries. This implies that central banks in developing countries need more caution while setting their monetary policy, as the inefficiencies in the real sector of their economy can modify standard results and alter the policy response. Among the class of

[51] For the other variables the effect is small and is not visible on the graphs.

monetary policy rules considered for comparison, the commitment rule is the best rule with the least welfare losses. Among the implementable rules it is seen that an optimal simple rule with terms of trade gap as one of the target variables (besides aggregate inflation and the output gap) reduces welfare losses significantly. As compared to a simple Taylor rule *without* terms of trade gaps, the optimal simple rule *with* terms of trade gap reduces welfare losses by 21% and 62% for a positive procurement shock and a negative productivity shock, respectively. Thus, a simple rule with terms of trade/ relative price gaps can be used by central banks in EMDEs to improve welfare outcomes.

References

[1] Acemoglu, D. & Robinson, J. (2001). Inefficient Redistribution. *American Political Science Review, 95*(3), 649-661.

[2] Anand, R., Prasad, E., & Zhang B. (2015). What measure of inflation should a developing country central bank target? *Journal of Monetary Economics, 74,* 102-116.

[3] Anand, R. Kumar, N., & Tulin V. (2016). *Understanding India's Food Inflation: The Role of Demand and Supply Factors.* IMF Working Paper, 02/2016. Asia and Pacific Department.

[4] Anderson, K., & Hayami, Y. (1986). *The Political Economy of Agricultural Protection: East Asia in International Perspective.* Sydney and London: Allen & Unwin.

[5] Aoki, K. (2001). Optimal monetary policy responses to relative price changes. *Journal of Monetary Economics, 48* (1), 55-80.

[6] Benjamin N. & Talab, I. (2011) . Agricultural Distortions, Structural Change and Economic Growth: A Cross-country Analysis. *American Journal of Agricultural Economics, 93*(3): 885–905.

[7] Basu, K. (2011). India's Food grain Policy: An Economic Theory Perspective. *Economic & Political Weekly, Special Issue, 46* (5), 37-45.

[8] Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of International Economics, 63* (2), 293-320.

[9] Bhattarai, S., Eggertsson, G., & Schoenle R. (2014). *Is increased price flexibility stabilizing? Redux.* NBER Working Paper 19886.

[10] Census of India (2011). Rural-urban distribution of population. Ministry of Home Affairs.

[11] Cuevas, A. & Topak S. (2008). *Monetary Policy and Relative Price Shocks in South Africa and Other Inflation Targeters.* IMF Working Paper, 289/2008. Washington, DC: International Monetary Fund.

[12] Dev, M., & Rao, N. C. (2015). Improved Terms of Trade for Agriculture. Results from revised methodology. *Economic & Political Weekly, Commentary, 50* (15), 19-22.

[13] Dewbre, J., Anton, J., & Thompson W. (2001). The Transfer Efficiency and Trade Effects of Direct Payments, *American Journal of Agricultural Economics*, 83(5):1204-1214.

[14] Erceg, C., & Levin, A. (2006). Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics, 53*(7), 1341-1359.

[15] FAO (Food and Agriculture Organization) *Macro Indicators database.* (Link: http://www.fao.org/faostat/en/#data/MK) Accessed in June, 2017.

[16] Fuhrer, J. C. (1997). Inflation/output variance trade-offs and optimal monetary policy. *Journal of Money, Credit and Banking 29*(2), 214-234.

[17] Gali, J., & Monacelli T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies, 72* (3), 707–734.

[18] Gali, J., Gertler, M., & Lopez-Salido, J. D. (2007). Markups, Gaps, and the Welfare Costs of Business Fluctuations. The Review of Economics and Statistics, 89(1), 4459.

[19] Ghate, C., Gupta, S., & Mallick D. (2018). Terms of Trade Shocks and Monetary Policy in India. *Computational Economics 51*(1), 75-121.

[20] Ghate, C. & Kletzer K. M. ed., (2016). *Monetary Policy in India: A Modern Macroeconomic Perspective.* Springer India.

[21] Gilchrist, S., Ortiz, A., & Zakrajsek, E. (2009). Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model. Federal Reserve Financial Market and Monetary Policy Conference.

[22] Hammond, G., Kanbur, R., & Prasad E. ed., (2009). *Monetary Policy Frameworks for Emerging Markets.* Edward Elgar Publishing Limited.

[23] Huang, K. X. D., & Liu, Z. (2005). Inflation targeting: what inflation rate to target? *Journal of Monetary Economics, 52* (8), 1435-1462.

[24] Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2013). Is There a Trade-Off between Inflation and Output Stabilization? American Economic Journal: Macroeconomics, 5(2), 131.

[25] Levine, P., Vasco, G. J., Pearlman, J., & Yang, B. (2012). An estimated DSGE model of the Indian economy. In C. Ghate (Ed.), *The Oxford handbook of the Indian economy* (pp. 835–890). New York: Oxford University Press.

[26] National Sample Survey Report 68^{th} Round. June (2013) Key indicators of household consumer expenditure in India. Ministry of Statistics and Programme Implementation, National Sample Survey Office. Report No. 558 (68/1.0/2).

[27] OECD(2009) *Agricultural Policies in Emerging Economies: Monitoring and Evaluation,* OECD publishing, Paris. DOI: http://dx.doi.org/10.1787/agr_emerging-2009-en

[28] OECD (2016*a*) *Producer and Consumer Support Estimates*, OECD Agriculture statistics (database). DOI: dx.doi.org/10.1787/agr-pcse-data-en (Accessed on 16 June, 2017)

[29] OECD(2016*b*) *Agricultural Policy Monitoring and Evaluation 2016*, OECD publishing, Paris. DOI: http://dx.doi.org/10.1787/agr_pol-2016-en

[30] Ramaswamy, B., Seshadri, S., & Subramanian, K.V. (2014). A Framework for Analysing Food Policy in India. Mimeo. Indian Statistical Institute, Delhi Centre.

[31] Rotemberg, J.J., Woodford, M., (1997). An optimization-based econometric framework for the evaluation of monetary policy. In: Bernanke, B.S., Rotemberg, J.J. (Eds.), *NBER Macroeconomic Annual.* MIT Press, Cambridge, pp. 297–346.

[32] Rotemberg, J., & Woodford M. (1999). Interest Rate Rules in an Estimated Sticky Price Model. In J. B. Taylor (ed.), *Monetary Policy Rules,* University of Chicago Press, Chicago, IL.

[33] Sharma, V.P., & Alagh, M. (2013). Food Subsidy in India: Its Components, Trends, Causes and Reforms for Public Policy. *Indian Journal of Agricultural Economics*, 68(2).

[34] Taylor, J. (1993). Discretion versus policy rules in practice. *Carnegie–Rochester Conference Series on Public Policy*, 39, 195–214.

[35] Taylor, J. (1999). *Monetary Policy Rules.* University of Chicago Press: Chicago, IL.

[36] Timmer, C.P. (1989). Food price policy: the rationale for government intervention. *Food Policy, 14*, 17-42.

[37] WEO (World Economic Outlook), 2016. *Subdued Demand: Symptoms and Remedies, October, 2016.* (Link: http://www.imf.org/external/pubs/ft/weo/2016/02/)

[38] Woodford, M. (1999). *Optimal Monetary Policy Inertia*, NBER Working Paper No. 7261.

