A Markov-Switching Vector Error Correction Model of the Indian Stock Price and Trading Volume

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Abstract

Using weekly data from the Indian stock market, we examine the relationship between stock price and trading volume using a Markov Switching-Vector Error Correction Model (MS-VECM), where deviations from the long run equilibrium are characterized by different rates of adjustment depending on the state of a hidden Markov chain. We justify the use of nonlinearity by the Brock, Dechert, and Scheinkman (BDS) test and the information criteria. The long run dynamics are characterized by one cointegrating vector relating the price to trading volume. We find stock price is weakly exogenous. The MS-VECM with two regimes provides a good characterization of the Indian stock market and performs well relative to other linear and non-linear models.

Key Words: Trading Volume, Cointegration, Error Correction, Regime Shift, Markov switching.
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1 Introduction

Testing for nonlinearity in financial variables has increasingly become an important research area in recent years. A substantial amount of research in the market microstructure literature focused on the link between stock price and trading volume. Much of the existing literature, Tauchen and Pitts (1983), Lamoureaux and Lastrapes (1990), Gallant et al., (1992), assumes that the stock price-volume change relationship is monotonic and linear. Such linearity assumptions may not be true and possibly may lead to model mis-specification and ultimately result in unreliable inference. For example, Heimstra and Jones (1994) point out significant nonlinear dynamics between stock trading volume and prices which they cite as evidence against limiting the class of relationships of interest to the linear set. Given this, we in this paper, examine the nonlinear evidence on the stock price and volume relationship in the Indian stock market.

The traditional linear models do not allow the parameters to adjust for the structural changes. This assumption is inappropriate given that in the case of financial markets the number and composition of transactors, as well as the market microstructre and even the securities that are traded, are likely to change. Potential arbitrage opportunities implied by causality tests are unlikley to persist for more than few hours. For structural changes, previous studies adopt the Chow test or event study methodology. However, as pointed by Lamourex and Lastrapes (1990), failure to allow for regime shifts or structural changes leads to an overstatement of the persistence of the variances of a series. Given this, numerous studies have analyzed the relationships between variables using Markov switching framework. Kim (1993) used the state space model to analyze the relationships between inflation and inflation uncertainty. Krolzig and Toro (2000) and Krolzig et al. (2002) used the MSVECM to study the dynamic adjustment of employment and its relationship with the business cycle in the UK labor market. Bhar and Hamori (2004) used the Markov switching heteroscedasticity model to analyze the interaction between inflation rate and its uncertainty over both the short run and long run for G7 countries. Psarakadakis et al. (2004) used the Markov error correction models to analyze the dynamic relation between US stock price and dividends. Sarno and Valente (2005) proposes a VECM of stock returns that exploits the information in the future markets, while allowing for regime-switching behaviour and international spillovers across stock market indices.

Our paper, thus, can be viewed as an additional evidence examining the relationship between stock return and trading volume. Our paper contributes to the existing literature in the following way. To the best of our knowledge, this study serves as the first that adopts a Markov Switching-Vector Error Correction Model (MS-VECM) to estimate relationships between stock price and trading volume. The use of MS-VECM model is justified based on the changes related to rolling settlement in the Indian stock market as well as other major domestic and international events. Thus the result has implications regarding market efficiency and the effect of various changes in the Indian stock market on the stock price-volume relation. Second, the standard Vector Error Correction Model (VECM) model assumes a constant co-integration space. We relax this assumption by implementing a regime switching VECM that allows for shifts in both: the drift term and as well as in the long-run equilibrium. Third, using recently available data for individual stocks traded on the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE), we estimate a MS-VECM model and test this against the linear VECM. Using MS-VECM we are able to simultaneously estimate long run and short run dynamics. We document that two regime model with changing intercept and variances turns out to be good description of the Indian Stock Market.

The rest of the paper is organized as follows. Section 2 provides a literature review. In Section 3, we outline the data and its properties motivating the econometrics methodology. We describe the methodology in Section 4. Section 5 presents the estimation results. Finally we present our conclusion in Section 6.

2 Literature Review

The price-volume relationship depends on the rates of information flow and its diffusion to the market, the extent to which markets convey information, the size of the market and the existence of short selling constraints. While price change can be interpreted as the evaluation of new information, volume is an indicator to which the investors disagrees about this information. There are two stylized facts related to stock price and volume: (i) volume is relatively heavy in bull market and light in bear market implying positive correlation between volume and returns, (ii) it takes volume to make price moves implying a positive correlation between volume and the magnitude of return. Earlier research mainly focuses on the contemporaneous relationship between price changes and volume (Karpoff

(1987), Gallant et al. (1992)). As pointed out by Karpoff(1987)[1], empirical relations between price changes and volume can help to discriminate between different hypotheses about market structure, viz.: the mixture of distribution hypothesis and sequential information hypothesis.

The sequential information arrival models of Copeland (1976), Morse (1980), Jennings, Starks and Fellingham (1981) suggest that new information reaching the market is not disseminated to all participants simultaneously but to one trader at a time. The sequential information hypothesis supports that final market equilibrium is established only after a sequence of transitional equilibria. Therefore, due to the sequential information flow, lagged trading volume provides information on current absolute stock returns and lagged absolute returns contain information on current trading volume.

The second explanation between the casual relations between returns and trading volume is based on the mixture of distributions models of Clark (1973) and Epps and Epps (1976) which posits that stock returns and trading volumes are jointly dependent on the same underlying latent information flow variable. It suggests that price change and trading volume bear a positive relationship due to their joint dependence on a common event. However, in Clark's model, there is no causal relationship from volume to returns. Epps and Epps (1976) use volume to measure disagreement among traders because traders revise their reservation prices after the arrival of new information and greater disagreement among investors cause the expected level of trading volume to increase further. A positive causality from volume to absolute stock returns is predicted in their model. Campbell et al. (1993) propose a model where a set of "noise" traders cause changes in trading volume which market makers observe if their expected return is higher. Blume, et. al. (1994), He and Wang (1995), Chordia and Swaminathan (2000) all predict causal relations from volume to return volatility. The possibility of a feedback where price movements might cause further changes in volume may not be ruled out.[2] Rogalski (1978), Smirlock and Starks (1988), and Jain and Joh (1988) report evidence of unidirectional Granger causality from returns to trading volume in case of US markets. More recently, Chen et. al. (2001) examine the dynamic relation between returns, volume and volatility of stock indices for nine countries and find mixed results. Lee and Rui (2002) also demonstrate that returns

[1]Karpoff(1987) point out that the relationship between stock market return and volume is important for four reasons.

