DO FINANCIAL MARKETS EXHIBIT CHAOTIC BEHAVIOUR? EVIDENCE FROM AN EMERGING ECONOMY

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Abstract

Chaotic deterministic models with sensitive dependence on initial conditions provide a powerful tool in understanding the apparently random movements in financial data. This study examines four financial markets in India, an emerging economy, for possible chaotic behavior. We employ four tests, viz. the BDS test on raw data, the BDS test on pre-filtered data, Correlation Dimension test and the Brock's Residual test. The financial markets considered are the stock market, the foreign exchange market, the money market and the government securities market. The results from these tests provide very weak evidence for the presence of chaos in Indian financial markets.

Keywords: Chaos, non linear dynamics, correlation dimension.

JEL Classification: G10, G14

1. Introduction

The bedrock of numerous studies in modern investment analysis is the efficient market hypothesis which conceptualizes the stock market as a stream of stochastic news that moves prices in random directions. Yet investors continue to exploit this "efficient" market to reap rich rewards. The emerging field of chaos theory provides valuable insights into understanding processes which, despite apparent randomness, have an inherent order. Chaos theory, which is the study of complex non linear dynamical systems, is a new tool made available to researchers looking for deterministic patterns in apparently random series of data like the time series data of a stock market index (Trippi, 1995). Chaos refers to bounded steady-state behavior that is neither a point equilibrium nor periodic or quasi-periodic (Barkoulas and Travlos, 1998). The distinctive feature of chaotic processes is sensitive dependence on initial conditions (popularly known as the Butterfly effect) which means that two points in a system that have very similar initial conditions may exhibit substantially different trajectories.

Most traditional tests to detect linear or non-linear deterministic patterns in series fail to detect the presence of chaos. One reason for this is the breakdown of forecasting capability in chaotic series beyond limited time periods. But a range of tests developed by researchers over the last two and a half decades for detection of chaos in mathematical and physical sciences, when suitably modified, can be advantageously applied to study the presence of chaos in stock markets. The advantage of these tests is that they not only detect chaos if it is present; even in its absence they provide insight into the deterministic (linear and non-linear) or stochastic nature of the time series under consideration. Thus these tests can serve as valuable tools in understanding the order, if any, in large time series data.

The search for chaos in financial markets has been mostly restricted to stock markets and that too in developed countries (e.g. Scheinkman and LeBaron, 1989; Copeland et. al., 1995; Olmeda and Perez, 1995). However given the very different institutional features of financial markets in developing countries, it is important to explore the possibilities of such markets exhibiting chaotic behavior. Financial markets in developing countries are relatively less mature and deep as compared to those in developed countries, and the implications of complex non linear behavior could be significant for traders, institutional investors as well as policy makers. This paper undertakes a case study of India, an emerging economy, and employs several tests to detect chaos in various types of financial market, viz. the stock, forex, bond and money markets. While this is one of the very few studies on chaos for a developing economy, it is to our knowledge the first study that analyzes several financial markets employing a battery of tests for chaos.

The motivation for undertaking this study is not only the dearth of research in this domain but also the potential implications of such a study for players in these markets. Detection of a deterministic model would mean an opportunity for hedgers, speculators as well as arbitrageurs to play the markets better. Given the fact that the Indian growth story has started unfolding in the last few years and India is currently one of the fastest growing economies in the world today, most of India's financial markets are booming and the implications of such modeling are indeed immense.

The paper is organized as follows. Section 2 introduces certain basic concepts in chaos. Section 3 discusses the extant evidence in the empirical literature. Section 4 outlines the tests used in the paper for detection of chaos and Section 5 introduces the data. This is followed by Section 6 that presents and discusses the results obtained. Finally, section 7 concludes the paper.

2. Detecting Chaos: Some Basic Concepts

The equilibrium state to which a system evolves over time is called its attractor. A *point attractor* exists if a system's equilibrium tends to a point. A falling pebble or a damped pendulum is a common instance of a system with a point attractor. In case a *limit cycle attractor* exists, the system, over time, tends to a repeating sequence of states – a periodic orbit. A simple pendulum with periodic replenishments of energy is an instance of such a system.

While the above two types of attractors are easy to detect, a third type of attractor, the *chaotic attractor*, has generated a lot of interest in recent times. Any chaotic time-series has chaotic attractors. Thus detecting chaotic attractors would indicate the presence of chaos. With a chaotic attractor, equilibrium applies to a region, rather than a particular point or orbit; equilibrium becomes dynamic (Peters, 1991). The chaotic attractor is a set of states such that if a system starts with its initial condition in the attractor's basin of attraction, it eventually ends up in the set. Also once a system is on an attractor, nearby states diverge from each other exponentially fast. Thus any noise or error in measurement gets amplified rapidly and beyond a point the system becomes unpredictable. And most often, chaotic attractors display elegant symmetric structures with self-similarity at different scales (fractals).

In more formal terms, a map has a chaotic attractor if it displays a sensitive dependence to initial conditions or has at least one positive Lyapunov exponent (discussed later). These attractors have certain unique characteristics that make their empirical detection possible.

Firstly, they exhibit a non-integer fractal dimension[1]. Algorithms have been developed to measure the fractal dimension of an attractor directly, e.g. Box counting algorithms, or its substitute measures, most notably, the correlation dimension measure developed by Grassberger and Procaccia (1983**).** The tests that we employ in this paper are based on the latter.

Secondly, the sensitive dependence on initial conditions can be checked by measuring the Lyapunov exponents of the attractor. This exponent of a dynamical system characterizes the rate of separation of infinitesimally close trajectories (Trippi, 1995). Further, topological approaches developed to detect chaos analyze the organization of a chaotic attractor and the mechanisms (stretching and compressing) responsible for its existence.[2]

3. Existing Evidence

Scheinkman and LeBaron (1989) was the first attempt at applying the tools of non-linear dynamics to stock market returns.[3] They reported some evidence of chaos in daily and weekly stock returns for U.S. markets. Empirical tests on the existence of deterministic chaos in economic series have proliferated only in recent years (see Sayers, 1991 for a survey) and there has been a surfeit of studies of chaos in stock markets in various countries. Willey (1992) studied the daily closing prices of the S&P Composite Index and the NASDAQ-100 Index but failed to detect any deterministic chaos. Pandey et. al. (1998) do not find low-dimensional deterministic chaos in five major European and U.S. stock markets. Copeland et. al. (1995) provide evidence for non linearity in the U.K. FTSE-100 Index and employ Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models to explain some of the non linearity.

Compared to the number of studies of chaos in stock markets, research for chaos in the other

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[1] Imagine a square decomposed into 4 self-similar, identical sub-squares. Each sub-square would need to be magnified by a factor of 2 to get the original square. Intuitively, the ratio 4 / 2 is the fractal dimension of the square. For a formal definition of fractal dimension, refer Gulick (1992).

[2] Refer Gilmore (1993) for further elaboration on the topological approach.

