GREEKS BASED RISK MANAGEMENT OF USD/INR EUROPEAN CURRENCY OPTION PORTFOLIO

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Abstract

Indian Banks have been allowed to trade in the simplest non-linear derivative products in a limited way by RBI only in USD/INR currency pair. Most of the volumes in Non-linear structured derivative products are driven by Foreign Banks running their trading portfolio from foreign shores. Indian clients take position in these products either through the foreign banks or through Indian banks who run a back-to-back position with the foreign bank (since Indian Banks cannot trade these structured non-linear derivatives). High bid-offer spreads usually result in addition to large margins enjoyed by foreign banks due to lack of Indian competition (and a consequent lack of fair prices and high volumes). Thus Indian corporates cannot avail of these hedging instruments with efficient pricing. The current work shows how Greek limits can be set up for trading USD/INR European currency options. The aim is to promote research and practice of Risk Management in this field so that the RBI and the Indian Banking industry can comfortably take steps in future to adopt non-linear structured derivatives trading.

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Foreigners buy US assets because of the failure of their own financial systems to generate financial assets of a suitable quality.

-Ricardo Caballero, MIT

§ 1 Introduction

In its Circular No. AP (DIR Series) No. 108 dated June 21, 2003, the RBI permitted banks to undertake USD/INR options on market making (trading) basis. Banks need to set up adequate risk management framework and obtain RBI's approval for trading. Setting up risk limits for options involves quantification of non-linear risk from multiple factors. The current work demonstrates how Greek limits can be set up for such non-linear portfolio by taking into account the pecking order of the many factors and the order of non-linearity in such a manner that efficiency of computation, simplicity of approach and integrity of results can be maintained.

§ 1.1 Literature Survey

Rouvinez, C (1997) and Mark Britten-Jones, Stephen M. Schaeffer (1999) made the first attempt at tackling the non-linear Greek risk arising out of options portfolios. The methodology developed by them is widely known as the Delta-Gamma VaR method. Jaschke (2002) examines Cornish-Fisher expansion in detail and concludes that the expansion works well if the distribution is close to normal. Jaschke points out that Cornish-Fisher expansion has some qualitative shortcomings and bad worst-case behavior but achieves sufficient accuracy faster and simpler than Fourier inversion and partial Monte Carlo. He recommends frequent use of full Monte Carlo simulation to check the suitability of the quadratic approximation. Castellacci and Siclari (2003) implement five different VaR methods and compare results with the results from full Monte Carlo simulation. In their comparisons, Castellacci and Siclari use five test portfolios consisting of 1-4 options. They conclude that delta gamma normal VaR can even produce less accurate VaR figures than simpler delta-normal method. They remark that delta-normal, delta-gamma-normal and delta-gamma using Cornish Fisher expansion tend to over-predict VaR. In their words: " The Delta and Delta-Gamma method exhibits incorrect behaviour in the hedged situation. …The Cornish-Fisher method slightly over-predicts the VaR and the Delta-Gamma Monte-Carlo method slightly under-predicts the VaR." The present work uses simulation techniques on Greeks on an aggregated portfolio basis.

§ 1.2 Risk Factors

The market risk of a Currency Options Portfolio depends on the following factors:

(i) USD/INR Spot

(ii) Implied Volatility Surface

(iii) USD/INR Forward Term structure

(iv) USD Libor term structure

(v) Time to Expiry

(vi) Characteristics of the Portfolio in terms of it's Greeks

These factors include market variables and others. Amongst the purely Market variables the maximum risk arises out of the following two variables (in the order of priority):

1. USD/INR Spot &
2. Implied Volatility Surface

Among the others, it is seen that the impact of various Greeks and the management of Greeks influence valuation of an options portfolio.

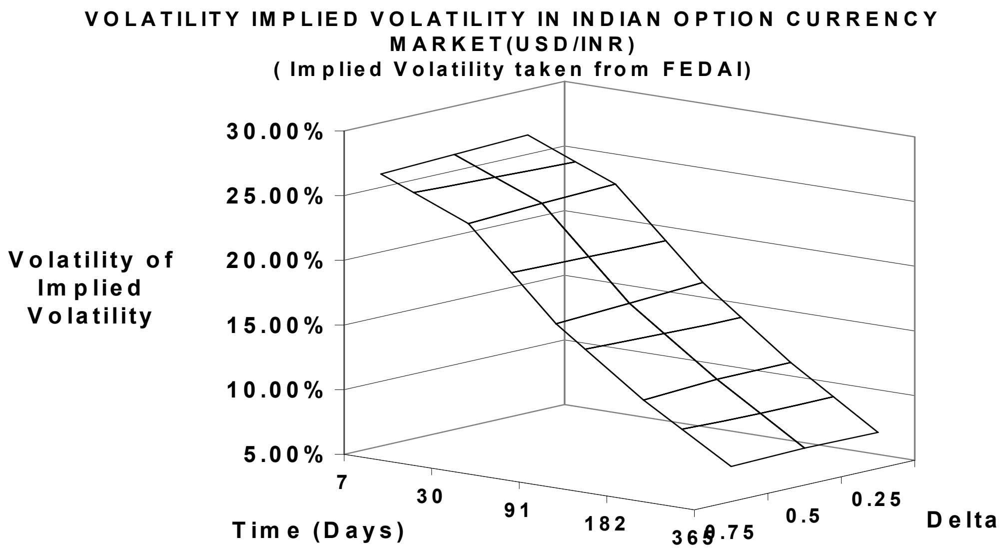
§ **1.3 Our Studies**

As the complexity of handling every additional variable increases exponentially we have adopted the following sequence of analysis:

1. Univariate Treatment: (Britten Jones & Schaeffer, 1999)

In this approach only the USD/INR Spot is considered as the source of risk. Though rather simplistic in approach, it is a very good rough-cut estimate for Delta and Gamma.