[2]See Hiemstra and Jones (1994), and Chen et al. (2001).

do Granger-cause trading volume in the US and Japanese markets but not in the UK market. They also show that trading volume does not Granger-cause stock market returns for the three stock exchanges.

Most of the above studies focus almost exclusively on the well-developed financial markets, usually the U.S. markets. Few exceptions exist: for example, Moosa and Al-Loughani (1995),[3] Basci et al. (1996) in case of Turkey, and Saatcioglu and Starks (1998) in case of Latin American countries.[4] The results obtained are again mixed in nature.

Most of the above-mentioned studies suffer from the linearity assumptions, with the exception of Heimstra and Jones (1994), Silvapulle and Choi (1999), Ratner and Leal (2001), Pant (2002) and Cetin (2002). Silvapulle and Choi documents presence of bi-directional linear and nonlinear causality between stock returns and volume changes in case of Korea. Pant (2002) found no evidence of linear or non-linear causality between returns and volume change in either direction using data from India.[5] Cetin (2002) finds significant bilateral non-linear causality between daily return and trading volume on the Toronto Stock Exchange (TSE) and points out that predictive power of volume for price variablity disappears after the full automation of the TSE.

Our study differs from the above in following ways: we examine the dynamic relationship between the two variables using not only the linear VECM but also using the MS-VECM framework. We also analyze the cointegration properties of the data using Johansen (1995) maximum likelihood procedure. We find the presence of one cointegrating relation between the stock price and the trading volume. We introduce the possibility of switches in the long-run equilibrium in a cointegrated VAR by allowing both the covariance matrix and weighting matrix in the error-correction term to switch. We find that two regime Markov Switching model with changing intercept and variance turns out to be good description of the data. We also obtain the evidence of price being weakly exogenous in the MS-VECM framework.

The hypothesis of structural change is reasonable in the Indian stock market. The Indian stock market had undergone changes in terms of clearing and settlement rules. On July 2001, all exchanges moved to compulsory rolling settlement (CRS)

[3] They examine the price-volume relation for four emerging Asian stock markets, namely, Malaysia, Philippines, Singapore and Thailand.

[4] They focus on Argentina, Brazil, Chile, Columbia, Mexico and Venezuela.

[5] This finding is true for rolling settlement period. We explain the rolling settlement and the associated concepts later.

for the largest stocks in the country. Earlier the BSE and the NSE followed two different settlement weeks (Monday to Friday in case of the BSE and Wednesday to next Tuesday in case of the NSE). This provided arbitrage opportunity for big players as they used to shift their position from one exchange to another depending on the end of settlement week. In the CRS, traders/buyers cannot carry forward their position to the next day. Till 30th June 2001, the trades carried out were settled by payment of money and delivery of securities in the following week. In CRS, the trading period (T) is one day and obligations have to be settled on the 5th working day (in case of $T + 5$ rolling settlement scheme). Typically a trade done on Monday would be settled on the following Monday. As an immediate response to the CRS, the liquidity dropped sharply in 2001 with impact cost going up by one percent, however, the fruits of this important reform were visible in 2002 when the trading volume attended the highest level. With a view to meet the best international trading practices, the Securities Exchange Board of India further directed the stock exchanges that trades in all scrips with effect from April 1, 2002 should be settled on $T+3$ basis. Later on, the settlement cycle was further shortened from the existing $T + 3$ settlement cycle to $T + 2$ settlement cycle with effect from April 1, 2003.

3 Data and Summary Statistics

In this paper, we have used the data from the Indian Stock market. The data used in the study are based on time series of daily trades and price data for individual stocks listed in the Bombay Stock Exchange **(BSE)** and the National Stock Exchange **(NSE)** during the period 1996-2003. We select the stocks based on the number of trading days. The data for the individual stocks listed in BSE and NSE are collected from PROWESS database provided by the 'Center for Monitoring Indian Economy' (CMIE). For our analysis we have taken only those stocks which have traded at least 75% of the total trading days.[6] Thus, we got 591 stocks for the analysis of BSE and 656 stocks for the analysis of NSE. We have constructed the weekly turnover and the price series using the time aggregation procedures. In total we get 396 weekly data points for both the BSE and the NSE. Instead of focusing on the behavior of the time series of individual stocks' volume, we focus on value weighted turnover and price series. The portfolio turnover (price) is

[6] Our results reported in the paper remains qualitatively unchanged if we use data for at least 90% of the total trading days or at least 60% of the total trading days.

the weighted average of the individual stock turnover (price) for the stocks that comprises the portfolio.

Figure 1 and 2 graphically display the time series of weekly value-weighted turnover (and, its first difference) and price(and, its first difference (return)) for our BSE and NSE portfolio respectively. As documented in Lo and Wang (2000) and in many other studies, aggregate turnover series seems to be non stationary. The value weighted turnover has increased dramatically until mid-2001, with a fall following the policy change related to settlement, followed by a slow increase again after mid-2001. The growth from 1996 through mid-2001 may be partly due to technological innovations that have lowered transaction costs. The year 2001-02 started in the backdrop of market turbulence. The volumes declined in the first quarter of 2001-02 following decisions affecting several structural changes in the market that included a shift to rolling settlement (initially in respect of major securities on $T + 5$ basis in July 2001, later for all securities) and ban on short sales. Such changes are usually accompanied by fall in volume.[7] We also observe a sharp increase in the price series for both the exchanges for some short periods (early 2000). Given this, we claim that the data generating process for the trading volume and the price series would thus be characterized by changing means implying different regimes. This is important because changing regimes is a source of nonlinearities in time series process.

Table 1 reports various summary statistics for the series over the sample period for BSE and NSE portfolio respectively. The empirical distributions of the turnover series and the price series are positively skewed and are not normal.[8] We also report the first 12 autocorrelations of turnover series and the price series and the corresponding Ljung-Box Q-statistics for $12th - order$ in Table 1. Both series are characterized by highly persistent behavior with autocorrelations decaying very slowly. This slow decay suggest some kind of non-stationarity.[9] This further motivates the use of Markov-switching heteroscedasticity model. The structural changes could effect the estimation of financial variables, hence time-varying pa-

[7]It has been shown that adverse information is incorporated into price more slowly when short sales are prohibited and consequently fewer trades takes place.