[39] Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Figures

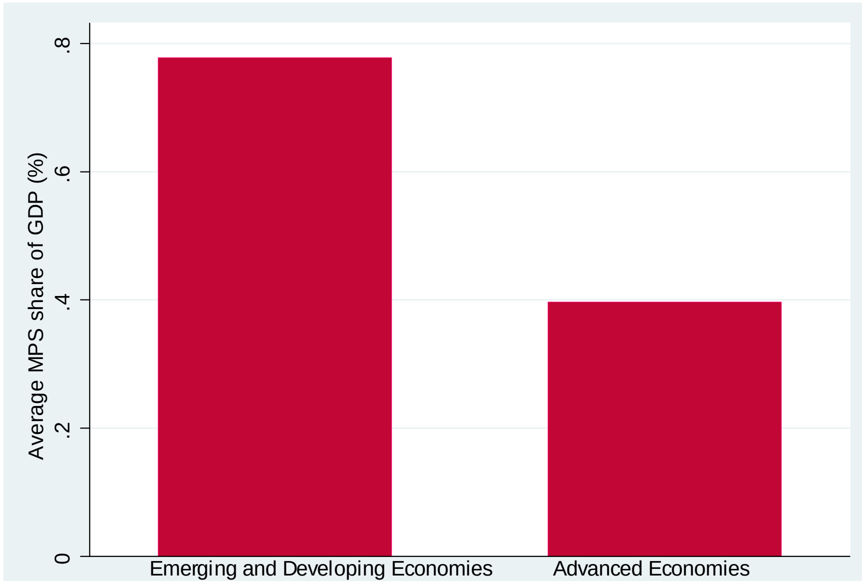

Figure 1a: Agricultural Market Price Support as a share of GDP between 2011-15

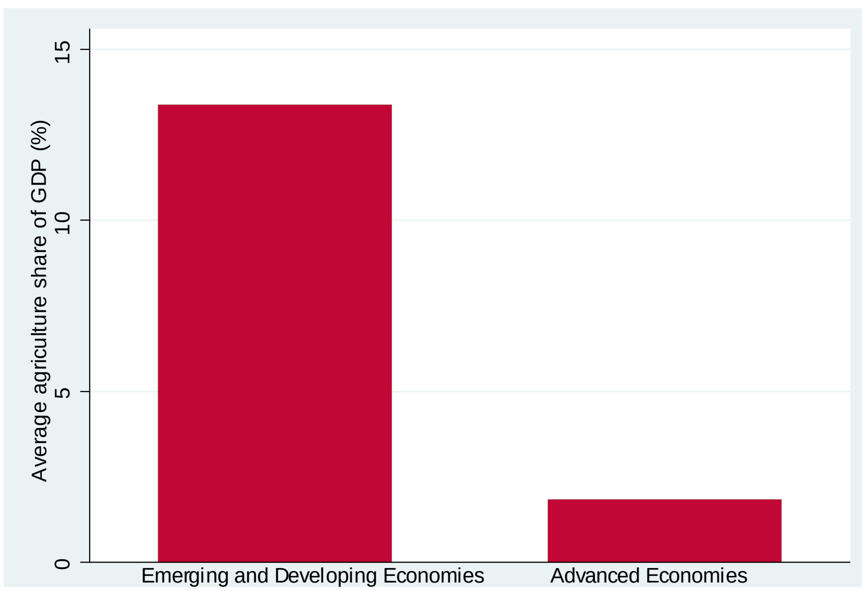

Figure 1b: Value of agriculture sector as a share of GDP between 2011-15

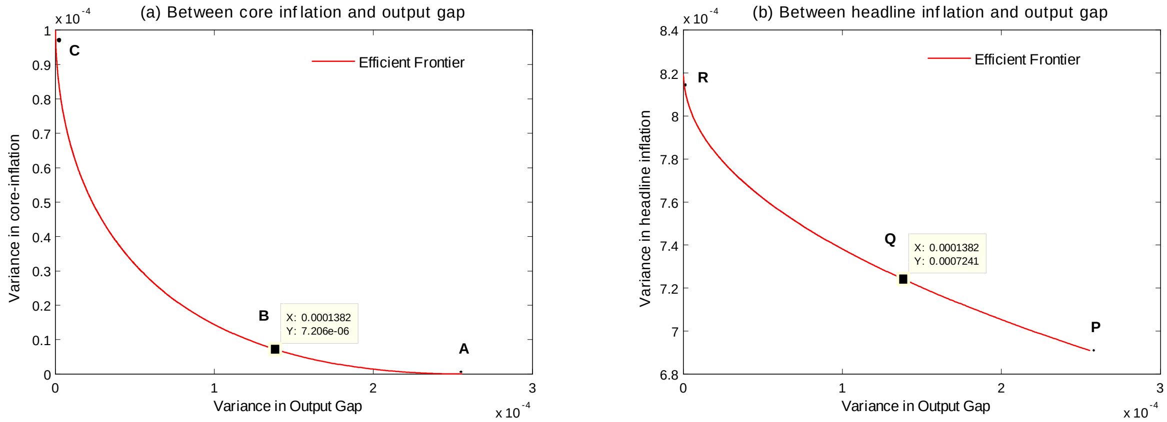

Figure 2: Trade-off between inflation and output stabilization

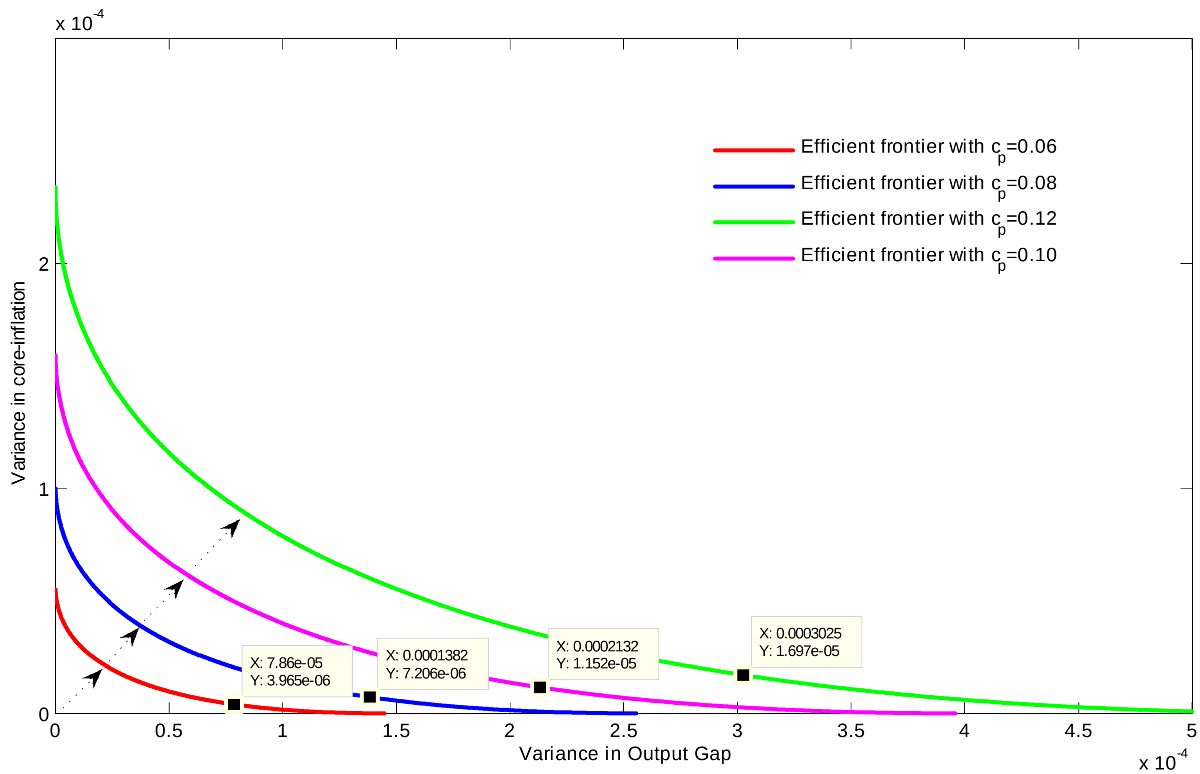

Figure 3: Trade-offs and varying procurement

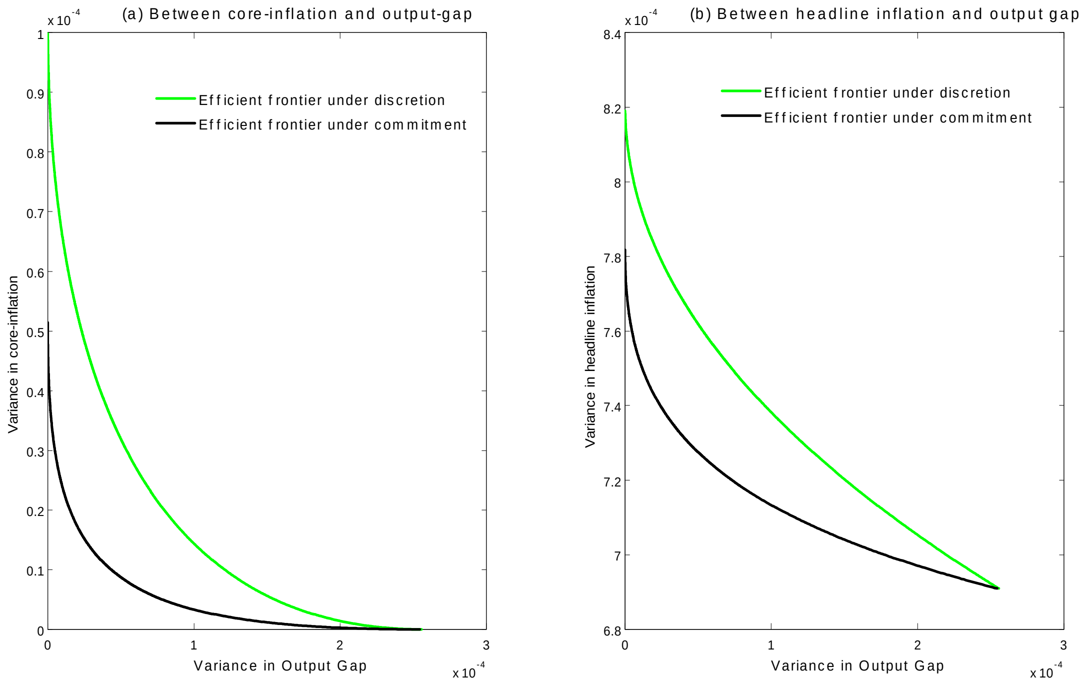

Figure 4: Trade-offs: Discretion Vs Commitment

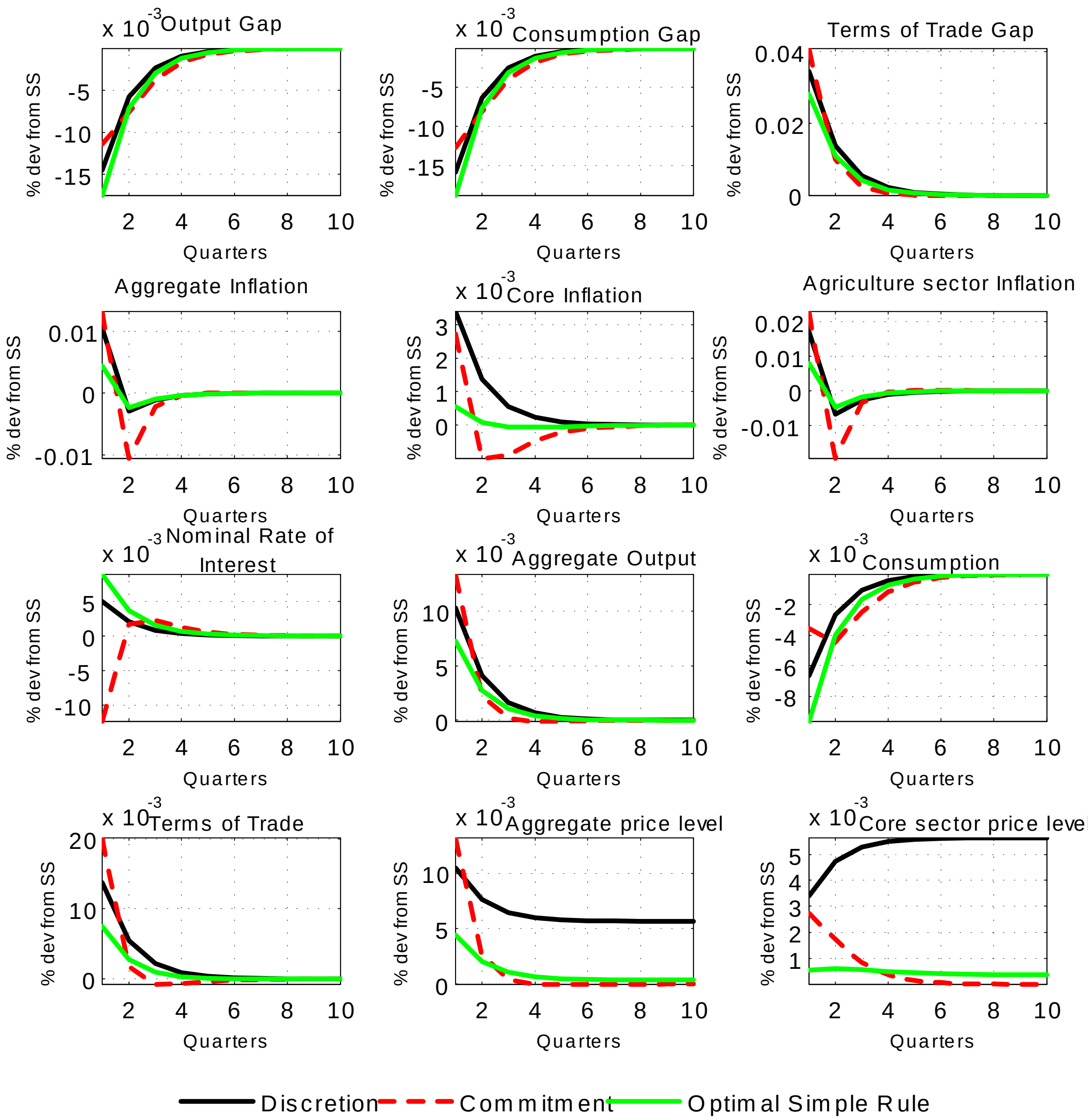

Figure 5a: IRFs comparing optimal monetary policies for procurement shock

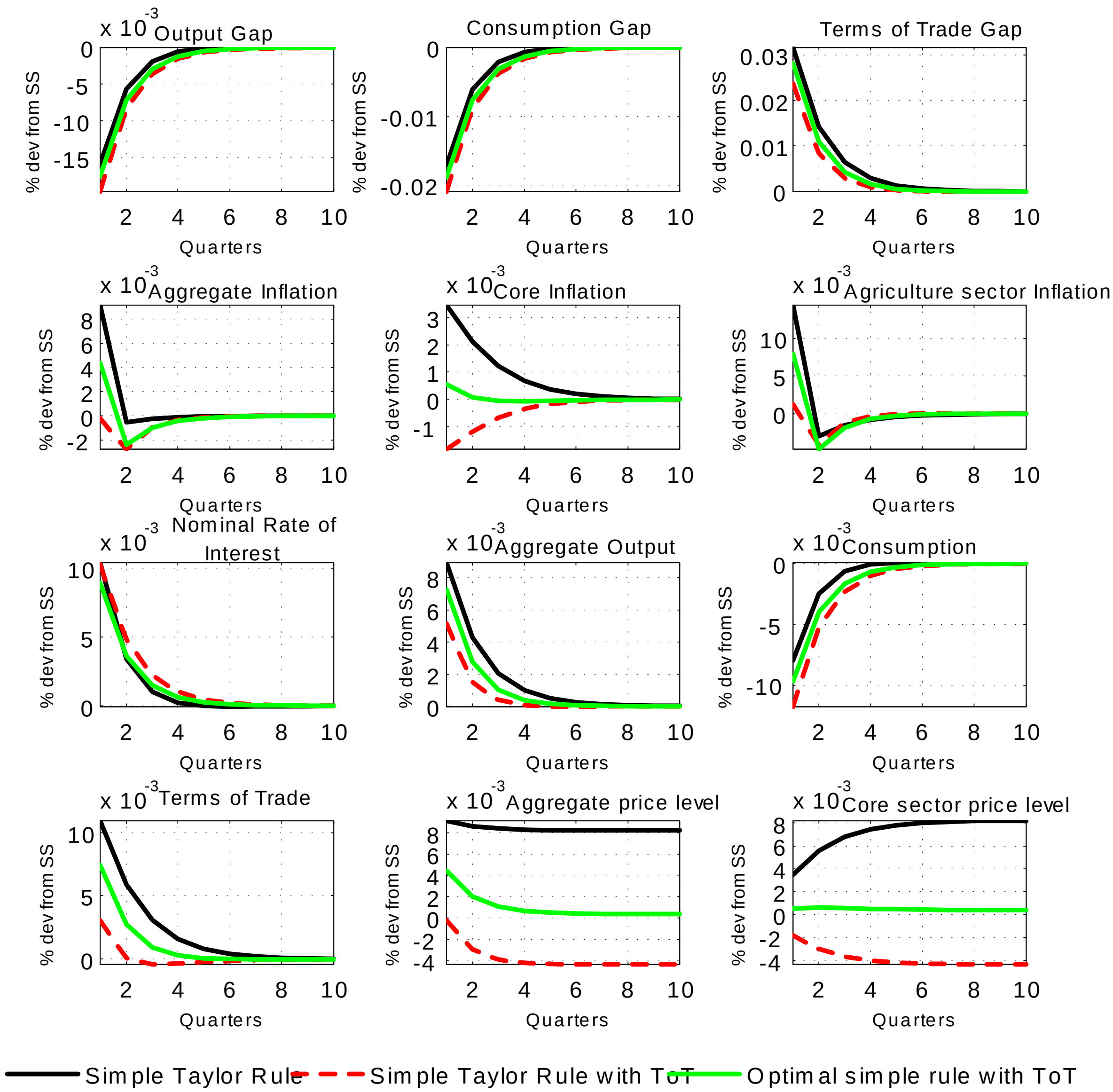

Figure 5b: IRFs comparing optimal simple rule and simple modified Taylor rules for procurement shock