2. **Bivariate Treatment**: Here we have considered the effects of both the Spot and the Volatility on the option book. It is similar to the EWMA approach of RBI in consultation with FEDAI (Vide Circular SPL-71/Revised VaR Model/2001.



However in this case the analysis is limited by the fact that we have not considered the effects of the Volatility term structure (the properties of the volatility surface). Nor have we considered the term structure effects of the volatility of the implied volatility (Please see the above Chart). Hence our analysis will be limited and would be based on simple models.

§ 2.1 **Univariate Treatment**

We shall consider the following stochastic process for the USD/INR Spot:

$d\ln S = (r - r_f - \frac{1}{2}\sigma^2)dt + \sigma dz$ ………. Equation 1.

where S = USD/INR Spot

r = Domestic Risk Free rate; assumed to be constant throughout; no term structure.[1]

r_f = Foreign Risk free rate; assumed to be constant throughout; no term structure.

σ = Implied Standard Deviation; Constant, No smile, no term structure.

dt= The incremental change of time

$dz = \varepsilon_t \sqrt{dt}$

$\varepsilon_t \sim N(\mu, \xi^2)$; Normality assumed

Obviously this model does not capture the GARCH/EWMA effects of Volatility. We shall introduce this in Section § 2.2.2.

§ 2.1.1 Calculation using Delta only

We will use some simplifications:

$r - r_f = f$ = Forward premium (in % age); Assumed constant since Term structure effects are not considered for both r and r_f. Using this notation we can rewrite Equation 1 as :

$d\ln S = (f-\sigma^2/2)dt + \sigma dz$ …….. Equation 2
(May also be rewritten as $dS = S(f-\sigma^2/2)dt + S\sigma dz$)

Hence the expected value: **$E(d\ln S) = (f-\sigma^2/2)\,dt$**

The Variance: **$Var(d\ln S) = \sigma^2\,dt$**

Using the concept of VaR at 99% Confidence interval

Linear VaR for **$d\ln S$** = **$(f-\sigma^2/2)\,dt - 2.33*\sigma^2\,dt$** {Assuming in Equation 1. $\varepsilon_t \sim N(\mu,\xi^2)$, $\mu=0$ and $\xi^2 = 1$ }

$$= \{(f-\sigma^2/2) - 2.33*\sigma^2\}dt$$

$$= \{(f - 2.83*\sigma^2\}dt$$

Hence Linear VaR for $dS = S * \{(f - 2.83*\sigma^2\}dt$

Now consider the Currency Option Portfolio **P.** By a first order Taylor's expansion:

$\delta P = dP/dS * \delta S = Delta * \delta S$

At 99% Confidence Interval VaR for **$\delta P = Delta * S * \{(f - 2.83*\sigma^2\}\delta t$**

For one day, if the Loss limit is = L Rupees then,

$Delta * S * [\,(f-\sigma^2/2)\,dt \pm 2.596*\sigma\sqrt{dt}\,] > L$

$\therefore L/ [S*\{(f-\sigma^2/2)\,dt + 2.596*\sigma\sqrt{dt}\}] < Delta < L/ [S*\{(f-\sigma^2/2)\,dt - 2.596*\sigma\sqrt{dt}\}]$

If L = -Rs.1 Lakh, S = 44.045 USD/INR, σ = 2.70%, f = 1.50%, δt = 1/365 (i.e. 1 day)

Then (681,198.46) < Delta < 697,992.91

i.e. The Open Delta position of the Portfolio should be between the bounds – \$ 0.68 Million & + \$ 0.697 Million to limit the loss of the portfolio to – Rs. 1 Lakh in a day. This approach does not take into account the nonlinear parameters of options. It also assumes a lognormal distribution of the USD/INR without incorporating skewness, flat tails (kurtosis) and stochastic jumps.

§ 2.1.2 Calculation using Delta & Gamma

Also called Non Linear VaR, this is one significant improvement over Linear VaR. We shall revisit Taylor's expansion to include the second order term:
$\delta P = dP/dS * \delta S + ½\, d^2P/dS^2 * \delta S^2 = Delta * \delta S + ½\, Gamma * \delta S^2$

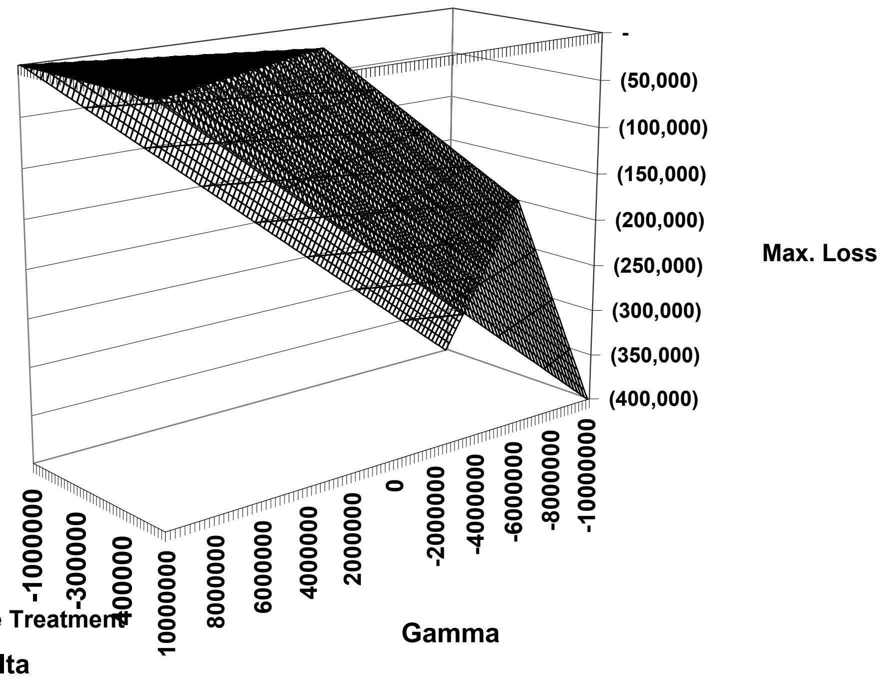
In usual notations, **$\delta P = \Delta * \delta S + ½\, \gamma * \delta S^2$**