[3] Hsieh (1990) provides a comprehensive exposition of various techniques to detect chaos in financial markets.

financial markets is scarce. Hsieh (1989) and Kugler and Lenz (1993) were one of the initial studies of nonlinearities in exchange rates and Diebold and Nason (1990) carried out nonparametric estimations of non-linear models. An important study in the interest rate and foreign exchange market was Wagner and Mahajan (1999) where interest rates for 11 countries (corresponding to Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the United States) as well as spot and forward foreign exchange rates for ten currencies over the period January 1974 to November 1991 was studied. With the use of the BDS statistic and a correlation dimension analysis, the paper's primary findings were that (1) foreign exchange markets have become increasingly complex and therefore less amenable to forecasting over time; (2) although forward exchange risk premia are statistically significant and display a deterministic structure, this structure is complex and therefore not easily discernible; and (3) innovations in real exchange rates are consistent with a Purchasing Power Parity equilibrium.

While there are a number of studies of chaos in developed financial markets, studies on emerging markets are comparatively limited though since the last decade there has been a flurry of publications. Barkoulas and Travlos (1998) was a study of the Athens stock exchange and Olmeda and Perez (1995) of the Spanish exchange. Antoniou et. al (1997) studied the Istanbul stock exchange for non-linearity and Bendel and Hamma’s (1996) was a study of persistence in South African financial series. Pandey et. al. (1997) undertook a study of non-linearities of equity markets in the major Asia Pacific rim countries.

There is a scarcity of empirical research in this field in the Indian context. Daterao and Madhusoodanan (1996) attempted to study chaos in the Reserve Bank of India Index Numbers of Security Price series for the time period 1953 to 1994 taking both weekly and monthly numbers. They employed correlation dimension and the Hurst exponent as measures of chaos. The correlation dimension of the series was found to be between 2 and 3 and the Hurst exponent was found as higher than 0.5 for daily data (indicating persistence) and lesser that 0.5 for the monthly data series. It was thus concluded that market was not efficient. Yet given the unusual data series chosen for studying Indian financial markets, the limited number of data points (504 data points for monthly series), the methodological shortcomings (the correlation dimensions calculated only up to embedding dimension of 6 for calculating correlation dimension of the series) and the limited number of tests undertaken, there is a great scope for improvement.

Another study by Thenmozhi (2000) also concluded that the BSE[4] (India) returns series is not random but non-linear in nature. Daily and weekly returns value of BSE from 1980 to 1997 were considered for the study and while the Hurst exponent was higher than 0.5 for the weekly data, for the daily data it was below the threshold. The BDS test rejected the null hypothesis of whiteness and the correlation dimension was calculated as between 2 and 3. Again the shortcomings of this study include the limited number of data points, the limited number of chaos tests applied, and the inference about correlation dimension of the series from calculations for a very limited number of embedding dimensions.

There is no major study on chaos in Indian forex, bond or money market. Understanding the inherent order of the Indian financial market indices through trying to detect deterministic chaos - if any - in them is the focus of this paper and the following series have been analyzed for chaos in this study:

- National Stock Exchange daily returns (stock market)
- Bombay Stock Exchange daily returns (stock market)
- Dollar Rupee Exchange rates (forex market)
- Call Money Rates (money market)
- Long term bond yields (bond market)

4. Empirical Strategy

The empirical analysis undertaken in this study aims at investigating the presence/ absence of chaos in Indian stock markets and its implications. For this purpose, tests are undertaken to detect a chaotic attractor in time series data taken from the stock market. Our empirical strategy is essentially as follows. We first employ the BDS test to ascertain whether the raw data is random. If the null of randomness is rejected, then the series may be either non linear

[4] Bombay Stock Exchange, Mumbai

stochastic or non linear deterministic. We then filter the series using the appropriate GARCH model and run the BDS test on the residuals. If the null of randomness is not rejected, then the series must be following the fitted process. Otherwise, the series may contain non linear deterministic process which may be chaotic. This is further investigated using the Correlation Dimension and Brock Residual tests of chaos. An overview of each test is presented below.

4.1. Test-1: BDS Test on raw data

The BDS (developed by Brock, Dechert and Scheinkman (1987)), is used to test the null hypothesis of whiteness (randomness) against an unspecified alternative using a nonparametric technique. The statistic based upon the correlation function is

$$W(N,m,\varepsilon) = \sqrt{N}\,\frac{C(N,m,\varepsilon) - C(N,1,\varepsilon)^m}{\hat{\sigma}(N,m,\varepsilon)}$$

where $\hat{\sigma}(N,m,\varepsilon)$ is an estimate of the asymptotic standard deviation of $C(N,m,\varepsilon) - C(N,1,\varepsilon)^m$. The formula for $\hat{\sigma}(N,m,\varepsilon)$ can be found in Brock, Dechert, et al. (1996). The BDS statistic is asymptotically standard normal under the whiteness null hypothesis.

The intuition behind the BDS statistic is as follows. The correlation function $C(N,m,\varepsilon)$ is an estimate of the probability that the distance between any two *m*-histories, X_t and X_s, of the series $\{x_t\}$ is less than έ. If $\{x_t\}$ were independent, then for $t \neq s$ the probability of this joint event equals the product of the individual probabilities. Moreover, if $\{x_t\}$ were also identically distributed, all of the *m* probabilities under the product sign would be the same. The BDS statistic therefore tests the null hypothesis that $C(N,m,\varepsilon) = C(N,1,\varepsilon)^m$, which is equivalent to the null hypothesis of whiteness.

Since the asymptotic distribution of the BDS test statistic is known under the null hypothesis of whiteness, the BDS test provides a direct statistical test for whiteness against general dependence, which includes both nonwhite linear and nonwhite nonlinear dependence.

4.2. Test-2: BDS Test on filtered series

This is a special application of the BDS test wherein a model is first fitted into the original series and the residual series is subjected to the BDS test. The residual series will be random if the fitted model is the correct specification and not random otherwise. Thus the BDS on residuals can be used to check if the best fit model for a given time series is a linear or non-linear model by fitting the best in class model into the original time series (for instance GARCH for non-linear in variance class of models) and applying the BDS test to check the fit of the model. Since financial time-series are known to exhibit clustering, we use appropriate GARCH models (based on information criteria) to filter the original series in all cases.

4.3. Test-3: The Correlation Dimension Test

Intuitively, imagine a shooter shooting at a point target located some distance away. After a large number of shots, if one were to check the target, some interesting inferences about the "sharpness" of the shooter could be drawn (sharpness being the anti-thesis of randomness). If the shots were completely random, a circle of radius 2 units drawn around the point would have 4 times the number of shots as a circle of radius 1 unit (and the shooter would need to go back to training!). A measure of the "sharpness" of the shooter could be the proportion value: the number of shots within a certain circular area as a fraction of the total number of shots, as the number of shots tends to infinity; the lesser the value of the fraction, the "sharper" the shooter. Now suppose instead of shots, one had a large number of data points and the distance between any two data points was measured and known. The proportion of data points with distances less than a pre-determined value could serve as a measure of the randomness in data; it is called the correlation integral.

Formally, the correlation integral C(g) for a time series is defined for different length scales g, by the equation,

$$C(g)=Lim_{n\rightarrow infinity}[1/N(N-1)]\sum_{i\neq j}^{N}(g,X_i,X_j)$$

where N is the sample size, X_i, X_j are observations in time series and (g,X_i,X_j) is the Heaveside function:

$$(g,X_i,X_j)=\begin{cases} 1 \text{ if } | X_i - X_j | < g \\ 0 \text{ if } | X_i - X_j | > g \end{cases}$$

Grassberger and Procaccia (1983) show that:

$$C(g) = Constant \times g^{d}$$

where the exponent d is called correlation dimension and is proposed as a measure of the fractal dimension. Thus correlation dimension, *for a particular embedding dimension*[5], (call it k), is the slope, of the regression: log C(g) versus log g for small values of g.