[8]The Jaque-Bera test rejects normality. We have also used Kernel density estimation in this context. The results, although not reported, however available on request, confirms the inferences based on the summary statistics. The presence of fatter tails in the kernel density functions also provides motivation for the use of regime switching model.

[9]These results are important for our study given the sample employed encompasses periods of significant changes relating to clearing and settlement, so that the presence of structural breaks cannot be ruled out apriori. We later in the paper have shown this.

rameters are assumed to capture structural changes.

4 Markov switching Model

The significance of models with structural change in economic series was proposed by Hamilton (1989) to analyze the growth rate in US GNP. Several extensions and generalizations have been further discussed in the literature. [10] The Markov switching models relax the restrictive assumption that all the observations on a particular series are drawn from a Gaussian distribution with constant mean and variances throughout the sample period. The basic idea idea is to decompose a series into a finite sequences of distinct stochastic processes, or regimes. The parameters of the underlying DGP of the observed time series y_t depend upon the unobservable regime or state variable s_t, which represent the probability of being in a different state of the world. As the state variable s_t cannot be directly observed, its realization is governed by a Markov Chain. The main idea behind such kind of process is to have a mixture of distributions with different characteristics. For our empirical applications, it might be more helpful to use a more generalized model where the autoregressive parameters, as well as the mean are regime dependent and where the error term is heteroskedastic. The Markov Switching Vector Autoregression (MS-VAR) model allows for a variety of specifications and it can be considered as generalizations of the basic finite order VAR model of order p. The MS-VAR model characterizes a non-linear DGP as piecewise linear by restricting the process to be linear in each regime (or, state), where the regime is itself unobservable. Consider the $p-th$ order autoregression for the K-dimensional time series vector $y_t = (y_{1t} \dots y_{Kt})^{'}$, $t \dots T$

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \qquad u_t \sim IID(0, \Sigma) \tag{1}$$

Denote $A(L) = I_k - A_1 L - \dots - A_p L^p$ as the $(K \times K)$ dimensional lag polynomial. We assume that the roots lie outside the unit circle$\mid A(z) \mid \neq 0$ for $\mid z \mid \nleq 1$ where L is the lag operator, so that $y_{t-k} = L^j y_t$. If we assume that error term is normally distributed, $u_t \sim NID(0, \Sigma)$, equation(1) is known as the intercept form of a stable Gaussian VAR(p) model. We can re parameterize equation (1) into the following

[10] See Kim and Nelson (1999) in this context.

mean adjusted form of a VAR model:

$$y_t - \mu = A_1(y_{t-1} - \mu) + \cdots + A_p(y_{t-p} - \mu) + u_t \tag{2}$$

where $\mu = (I_K - \sum_{j=1}^{p} A_j)^{-1}\nu$ is the $(K \times 1)$ dimensional means of y_t. However if the time series are subjects to shifts in regime, the stable VAR might be restrictive. Hence to accommodate regime-switching framework we assume that the parameters of the underlying DGP of the observed time series vector y_t depend upon the unobservable variable s_t. The unobservable realization of the regime $s_t \in (1 \dots M)$ is governed by a discrete time, discrete state Markov stochastic process which is defined by the following transition probabilities:

$$p_{ij} = Pr(s_{t+1} = j \mid s_t = i), \qquad \sum_{j=1}^{M} p_{ij} = 1 \forall i, j \in (1 \dots M). \tag{3}$$

We assume that s_t follows an irreducible ergodic M state Markov process with the transition matrix given by:

$$\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ . & . & \cdots & . \\ p_{i1} & p_{i2} & \cdots & p_{iM} \end{pmatrix}$$

where $p_{iM} = 1 - p_{i1} - \ldots - p_{i,M-1}$ for $i = 1, \ldots, M$.

Thus we can extend the equation (2) to Markov-switching vector autoregression of order p and M regimes:

$$y_t - \mu(s_t) = A_1(s_t)(y_{t-1} - \mu(s_{t-1})) + \cdots + A_p(s_t)(y_{t-p} - \mu(s_{t-p})) + u_t \tag{4}$$

where $u_t \sim (0, \Sigma s_t)$ and $\mu(s_t)$, $A_1(s_t)$,...., $A_p(s_t)$, Σs_t are shift functions describing the dependence of the parameters on the realized regimes s_t. However in equation (4), there is an immediate one time jump in the process mean as we move from one state to another. Hence, it may be more reasonable to assume that the mean of the process approaches to new level smoothly after the transition from one state to another and in such a situation we formulate the following model with regime dependent intercept term $\nu(s_t)$:

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \tag{5}$$

It should be noted that the mean adjusted form (in equation (4)) and the intercept form (in equation (5)) of an $MS(M)-VAR(p)$ model are not equivalent. While a permanent regime shift in the mean $\mu(S_t)$ causes an immediate jump of the observed time series onto its new level, corresponding shift in the intercept term $\nu(S_t)$ causes a shift in the white noise error series u_t. Given this, the most general specification of an $MS-VAR$ model where all the parameters are conditioned on the state s_t of the Markov chain can be expressed as: :

$$y_t = \begin{cases} \nu_1 + A_{11}y_{t-1} + ... + A_{p1}y_{t-p} + \Sigma_1^{1/2}u_t & \text{if } s_t = 1 \\ \nu_M + A_{1M}y_{t-1} + ... + A_{pm}y_{t-p} + \Sigma_M^{1/2}u_t & \text{if } s_t = M \end{cases}$$

The above formulation allows for a great variety of specifications and following the notation for each model due to Krolzig (1997), we specify the general $MS(\mathbf{M})$ term as: M for Markov-switching mean, I for Markov-switching intercept term, A for Markov-switching auto-regression parameters, H for Markov-switching heteroscedasticity. The unknown parameters are estimated by maximum log likelihood function via Expected Maximum (EM) algorithm.