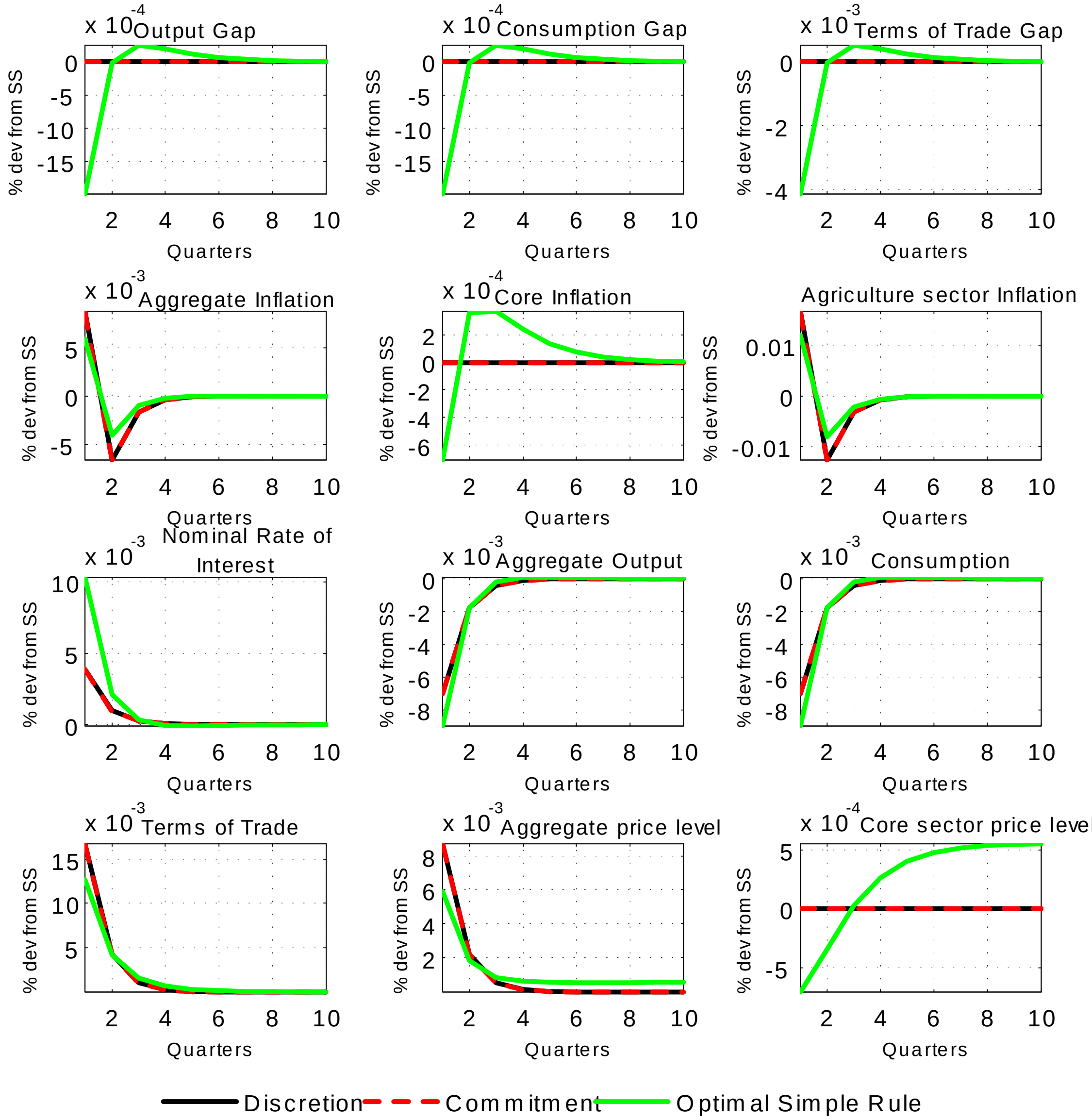

Figure 6a: IRFs comparing optimal monetary policies for productivity shock without procurement distortion

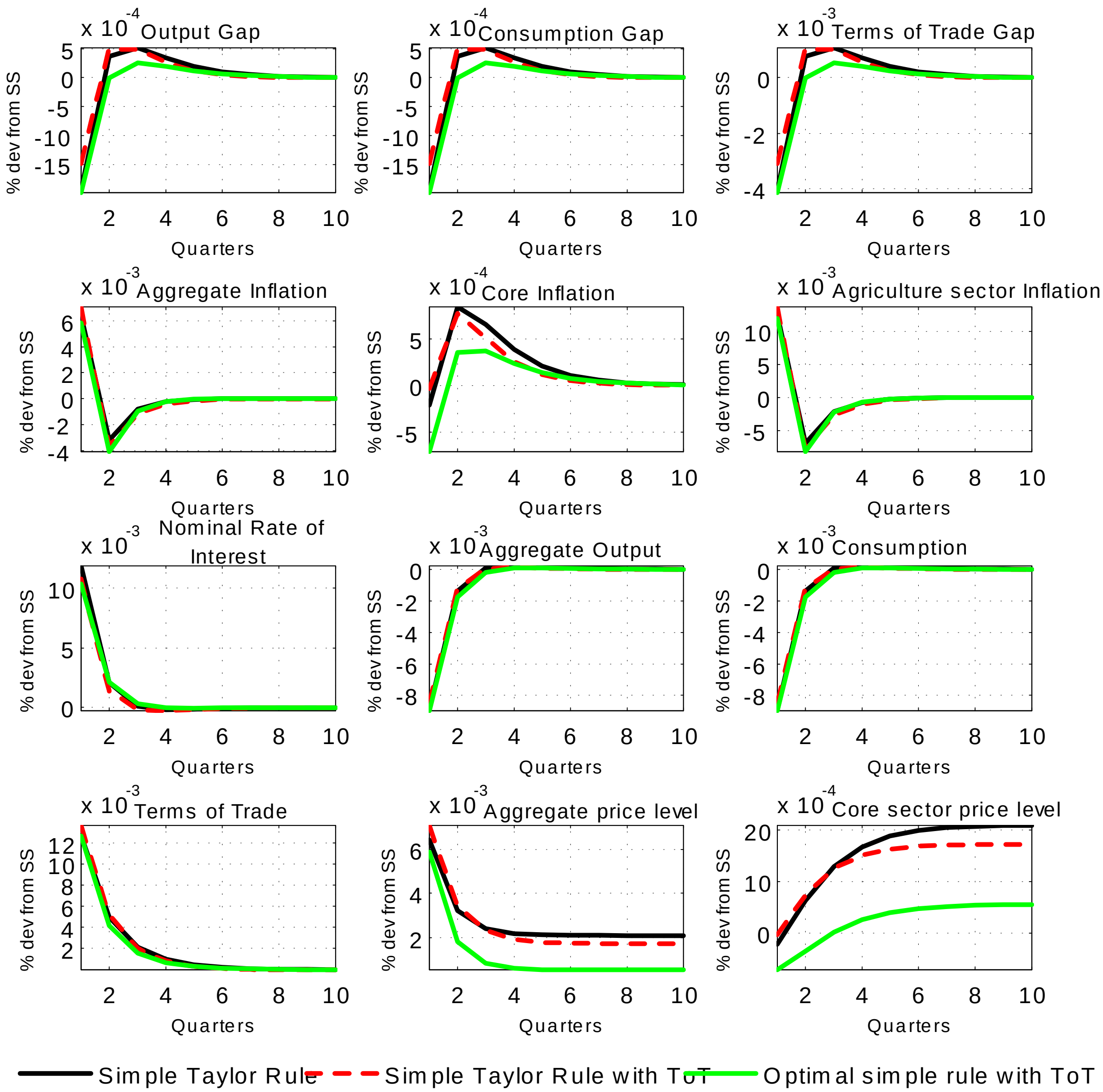

Figure 6b: IRFs comparing optimal simple rule and simple modified Taylor rules for productivity shock without procurement distortion

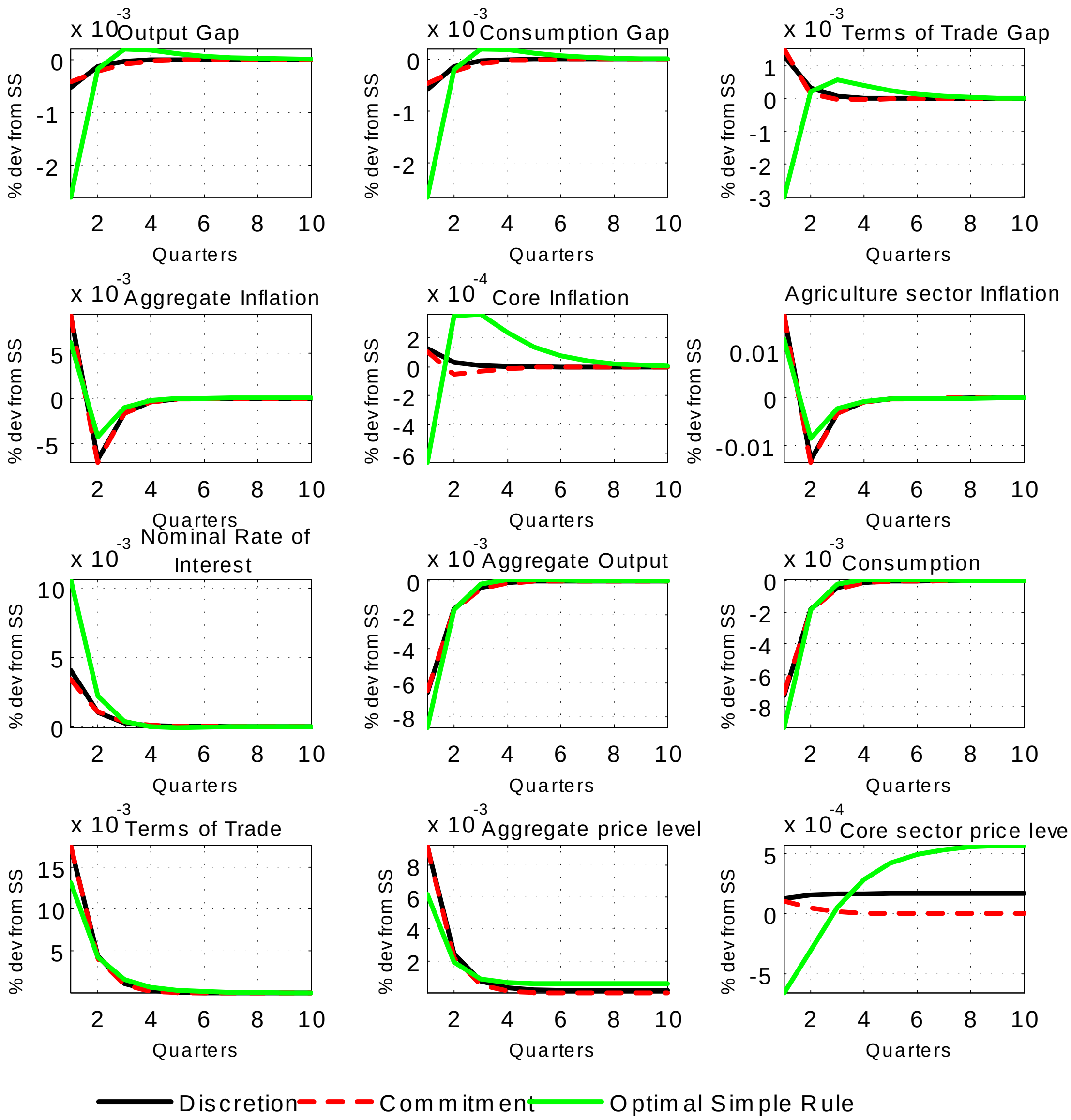

Figure 7a: IRFs comparing optimal monetary policies for productivity shock with procurement distortion