Now as per equation 2

$dS = S\{(f-\sigma^2/2)dt + \sigma dz\}$ i.e. $\delta S = S\{(f-\sigma^2/2)\,\delta t + \sigma\delta z\}$

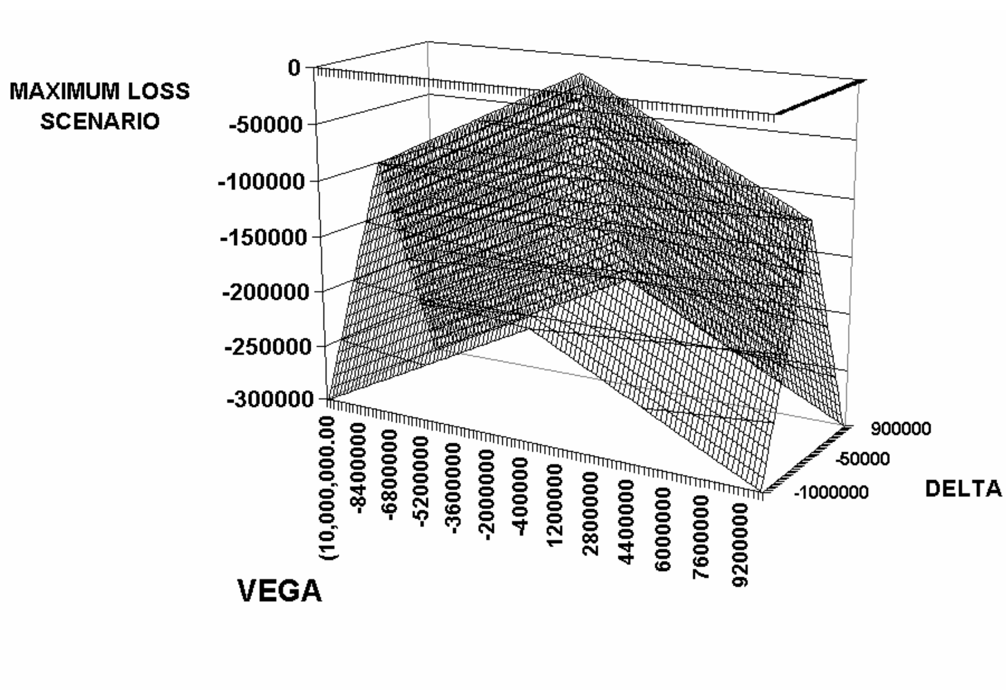
Thus, $\delta P = \Delta * S\{(f-\sigma^2/2)\,\delta t + \sigma\delta z\} + ½\, \gamma * [S\{(f-\sigma^2/2)\,\delta t + \sigma\delta z\}]^2$

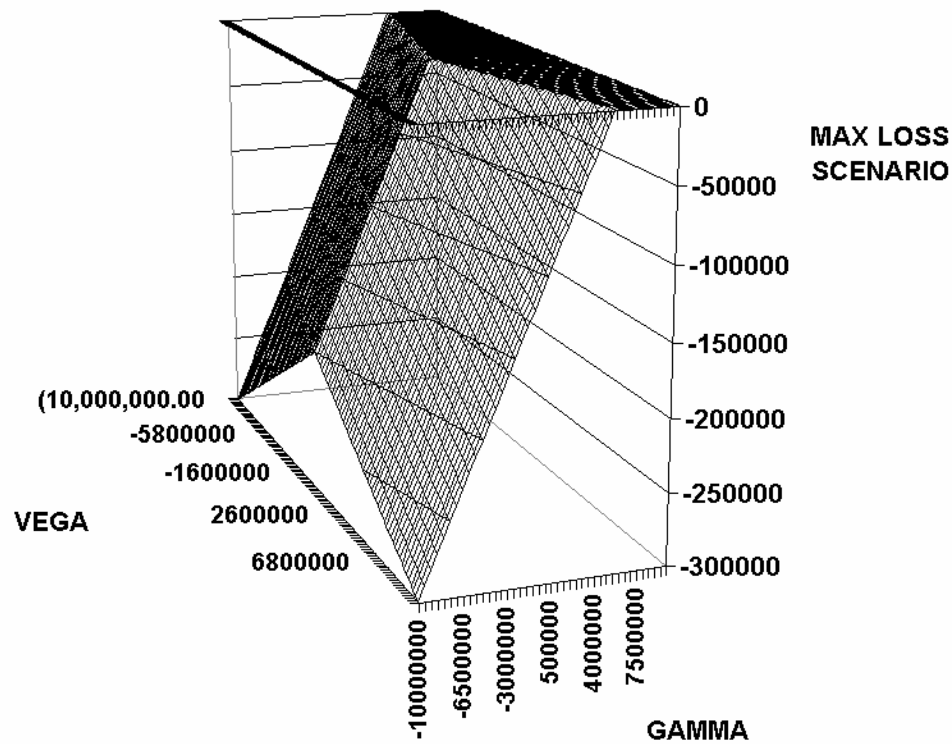
Figure: Maximum Loss Profile for +/- 20 Paise Movement in USD/INR Spot