To estimate the *true* correlation dimension of a chaotic attractor from a single variable time series data, it is embedded in successively higher dimensions till k converges to a stable value (plateau-ing of the graph) which is the *true* correlation dimension value.

In case of white noise, as the number of embedding dimensions increases, the correlation dimension increases at the same rate forever (the slope is 1 and the correlation dimension is equal to the embedding dimension) since white noise, being random, fills whatever space is available to it. If the correlation dimension increases (as the embedding dimension is increased) but at a much slower rate (slope much lesser than one), it suggests a deterministic system which is not chaotic. The importance of the correlation dimension arises from the fact that the minimum number of variables to model a chaotic attractor is the smallest integer greater than it.

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[5] Embedding dimension or Euclidean dimension represents the number of coordinates necessary to define a point.

4.4. Test-4: Brock's Residual Test

Brock (1986) shows that the estimated correlation dimension of the residuals from the best fitting serial generator model must be the same as that of the original data if the data is chaotic (or has a chaotic attractor). If the data are stochastic, the dimension of the residuals will increase since they have less structure than the original data (Yang and Brorsen, 1993). The key is to first make the residuals as close to white noise as possible by filtering with traditional linear or non linear stochastic models and then check the residual series for chaos.

To confirm deterministic chaos, diagnostics should meet the saturation condition as well as Brock's residual test. That is, beyond some embedding dimension, estimated correlation dimension for both raw and residual data should be the same and be stable with embedding dimension (Yang and Brorsen, 1993).

5. Data

We discuss the data used in this study as follows.

Stock Market Data

Two major Indian Stock markets are studied for chaos:

- <u>National Stock Exchange, Mumbai</u>

  For NSE, the input data for this study was the daily closing index value of the S&P CNX Nifty, National Stock Exchange, for the period April 02, 1993 to September 20, 2005.[6] This gave us 3067 data points.

- <u>Bombay Stock Exchange, Mumbai</u>

  The input data in this case was the daily adjusted closing index value of the BSE index, for the period July 01, 1997 to January 20, 2006.[7] This gave us 2120 data points.

Closing index value was preferred over returns or logarithmic first difference values since the autocorrelations in the original series would be lost otherwise (Peters, 1991). The index values

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[6] Available at http://www.nse-india.com/, accessed on 25th October, 2005.

[7] Available at http://finance.yahoo.co.in/, accessed on 21st January, 2006.

were not filtered for growth of the economy as doing so would remove a significant linear component of the data which is not desirable. Furthermore, ordinary investors trade on the basis of the actual indices and not filtered values.

Exchange Rate Data

The daily dollar-rupee inter-bank exchange rate (as released by Reserve Bank of India) for the period January 01, 1999 to December 16, 2005 was used for the study.[8] This provided 2542 data points.

Money Market Data

The average of daily high and low call money rate (expressed in percentage, released by Reserve Bank of India) for the period April 02, 2000 to January 25, 2006 was used for the study.[9] This provided 2069 data points.

Long Term Bond Rate

In 1997, the NSE created a well defined bond index to measure returns in the bond market called the NSE Government Securities Index. Movements on this index reflect returns to an investor on account of change in interest rate only, and not those arising on account of the impact of idiosyncratic factors. It is used as a benchmark for portfolio management and also for designing index funds and derivative products. For our study, the portfolio yield to maturity (YTM) for NSE Gsec Index 8+ (i.e. of duration greater that 8 years) for the period January 01, 1997 to January 27, 2006 was used.[10] This provided 2629 data points.

Eviews version 5.0 was used for the BDS test. The Visual Recurrence Analysis software[11] was used for estimation of the correlation dimension and time delay. For the Brock's Residual test and the BDS on residuals test, the model fitted into the original data series was the GARCH model. SAS version 9.1 was used to fit plain vanilla GARCH to the time series and

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[8] Available at http://www.oanda.com/convert/fxhistory , accessed on 18th January, 2006.

[9] Available at https://reservebank.org.in/cdbmsi/servlet/login/ , accessed on 25th January, 2006.

[10] Available at http://www.nseindia.com/, accessed on 29th January, 2006.

[11] Visual Recurrence Analysis, available at http://home.netcom.com/~eugenek/download.html

residuals were obtained. Specifically GARCH(1,1) was selected on the basis of information criteria.

6. Results and Discussion

6.1. Stock Market Data

a) NSE Returns

BDS Test on Raw Data

The results of the *BDS test* on the original time series are summarized in Table-1

Table 1 BDS Test Output (on returns data)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.199694	0.001738	114.8747	0
3	0.339906	0.002766	122.8985	0
4	0.438038	0.003298	132.8292	0
5	0.506523	0.003442	147.1615	0
6	0.554027	0.003324	166.6684	0
7	0.586814	0.003051	192.358	0
8	0.60929	0.0027	225.6378	0
9	0.624471	0.002327	268.3952	0
10	0.634529	0.001963	323.1776	0

The null hypothesis (original series is random) is rejected for all measured dimensions. Thus the series is non-white.

BDS Test on Residuals

The results of the BDS test on GARCH residuals are summarized in Table 2. The null hypothesis (residual series is random) is rejected for all dimension values measured. Since the GARCH residual series is not random, GARCH is not an appropriate fit model for the original series.

Table 2 BDS Test Output (on residuals)

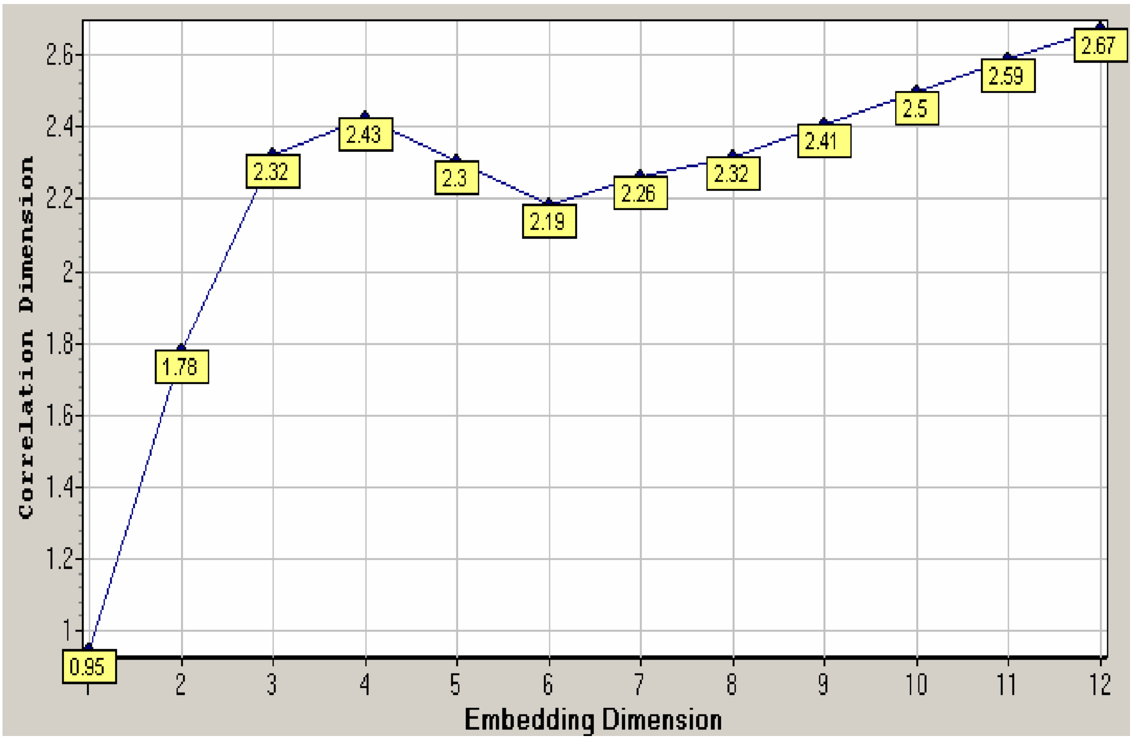
Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.021474	0.001531	14.02391	0
3	0.044081	0.002428	18.15845	0
4	0.059091	0.002884	20.49088	0
5	0.066554	0.002998	22.19634	0
6	0.070105	0.002885	24.30342	0
7	0.070923	0.002637	26.89612	0
8	0.068628	0.002325	29.51919	0
9	0.065004	0.001995	32.58077	0
10	0.060418	0.001677	36.03047	0