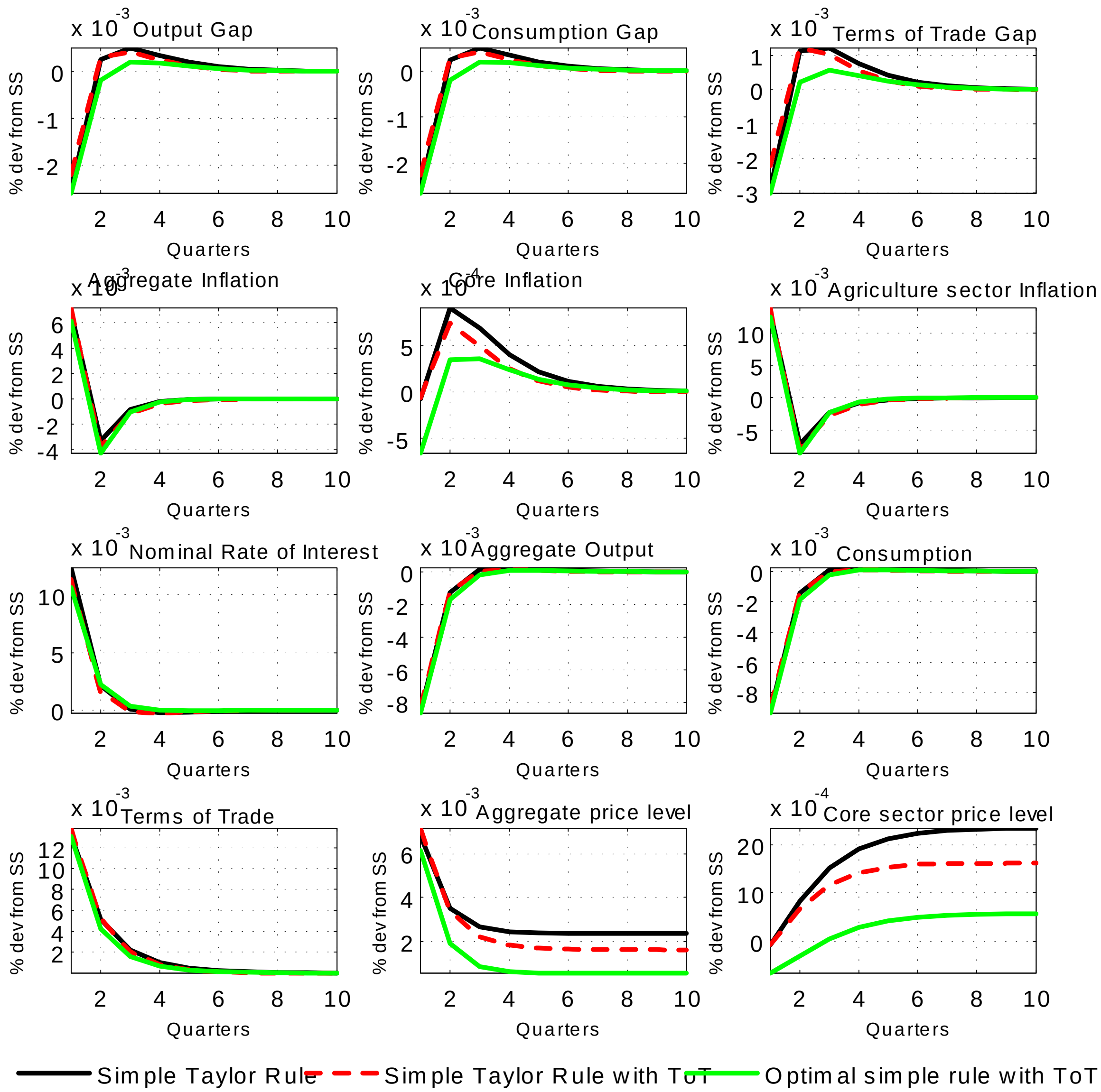

Figure 7b: IRFs comparing optimal simple rule and simple modified Taylor rules for productivity shock with procurement distortion

Technical Appendix

Part A. Derivation of welfare loss function

The average utility flow at time t, is defined as

$$w_t = U(C_t) - \int_0^1 v(N_t(i))di \tag{38}$$

where $U(C_t)$ is the utility from the aggregate consumption bundle C_t and $v(N_t(i))$ is the disutility of supplying labor $N_t(i)$ by the i^{th} household. The welfare function would then become,

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C \; C} \right) \tag{39}$$

Alternatively, the welfare loss function would become

$$W = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C \; C} \right) \tag{40}$$

We take second order approximation to the $U(C_t,)$,

$$U(C_t,) \approx U_c \; C \left(\frac{C_t - C}{C} \right) + U_{cc} \; C^2 \left(\frac{C_t - C}{C} \right)^2$$

using $\frac{Z_t - Z}{Z} \approx \widehat{Z}_t + \frac{1}{2}\widehat{Z}_t^2$ where $\widehat{Z}_t = \ln Z_t - \ln Z$

$$U(C_t,) \approx U_c \; C \left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 \right)^2$$

$$U(C_t,) \approx U_c \; C \left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \widehat{C}_t^2 + \|O\|^3$$

using $\sigma = -\frac{U_{cc}}{U_c} C$

$$U(C_t,) \approx U_c \; C \left[\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2 \right] + \|O\|^3 \tag{41}$$

Now we take the second order approximation to $v(N_t(i))$. An i^{th} household supplies labor to three sectors, i.e. grain (G), vegetable (V), manufacturing (M)

$$v(N_t(i)) = v\left(N_t^V(i)\right) + v(N_{G,t}(i)) + v(N_{M,t}(i))$$

Now $v(N_{V,t}(i))$ can be rewritten as $V(Y_{V,t}(i), A_{V,t})$, since $Y_{V,t}(i) = A_{V,t}N_{V,t}(i)$. Similarly $v(N_{M,t}(i))$ and $v(N_{G,t}(i))$ can be rewritten as $V(Y_{M,t}(i), A_{M,t})$ and $V\left(Y_t^{OG}(i), Y_{PG,t}, A_{G,t}\right)$ respectively. Consider second order approximation to $v\left(N_t^V(i)\right)$, since $Y_{V,t}(i) = A_{V,t}N_t^V(i)$, $v\left(N_t^V(i)\right) = V(Y_{V,t}(i), A_{V,t})$,

$$\begin{aligned}
V(Y_{V,t}(i), A_{V,t}) \approx\ & V(Y_V, A_V) + V_Y(Y_{V,t}(i) - Y_V) + V_A(A_{V,t} - A_V) + \frac{V_{AA}}{2}(A_{V,t} - A_V)^2 \\
& + V_{YA}(Y_{V,t}(i) - Y_V)(A_{V,t} - A_V) + \frac{V_{YY}}{2}(Y_{V,t}(i) - Y_V)^2 + \|O\|^3
\end{aligned}$$

$$\begin{aligned}
V(Y_{V,t}(i), A_{V,t}) \approx\ & V_Y Y_V\left(\widehat{Y}_{V,t}(i) + \frac{1}{2}\left(\widehat{Y}_{V,t}(i)\right)^2\right) + V_A A_V\left(\widehat{A}_{V,t} + \frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right) \\
& + V_{YA}Y_V A_V\left(\widehat{Y}_{V,t}(i) + \frac{1}{2}\left(\widehat{Y}_{V,t}(i)\right)^2\right)\left(\widehat{A}_{V,t} + \frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right) \\
& + \frac{V_{YY}}{2}Y_V Y_V\left(\widehat{Y}_{V,t}(i) + \frac{1}{2}\left(\widehat{Y}_{V,t}(i)\right)^2\right)^2 \\
& + \frac{V_{AA}}{2}A_V A_V\left(\widehat{A}_{V,t} + \frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right)^2 + \|O\|^3 + t.i.p.
\end{aligned}$$

$$\begin{aligned}
V(Y_{V,t}(i), A_{V,t}) \approx\ & V_Y Y_V\left(\widehat{Y}_{V,t}(i) + \frac{1}{2}\left(\widehat{Y}_{V,t}(i)\right)^2\right) + V_{YA}Y_V A_V\left(\widehat{Y}_{V,t}(i)\,\widehat{A}_{V,t}\right) \\
& + \frac{V_{YY}}{2}Y_V Y_V\left(\widehat{Y}_{V,t}(i)\right)^2 + \|O\|^3 + t.i.p.
\end{aligned}$$

Assuming the steady state to shocks is 1, i.e. $A_V = A_G = A_M = 1$ and let $g_{V,t} = -\frac{V_{YA}\widehat{A}_{V,t}}{V_{YY}Y_V}$

$$\begin{aligned}
V(Y_{V,t}(i), A_{V,t}) \approx\ & V_Y Y_V\left(\widehat{Y}_{V,t}(i) + \frac{1}{2}\left(\widehat{Y}_{V,t}(i)\right)^2\right) - g_{V,t}V_{YY}Y_V Y_V\left(\widehat{Y}_{V,t}(i)\right) \\
& + \frac{V_{YY}}{2}Y_V Y_V\left(\widehat{Y}_{V,t}(i)\right)^2 + \|O\|^3 + t.i.p.
\end{aligned}$$

Using $V_{YY} = \psi\frac{V_Y}{Y_V}$

$$V(Y_{V,t}(i), A_{V,t}) \approx V_Y Y_V\left[\widehat{Y}_{V,t}(i) - \psi g_{V,t}\left(\widehat{Y}_{V,t}(i)\right) + \left(\frac{\psi + 1}{2}\right)\left(\widehat{Y}_{V,t}(i)\right)^2\right] + \|O\|^3 + t.i.p. \tag{42}$$

Similarly for the manufacturing sector,

$$V\left(Y_{M,t}(i), A_{M,t}\right) \approx V_Y Y_M \left[\widehat{Y}_{M,t}(i) - \psi g_{M,t}\left(\widehat{Y}_{M,t}(i)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{M,t}(i)\right)^2\right] + \|O\|^3 + t.i.p. \tag{43}$$

where $g_{M,t} = -\frac{V_{YA}\widehat{A}_{M,t}}{V_{YY}Y_M}$. For the grain sector, consider second order approximation to $v\left(N_{G,t}(i)\right)$,

$$\text{since } Y_{G,t}(i) = Y_{OG}(i) + Y_{PG,t} = A_{G,t}N_{G,t}(i)$$

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V\left(Y_{OG}, Y_{PG,t}, A_G\right) + V_Y\left(Y_{OG,t}(i) - Y_{OG}\right) + V_Y\left(Y_{PG,t} - Y_{PG}\right) \\ & + V_A\left(A_{G,t} - A_G\right) + V_{YA}\left(Y_{OG,t}(i) - Y_{OG}\right)\left(A_{G,t} - A_G\right) \\ & + V_{YA}\left(Y_{PG,t} - Y_{PG,t}\right)\left(A_{G,t} - A_G\right) + \frac{V_{AA}}{2}\left(A_{G,t} - A_G\right)^2 \\ & + V_{YY}\left(Y_{OG,t}(i) - Y_{OG}\right)\left(Y_{PG,t} - Y_{PG,t}\right) + \frac{V_{YY}}{2}\left(Y_{OG,t}(i) - Y_{OG}\right)^2 \\ & + \frac{V_{YY}}{2}\left(Y_{PG,t} - Y_{PG,t}\right)^2 + \|O\|^3 \end{aligned}$$

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i) + \frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right) + \frac{V_{YY}}{2} Y_{OG} Y_{OG}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\ & + V_{YA} Y_{OG} A_G\left(\widehat{Y}_{OG,t}(i)\,\widehat{A}_{G,t}\right) + V_{YY} Y_{OG} Y_{PG}\left(\widehat{Y}_{OG,t}(i)\,\widehat{Y}_{PG,t}\right) \\ & + \|O\|^3 + t.i.p. \end{aligned}$$

Assuming the steady state to shocks is 1, i.e. $A_V = A_G = A_M = 1$ and let $g_{OG,t} = -\frac{V_{YA}\widehat{A}_{G,t}}{V_{YY}Y_{OG}}$ and $g_{PG,t} = -\widehat{Y}_{PG,t}$

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i) + \frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right) + \frac{V_{YY}}{2} Y_{OG} Y_{OG}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\ & - g_{OG,t} V_{YY} Y_{OG} Y_{OG} \widehat{Y}_{OG,t}(i) - g_{PG,t} V_{YY} Y_{OG} Y_{PG} \widehat{Y}_{OG,t}(i) + \|O\|^3 + t.i.p. \end{aligned}$$

Using $V_{YY} = \psi\frac{V_Y}{Y_G} = \psi\frac{V_Y}{Y_{OG}+Y_{PG,t}}$

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}[\widehat{Y}_{OG,t}(i) + \frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2 + \psi\frac{Y_{OG}}{2\left(Y_{OG} + Y_{PG,t}\right)}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\ & - g_{OG,t}\psi\frac{Y_{OG}}{Y_{OG} + Y_{PG}}\widehat{Y}_{OG,t}(i) - g_{PG,t}\psi\frac{Y_{PG}}{Y_{OG} + Y_{PG}}\widehat{Y}_{OG,t}(i)] + \|O\|^3 + t.i.p. \end{aligned}$$

Since $c_p = \frac{Y_{PG}}{Y_{PG}+Y_{OG}}$,

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) &\approx V_Y Y_{OG}[\widehat{Y}_{OG,t}\left(i\right) + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left(\widehat{Y}_{OG,t}\left(i\right)\right)^2 \\ &\quad -\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\left(i\right)] + \left\|O\right\|^3 + t.i.p. \end{aligned} \tag{44}$$

Therefore,

$$V\left(N_t\left(i\right)\right) = V\left(Y_{V,t}(i), A_{V,t}\right) + V\left(Y_{M,t}(i), A_{M,t}\right) + V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right)$$

In the second order,

$$v\left(N_t\left(i\right)\right) \approx (42) + (43) + (44)$$

$$\begin{aligned} v\left(N_t\left(i\right)\right) &\approx V_Y Y_V\left[\widehat{Y}_{V,t}\left(i\right) - \psi g_{V,t}\left(\widehat{Y}_{V,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right] \\ &\quad + V_Y Y_M\left[\widehat{Y}_{M,t}\left(i\right) - \psi g_{M,t}\left(\widehat{Y}_{M,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{M,t}\left(i\right)\right)^2\right] \\ &\quad + V_Y Y_{OG}\left[\widehat{Y}_{OG,t}\left(i\right) + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left(\widehat{Y}_{OG,t}\left(i\right)\right)^2\right. \\ &\quad \left. -\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\left(i\right)\right] \\ &\quad + \left\|O\right\|^3 + t.i.p. \end{aligned}$$