§ 1.2 Bivariate Treatment

Figure: Maximum Loss Profile for +/- 20 Paise Movement in USD/INR and +/- 1% Movement in Vols





§ 3. Deriving Greek Limits

The five option Greeks that are important to the valuation of the option portfolio are: ***Delta, Gamma, Vega, Rho*** and ***Phi***. However, the Greeks that are of utmost importance are the first three, i..e. Delta, Gamma, and Vega.

§ 3.1.2 Gamma Limits - Gamma limits are most critical. Gamma for options behaves differently for In-the-money, Out-of-the-money and At-the-money options. The following properties about the Gamma of an option are to be noted:

1. $Gamma_{Call}$ = $Gamma_{Put}$ for same strike, maturity, notional principal and bought/sold.

2. For a bought Option Gamma = Positive and
   Sold Option Gamma = Negative

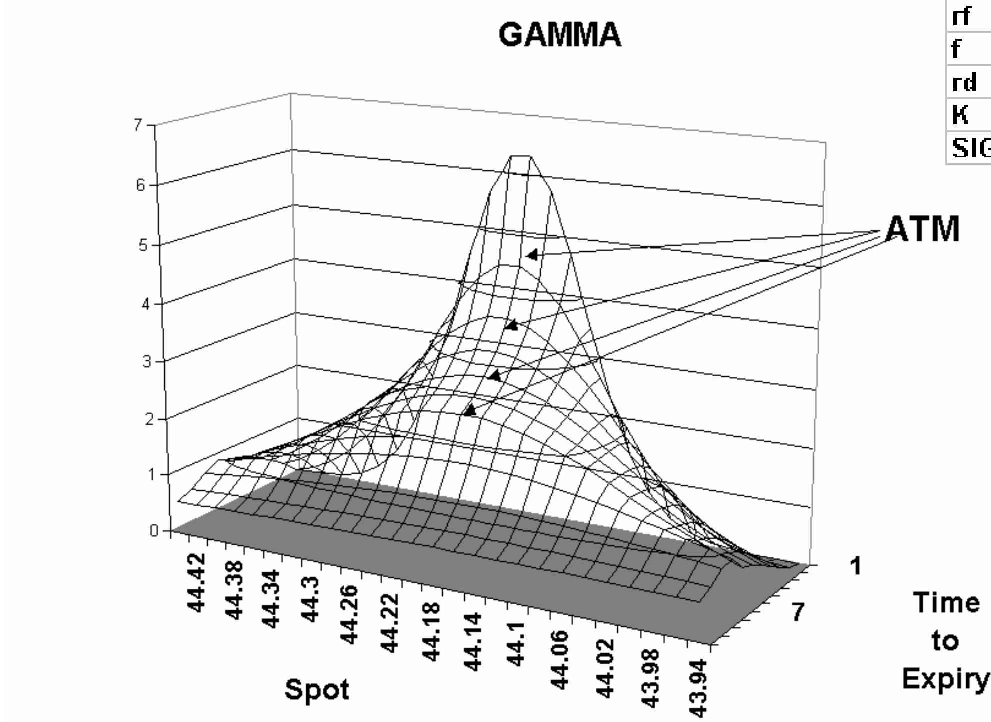
3.Gamma of an option that is exactly AT THE MONEY is higher than an option that is in the money or out of the money.

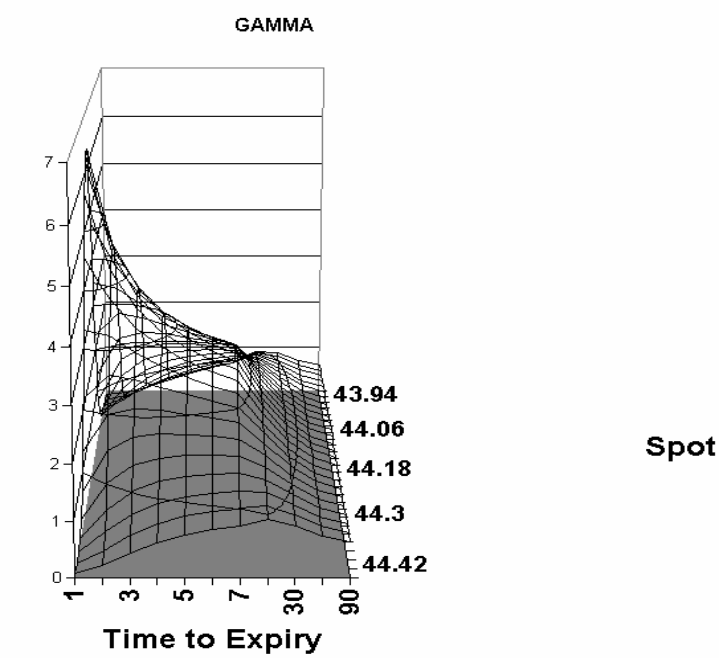
4. The gamma for both OTM and ITM options decrease with time.