Correlation Dimension Test and Brock's Residual Test

Figure-1 shows the plot of correlation dimension for increasing embedding dimensions, obtained for the original time-series (S&P, CNX Nifty, April 02, 1993 to September 20, 2005).

Figure-1

Plot of Correlation dimension for increasing Embedding dimensions for S&P CNX Nifty [12]

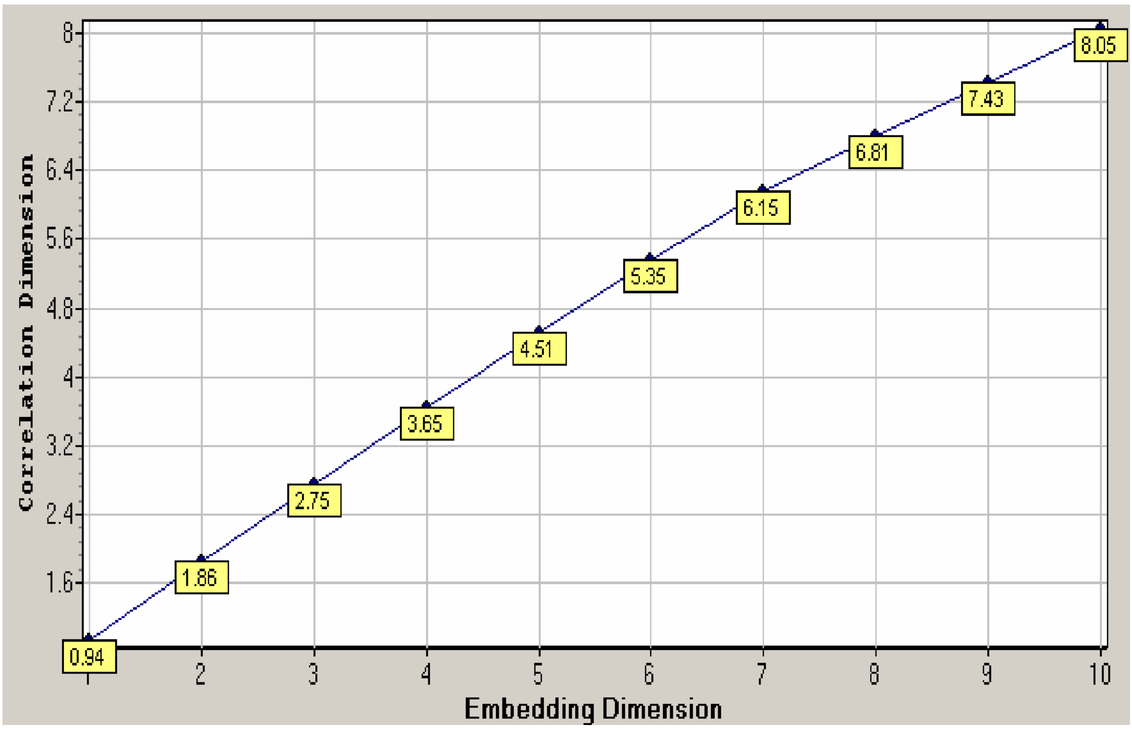


A plain vanilla GARCH (GARCH(1,1)) was fitted into the S&P CNX Nifty time series to obtain the residuals for Brock's Residual test. The residuals of the GARCH(1,1) model were tested for their correlation dimension and Figure-2 shows the plot .

[12] Plot obtained from Visual Recurrence Analysis, the settings used were 'Max radius=better saturation', 'Accuracy vs Speed = more accurate' and 'Time delay = 35'. The Time delay value was obtained from the Mutual Information plot with setting 'Detail = more'.

Figure-2

Plot of Correlation dimension for increasing Embedding dimensions for GARCH(1,1) residuals (NSE)[13]



Interpretation of Test Results

The BDS test unequivocally rejects the null hypothesis of whiteness for the Nifty series. To determine whether the generator is non-linear stochastic or deterministic, the BDS test is applied to the GARCH residuals. GARCH (1, 1) is the best-in-class representative for non-linear in variance models. Since the null hypothesis is rejected for all tested dimensions it can be concluded that the series generator may be non-linear deterministic.

[13] The settings of the software were the same as for S&P CNX Nifty, except that the Time delay was set to 4, as obtained from the Mutual Information plot.

Next we test for the presence of chaos in the series. From figure-1 it can be seen that the correlation dimension value does not plateau for increasing embedding dimensions as would happen for a series with a chaotic attractor. Thus the S&P CNX Nifty is not chaotic.

On the other hand the slope of the graph is much lesser that 45 degrees and the correlation dimension remains well below the embedding dimension even as the embedding dimension increases (correlation dimension equals 2.5, 2.59, 2.67 for embedding dimensions 10, 11, 12 respectively). This means the S&P CNX Nifty time series is not stochastic but deterministic. (As mentioned earlier, for a stochastic time series, correlation dimension is equal to embedding dimension for increasing embedding dimension values.) This indicates a non-chaotic deterministic series.

That the series is not chaotic can also be concluded from figure-2 wherein it is clear that the correlation dimension plot of the residuals of S&P CNX Nifty does not coincide with that of the original plot. Thus the series fails the Brock's residual test and this indicates the absence deterministic chaos.

b) BSE Returns

The results of the tests on the BSE time series is summarized in Tables 3-4 and Figures 3-4.