4.1 Markov Switching Vector Error Correction Model

To analyze the relationship between multiple time series variables, we now define a Markov-switching vector error correction model(MS-VECM). A MS-VECM is a vector error correction model with shifts in some of parameters. For our analysis, we concentrate on MSIH-VECM (See Krolzig, 1997) where MSIH refers to a Markov-switching Intercept Heteroscedasticity and VECM refers to Vector error correction model. The MSIH-VECM exhibits error correction mechanism: errors arising from regime shifts themselves are corrected towards the stationary distribution of the regimes. The assumed properties of the Markov chain have important implications for the analysis of the long-run properties of the system. Markov switching error correction model account for periods of temporary divergence from the long run equilibrium relationship. Following Krolzig (1997), we consider the VECM for the I(1) variables :

$$\Delta x_t = \nu(s_t) + \alpha(s_t)(\beta x_{t-1}) + \sum_{k=1}^{p-1} \Gamma_k(\Delta x_{t-k}) + u_t \qquad (6)$$

where Δx_t is an m-dimensional vector of differenced variables of interest, $\nu(s_t)$ is regime dependent intercept term, Γ_k are parameter matrices and the error variance is allowed to change across states $u_t \sim (0, \Sigma(s_t))$. As in (3), the unobservable regime variable s_t is governed by a Markov chain with a finite number of state defined by the transition probabilities p_{ij}. Here, $\alpha(s_t)$ is the matrix of adjustment parameters and β is the matrix of long run parameters(cointegrating vectors). Each regime is characterized by a particular attractor defined by $\delta(s_t)$ and $\mu(s_t)$:

$$\Delta x_t - \delta(s_t) = \alpha(\beta x_{t-1} - \mu(s_t)) + \sum_{k=1}^{p-1} \Gamma_k(\Delta x_{t-k} - \delta(s_t)) + u_t \qquad (7)$$

Here, both Δx_t and βx_t are expressed as deviations about their regime and time dependent means $\delta(s_t)$ and $\mu(s_t)$. For an ergodic and irreducible Markov chain, regime shifts are persistent (if $p_{ij} \neq p_{ii}$ for some i,j) but not permanent (if $p_{ii} \neq 1$ for all i). We now estimate the MSIH-VECM in (7) for the analysis of our price and turnover series.

5 Estimation Results

Following the two-stage procedure by Krolzig (1997), we start our analysis with the linear VECM using maximum likelihood techniques in the first stage. In the second stage given the estimated cointegrated matrix, we estimate the MS-VECM using the EM algorithm. The section is divided in three parts: sub-section 1 reports the results from the cointegration analysis, where as the results from Markov Switching VECM model is presented in the next sub-section.

5.1 Cointegration analysis

We employ a three-step procedure in our empirical analysis. First, unit root tests are undertaken to see if price and volume series are integrated of order one. Then, level regressions are performed to test whether price and volume series are cointegrated. The cointegration properties of the data are also studied within a linear VAR representation using the maximum likelihood procedure of Johansen (1995). Finally, lagged values of the residuals from the level regression are utilized in the error correction models for price and volume series.

We test for the stationarity of the log transformation of the two series using the Augmented Dickey Fuller (ADF) test, and the Kwaitkowski et al. (KPSS) (1992), test. However, the ADF test is not reliable when the sample period has structural breaks and failure to consider it properly can lead to erroneous conclusions. Given the evidence of structural break, we have also performed Zivot-Andrews (1992) unit root test allowing for a single break both in intercept and trend.[11] The volume and price series are found to be non-stationary. Next, Johanen's cointegration analysis was applied to a VAR with 4 lags(BSE) and 2 lags(NSE) as follows:

$$x_t = \mu + \sum_{i=1}^{p} A_i x_{t-i} + u_t \tag{8}$$

where $x_t = [p_t : v_t]$. The results of the Johansen's cointegration tests are shown in Tables 2 and 5. We accept the hypothesis of one cointegrating relationship at the 2.5 percent level for the BSE and at 1 percent level for the NSE. Hence, the cointegration test based on Johansen's procedure provide empirical support for long run relationship between stock prices and trading volumes. To estimate the cointegrating vector, we use fully modified least square (FMOLS) method of Phillips and Hansen(1990) (henceforth, PH). The estimated cointegrating vector for the BSE and the NSE are reported in Tables 3 and 6 respectively. For robustness of our findings in terms of presence of cointegration, we also report Z_α, Z_t and P_Z residual based tests for cointegration due to Phillips and Oularis (1990) in Tables 3 and 6 respectively. The results support the findings of the Johansen cointegration test.

Given that the variables are cointegrated, we next estimate the linear VECM model.[12] Tables 4 and 7 report the estimates of the linear VECM for the BSE and the NSE respectively. From the tables we notice that the coefficient of the error component term for the price equation is insignificant for both the BSE and the NSE implying that price seems to be weakly exogenous.[13]

Given our main motivation of the paper, we perform a test of nonlinearity on the residuals of the linear VECM following Brock et al. (BDS). The BDS test [14] confirms the presence of non-linearity in the residuals. We also report the

[11] We find the series to be non-stationary subject to a break at March 2001 corresponding to the ban on short sales. The results of the tests are not reported here but available from the author on request.

[12] We have used the lagged residual term estimated using the FMOLS procedure in the VECM model. The optimal lag order in VECM have been chosen using the AIC.

[13] We also perform the linear Granger causality test. We find no evidence of linear causality in either direction for both the stock markets. The results of Granger causality tests are not reported but are available from the author on request. We also carried out the modified Baek and Brock (1992) test to examine the nonlinear causality relationship. We find the absence of nonlinear Granger causality for the two variables. Results of nonlinear Granger causality are not reported here but are available from the author on request.

[14] We perform the BDS test with embedding dimension equal to 2 and metric bound equal to

outcomes of the test of parameter instability for the linear VECM due to Hansen (1992a), Andrews (1993) and Andrews and Ploberger (1994) as outlined below respectively[15]:

$$AvgLR = \int_{\omega_1}^{\omega_2} LR(\omega)\, d\omega. \tag{9}$$

$$ExpLR = ln \int_{\omega_1}^{\omega_2} exp[LR(\omega)]/2\, d\omega. \tag{10}$$

$$SupLR = sup_{\omega \in (\omega_1, \omega_2)} LR(\omega) \tag{11}$$

where $LR(\omega)$ stands for the LR statistics for a single break at a fraction ω through the sample.[16]

Tests of parameter instability yield evidence against linear VAR models for both the BSE and the NSE. Hence to account for the non-linearity in the variables, the cointegrating vectors are estimated using the MS-VECM framework in the next section.

5.2 MS-VECM

On the basis of the evidence against the linear VECM reported in the previous subsection, we proceed to estimate a MS-VECM to examine the relationship between stock prices and trading volume series. We have used a combinations of tests to find the correct specification of the Markov Switching model.[17] To determine the number of regimes, we have used the tests based on information criteria (AIC/HQ). We have estimated MSIH-VECM with 2 regimes and 3 (1) lags models for BSE (NSE), with shift in the intercepts and in the error variances. The models

the standard deviations of the residuals.