5. Gamma for ATM options increases with decrease in time to expiry, and for very short maturities, the gamma simply explodes. Thus, Gamma of an option that is near expiry (within a week) and at the money has a Gamma that is very high. Thus Gamma risk varies with Strike Price and time to maturity.

Figure. Gamma of an ATM & near Maturity Option Explodes

S	44.2
rf	3.90%
f	1%
rd	4.900%
K	44.21
SIGMA	2.70%





We conducted a study to track the behavior of ATM options with decrease in time. Thus, we calculated the Gamma of ATM options with different maturities. We also examined the impact of change in volatility levels for these options, since the Gamma decreases with increase in option volatility. The following table presents the Gamma (in terms of multiple of the notional amount) for each of the maturities at various volatilities.

Tenor	**Gamma For ATM Option**		
	Vols=1.50%	Current Vols	Vols = 5%
1 year	0.56	0.28	0.17
6 month	0.81	0.40	0.24
3 month	1.12	0.54	0.34
2 month	1.39	0.64	0.42
1 month	1.84	0.81	0.55
2 week	2.65	1.26	0.79
1 week	3.39	1.65	1.02
1 day	4.25	2.13	1.28

The above table shows the Gamma of an ATM option as the multiple of the notional. So, as time reduces, the Gamma of an ITM option can theoretically go upto 4.25 times of the notional. Using this study as the base, we need to estimate the following:

1. Projected volumes of options
2. Distribution of options in various maturity buckets
3. Ratio of bought options and sold options (for net Gamma)
4. Ratio of OTM, ATM and ITM options

Based on various such assumptions, we need to estimate the Gamma limits for the portfolio. The sum of net bought and sold options for each time bucket should give us the Gamma of the portfolio. Thus, corresponding volumes are to be estimated for each time bucket, along with the distribution of strikes in terms of OTM, ATM and ITM options, to arrive at the Gamma limits for the portfolio.

§ 3.1.3 Vega Limits – Vega is another important Greek, which has a crucial impact on the value of the portfolio. Vega of an option has the following characteristics.

1. $Vega_{Call}$ = $Vega_{Put}$ for same strike, maturity, notional principal and bought/sold.

2. For a bought Option Vega = Positive and
   Sold Option Vega = Negative

3. Vega of an option that is exactly AT THE MONEY is higher than an option that is in the money or out of the money.

4. The Vega for both OTM and ITM options decrease with decrease in time to expiry.

5. Vega for ATM options increases with increase in time to expiry, and is highest for options with longest tenor.

We conducted a similar study for Vega of the options, and the following table depicts the Vega of an option (in terms of multiples of the notional Principal) for various maturities.

Tenor	Vega
1 year	0.17
6 month	0.13
3 month	0.09
2 month	0.08
1 month	0.06
2 week	0.04
1 week	0.03
1 day	0.02

Thus, the Vega is highest for an ATM option for 1 year. Since we propose to offer options upto 1 year initially, the highest Vega would be 0.17 of notional. Since Vega for bought and sold options have a netting effect, we have to estimate the volumes again, and estimate the Vega limits.

As regards the other Greeks, the following are our views -

§ 3.1.4 Theta - It is an important Greek, which needs to be monitored on a regular basis, because it will determine the value that we gain/lose on the book on a daily basis.

§ 3.1.5 Rho, Phi - In the Indian context, Rho is also important, due to the observed volatility in MIFOR rates. The risk of Rho can be hedged through forwards. However, no decision has been taken on monitoring Rho limits. Since Rho and Phi would be in opposite direction, their impact on the portfolio P&L would be reduced to that extent.

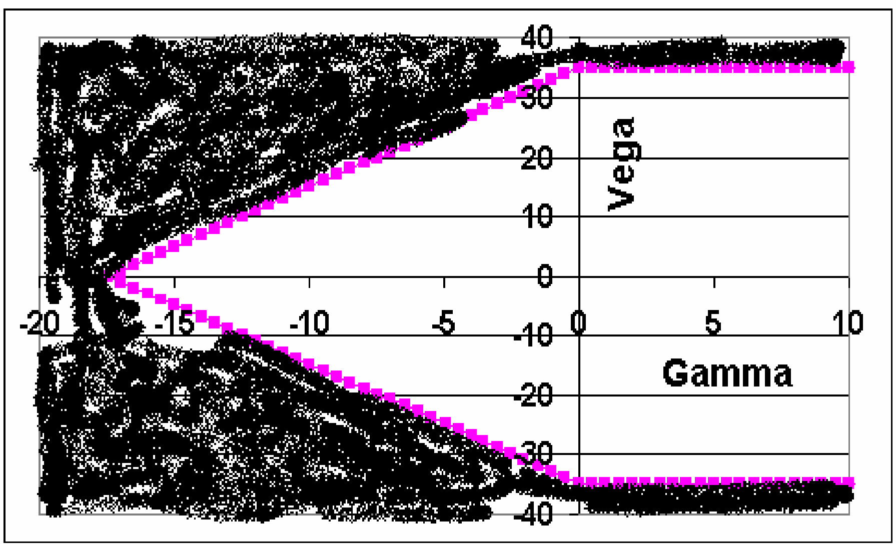
§ 3.2 Greeks - The Trade-Off

Ostensibly there's a trade-off involved in selecting the bounds of the Greeks of the Option portfolio. One can do with a relatively high Gamma and Vega limit provided there is a guarantee that the Delta of the portfolio will never be allowed to exceed 0.625 Million or even lower. As the size of the portfolio grows, the need for wider Gamma Limit increases. Then Delta adjustments via spot/forward hedges must be tighter, more frequent and more robust. This not only increases the Gamma limit but also the Vega limit.