Table 3 BDS Test Output (on returns data)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.200277	0.001984	100.9296	0
3	0.340653	0.003154	107.9964	0
4	0.438793	0.003757	116.7793	0
5	0.507174	0.003918	129.4518	0
6	0.554559	0.00378	146.7133	0
7	0.587219	0.003465	169.4578	0
8	0.609602	0.003064	198.9517	0
9	0.624714	0.002637	236.8789	0
10	0.634689	0.002223	285.5038	0

Table 4 BDS Test Output (on residuals)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.023435	0.001834	12.77496	0
3	0.045381	0.002913	15.57949	0
4	0.061084	0.003466	17.62534	0
5	0.069794	0.003609	19.33821	0
6	0.074365	0.003477	21.38457	0
7	0.075145	0.003184	23.60215	0
8	0.07329	0.002811	26.06863	0
9	0.069968	0.002416	28.95482	0
10	0.066141	0.002034	32.5168	0

Figure-3

Plot of Correlation dimension for increasing Embedding dimensions for BSE Sensex(original series)

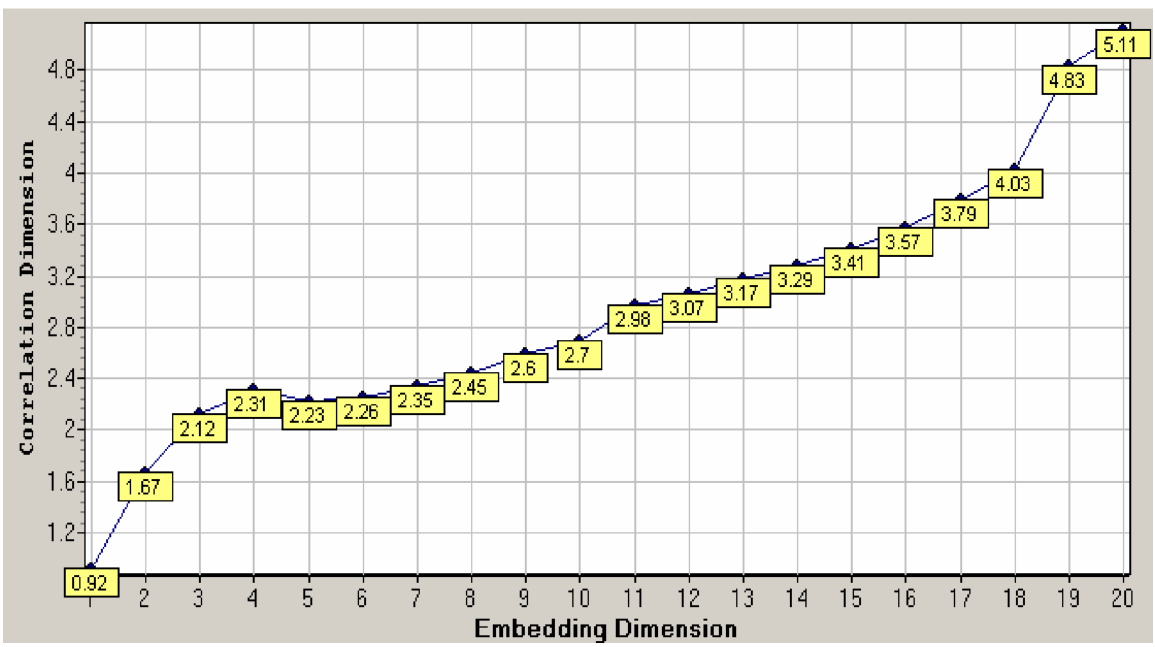
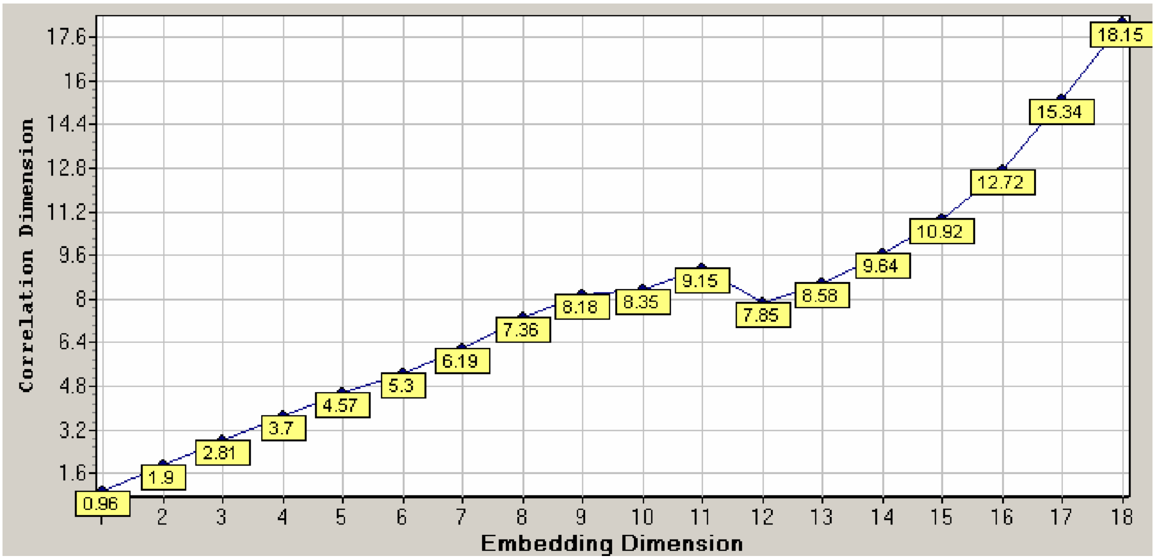


Figure-4

Plot of Correlation dimension for increasing Embedding dimensions for GARCH(1,1) residuals (BSE)



Interpretation of Test Results

Using similar reasoning - as in the case of the NSE series - the following conclusions can be drawn for the BSE series:

- From the BDS test result(Table 3), since the null hypothesis is rejected for all dimensions
  - o *The series is not random.*
- From the result of BDS test on GARCH residuals (Table 4), since the null hypothesis is rejected for all dimensions
  - o *GARCH is not an appropriate model for this series. If GARCH is taken as a representative of non-linear in variance models, then this result may be interpreted as the series having a non-linear deterministic generator.*
- From the Correlation Dimension test (Figure 3) result where the correlation dimension value does not plateau off and,
- from the Brock Residual test (Figure 4) since the Correlation Dimension plot of the residuals does not coincide with the Correlation Dimension plot of the original series,

- *The series does not have a chaotic generator*.

6.2. Exchange Rate Data

The results of the tests on the Exchange Rate time series is summarized in Tables 5-6 and Figures 5-6.