Aggregating disutility over all households,

$$\begin{aligned} \int_0^1 v\left(N_t\left(i\right)\right)\, di &\approx V_Y Y_V\left[E_i\left\{\widehat{Y}_{V,t}\left(i\right)\right\} - \psi g_{V,t} E_i\left\{\widehat{Y}_{V,t}\left(i\right)\right\} + \left(\frac{\psi+1}{2}\right) E_i\left\{\widehat{Y}_{V,t}\left(i\right)^2\right\}\right] \\ &\quad + V_Y Y_M\left[E_i\left\{\widehat{Y}_{M,t}\left(i\right)\right\} - \psi g_{M,t} E_i\left\{\widehat{Y}_{M,t}\left(i\right)\right\} + \left(\frac{\psi+1}{2}\right) E_i\left\{\widehat{Y}_{M,t}\left(i\right)^2\right\}\right] \\ &\quad + \left[V_Y Y_{OG} E_i\left\{\widehat{Y}_{OG,t}\left(i\right)\right\} + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right) E_i\left\{\widehat{Y}_{OG,t}\left(i\right)^2\right\}\right. \\ &\quad \left. -\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right) E_i\left\{\widehat{Y}_{OG,t}\left(i\right)\right\}\right] \\ &\quad + \left\|O\right\|^3 + t.i.p. \end{aligned}$$

Since $Var\left(X\right) = E\left(X^2\right) - \left(E\left(X\right)\right)^2$

$$\begin{aligned}
\int_0^1 v\left(N_t\left(i\right)\right)\ di &\approx V_Y Y_V \left[\left(1-\psi g_{V,t}\right) E_i\left\{\widehat{Y}_{V,t}\left(i\right)\right\} + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}\left(i\right)\right\}\right.\right. \\
&\quad \left.\left. + \left[E_i\left\{\widehat{Y}_{V,t}\left(i\right)\right\}\right]^2\right]\right] + V_Y Y_M \left[\left(1-\psi g_{M,t}\right) E_i\left\{\widehat{Y}_{M,t}\left(i\right)\right\}\right. \\
&\quad \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}\left(i\right)\right\} + \left[E_i\left\{\widehat{Y}_{M,t}\left(i\right)\right\}\right]^2\right]\right] \\
&\quad + V_Y Y_{OG}\left[\left(1-\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\right) E_i\left\{\widehat{Y}_{OG,t}\left(i\right)\right\}\right. \\
&\quad \left. + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}\left(i\right)\right\} + \left[E_i\left\{\widehat{Y}_{OG,t}\left(i\right)\right\}\right]^2\right]\right] \\
&\quad + \left\|O\right\|^3 + t.i.p.
\end{aligned}$$

It can be shown that (see Woodford (2003) and Gali and Monacelli (2005)),

$$\begin{aligned}
\widehat{Y}_{V,t} &= E_i\left\{\widehat{Y}_{V,t}\left(i\right)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}\left(i\right)\right\} \\
\widehat{Y}_{M,t} &= E_i\left\{\widehat{Y}_{M,t}\left(i\right)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}\left(i\right)\right\} \\
\widehat{Y}_{OG,t} &= E_i\left\{\widehat{Y}_{OG,t}\left(i\right)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}\left(i\right)\right\}
\end{aligned}$$

Therefore

$$\begin{aligned}
\int_0^1 v\left(N_t\left(i\right)\right)\ di &\approx V_Y Y_V \left[\left(1-\psi g_{V,t}\right)\left[\widehat{Y}_{V,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}\left(i\right)\right\}\right] + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}\left(i\right)\right\}\right.\right. \\
&\quad \left.\left. + \widehat{Y}_{V,t}^2\right]\right] + V_Y Y_M\left[\left(1-\psi g_{M,t}\right)\left[\widehat{Y}_{M,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}\left(i\right)\right\}\right]\right. \\
&\quad \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}\left(i\right)\right\} + \widehat{Y}_{M,t}^2\right]\right] \\
&\quad + V_Y Y_{OG}\left[\left(1-\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\right)\left[\widehat{Y}_{OG,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}\left(i\right)\right\}\right]\right. \\
&\quad \left. + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}\left(i\right)\right\} + \widehat{Y}_{OG,t}^2\right]\right] + \left\|O\right\|^3 + t.i.p.
\end{aligned}$$

Using a result in Woodford (2003) , since the manufacturing sector has sticky prices in place,

$$Var\left\{\widehat{Y}_{M,t}\left(i\right)\right\} = \theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\}.$$

Similarly for the grain and vegetable sectors are flexible price sectors,

$$
\begin{aligned}
Var\left\{\widehat{Y}_{V,t}(i)\right\} &= \theta^2 Var\left\{\widehat{P}_{V,t}(i)\right\} = 0 \\
Var\left\{\widehat{Y}_{OG,t}(i)\right\} &= \theta^2 Var\left\{\widehat{P}_{OG,t}(i)\right\} = 0
\end{aligned}
$$

On simplifying we get,

$$
\begin{aligned}
\int_0^1 v(N_t(i))\ di \approx\ & V_Y Y_V \left[\widehat{Y}_{V,t} - \psi g_{V,t}\widehat{Y}_{V,t} + \left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right] \\
& + V_Y Y_M \left[\widehat{Y}_{M,t} - \psi g_{M,t}\widehat{Y}_{M,t} + \frac{1}{2}\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right. \\
& \left. + \left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2\right] + V_Y Y_{OG}\left[\widehat{Y}_{OG,t} - \psi\left(g_{OG,t}(1-c_p) + g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\right. \\
& \left. + \left(\frac{1+\psi(1-c_p)}{2}\right)\widehat{Y}_{OG,t}^2\right] + \|O\|^3 + t.i.p.
\end{aligned} \tag{45}
$$

From the first order condition of consumption-leisure choice at steady state,

$$
\frac{V_Y}{U_C} = \frac{W}{P}
$$

Note here $P = P_A^{\delta} P_M^{1-\delta} = P_{OG}^{(1-\mu)\delta} P_V^{\mu\delta} P_M^{1-\delta}$. Using the technical appendix of Ghate et al. (2018),

$$
P_A = \frac{\theta(1-c_p)}{(\theta-1)(1-c_p)-c_p}W;\ P_M = P_V = \frac{\theta}{\theta-1}W
$$

$$
P = \gamma^{-(1-\mu)\delta}\left(\frac{\theta-1}{\theta}\right)W
$$

We assume that government provides employment subsidy, $(1-\tau)$, to do away with inefficiency due to monopolistic competition. Here $(1-\tau) = \frac{\theta-1}{\theta}$. This implies,

$$
\frac{V_Y}{U_C} = \gamma^{(1-\mu)\delta}
$$

Again using the technical appendix of Ghate et al. (2018),

$$\begin{aligned}
\frac{C_M}{C} &= (1-\delta)\,\gamma^{-(1-\mu)\delta} \\
\frac{C_V}{C} &= \mu\delta\gamma^{-(1-\mu)\delta} \\
\frac{C_{OG}}{C} &= (1-\mu)\,\delta\gamma^{-(1-\mu)\delta+1}
\end{aligned}$$

Replacing Y_M, Y_V, Y_{OG} and V_Y in equation(45) with C_M, C_V, C_{OG} and $U_C\ \gamma^{(1-\mu)\delta}$ respectively we get,

$$\begin{aligned}
\int_0^1 v\left(N_t\left(i\right)\right)\ di \ \approx\ & U_C\ C\left[\mu\delta\left[\widehat{Y}_{V,t}-\psi g_{V,t}\widehat{Y}_{V,t}+\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right]\right. \\
& +(1-\delta)\left[\widehat{Y}_{M,t}-\psi g_{M,t}\widehat{Y}_{M,t}+\frac{1}{2}\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\}\right. \\
& \left.+\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2\right]+(1-\mu)\,\delta\gamma\left[\widehat{Y}_{OG,t}-\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\right. \\
& \left.\left.+\left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\widehat{Y}_{OG,t}^2\right]\right]+\left\|O\right\|^3+t.i.p.
\end{aligned} \tag{46}$$

Now, we know that

$$w_t = U\left(C_t\right)-\int_0^1 v\left(N_t\left(i\right)\right)\ di$$

Now, combining the second order approximation of utility from consumption (equation (41)) and the second order approximation of aggregated disutility from labour supply (equation (46)) in the average utility function (equation (38)), and using $\mu\delta\widehat{Y}_{V,t}+(1-\delta)\,\widehat{Y}_{M,t}+(1-\mu)\,\delta\widehat{Y}_{OG,t}=\widehat{C}_t$ we get,

$$\begin{aligned}
w_t \ \approx\ & U_C\ C\left[\widehat{C}_t+\frac{1}{2}(1-\sigma)\,\widehat{C}_t^2-\widehat{C}_t+(1-\mu)\,\delta\,(1-\gamma)\,\widehat{Y}_{OG,t}+\mu\delta\psi g_{V,t}\widehat{Y}_{V,t}\right. \\
& -\mu\delta\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2+(1-\delta)\,\psi g_{M,t}\widehat{Y}_{M,t}-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\} \\
& -(1-\delta)\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2+(1-\mu)\,\delta\gamma\psi\left(g_{OG,t}\left(1-c_p\right)-g_{PG,t}c_p\right)\widehat{Y}_{OG,t} \\
& \left.-(1-\mu)\,\delta\gamma\left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\widehat{Y}_{OG,t}^2\right]+\left\|O\right\|^3+t.i.p.
\end{aligned} \tag{47}$$

Simplifying further, we get

$$
\begin{aligned}
w_t \approx\ & U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2+\alpha_{1V}\widehat{Y}_{V,t}-\alpha_{2V}\widehat{Y}_{V,t}^2\right. \\
& +\alpha_{1M}\widehat{Y}_{M,t}-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}-\alpha_{2M}\widehat{Y}_{M,t}^2 \\
& \left.+\alpha_{1G}\widehat{Y}_{OG,t}-\alpha_{2G}\widehat{Y}_{OG,t}^2\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

where,

$$
\begin{aligned}
\alpha_{1V}\ (\text{coefficient of } \widehat{Y}_{V,t}) &= \mu\delta\psi g_{V,t} \\
\alpha_{2V}\ (\text{coefficient of } \widehat{Y}_{V,t}^2) &= \mu\delta\left(\frac{\psi+1}{2}\right) \\
\alpha_{1M}\ (\text{coefficient of } \widehat{Y}_{M,t}) &= (1-\delta)\psi g_{M,t} \\
\alpha_{2M}\ (\text{coefficient of } \widehat{Y}_{M,t}^2) &= (1-\delta)\left(\frac{\psi+1}{2}\right) \\
\alpha_{1G}\ (\text{coefficient of } \widehat{Y}_{OG,t}) &= (1-\mu)\delta\left(\gamma\psi\left(g_{OG,t}(1-c_p)-g_{PG,t}c_p\right)+(1-\gamma)\right) \\
\alpha_{2G}\ (\text{coefficient of } \widehat{Y}_{OG,t}^2) &= (1-\mu)\delta\gamma\left(\frac{1+\psi(1-c_p)}{2}\right)
\end{aligned}
$$

Now substituting,

$$
\begin{aligned}
\widehat{Y}_{M,t} &= \widehat{C}_t+\delta\widehat{T}_{AM,t} \qquad (48) \\
\widehat{Y}_{V,t} &= \widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}+(1-\mu)\widehat{T}_{OGV,t} \\
\widehat{Y}_{OG,t} &= \widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t}
\end{aligned}
$$

$$
\begin{aligned}
w_t \approx\ & U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right. \\
& +(\alpha_{1V}+\alpha_{1M}+\alpha_{1G})\widehat{C}_t+(\alpha_{1M}\delta-\alpha_{1V}(1-\delta)-\alpha_{1G}(1-\delta))\widehat{T}_{AM,t} \\
& +(\alpha_{1V}(1-\mu)-\alpha_{1G}\mu)\widehat{T}_{OGV,t}-(\alpha_{2V}+\alpha_{2M}+\alpha_{2G})\widehat{C}_t^2-\left[\alpha_{2M}\delta^2+\alpha_{2V}(1-\delta)^2\right. \\
& \left.+\alpha_{2G}(1-\delta)^2\right]\widehat{T}_{AM,t}^2-\left(\alpha_{2V}(1-\mu)^2+\alpha_{2G}\mu^2\right)\widehat{T}_{OGV,t}^2 \\
& -(2\alpha_{2M}\delta-2\alpha_{2V}(1-\delta)-2\alpha_{2G}(1-\delta))\widehat{C}_t\widehat{T}_{AM,t} \\
& \left.-(2\alpha_{2V}(1-\mu)-2\alpha_{2G}\mu)\widehat{C}_t\widehat{T}_{OGV,t}-(2\alpha_{2G}(1-\delta)\mu-2\alpha_{2V}(1-\delta)(1-\mu))\widehat{T}_{AM,t}\widehat{T}_{OGV,t}\right] \\
& +\|O\|^3+t.i.p.
\end{aligned}
$$