Figure: (Illustrative Example Contd…..) For a Delta Limit of +/- 0.625 Million US

				GAMMA			
		(18,750,000.00)	(18,500,000.00)	(18,250,000.00)	(18,000,000.00)	(17,750,000.00)	(17,500,000.00)
	(1,800,000)					-498000	-493000
	(1,700,000)					-497000	-492000
	(1,600,000)					-496000	-491000
	(1,500,000)				-500000	-495000	-490000
	(1,400,000)				-499000	-494000	-489000
	(1,300,000)				-498000	-493000	-488000
	(1,200,000)				-497000	-492000	-487000
	(1,100,000)				-496000	-491000	-486000
	(1,000,000)			-500000	-495000	-490000	-485000
	(900,000)			-499000	-494000	-489000	-484000
	(800,000)			-498000	-493000	-488000	-483000
	(700,000)			-497000	-492000	-487000	-482000
	(600,000)			-496000	-491000	-486000	-481000
	(500,000)		-500000	-495000	-490000	-485000	-480000
	(400,000)		-499000	-494000	-489000	-484000	-479000
	(300,000)		-498000	-493000	-488000	-483000	-478000
	(200,000)		-497000	-492000	-487000	-482000	-477000
	(100,000)		-496000	-491000	-486000	-481000	-476000
VEGA	-	-500000	-495000	-490000	-485000	-480000	-475000
	100,000		-496000	-491000	-486000	-481000	-476000
	200,000		-497000	-492000	-487000	-482000	-477000
	300,000		-498000	-493000	-488000	-483000	-478000
	400,000		-499000	-494000	-489000	-484000	-479000
	500,000		-500000	-495000	-490000	-485000	-480000
	600,000			-496000	-491000	-486000	-481000
	700,000			-497000	-492000	-487000	-482000
	800,000			-498000	-493000	-488000	-483000
	900,000			-499000	-494000	-489000	-484000
	1,000,000			-500000	-495000	-490000	-485000
	1,100,000				-496000	-491000	-486000
	1,200,000				-497000	-492000	-487000
	1,300,000				-498000	-493000	-488000
	1,400,000				-499000	-494000	-489000
	1,500,000				-500000	-495000	-490000

Figure: Gamma & Vega Bounds for Maximum 1 Day Loss of - Rs.5 Lakhs under Extreme Stress Scenario



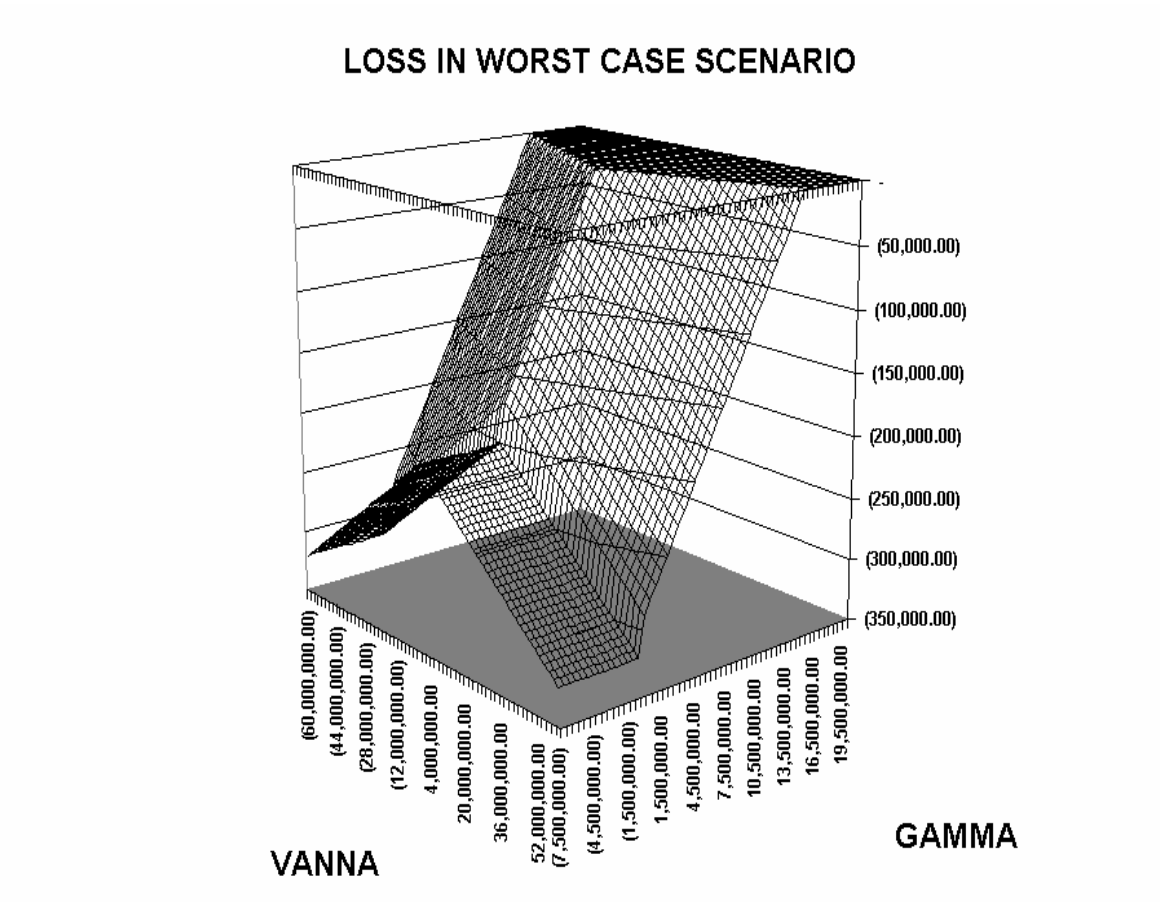
(The Darkened Zone is forbidden Territory. Operations allowed only within the White Zone)

If we have to keep our Daily Losses under Extreme Stress below a certain predetermined level, we have to do a balancing act between Delta, Gamma and Vega. In this illustrative case we have taken the Delta Limit to be +/- 0.625 Million USD. Using an Extreme Stress Scenario and assuming the maximum 1-Day loss to – Rs. 5 Lakh, we derive the Gamma and Vega Limits as shown in the above Table.