Table 5 BDS Test Output (on original exchange rate series)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.20352	0.00082	248.2379	0
3	0.346481	0.001294	267.7348	0
4	0.44644	0.00153	291.8732	0
5	0.516067	0.001582	326.2313	0
6	0.56439	0.001513	372.916	0
7	0.597752	0.001376	434.522	0
8	0.620578	0.001206	514.6532	0
9	0.63603	0.001029	618.2929	0
10	0.64633	0.000859	752.1291	0

Table 6 BDS Test Output (on residuals)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.037041	0.002484	14.91075	0
3	0.062526	0.00396	15.7906	0
4	0.080045	0.004732	16.91443	0
5	0.091335	0.004952	18.44422	0
6	0.097068	0.004796	20.24127	0
7	0.096877	0.004414	21.94915	0
8	0.094143	0.003919	24.02405	0
9	0.08945	0.003387	26.40817	0
10	0.082943	0.002868	28.92217	0

Figure-5

Plot of Correlation dimension for increasing Embedding dimensions for BSE Sensex(original series)

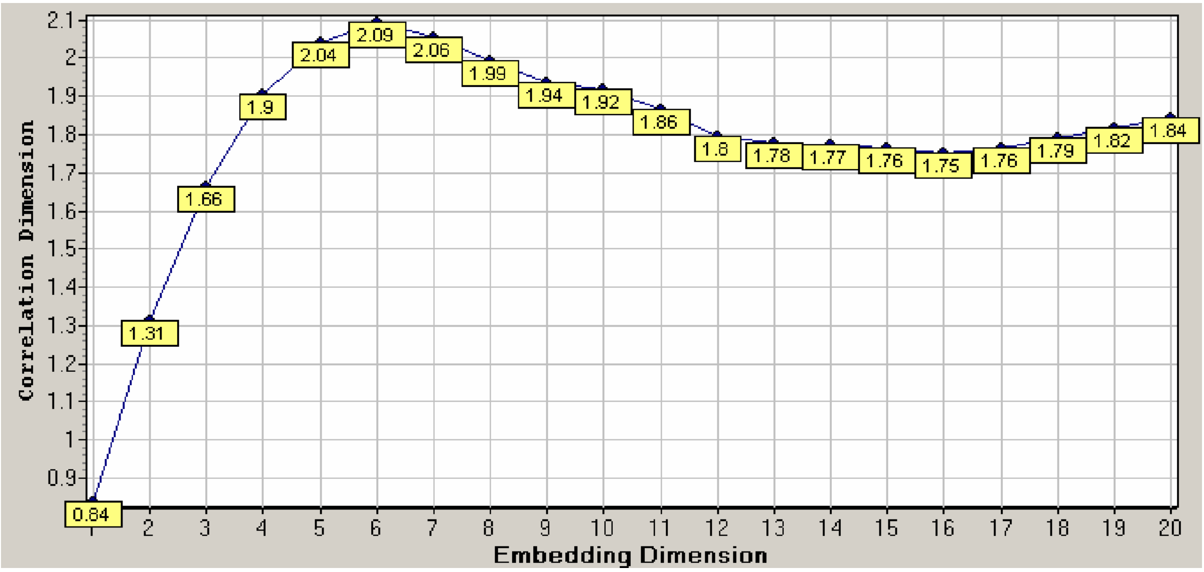
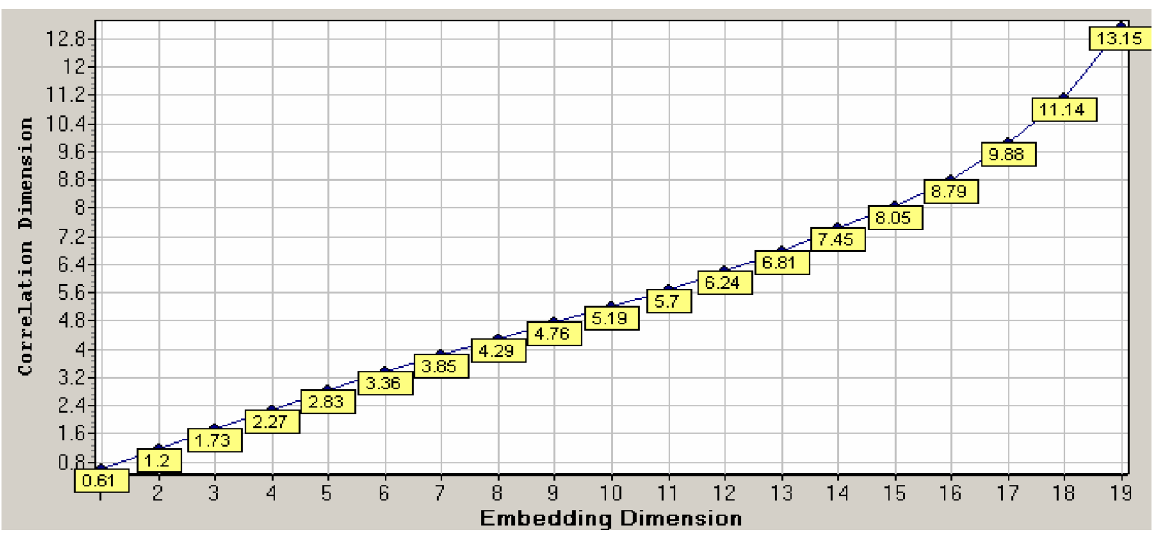


Figure-6

Plot of Correlation dimension for increasing Embedding dimensions for GARCH(1,1) residuals (BSE)



Interpretation of Test Results

Using similar reasoning - as in the case of the NSE series - the following conclusions can be drawn for the BSE series:

- From the BDS test result (Table 5), since the null hypothesis is rejected for all dimensions
  - *The series is not random.*
- From the result of BDS test on GARCH residuals (Table 6), since the null hypothesis is rejected for all dimensions,
  - *GARCH is not an appropriate model for this series. If GARCH is taken as a representative of non-linear in variance models, then this result may be interpreted as the series having a non-linear deterministic generator.*
- The Correlation Dimension plot for the original series (Figure 5) shows that the correlation dimension value hovers below 2. From embedding dimension 13 to dimension 20, the increment in correlation dimension is only 0.06. Yet since the correlation dimension values don’t saturate, nothing definite can be inferred about chaotic/non-chaotic nature of the series generator.
- Further, from the Brock Residual test the Correlation Dimension plot of the residuals does not coincide with the Correlation Dimension plot of the original series suggesting that the series does not have a chaotic generator. But since the Brock residual is not a very powerful test, especially with less that four to five thousand data points, it would not be prudent to outright reject the possibility of a weak chaotic generator for the series.

6.3. Call Money Rates

The results of the tests on the Call Money Rate series is summarized in Tables 7-8 and Figures 7-8.

Table 7 BDS Test Output (on original call money rate series)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.17125	0.001936	88.45995	0
3	0.289421	0.003064	94.47228	0
4	0.368682	0.003633	101.4852	0
5	0.420591	0.003771	111.5399	0

6	0.45328	0.003621	125.1666	0
7	0.472705	0.003305	143.0335	0
8	0.482919	0.002909	166.0167	0
9	0.487134	0.002492	195.463	0
10	0.487136	0.002091	232.9545	0

Table 8 BDS Test Output (on residuals)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.058214	0.002595	22.4299	0
3	0.098087	0.004136	23.71399	0
4	0.121966	0.004942	24.67936	0
5	0.132718	0.00517	25.67255	0
6	0.132143	0.005004	26.40518	0
7	0.125683	0.004604	27.29823	0
8	0.117779	0.004086	28.82623	0
9	0.107374	0.00353	30.41825	0
10	0.09669	0.002987	32.36996	0

Figure-7

Plot of Correlation dimension for increasing Embedding dimensions for Call Money rate rate (original series)