Now we use the fact that $\widehat{Y}_{OG,t}$, $\widehat{T}_{OGV,t}$, $\widehat{Y}_{V,t}$, are $t.i.p.$ as they are natural levels.

$$
\begin{aligned}
\widehat{C}_t\widehat{T}_{AM,t} &= \left(\widehat{Y}_{V,t}+(1-\delta)\widehat{T}_{AM,t}-(1-\mu)\widehat{T}_{OGV,t}\right)\widehat{T}_{AM,t} \\
&= \widehat{Y}_{V,t}\widehat{T}_{AM,t}+(1-\delta)\widehat{T}^2_{AM,t}-(1-\mu)\widehat{T}_{OGV,t}\widehat{T}_{AM,t}
\end{aligned}
$$

$$
\begin{aligned}
w_t \approx\ & U_C C\Bigg[-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} \\
&+\left[(\alpha_{1V}+\alpha_{1M}+\alpha_{1G})-(2\alpha_{2V}(1-\mu)-2\alpha_{2G}\mu)\widehat{T}_{OGV,t}\right]\widehat{C}_t+\left[(\alpha_{1M}\delta-\alpha_{1V}(1-\delta)\right. \\
&-\alpha_{1G}(1-\delta))-(2\alpha_{2M}\delta-2\alpha_{2V}(1-\delta)-2\alpha_{2G}(1-\delta))\left(\widehat{Y}_{V,t}-(1-\mu)\widehat{T}_{OGV,t}\right) \\
&\left.-(2\alpha_{2G}(1-\delta)\mu-2\alpha_{2V}(1-\delta)(1-\mu))\widehat{T}_{OGV,t}\right]\widehat{T}_{AM,t}+(\alpha_{1V}(1-\mu)-\alpha_{1G}\mu)\widehat{T}_{OGV,t}- \\
&\left[-\frac{1}{2}(1-\sigma)+(\alpha_{2V}+\alpha_{2M}+\alpha_{2G})\right]\widehat{C}_t^2-\left[\left(\alpha_{2M}\delta^2+\alpha_{2V}(1-\delta)^2+\alpha_{2G}(1-\delta)^2\right)\right. \\
&\left.+(1-\delta)(2\alpha_{2M}\delta-2\alpha_{2V}(1-\delta)-2\alpha_{2G}(1-\delta))\right]\widehat{T}^2_{AM,t}-\left(\alpha_{2V}(1-\mu)^2+\alpha_{2G}\mu^2\right)\widehat{T}^2_{OGV,t}\Bigg] \\
&+\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
w_t \approx\ & U_C\ C\Bigg[-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+\beta_{1C}\widehat{C}_t \\
&+\beta_{ITAM}\widehat{T}_{AM,t}+\beta_{1TOGV}\widehat{T}_{OGV,t}-\beta_{2C}\widehat{C}_t^2-\beta_{2TAM}\widehat{T}^2_{AM,t}-\beta_{2TOGV}\widehat{T}^2_{OGV,t}\Bigg] \\
&+\|O\|^3+t.i.p.
\end{aligned}
$$

where,

$$
\begin{aligned}
\beta_{1C}\text{ (coefficient of }\widehat{C}_t\text{),} &= (\alpha_{1V}+\alpha_{1M}+\alpha_{1G})-(2\alpha_{2V}(1-\mu)-2\alpha_{2G}\mu)\widehat{T}_{OGV,t} \\
\beta_{2C}\text{ (coefficient of }\widehat{C}_t^2\text{)} &= -\frac{1}{2}(1-\sigma)+(\alpha_{2V}+\alpha_{2M}+\alpha_{2G}) \\
\beta_{1TAM}\text{ (coefficient of }\widehat{T}_{AM,t}\text{)} &= (\alpha_{1M}\delta-\alpha_{1V}(1-\delta)-\alpha_{1G}(1-\delta)) \\
&\quad -(2\alpha_{2M}\delta-2\alpha_{2V}(1-\delta)-2\alpha_{2G}(1-\delta))\left(\widehat{Y}_{V,t}-(1-\mu)\widehat{T}_{OGV,t}\right) \\
&\quad -(2\alpha_{2G}(1-\delta)\mu-2\alpha_{2V}(1-\delta)(1-\mu))\widehat{T}_{OGV,t} \\
\beta_{2TAM}\text{ (coefficient of }\widehat{T}^2_{AM,t}\text{)} &= \left(\alpha_{2M}\delta^2+\alpha_{2V}(1-\delta)^2+\alpha_{2G}(1-\delta)^2\right) \\
&\quad +(1-\delta)(2\alpha_{2M}\delta-2\alpha_{2V}(1-\delta)-2\alpha_{2G}(1-\delta)) \\
\beta_{1TOG}\text{ (coefficient of }\widehat{T}_{OG,t}\text{)} &= \alpha_{1V}(1-\mu)-\alpha_{1G}\mu \\
\beta_{2TOG}\text{ (coefficient of }\widehat{T}^2_{OG,t}\text{)} &= \alpha_{2V}(1-\mu)^2+\alpha_{2G}\mu^2
\end{aligned}
$$

$$
\begin{aligned}
w_t \;\approx\; & -\frac{U_C\ C}{2}\Big[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}-2\beta_{1C}\widehat{C}_t \\
& -2\beta_{ITAM}\widehat{T}_{AM,t}-2\beta_{2TOGV}\widehat{T}_{OGV,t}+2\beta_{2C}\widehat{C}_t^2+2\beta_{2TAM}\widehat{T}_{AM,t}^2+2\beta_{2TOGV}\widehat{T}_{OGV,t}^2\Big] \\
& +\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
w_t \;\approx\; & -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+2\beta_{2C}\left(\widehat{C}_t^2-\frac{\beta_{1C}}{\beta_{2C}}\widehat{C}_t\right)+\right. \\
& \left.+2\beta_{2TAM}\left(\widehat{T}_{AM,t}^2-\frac{\beta_{ITAM}}{\beta_{2TAM}}\widehat{T}_{AM,t}\right)+2\beta_{2TOGV}\left(\widehat{T}_{OGV,t}^2-\frac{\beta_{TOGV}}{\beta_{2TOGV}}\widehat{T}_{OGV,t}\right)\right] \\
& +\|O\|^3+t.i.p.
\end{aligned}
$$

Note here $\beta_{1C}, \beta_{1TAM}, \beta_{1TOGV}$ are functions of shocks and $\beta_{2C}, \beta_{2TAM}, \beta_{2TOGV}$ are constants.

$$
\begin{aligned}
w_t \;\approx\; & -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+2\beta_{2C}\left(\widehat{C}_t-\widehat{C}_t^*\right)^2+\right. \\
& \left.+2\beta_{2TAM}\left(\widehat{T}_{AM,t}-\widehat{T}_{AM,t}^*\right)^2+2\beta_{2TOGV}\left(\widehat{T}_{OGV,t}-\widehat{T}_{OGV,t}^*\right)^2\right] \\
& +\|O\|^3+t.i.p.
\end{aligned}
$$

where $\frac{\beta_{1C}}{2\beta_{2C}}=\widehat{C}_t^*$, $\frac{\beta_{ITAM}}{2\beta_{2TAM}}=\widehat{T}_{AMt}^*$, $\frac{\beta_{TOGV}}{2\beta_{2TOGV}}=\widehat{T}_{OGV,t}^*$.The welfare function reduces to,

$$
\begin{aligned}
w_t \;\approx\; & -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+2\beta_{2C}\left(\widetilde{C}_t^*\right)^2+\right. \\
& \left.+2\beta_{2TAM}\left(\widetilde{T}_{AM,t}^*\right)^2+2\beta_{2TOGV}\left(\widetilde{T}_{OGV,t}^*\right)^2\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

where $\widehat{C}_t-\widehat{C}_t^*=\widetilde{C}_t^*$, $\widehat{T}_{AM,t}-\widehat{T}_{AM,t}^*=\widetilde{T}_{AM,t}^*$. Since $\widehat{T}_{OGV,t}=\widehat{T}_{OGV,t}^n$, and $\widehat{T}_{OGV,t}^n$ & $\widehat{T}_{OGV,t}^*$ are functions of shocks, it is t.i.p. Lifetime welfare function,

$$
\begin{aligned}
W_t \;=\; & E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{w_t}{U_C\ C}\right)\approx-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+2\beta_{2C}\left(\widetilde{C}_t^*\right)^2+\right. \\
& \left.+2\beta_{2TAM}\left(\widetilde{T}_{AM,t}^*\right)^2\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

Using the following result from Woodford (2003),[52]

$$
E_0\sum_{t=0}^{\infty}\beta^t Var\left\{\widehat{P}_{M,t}(i)\right\}=\frac{\alpha_M}{(1-\beta\alpha_M)(1-\alpha_M)}E_0\sum_{t=0}^{\infty}\beta^t\pi_{M,t}^2
$$

[52] Please refer Chapter 6 of the book.

$$
\begin{aligned}
W_t &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \ C} \right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + \lambda_{\widetilde{C}} \left(\widetilde{C}_t^* \right)^2 + \right. \\
& \left. + \lambda_{\widetilde{TAM}} \left(\widetilde{T}_{AM,t}^* \right)^2 \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

where $\lambda_{\pi M} = \frac{\alpha_M (1-\delta) \left(\theta^{-1} + \psi \right) \theta^2}{(1-\beta \alpha_M)(1-\alpha_M)}$, $\lambda_{\widetilde{C}} = 2\beta_{2C}$ and $\lambda_{\widetilde{TAM}} = 2\beta_{2TAM}$. Finally,

$$
\begin{aligned}
W_t &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \ C} \right) \approx -\frac{1}{2} \lambda_{\pi M} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{M,t}^2 + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}} \left(\widetilde{C}_t^* \right)^2 + \right. \\
& \left. + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}} \left(\widetilde{T}_{AM,t}^* \right)^2 \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

With special case when $c_p = 0$, continuing from equation (47) and using the fact that $\gamma = 1$ when $c_p = 0$, we get,

$$
\begin{aligned}
w_t &\approx U_C \ C \left[\frac{1}{2} (1 - \sigma) \widehat{C}_t^2 + \mu\delta\psi g_{V,t} \widehat{Y}_{V,t} + \mu\delta \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{V,t}^2 \right. \\
& + (1 - \delta) \psi g_{M,t} \widehat{Y}_{M,t} - \frac{1}{2} (1 - \delta) \left(\theta^{-1} + \psi \right) \theta^2 Var \left\{ \widehat{P}_{M,t} (i) \right\} - (1 - \delta) \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{M,t}^2 \\
& \left. + (1 - \mu) \delta\psi g_{OG,t} \widehat{Y}_{OG,t} - (1 - \mu) \delta \left(\frac{1 + \psi}{2} \right) \widehat{Y}_{OG,t}^2 \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

Using demand functions and simplifying further we get,

$$
\begin{aligned}
w_t &\approx -\frac{1}{2} U_C \ C \left[((\sigma + \psi)) \left(\widehat{C}_t - \widehat{C}_t^n \right)^2 + (\psi + 1) (1 - \delta) \delta \left(\widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n \right)^2 \right. \\
& + (\psi + 1) \mu\delta (1 - \mu) \left(\widehat{T}_{OGV,t} - \widehat{T}_{OGV,t}^n \right)^2 \\
& \left. + (1 - \delta) \left(\theta^{-1} + \psi \right) \theta^2 Var \left\{ \widehat{P}_{M,t} (i) \right\} \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

where,

$$
\begin{aligned}
\widehat{C}_t^* &= \widehat{C}_t^n = \frac{(\mu\delta\psi g_{V,t} + (1 - \delta) \psi g_{M,t} + (1 - \mu) \delta\psi g_{OG,t})}{(\sigma + \psi)}, \\
\widehat{T}_{AM,t}^* &= \widehat{T}_{AM,t}^n = \frac{(1 - \delta) \delta\psi (g_{M,t} - \mu g_{V,t} - (1 - \mu) g_{OG,t})}{(\psi + 1) (1 - \delta) \delta}, \\
\widehat{T}_{OGV,t}^* &= \widehat{T}_{OGV,t}^n = \frac{\psi\mu\delta (1 - \mu) (g_{V,t} - g_{OG,t})}{(\psi + 1) \mu\delta (1 - \mu)}.
\end{aligned}
$$

Since $c_p = 0$, $\widehat{C}_t = \widehat{Y}_t$,

$$\therefore \quad w_t \approx -\frac{1}{2} U_C \ C \left[(\sigma + \psi) \left(\widetilde{Y}_t \right)^2 + (\psi + 1)(1 - \delta) \delta \left(\widetilde{T}_{AM,t} \right)^2 + \right.$$

$$\left. (1 - \delta) \left(\theta^{-1} + \psi \right) \theta^2 Var \left\{ \widehat{P}_{M,t}(i) \right\} \right] + \|O\|^3 + t.i.p.$$

where $\widetilde{Y}_t \ = \ \widehat{Y}_t - \widehat{Y}_t^n, \ \widetilde{T}_{AM,t} = \widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n$ and $\widetilde{T}_{OGV,t} = \widehat{T}_{OGV,t} - \widehat{T}_{OGV,t}^n = 0$.