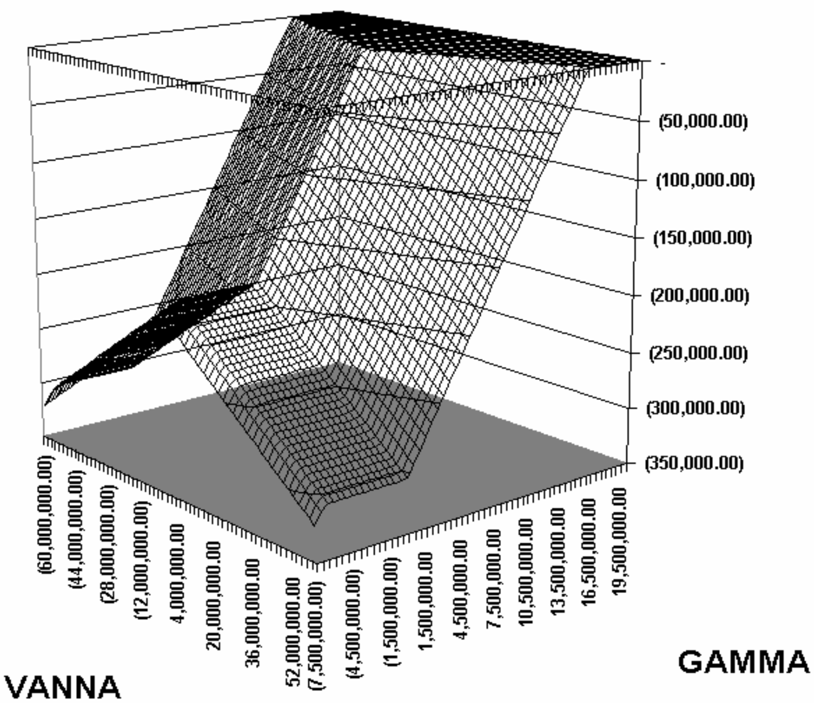
Now we examine the effect of Vanna on the portfolio. If we map the above limits of Delta, Vega and Gamma on Vanna and use the same Worst-Case Loss technique we derive the effect of Vanna on the loss profile.

Vega is taken as per the estimates given above for every level of Gamma. Thence the Loss in the worst case scenario is worked out for various levels of Vanna, Gamma and Vega.