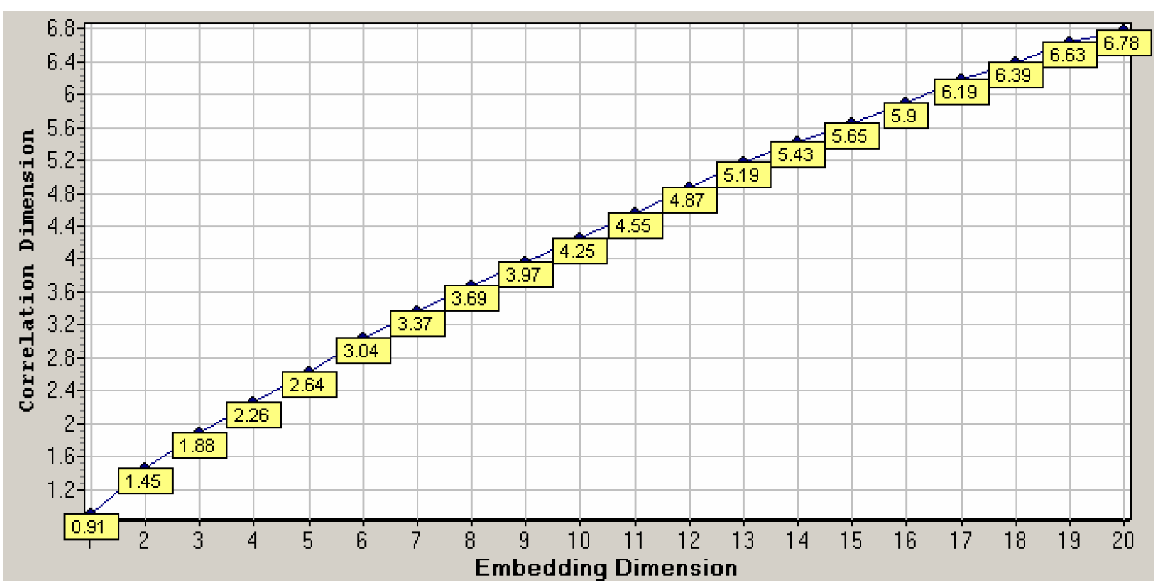
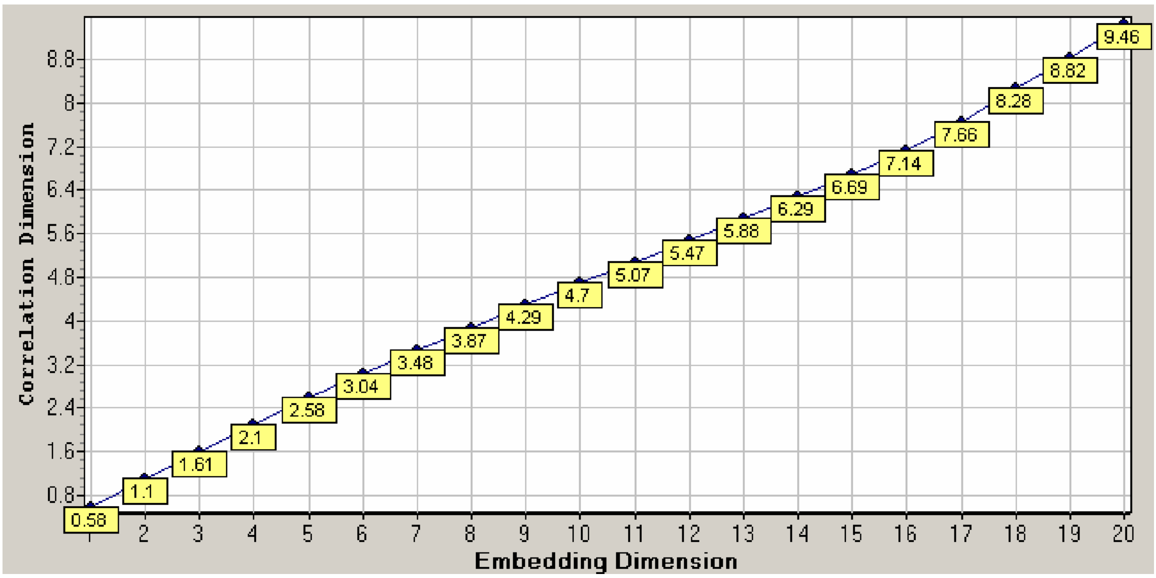


Figure-8

Plot of Correlation dimension for increasing Embedding dimensions for GARCH(1,1) residuals (Call Money series)



Interpretation of Test Results

- From the BDS test result(Table 7),
  - *The series is not random.*
- From the result of BDS test on GARCH residuals (Table 8),
  - *GARCH is not an appropriate model for this series. If GARCH is taken as a representative of non-linear in variance models, then this result may be interpreted as the series having a non-linear deterministic generator.*
- From the Correlation Dimension test (Figure 7) result where the correlation dimension value does not plateau off and,
- from the Brock Residual test (Figure 8) since the Correlation Dimension plot of the residuals does not coincide with the Correlation Dimension plot of the original series,
  - *The series does not have a chaotic generator.*

6.4. Long Term Bond Rates

Table 9 BDS Test Output (on original call money rate series)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.200624	0.000784	255.84	0
3	0.342278	0.001235	277.0557	0
4	0.441358	0.001457	302.8496	0
5	0.510352	0.001504	339.2658	0
6	0.558106	0.001436	388.5493	0
7	0.590988	0.001303	453.539	0
8	0.613526	0.00114	538.203	0
9	0.628836	0.000971	647.8848	0
10	0.639147	0.000809	789.8303	0

Table 10 BDS Test Output (on residuals)

Dimension	BDS Statistic	Standard Error	z-Statistic	P-value
2	0.038571	0.002165	17.81675	0
3	0.074688	0.00345	21.64908	0
4	0.100829	0.004121	24.46604	0
5	0.115673	0.00431	26.83931	0
6	0.123939	0.004171	29.71555	0
7	0.127496	0.003836	33.23825	0
8	0.126724	0.003403	37.24147	0
9	0.123317	0.002939	41.96502	0
10	0.117565	0.002486	47.30023	0

Figure-9

Plot of Correlation dimension for increasing Embedding dimensions for Long Term Bond YTM (original series)