Lifetime welfare function thus becomes,

$$\begin{aligned} W_t \ &= \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \ C} \right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + (\sigma + \psi) \left(\widetilde{Y}_t \right)^2 \right. \\ &\left. + (\psi + 1)(1 - \delta) \delta \left(\widetilde{T}_{AM,t} \right)^2 \right] + \|O\|^3 + t.i.p. \end{aligned}$$

Part B. Derivation of aggregate goods market condition, NKPC and dynamic-IS

Re-writing manufacturing sector NKPC, aggregate goods market condition and DIS in terms of gaps from the welfare relevant levels instead of natural levels.

Aggregate goods market clearing condition,

$$\widetilde{Y}_t = (1 - \lambda_c) \widetilde{C}_t + \lambda_c (1 - \delta) \widetilde{T}_{AM,t}$$

Adding and subtracting relevant the welfare relevant levels, we get

$$\begin{aligned} \left(\widehat{Y}_t - \widehat{Y}_t^n \right) \pm \widehat{Y}_t^* \ &= \ (1 - \lambda_c) \left(\widehat{C}_t - \widehat{C}_t^n \right) \pm (1 - \lambda_c) \widehat{C}_t^* \\ &+ \lambda_c (1 - \delta) \left(\widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n \right) \pm \lambda_c (1 - \delta) \widehat{T}_{AM,t}^*. \end{aligned}$$

$$\begin{aligned}
\widetilde{C}_t^* &= \frac{1}{(1-\lambda_c)}\widetilde{Y}_t^* - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)}\widetilde{T}_{AM,t}^* - \left(\widehat{C}_t^* - \widehat{C}_t^n\right) \\
&\quad - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)}\left(\widehat{T}_{AM,t}^* - \widehat{T}_{AM,t}^n\right) + \frac{1}{(1-\lambda_c)}\left(\widehat{Y}_t^* - \widehat{Y}_t^n\right) \\
&= \frac{1}{(1-\lambda_c)}\widetilde{Y}_t^* - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)}\widetilde{T}_{AM,t}^* + z_{1,t}^* \qquad (49) \\
\text{where } z_{1,t}^* &= \frac{1}{(1-\lambda_c)}\left(\widehat{Y}_t^* - \widehat{Y}_t^n\right) - \left(\widehat{C}_t^* - \widehat{C}_t^n\right) - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)}\left(\widehat{T}_{AM,t}^* - \widehat{T}_{AM,t}^n\right).
\end{aligned}$$

Manufacturing sector NKPC,

$$\pi_{M,t} = \beta E_t\left\{\pi_{M,t+1}\right\} + \lambda_M\left(\sigma + \psi\Theta_1\right)\widetilde{C}_t + \lambda_M\delta\widetilde{T}_{AM,t}$$

Adding and subtracting relevant the welfare relevant levels, we get

$$\pi_{M,t} = \beta E_t\left\{\pi_{M,t+1}\right\} + \lambda_M\left(\sigma+\psi\Theta_1\right)\left(\widehat{C}_t - \widehat{C}_t^n\right) \pm \lambda_M\left(\sigma+\psi\Theta_1\right)\widehat{C}_t^* + \lambda_M\delta\left(\widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n\right) \pm \lambda_M\delta\widehat{T}_{AM,t}^*$$

$$\begin{aligned}
\pi_{M,t} &= \beta E_t\left\{\pi_{M,t+1}\right\} + \lambda_M\left(\sigma+\psi\Theta_1\right)\widetilde{C}_t^* + \lambda_M\delta\widetilde{T}_{AM,t}^* + z_{2,t}^* \\
\text{where } z_{2,t}^* &= \lambda_M\left(\sigma+\psi\Theta_1\right)\left(\widehat{C}_t^* - \widehat{C}_t^n\right) + \lambda_M\delta\left(\widehat{T}_{AM,t}^* - \widehat{T}_{AM,t}^n\right).
\end{aligned}$$

DIS equation,

$$\widetilde{Y}_t = E_t\{\widetilde{Y}_{t+1}\} - \frac{(1-\lambda_c)}{\sigma}\left[\widehat{R}_t - E_t\left\{\pi_{t+1}\right\} - \widehat{r}_t^n\right] - \lambda_c\left(1-\delta\right)E_t\left\{\Delta\widetilde{T}_{AM,t+1}\right\}$$

Adding and subtracting the welfare relevant levels, we get

$$\begin{aligned}
\left(\widehat{Y}_t - \widehat{Y}_t^n\right) \pm \widehat{Y}_t^* &= E_t\left\{\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^n\right\} \pm E_t\left\{\widehat{Y}_{t+1}^*\right\} - \frac{(1-\lambda_c)}{\sigma}\left[\widehat{R}_t - E_t\left\{\pi_{t+1}\right\} - \widehat{r}_t^n\right] \\
&\quad -\lambda_c\left(1-\delta\right)E_t\left\{\Delta\widehat{T}_{AM,t+1} - \Delta\widehat{T}_{AM,t+1}^n\right\} \mp \lambda_c\left(1-\delta\right)E_t\left\{\Delta\widehat{T}_{AM,t+1}^*\right\}
\end{aligned}$$

Re-arranging and substituting $\pi_{t+1} = \pi_{M,t+1} + \delta \Delta \widehat{T}_{AM,t+1}$ as $P_t = P_{A,t}^{\delta} P_{M,t}^{1-\delta}$,

$$\begin{aligned}
\widetilde{Y}_t^* &= E_t\left\{\widetilde{Y}_{t+1}^*\right\} - \frac{(1-\lambda_c)}{\sigma}\left[\widehat{R}_t - E_t\left\{\pi_{M,t+1}\right\} - \widehat{r}_t^*\right] + \left(\frac{(1-\lambda_c)\delta}{\sigma} - \lambda_c\left(1-\delta\right)\right) E_t\left\{\Delta \widetilde{T}_{AM,t+1}^*\right\} \\
\text{where } \widehat{r}_t^* &= \widehat{r}_t^n + E_t\left\{\delta \Delta \widehat{T}_{AM,t+1}^*\right\} - \frac{\lambda_c \sigma\left(1-\delta\right)}{(1-\lambda_c)} E_t\left\{\Delta \widehat{T}_{AM,t+1}^* - \Delta \widehat{T}_{AM,t+1}^n\right\} \\
&\quad - \frac{\sigma}{(1-\lambda_c)}\left(\widehat{Y}_t^* - \widehat{Y}_t^n\right) + \frac{\sigma}{(1-\lambda_c)} E_t\left\{\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^n\right\}
\end{aligned}$$

Part C. Optimal monetary policy under discretion

Minimize welfare loss function subject to constraint the aggregate NKPC. Lagrangian,

$$\begin{aligned}
L_t &= \min \frac{1}{2}\left[\pi_{M,t}^2 + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}}\left(\widetilde{C}_t^*\right)^2 + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}}\left(\widetilde{T}_{AM,t}^*\right)^2\right] \\
&\quad - \phi_1\left[\pi_{M,t} - \lambda_M\left(\sigma + \psi\Theta_1\right)\widetilde{C}_t^* - \lambda_M \delta \widetilde{T}_{AM,t}^* - z_{2,t}^*\right]
\end{aligned}$$

First order conditions,

$$\begin{aligned}
\frac{\partial L_t}{\partial \pi_{M,t}} &= \pi_{M,t} - \phi_1 = 0 \\
\frac{\partial L_t}{\partial \widetilde{C}_t^*} &= \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}}\left(\widetilde{C}_t^*\right) + \phi_1 \lambda_M\left(\sigma + \psi\Theta_1\right) = 0 \\
\frac{\partial L_t}{\partial \widetilde{T}_{AM,t}^*} &= \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}}\left(\widetilde{T}_{AM,t}^*\right) + \phi_1 \lambda_M \delta = 0
\end{aligned}$$

This implies,

$$\pi_{M,t} = -\frac{\lambda_{\widetilde{TAM}}}{\delta \lambda_M \lambda_{\pi M}} \widetilde{T}_{AM,t}^* \tag{50}$$

$$\widetilde{C}_t^* = -\frac{\lambda_M\left(\sigma + \psi\Theta_1\right)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} \pi_{M,t} \tag{51}$$

We know that,

$$\widetilde{C}_t^* = \frac{1}{(1-\lambda_c)} \widetilde{Y}_t^* - \frac{\lambda_c(1-\delta)}{(1-\lambda_c)} \widetilde{T}_{AM,t}^* + z_{1,t}^*$$

Substituting for $\widetilde{C}_t^*$ in first order conditions, we get

$$\widetilde{Y}_t^* = -\left[\frac{\lambda_M\left(\sigma+\psi\Theta_1\right)(1-\lambda_c)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \frac{\lambda_c(1-\delta)\delta\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right]\pi_{M,t} - (1-\lambda_c)z_{1,t}^*$$

Let $\left[\frac{\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \frac{\lambda_c(1-\delta)\delta\lambda_M\lambda_{\pi M}}{(1-\lambda_c)\lambda_{\widetilde{TAM}}}\right] = X_1$, such that

$$\pi_{M,t} = -\frac{1}{X_1(1-\lambda_c)}\widetilde{Y}_t^* - \frac{1}{X_1}z_{1,t}^*$$

Since $\pi_t = \pi_{M,t} + \delta\Delta\widehat{T}_{AM,t}$,

$$\pi_{M,t} = \pi_t - \delta\Delta\widetilde{T}_{AM,t}^* - z_{3,t}^* \tag{52}$$

where, $z_{3,t}^* = \Delta\widehat{T}_{AM,t}^*$. Substituting $\widetilde{T}_{AM,t}^*$ from FOC, we get, it in above equation,

$$\begin{aligned}
\pi_{M,t} &= \pi_t + \frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\Delta\pi_{M,t} - z_{3,t}^* \\
\pi_t &= -\left(1-\frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right)\left[\frac{1}{X_1(1-\lambda_c)}\widetilde{Y}_t^* + \frac{1}{X_1}z_{1,t}^*\right] - \frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\pi_{M,t-1} + z_{3,t}^*
\end{aligned}$$

Let $\left(1-\frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right) = X_2$

$$\pi_t = -\frac{X_2}{X_1(1-\lambda_c)}\widetilde{Y}_t^* - \frac{X_2}{X_2X_1}z_{1,t}^* + z_{3,t}^* - \frac{\delta^2\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\pi_{M,t-1}$$

To get the optimal value of manufacturing sector inflation, $\pi_{M,t}$, consumption gap, $\widetilde{C}_t^*$,output gap, $\widetilde{Y}_t^*$, terms of trade gap, $\widetilde{T}_{AM,t}^*$ and aggregate inflation, π_t, we first substitute value of $\widetilde{C}_t^*$ and $\widetilde{T}_{AM,t}^*$ from equations (50) and (51) into the NKPC, we get

$$X_3\pi_{M,t} = \beta E_t\left\{\pi_{M,t+1}\right\} + z_{2,t}^*$$

where $X_3 = \left[1 + \lambda_M\left(\sigma+\psi\Theta_1\right)\frac{\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \frac{\delta^2\lambda_M^2\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right]$. Thus optimal level $\pi_{M,t}^*$ is

$$\pi_{M,t} = \frac{1}{X_3}\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z_{2,t+j}^*$$

Substituting this in the first two FOC's, we get optimal value $\widetilde{T}^*_{AM,t}$ and $\widetilde{C}^*_t$ as,

$$\begin{aligned}\widetilde{T}^*_{AM,t} &= -\frac{\lambda_{\pi M}\lambda_M\delta}{\lambda_{\widetilde{TAM}}}\frac{1}{X_3}\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+j} \\ \widetilde{C}^*_t &= -\frac{\lambda_M\left(\sigma+\psi\Theta_1\right)\lambda_{\pi M}}{\lambda_{\widetilde{C}}}\frac{1}{X_3}\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+j}\end{aligned}$$

Substituting this in equation (49), we get following optimal value $\widetilde{Y}^*_t$,

$$\widetilde{Y}^*_t = -(1-\lambda_c)\left[\frac{X_1}{X_3}\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+j} + z^*_{1,t}\right]$$

Substituting value in equation (52), we get optimal value of aggregate inflation, π_t

$$\pi_t = \frac{X_2}{X_1}\left[\frac{X_1}{X_3}\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+j} + z^*_{1,t}\right] - \frac{1}{X_1}z^*_{1,t} + z^*_{3,t} - \frac{\delta^2\lambda_M\lambda_{\pi M}}{X_3\lambda_{\widetilde{TAM}}}\left[\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t-1+j}\right]$$