[15]The tests are based on functions of the sequence of LR (Likelihood Ratio) statistics that tests the null hypothesis of parameter stability against the alternatives of a one time break at all possible break-points in the sample period under study.

[16]The tests has been implemented with $\omega_1 = 1 - \omega_2 = 0.15$In our analysis the estimated fraction was found to be observation number 223 (March 2000), which corresponds with the budget announcement by the Union Finance Minister.

[17]The main problem with the determination of the appropriate specification for a Markov Switching model is the determination of the number of regimes. Tests used to determine the null hypothesis of $n-1$ regimes against the alternative hypothesis of n regimes do not have a standard distribution, since the null hypothesis is not identified due to the presence of nuisance parameters, hence the likelihood ratio test is not valid.

are estimated by using EM algorithm.[18] The resulting model is :

$$\Delta x_t = \nu(s_t) + \alpha(\beta x_{t-1}) + \sum_{k=1}^{p-1} \Gamma_k(\Delta x_{t-k}) + u_t \tag{12}$$

where $\Delta x_t = [\Delta p_t, \Delta v_t]$ and $\alpha\beta$ is the mean adjusted error correction term and $u_t \sim (0, \Sigma(s_t))$. A comparison of the log-likelihood, the AIC and the HQ criteria for the MS-VECM and its linear counterpart supports the significance of the regime shifts. The estimated parameters along with the t-statistics of the MSIH(2)-VECM model are presented in Table 8 and Table 9 for BSE and NSE respectively. From the Tables, it is clear that not only the estimated intercepts differ across regimes, but are also changes in variances. Moreover the correlations between the variables conditional on the past, differ across regimes. Note that the error correction term is not significant with the price regressions, however, the trading volume partially adjusts towards the equilibrium. The resulting regime probabilities for both the stock markets are plotted in Figures 3 and 4. The filtered probability represents the conditional probability based on the information contained in he information set and observed up to date t. The smoothed probability, on the other hand, represents the conditional probability based on the information available throughout the whole sample at future date T. In markov switching models, the classification of the regimes and dating amounts to assigning every observation to a regime s_t. At each point in time, the smoothed regime probabilities are calculated. In the case of two regimes, the classification rule simplifies to assigning the observation to the first regime if $Pr(s_t = 1 \mid Y_T) > 0.5$ and to the second if $Pr(s_t = 1 \mid Y_T) < 0.5$.

If we look at the regime probabilities, we observe almost similar regime properties for both the stock markets. For both the stock markets, regime 1 characterizes the period of episodes (events) mainly like February-March 1999 (Finance Minister Mr. Jaswant Sinha's second budget), September/October 1999 to October 2000 (consisting of major events like the general elections in 1999, rising oil prices), April 2001 (ban on short sales and introduction of rolling settlement), September 2001 (World Trade Center Disaster), and March-April 2003 (changes in rolling settlement from $T + 3$ to $T + 2$, and US led March on Iraq). Such events and macro-economic announcements are said to have important impact on the Indian stock market which have been captured in regime 1. Note that these are domestic as well as international events. Regime 2 depicts the remaining stable period.

[18]We have used MS-VAR software by Hans Martin Krolzig in OX language.

Table 10 and 12 report the transition probabilities for the BSE and the NSE respectively where as Table 11 and 13 shows the number of observations in each regimes along with the unconditional probability (or ergodic probability) and the half-life or expected durations for each regime. The ergodic probability of being in state $s_t = 1$ is given by :

$$\pi_1 = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} \tag{13}$$

Expected duration of the first regime is calculated as :

$$d_1 = \frac{1}{1 - p_{11}} \tag{14}$$

Similarly we have calculated the ergodic probability π_2 and expected duration d_2 for the second regime. We note that the unconditional probability of being in state $s_t = 1$ is 0.22 (0.29) for the BSE (the NSE) and regime 1 has estimated durations of 14.8 (18.55) weeks for the BSE (the NSE).

Given the above findings, we have also performed the regime classification measure recently by Ang and Bekaert (2002) to ascertain the performance of the model. We define this for m regimes as:

$[RCM(m) = 100m^2 \times 1/T \sum_{t=1}^{T} \prod_{i=1}^{m} p_{i,t}]$, where $p_{i,t}$ is the ex-ante smoothed regime probability. Good regime classification is associated with a low value of the measure. The regime classification measure stands at 13.9 for the BSE and 15.3 for the NSE.[19] In sum, we conclude that the use of MS-VECM is appropriate for the Indian stock market to examine the relationship between price and volume.

6 Conclusion

Using the the Markov Switching model, this paper have examined the joint dynamics of stock price and trading volume. To test for Markov switching model, we have utilized two-step approach which involves first testing for cointegration under the assumption of linear adjustment and then testing for Markov switching behavior in the dynamics of the error-correction model. Thus, we first investigated linear relationships between two variables using the standard VECM. Results of the Johansen's technique support the evidence of one cointegrating vector. Using

[19] Additional evidence in favor of the MS-VECM is provided by the statistical properties of the normalized residuals in Figures 5 and 6. We find the residuals to be non-autocorrelated, homoscedastic, and normally distributed.

standard linear VECM, we find the evidence of stock price being weakly exogenous for both the stock market. However, the test on residuals from the linear model support the presence of nonlinear pattern in the two variables. To control for nonlinearity, we implement a regime switching VECM that allows for shifts in both the drift and the long-run equilibrium and tested this model against the linear VECM. Regime switching VECM model thus accounts for situations where adjustment towards the long-run equilibrium occurs at all time, but at different rates under the two regimes. The evidence of stock price being weakly exogenous for both the stock market is found in the Markov switching case. We have demonstrated that a Markov-switching model of the joint process of the stock price and trading volume seems to be a well suited tool for the Indian stock market.