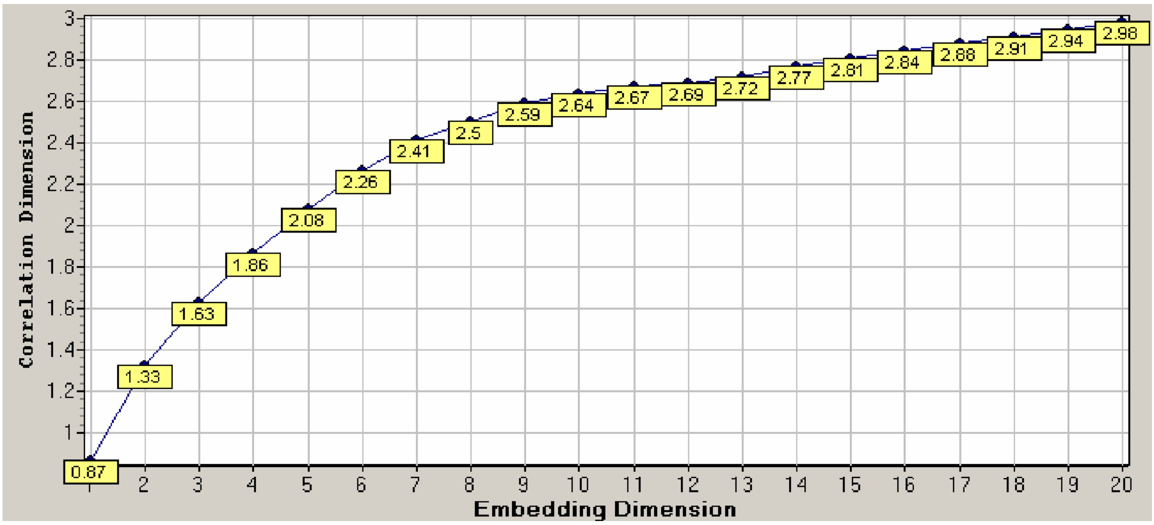
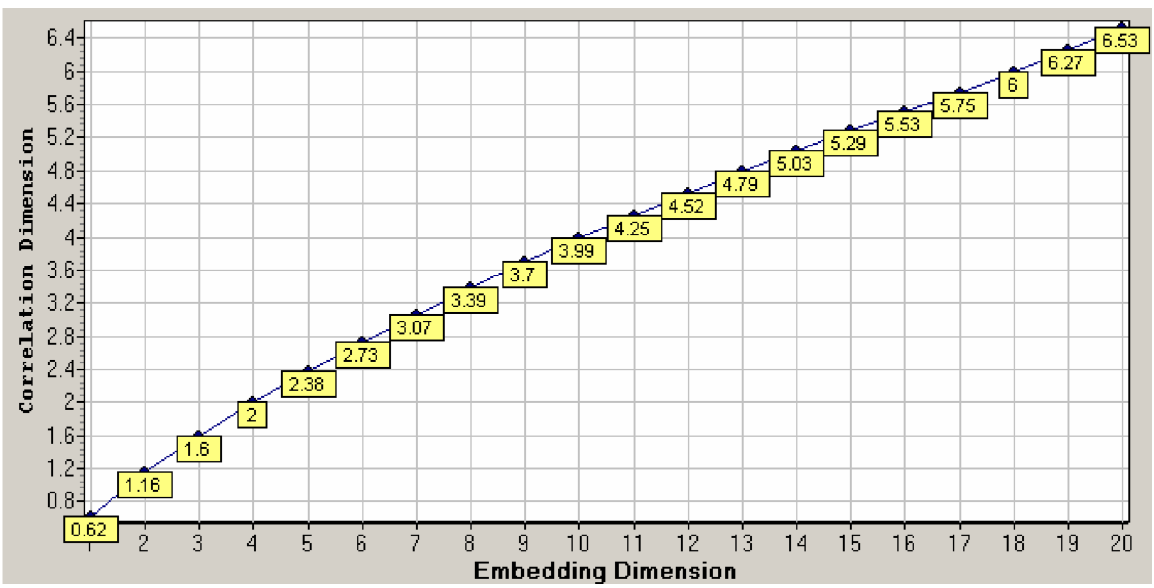


Figure-10

Plot of Correlation dimension for increasing Embedding dimensions for GARCH(1,1) residuals (Long Term Bond YTM series)



Interpretation of Test Results

The following can be inferred from the results of the test :

- From the BDS test result (Table 9),
    - *The series is not random.*
- From the result of BDS test on GARCH residuals (Table 10),
    - *GARCH is not an appropriate model for this series. If GARCH is taken as a representative of non-linear in variance models, then this result may be interpreted as the series having a non-linear deterministic generator.*
- The Correlation Dimension plot for the original series (Figure 9) shows that the correlation dimension inches up to 3 with increasing embedding dimension. Yet since the correlation dimension values don't saturate (at least till embedding dimension 20) nothing definite can be inferred about chaotic/non-chaotic nature of the series generator.
- Further, from the Brock Residual test, the Correlation Dimension plot of the residuals does not coincide with the Correlation Dimension plot of the original series suggesting that the series does not have a chaotic generator. But since the Brock residual is not a powerful test, especially with less that four to five thousand data points, it would not be prudent to outright reject the possibility of a weak chaotic generator for the series.

7. Conclusion and Implications

The results from the paper can be summarized as follows.

Stock Markets:

What are the potential implications of linear deterministic stock markets (BSE or NSE) in India which are not chaotic?

- First, a potential windfall for intelligent investors (*hedgers and speculators*) who can use a number of investment strategies and financial models like trend analysis, market timing, value investing, tactical asset allocation etc. which would fail in efficient markets. Also equilibrium models assuming efficient markets (like the Capital Asset Pricing Model or the Arbitrage Pricing Theory) or investment strategies based on them would become suspect.
- Second, since forecasting does not become increasingly unpredictable into the future

(as would happen in case markets were chaotic), long term forecasting is possible. This again is a window of opportunity for investors trying to gain off the market.

- Finally, the market, being predictable, is vulnerable to manipulators: a wake-up call for regulators who aim to keep the markets efficient.

A ray of hope might be the fact that as more and more models are built to exploit these inefficiencies, the market will attain efficiency through the utilization of these opportunities by the investors.

Exchange Rates:

Again, the major implications of a deterministic generator for exchange rate series would be the host of opportunities it would offer to speculators, hedgers and arbitrageurs, though if the generator is chaotic (as the Correlation Dimension test results of the study tends to suggest) the long term benefits would be limited (due to the unpredictability of a chaotic series in the long run).

- First, a potential gain for players in the currency derivatives (forwards/futures/options) market. Determinism implies that the spot rate can be predicted into the future and can be compared to available forward/future price and depending on the variance, an appropriate position may be taken to make a killing.
- Second, hedge ratios and the amount of capital needed to cover possible losses during time a futures position is held will come down dramatically since the probability distribution of changes in futures prices could be estimated. This is especially important since futures are marked to market every day.

Call Money Rate and Long Term Bond Yield rates:

- The implication of a deterministic call money rate series is potentially limited since it is rarely used for purposes of speculation or hedging - the main benefactors of predictability - and the call money based derivative instruments are rare in India.
- A deterministic long term bond yield rate series can be used by intelligent investors to take appropriate positions in the bond market to make gains. For instance, predicted falling interest rates would be a precursor to the sale of long term bonds and vice-versa. Such models would be especially useful for tactical allocation (a form of conditional asset allocation which consists of rebalancing portfolios around asset

weights depending on conditional information. Again, a chaotic generator (as suggested by the result of the Correlation Dimension test) would curtail the predictability of the generator to limited time periods thus constraining its utility.

- Overall, given the small size and limited innovations in the interest rate derivatives market in India, the financial impact and the opportunities unlocked by the discovery of a deterministic or chaotic generator in the interest rate time series is currently limited. As and when these instruments develop (as is bound to happen with the maturing of the economy), the importance of these models will increase.

This study shows that recent time series data from Indian financial markets is not random. This implies the rejection of efficient market hypothesis for Indian markets and a scope for hedging and gains through speculation. Also given the non-chaotic deterministic nature of these series it should be possible to forecast into the long term with minimal loss of predictability. Given the large trade volumes, the immediate gain for investors from use of such models would be maximal in the stock and forex markets. But in the long run as more innovative instruments are created in the bond and money markets such models would be equally useful in making gains in them.

But that old maxim of the market would still hold. As more and more such non-chaotic deterministic models are used to forecast the market, the market will start to correct itself to attain efficiency. And as the maxim goes, “The more and more you exploit, the more and more efficient it becomes.”

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