To get the optimal instrument rule, $\widehat{R}^*_t$ we will substitute optimal values $\widetilde{Y}^*_t$, $\widetilde{T}^*_{AM,t}$ and $\pi_{M,t}$ in DIS equation, we get,

$$\begin{aligned}\widehat{R}^*_t &= \widehat{r}^*_t - \sigma E_t\left\{\Delta z^*_{1,t+1}\right\} + \frac{1}{X_3}\left[E_t\left\{\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+1+j}\right\}\right. \\ &\quad \left. -\left(\sigma X_1 + \frac{\lambda_{\pi M}\lambda_M\delta}{\lambda_{\widetilde{TAM}}}\frac{\sigma}{(1-\lambda_c)}\left(\frac{(1-\lambda_c)\delta}{\sigma} - \lambda_c\left(1-\delta\right)\right)\right)E_t\left\{\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j \Delta z^*_{2,t+1+j}\right\}\right]\end{aligned}$$

Let $\left(\sigma X_1 + \frac{\lambda_{\pi M}\lambda_M\delta}{\lambda_{\widetilde{TAM}}}\frac{\sigma}{(1-\lambda_c)}\left(\frac{(1-\lambda_c)\delta}{\sigma} - \lambda_c\left(1-\delta\right)\right)\right) = X_4$,

$$\widehat{R}^*_t = \widehat{r}^*_t + \frac{(1-X_4)}{X_3}E_t\left\{\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j z^*_{2,t+1+j}\right\} + \frac{X_4}{X_3}E_t\left\{\sum_{j=0}^{\infty}\left(\frac{\beta}{X_3}\right)^j \Delta z^*_{2,t+j}\right\} - \sigma E_t\left\{\Delta z^*_{1,t+1}\right\}$$

Part D. Optimal monetary policy under commitment

$$L_t = \min_{\{\pi_{M,t},\, \widetilde{C}_t^*,\, \widetilde{T}_{AM,t}^*, \widetilde{T}_{OGV,t}^*\}} -\frac{1}{2}\lambda_{\pi M} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{M,t}^2 + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}}\left(\widetilde{C}_t^*\right)^2 + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}}\left(\widetilde{T}_{AM,t}^*\right)^2\right.$$

$$\left. -\phi_t\left(\pi_{M,t} - \beta E_t\left\{\pi_{M,t+1}\right\} - \lambda_M\left(\sigma + \psi\Theta_1\right)\widetilde{C}_t^* - \lambda_M \delta \widetilde{T}_{AM,t}^* - z_{2,t}^*\right)\right]$$

First order conditions,

$$\frac{\partial L_t}{\partial \pi_{M,t}} = \pi_{M,t} - \phi_t + \phi_{t-1} = 0 \tag{i}$$

$$\frac{\partial L_t}{\partial \widetilde{C}_t^*} = \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}}\left(\widetilde{C}_t^*\right) + \phi_t \lambda_M\left(\sigma + \psi\Theta_1\right) = 0 \tag{ii}$$

$$\frac{\partial L_t}{\partial \widetilde{T}_{AM,t}^*} = \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}}\left(\widetilde{T}_{AM,t}^*\right) + \phi_t \lambda_M \delta = 0 \tag{iv}$$

From equations, (ii) and (iv),

$$\phi_t = -\frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}\lambda_M\left(\sigma + \psi\Theta_1\right)}\widetilde{C}_t^*$$

$$\text{or } \phi_t = -\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}\lambda_M \delta}\widetilde{T}_{AM,t}^*,$$

respectively, such that from (i) we get

$$\pi_{M,t} + \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}\lambda_M\left(\sigma + \psi\Theta_1\right)}\widetilde{C}_t^* - \frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}\lambda_M\left(\sigma + \psi\Theta_1\right)}\widetilde{C}_{t-1}^* = 0$$

$$\pi_{M,t} + \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}\lambda_M \delta}\widetilde{T}_{AM,t}^* - \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}\lambda_M \delta}\widetilde{T}_{AM,t-1}^* = 0$$

Re-writing, we get,

$$\widetilde{C}_t^* = \widetilde{C}_{t-1}^* - \frac{\lambda_{\pi M}\lambda_M\left(\sigma + \psi\Theta_1\right)}{\lambda_{\widetilde{C}}}\pi_{M,t}$$

$$\widetilde{T}_{AM,t}^* = \widetilde{T}_{AM,t-1}^* - \frac{\lambda_{\pi M}\lambda_M \delta}{\lambda_{\widetilde{TAM}}}\pi_{M,t}$$

Using value of $\widetilde{C}_t^*$ from the aggregate output in above equations, and putting,

$$\left(\lambda_c(1-\delta)\frac{\lambda_{\pi M}\lambda_M\delta}{\lambda_{\widetilde{TAM}}} + \frac{\lambda_{\pi M}\lambda_M(1-\lambda_c)\,(\sigma+\psi\Theta_1)}{\lambda_{\widetilde{C}}}\right) = \omega_1, \text{ we get}$$

$$\widetilde{Y}_t^* = \widetilde{Y}_{t-1}^* - \omega_1\pi_{M,t} - (1-\lambda_c)\left[z_{1,t}^* - z_{1,t-1}^*\right]$$

We now assume that $\phi_{-1} = 0$, such that,

$$\pi_{M,0} = \phi_0$$

which implies,

$$\begin{aligned}
\pi_{M,0} &= -\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\pi M}\lambda_M\delta}\widetilde{T}_{AM,0}^* \\
&= -\frac{\lambda_{\widetilde{C}}}{\lambda_{\pi M}\lambda_M\,(\sigma+\psi\Theta_1)}\widetilde{C}_0^* \\
\text{and } \widetilde{Y}_0^* &= -\omega_1\pi_{M,0} - (1-\lambda_c)z_{1,0}^*
\end{aligned}$$

Writing the above equation recursively,

$$\begin{aligned}
\widetilde{Y}_t^* &= -\omega_1\sum_{k=0}^{t}\pi_{M,t-k} - (1-\lambda_c)\left[\sum_{k=0}^{t} z_{1,t-k}^* - \sum_{k=0}^{t-1} z_{1,t-1-k}^*\right] \\
&= -\omega_1\widehat{\widehat{P}}_{M,t} - (1-\lambda_c)z_{1,t}^*
\end{aligned}$$

where $\widehat{\widehat{P}}_{M,t} = \widehat{P}_{M,t} - \widehat{P}_{M,-1}$. Similarly,

$$\begin{aligned}
\widetilde{C}_t^* &= -\omega_2\left(\widehat{P}_{M,t} - \widehat{P}_{M,-1}\right) = -\omega_2\widehat{\widehat{P}}_{M,t} \\
\widetilde{T}_{AM,t}^* &= -\omega_3\left(\widehat{P}_{M,t} - \widehat{P}_{M,-1}\right) = -\omega_3\widehat{\widehat{P}}_{M,t}
\end{aligned}$$

$$\text{where } \omega_2 = \frac{\lambda_{\pi M}\lambda_M\,(\sigma+\psi\Theta_1)}{\lambda_{\widetilde{C}}},\ \omega_3 = \frac{\lambda_{\pi M}\lambda_M\delta}{\lambda_{\widetilde{TAM}}}$$

To get the optimal values of variables we substitute the value of $\widetilde{C}_t^*, \widetilde{T}_{AM,t}^*$ in NKPC. Re-writing NKPC,

$$\begin{aligned}
\widehat{P}_{M,t} - \widehat{P}_{M,t-1} &= \beta E_t \left\{ \widehat{P}_{M,t+1} - \widehat{P}_{M,t} \right\} + \lambda_M (\sigma + \psi \Theta_1) \widetilde{C}_t^* + \lambda_M \delta \widetilde{T}_{AM,t}^* + z_{2,t}^* \\
\widehat{\widehat{P}}_{M,t} - \widehat{\widehat{P}}_{M,t-1} &= \beta E_t \left\{ \widehat{\widehat{P}}_{M,t+1} - \widehat{\widehat{P}}_{M,t} \right\} + \lambda_M (\sigma + \psi \Theta_1) \widetilde{C}_t^* + \lambda_M \delta \widetilde{T}_{AM,t}^* + z_{2,t}^*
\end{aligned}$$

substituting values from above,

$$\begin{aligned}
\widehat{\widehat{P}}_{M,t} &= \omega_4 \widehat{\widehat{P}}_{M,t-1} + \beta \omega_4 E_t \left\{ \widehat{\widehat{P}}_{M,t+1} \right\} + \omega_4 z_{2,t}^* \\
\text{where, } \omega_4 &= \frac{1}{1 + \beta + \lambda_M (\sigma + \psi \Theta_1) \omega_2 + \lambda_M \delta \omega_3}
\end{aligned}$$

Solving this differential equation,

$$\begin{aligned}
\widehat{\widehat{P}}_{M,t} - \omega_4 \widehat{\widehat{P}}_{M,t-1} - \beta \omega_4 E_t \left\{ \widehat{\widehat{P}}_{M,t+1} \right\} &= \omega_4 z_{2,t}^* \\
\widehat{\widehat{P}}_{M,t-1} \left[-\omega_4 + F - \beta \omega_4 F^2 \right] &= \omega_4 z_{2,t}^*
\end{aligned}$$

such that, $F^n X_t = X_{t+n}$. Let $\varkappa_1$ and $\varkappa_2$ be the roots of quadratic equation,

$$\varkappa_1 = \frac{1 - \sqrt{1 - 4\beta\omega_4^2}}{2\beta\omega_4} \text{ and } \varkappa_2 = \frac{1 + \sqrt{1 - 4\beta\omega_4^2}}{2\beta\omega_4}$$

Assuming, $\varkappa_2 > 1$,[53]

$$\widehat{\widehat{P}}_{M,t} = \varkappa_1 \widehat{\widehat{P}}_{M,t-1} + \frac{1}{\varkappa_2 \beta} \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2} \right)^k z_{2,t+k}^*$$

$$\widetilde{Y}_t^* = -\omega_1 \widehat{\widehat{P}}_{M,t} - (1 - \lambda_c) z_{1,t}^*$$

$$\widehat{\widehat{P}}_{M,t} = -\frac{1}{\omega_1} \widetilde{Y}_t^* - \frac{(1 - \lambda_c)}{\omega_1} z_{1,t}^*$$

Substituting value in the optimal price path above,

$$\widetilde{Y}_t^* = \varkappa_1 \widetilde{Y}_{t-1}^* - (1 - \lambda_c) \left[z_{1,t}^* - \varkappa_1 z_{1,t-1}^* \right] - \frac{\omega_1}{\varkappa_2 \beta} \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2} \right)^k z_{2,t+k}^*$$

[53] $\varkappa_2 > 1$ and $\varkappa_1 < 1$ has been verified for the calibrated values of parameters of the model.

Similarly,

$$\widetilde{T}^{*}_{AM,t} = \varkappa_1 \widetilde{T}^{*}_{AM,t-1} - \frac{\omega_3}{\varkappa_2 \beta} \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z^{*}_{2,t+k}$$

Putting these equations in following re-written DIS equation,

$$\begin{aligned}
\widehat{R}^{*}_t &= \widehat{r}^{*}_t + \omega_5 E_t \left\{ \widehat{\widehat{P}}_{M,t+1} - \widehat{\widehat{P}}_{M,t} \right\} - \frac{\sigma}{(1-\lambda_c)}(1-\lambda_c) E_t \left\{ z^{*}_{1,t+1} - z^{*}_{1,t} \right\} \\
\text{where, } \omega_5 &= \left[1 - \frac{\sigma}{(1-\lambda_c)}\omega_1 - \frac{\sigma}{(1-\lambda_c)} \left(\frac{(1-\lambda_c)\delta}{\sigma} - \lambda_c (1-\delta) \right) \omega_3 \right] \\
\therefore \quad \widehat{R}^{*}_t &= \widehat{r}^{*}_t + \omega_5 E_t \left\{ \widehat{\widehat{P}}_{M,t+1} - \widehat{\widehat{P}}_{M,t} \right\} - \frac{\sigma}{(1-\lambda_c)}(1-\lambda_c) E_t \left\{ z^{*}_{1,t+1} - z^{*}_{1,t} \right\}
\end{aligned}$$

Re-writing $\widehat{\widehat{P}}_{M,t}$

$$\begin{aligned}
\widehat{\widehat{P}}_{M,t} &= \varkappa_1 \widehat{\widehat{P}}_{M,t-1} + \frac{1}{\varkappa_2 \beta} \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z^{*}_{2,t+k} \\
&= \frac{1}{\varkappa_2 \beta} \sum_{j=0}^{t} \varkappa_1^j \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z^{*}_{2,t+k-j}
\end{aligned}$$

Therefore,

$$\widehat{R}^{*}_t = \widehat{r}^{*}_t + \omega_5 (\varkappa_1 - 1) \widehat{\widehat{P}}_{M,t} + \frac{\omega_5}{\varkappa_2 \beta} E_t \sum_{k=0}^{\infty} \left(\frac{1}{\varkappa_2}\right)^k z^{*}_{2,t+1+k} - \frac{\sigma}{(1-\lambda_c)}(1-\lambda_c) E_t \left\{ z^{*}_{1,t+1} - z^{*}_{1,t} \right\}$$