7 References

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Figure 1: BSE Trading Volume and Price(Levels and First Difference)

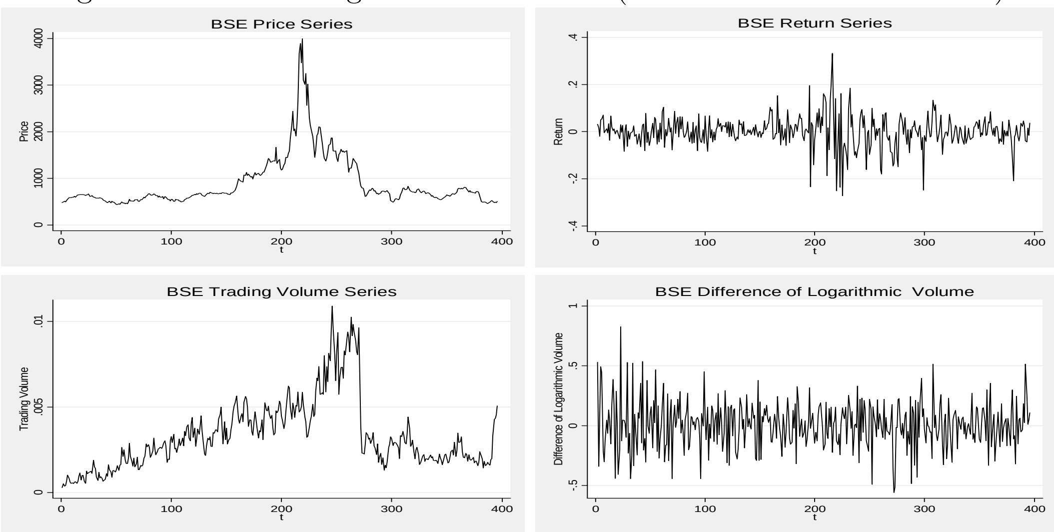

Figure 2: NSE Trading Volume and Price(Levels and First Difference)

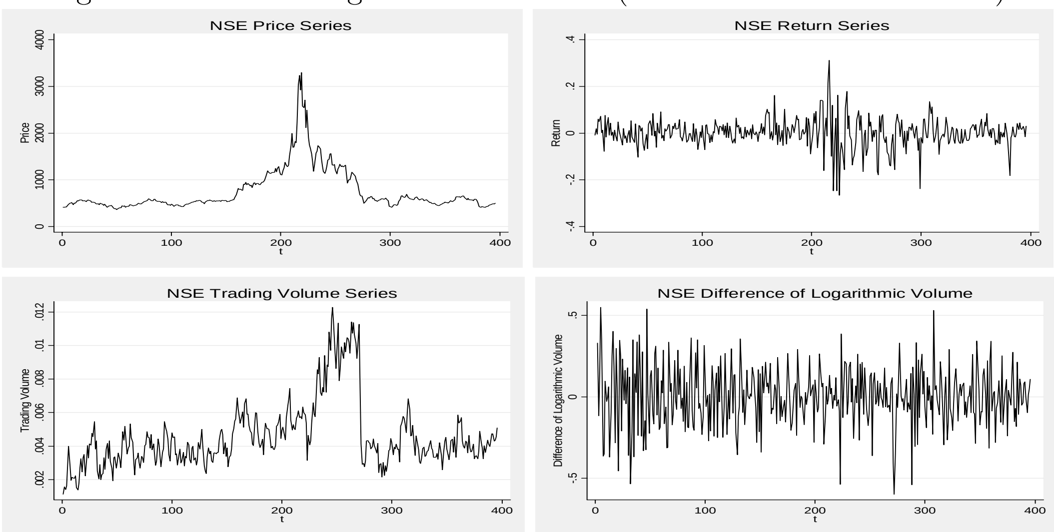

Figure 3: Estimated Probabilities (BSE)

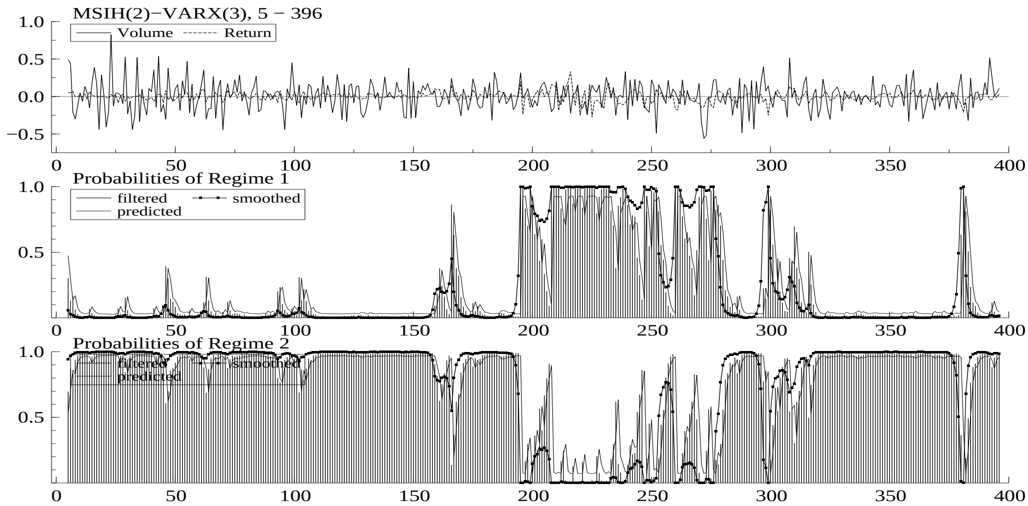

Figure 4: Estimated Probabilities (NSE)

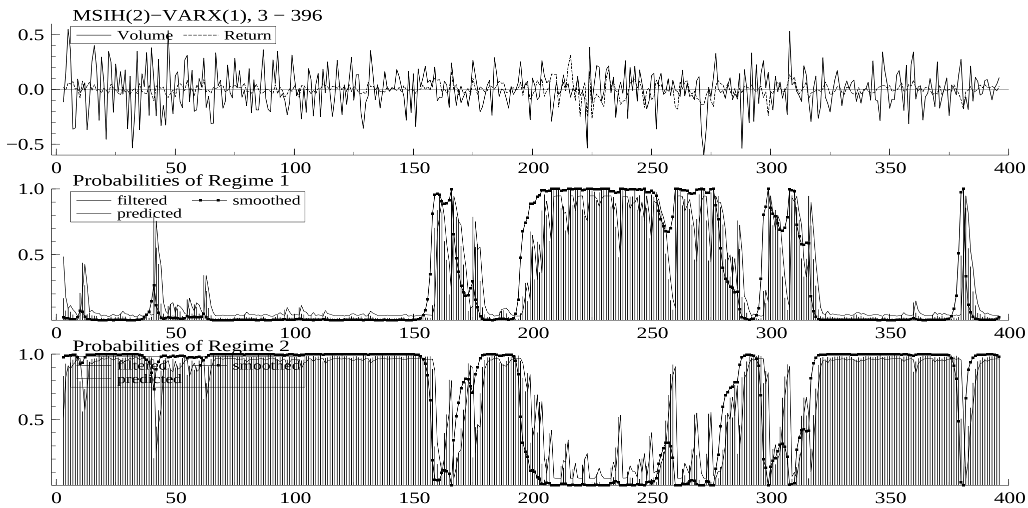

Figure 5: Statistical Properties of the normalized residuals(BSE)

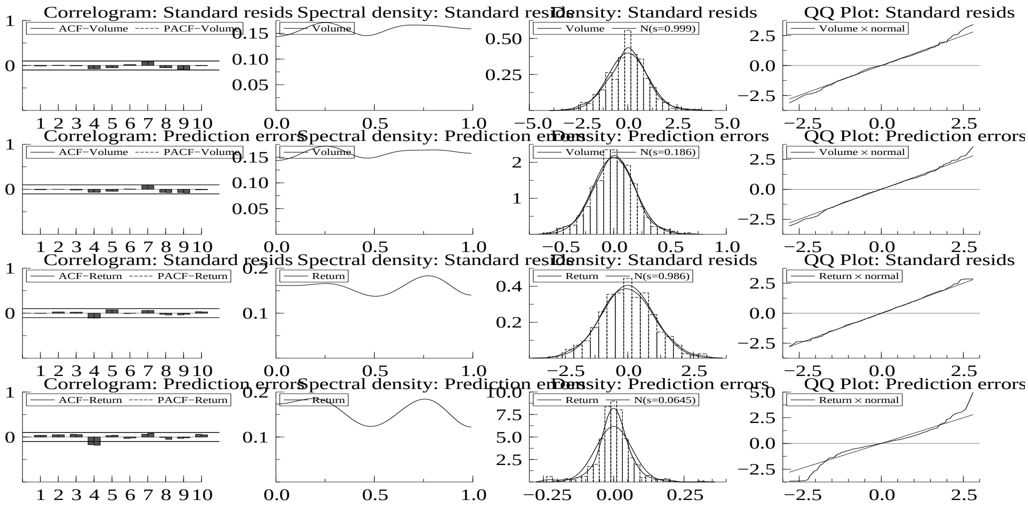

Figure 6: Statistical Properties of the normalized residuals(NSE)

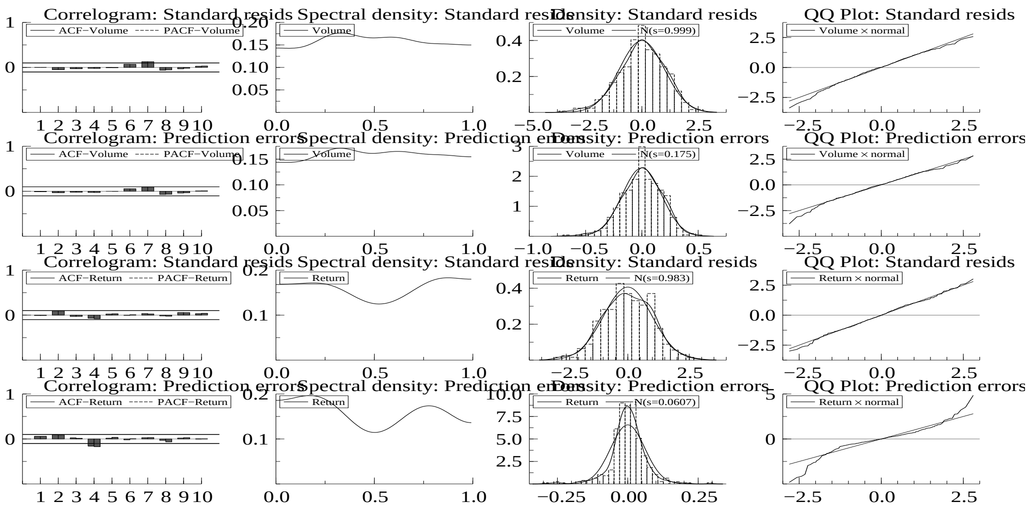

Table 1: **Summary Statistics**

Summary statistics for weekly Turnover and Price series of BSE and NSE stocks

	BSE		NSE	
Statistics	V_t	P_t	V_t	P_t
Mean	0.0033	898.9619	0.0046	753.8620
Std.dev	0.0020	561.3146	0.0020	465.8569
Skewness	1.2946 ‡	2.5700 ‡	1.6654‡	2.5632 ‡
Kurtosis	1.6800 ‡	8.1171 ‡	2.8391‡	8.0111 ‡
Jarque-Bera	157.1841‡	1523.0553 ‡	316.0513‡	1492.5555 ‡
Autocorrelations:				
ρ_1	0.9481	0.9768	0.9193	0.9790
ρ_2	0.9101	0.9558	0.8594	0.9587
ρ_3	0.8780	0.9273	0.8117	0.9334
ρ_4	0.8548	0.8938	0.7783	0.9012
ρ_5	0.8316	0.8757	0.7462	0.8821
ρ_6	0.8082	0.8537	0.7237	0.8609
ρ_7	0.7899	0.8363	0.6973	0.8405
ρ_8	0.7640	0.8148	0.6644	0.8190
ρ_9	0.7441	0.7919	0.6413	0.7974
ρ_{10}	0.7255	0.7709	0.6247	0.7755
ρ_{11}	0.7110	0.7483	0.6111	0.7564
ρ_{12}	0.7065	0.7301	0.6072	0.7401
Ljung-Box Q_{12}	3177.8308‡	3515.0256‡	2583.7600‡	3561.5069‡
Sample Period	Jan. 1996-July 2003	-	-	-

‡ indicates significance at 0.01 level of significance.

Table 2: **Johansen Cointegration Test(BSE)**

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistics Statistics	2.5 percent Critical Value	5 Percent Critical Value
None†	0.0348	18.07	17.52	15.41
At most 1	0.0106	4.18	4.95	3.76

‡ denotes rejection of the hypothesis at 2.5 percent level of significance.

Table 3: **Estimated Cointegration Vector and the Parameter Instability Test(BSE)**

PH	$ECM = v_t + 12.5163 - 0.9916 p_t$
Z_t	-3.8251
Z_{alpha}	-27.8003
P_Z	55.8650[5]
	Instability Test
SupLR	28.6770(0.0043)
ExpLR	9.4395(0.0304)
AvgLR	8.3133(0.2582)

For all tests, a constant is included in the regressions. PH refers to FMOLS due to Phillips and Hansen. p-values in parenthesis are due to Andrews.

Table 4: **ML estimation results for the Linear VECM Parameters (BSE)**

	Δv_t	Δp_t
Intercepts		
ν_1	0.0089 (0.9367)	0.0001 (0.0268)
Short run dynamics		
Δv_{t-1}	-0.1452 (-2.8077)	-0.0053 (-0.2972)
Δv_{t-2}	-0.0924 (-1.7872)	-0.0070 (-0.3908)
Δv_{t-3}	-0.0953 (-1.8987)	-0.0030 (-0.1742)
Δp_{t-1}	0.1907 (1.2814)	0.0995 (1.9377)
Δp_{t-2}	0.0605 (0.4074)	0.1367 (2.6658)
Δp_{t-3}	-0.0449 (-0.3013)	-0.0071(-0.1371)
Error correction		
ecm_{t-1}	-0.0694 (-3.2291)	0.0093 (1.2520)
Standard Errors		
σ	0.1936	0.0654
Correlation		
Δv_t	1.0000	0.1219
Δp_t	0.1219	1.0000
BDS Test	3.2804‡	8.0987‡
Loglik		626.847
AIC/HQ	-3.1013	-3.025

‡ indicates significance at 1 percent level of significance. t-statistics are given in parenthesis.

Table 5: **Johansen Cointegration Test(NSE)**

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistics	5 percent Critical Value	1 Percent Critical Value
None†	0.0851	37.98	15.41	20.04
At most 1	0.0075	2.95	3.76	6.65

‡ denotes rejection of the hypothesis at 5 and 1 percent level of significance.

Table 6: **Estimated Cointegration Vector and the Parameter Instability Test(NSE)**

PH	$ECM = v_t + 9.0089 - 0.54571 p_t$
Z_t	-3.7026
Z_{alpha}	-32.3805
P_Z	123.9825[5]
	Instability Test
SupLR	21.7910(0.0016)
ExpLR	5.8525(0.0128)
AvgLR	5.6081(0.0716)

For all tests, a constant is included in the regressions. PH refers to FMOLS due to Phillips and Hansen. p-values in parenthesis are due to Andrews.

Table 7: **ML estimation results for the Linear VECM Parameters (NSE)**

	Δv_t	Δp_t
Intercepts		
ν_1	0.0030 (0.3448)	0.0004 (0.1425)
Short run dynamics		
Δv_{t-1}	-0.0625 (-1.2402)	-0.0027 (-0.1574)
Δp_{t-1}	0.0606 (0.4139)	0.1704 (3.3683)
Error correction		
ecm_{t-1}	-0.1845 (-5.9440)	-0.0002 (-0.0162)
Standard Errors		
σ	0.1844	0.0612
Correlation		
Δv_t	1.0000	0.1478
Δp_t	0.1478	1.0000
BDS Test	2.6986‡	7.5651‡
Loglik		681.2933
AIC/HQ	-3.4025	-3.3585

‡ indicates significance at 1 percent level of significance. t-statistics are given in parenthesis.

Table 8: **ML estimation results for the MS(2)-VECM Parameters (BSE)**

	Δv_t	Δp_t
Regime Dependent Intercepts		
ν_1	0.0094 (0.4415)	-0.0141 (-0.9960)
ν_2	0.0087 (0.7858)	0.0042 (1.6317)
Short run dynamics		
Δv_{t-1}	-0.1424 (-2.6981)	0.0154 (1.2608)
Δv_{t-2}	-0.0907 (-1.7514)	-0.0064 (-0.5256)
Δv_{t-3}	-0.0950 (-1.9081)	-0.0131 (-1.1170)
Δp_{t-1}	0.1880 (1.2784)	0.0483 (0.9078)
Δp_{t-2}	0.0599 (0.4102)	0.1012 (2.1283)
Δp_{t-3}	-0.0420 (-0.2854)	-0.0561 (-1.0904)
Error correction		
ecm_{t-1}	-0.0697 (-3.2724)	0.0066 (1.2647)
Standard Errors		
σ_1	0.1824	0.1145
σ_2	0.1870	0.0382
Regime 1 correlation		
Δv_t	1.0000	0.0767
Δr_t	0.0767	1.0000
Regime 2 correlation		
Δv_t	1.0000	0.2200
Δp_t	0.2200	1.0000
Loglik/RCM	707.7626	13.9027
AIC/HQ	-3.4784	-3.3740

t-statistics are given in parenthesis.

Table 9: **ML estimation results for the MS(2)-VECM Parameters (NSE)**

	Δv_t	Δp_t
Regime Dependent Intercepts		
ν_1	0.0277 (1.6378)	-0.0043 (-0.4532)
ν_2	-0.0073 (-0.6683)	0.0023 (1.0728)
Short run dynamics		
Δv_{t-1}	-0.0412 (-0.8197)	0.0298 (2.6189)
Δp_{t-1}	0.0201 (0.1415)	0.0832 (1.6091)
Error correction		
ecm_{t-1}	-0.1955 (-6.1348)	-0.0079 (-0.9313)
Standard Errors		
σ_1	0.1700	0.0993
σ_2	0.1754	0.0332
Regime 1 Correlation		
Δv_t	1.0000	0.0715
Δp_t	0.0715	1.0000
Regime 2 Correlations		
Δv_t	1.0000	0.2843
Δp_t	0.2843	1.0000
Loglik/RCM	764.006	15.2818
AIC/HQ	-3.7868	-3.7148

t-statistics are given in parenthesis.

Table 10: **Matrix of Transition Probabilities(BSE)**

	Regime 1	Regime 2
Regime1	0.9281	0.0719
Regime2	0.0209	0.9271

$p_{ij} = Pr(s_{t+1} = j|s_t = i)$

Table 11: **Regimes and Duration(BSE)**

	No. of obs.	Ergodic Probability	Duration
Regime 1	89.3	0.2251	13.9
Regime 2	302.7	0.7749	47.86

Probability is the unconditional one.

Table 12: **Matrix of Transition Probabilities(NSE)**

	Regime 1	Regime 2
Regime1	0.9468	0.0532
Regime2	0.0220	0.9780

$p_{ij} = Pr(s_{t+1} = j|s_t = i)$

Table 13: **Regimes and Duration(NSE)**

	No. of obs.	Ergodic Probability	Duration
Regime 1	115.9000	0.2922	18.7900
Regime 2	278.1000	0.7078	45.5200

Probability is the unconditional one.