Figure: The Figure below is for a Vega that is taken as given in Figure



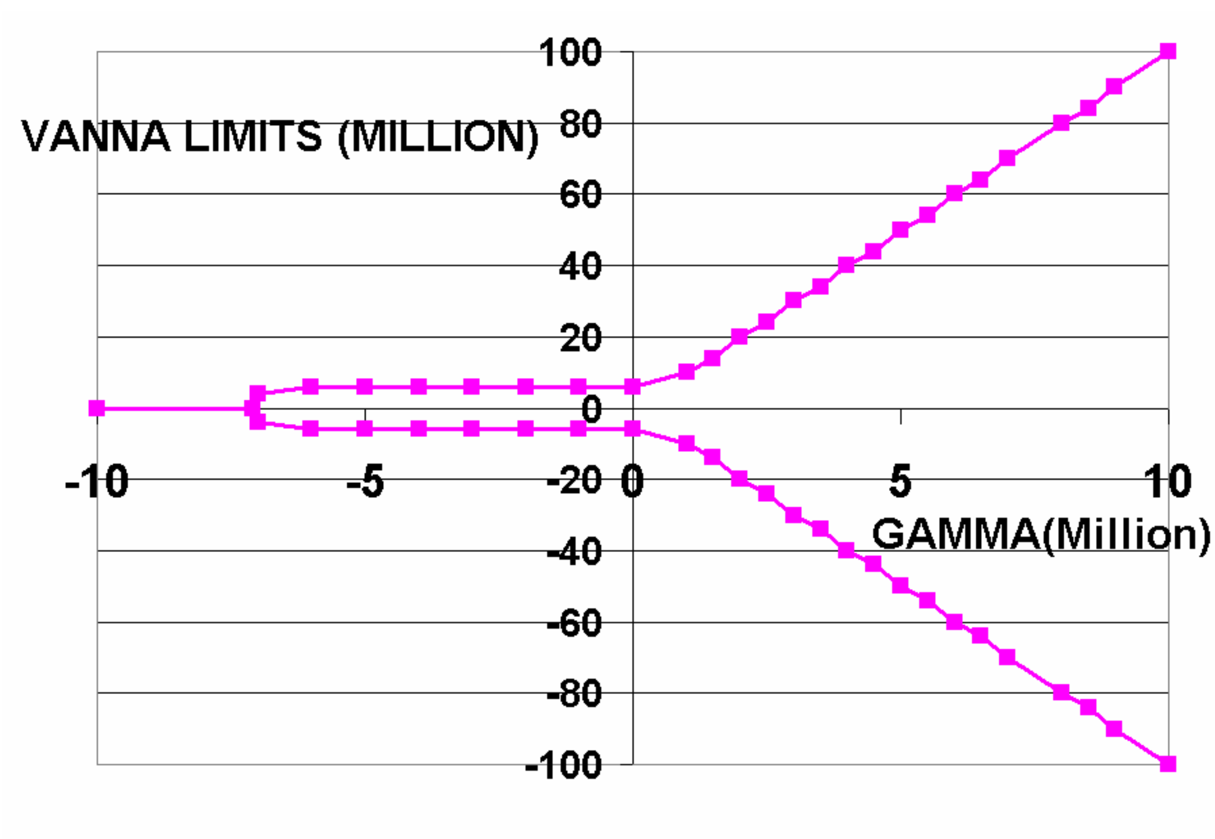
LOSS IN WORST CASE SCENARIO



	(7,500,000.00)	(7,000,000.00)	(6,500,000.00)	(6,000,000.00)	(5,500,000.00)	(5,000,000.00)
(28,000,000.00)						
(26,000,000.00)				GAMMA		
(24,000,000.00)						
(22,000,000.00)						
(20,000,000.00)						
(18,000,000.00)	VANNA					
(16,000,000.00)						
(14,000,000.00)						
(12,000,000.00)						
(10,000,000.00)						
(8,000,000.00)						
(6,000,000.00)			(197,000.00)	(197,000.00)	(197,000.00)	(197,000.00)
(4,000,000.00)		(198,000.00)	(193,000.00)	(193,000.00)	(193,000.00)	(193,000.00)
(2,000,000.00)		(194,000.00)	(189,000.00)	(189,000.00)	(189,000.00)	(189,000.00)
-	(200,000.00)	(190,000.00)	(185,000.00)	(185,000.00)	(185,000.00)	(185,000.00)
2,000,000.00		(194,000.00)	(189,000.00)	(189,000.00)	(189,000.00)	(189,000.00)
4,000,000.00		(198,000.00)	(193,000.00)	(193,000.00)	(193,000.00)	(193,000.00)
6,000,000.00			(197,000.00)	(197,000.00)	(197,000.00)	(197,000.00)
8,000,000.00						
10,000,000.00						
12,000,000.00						
14,000,000.00						
16,000,000.00						
18,000,000.00						
20,000,000.00						
22,000,000.00						

250,000.00		x		GAMMA							
			500,000.00	1,000,000.00	1,500,000.00	2,000,000.00	2,500,000.00	3,000,000.00	3,500,000.00	4,000,000.00	4,500,000.00
		70,000,000.00	(180,000.00)	(170,000.00)	(160,000.00)	(150,000.00)	(140,000.00)	(130,000.00)	(120,000.00)	(110,000.00)	(100,000.00)
		72,000,000.00	(184,000.00)	(174,000.00)	(164,000.00)	(154,000.00)	(144,000.00)	(134,000.00)	(124,000.00)	(114,000.00)	(104,000.00)
		74,000,000.00	(188,000.00)	(178,000.00)	(168,000.00)	(158,000.00)	(148,000.00)	(138,000.00)	(128,000.00)	(118,000.00)	(108,000.00)
		76,000,000.00	(192,000.00)	(182,000.00)	(172,000.00)	(162,000.00)	(152,000.00)	(142,000.00)	(132,000.00)	(122,000.00)	(112,000.00)
		78,000,000.00	(196,000.00)	(186,000.00)	(176,000.00)	(166,000.00)	(156,000.00)	(146,000.00)	(136,000.00)	(126,000.00)	(116,000.00)
		80,000,000.00	(200,000.00)	(190,000.00)	(180,000.00)	(170,000.00)	(160,000.00)	(150,000.00)	(140,000.00)	(130,000.00)	(120,000.00)
		82,000,000.00		(194,000.00)	(184,000.00)	(174,000.00)	(164,000.00)	(154,000.00)	(144,000.00)	(134,000.00)	(124,000.00)
		84,000,000.00		(198,000.00)	(188,000.00)	(178,000.00)	(168,000.00)	(158,000.00)	(148,000.00)	(138,000.00)	(128,000.00)
		86,000,000.00			(192,000.00)	(182,000.00)	(172,000.00)	(162,000.00)	(152,000.00)	(142,000.00)	(132,000.00)
		88,000,000.00			(196,000.00)	(186,000.00)	(176,000.00)	(166,000.00)	(156,000.00)	(146,000.00)	(136,000.00)
VANNA		**90,000,000.00**			(200,000.00)	(190,000.00)	(180,000.00)	(170,000.00)	(160,000.00)	(150,000.00)	(140,000.00)
		92,000,000.00				(194,000.00)	(184,000.00)	(174,000.00)	(164,000.00)	(154,000.00)	(144,000.00)
		94,000,000.00				(198,000.00)	(188,000.00)	(178,000.00)	(168,000.00)	(158,000.00)	(148,000.00)
		96,000,000.00					(192,000.00)	(182,000.00)	(172,000.00)	(162,000.00)	(152,000.00)
		98,000,000.00					(196,000.00)	(186,000.00)	(176,000.00)	(166,000.00)	(156,000.00)
		100,000,000.00					(200,000.00)	(190,000.00)	(180,000.00)	(170,000.00)	(160,000.00)
		102,000,000.00						(194,000.00)	(184,000.00)	(174,000.00)	(164,000.00)
		104,000,000.00						(198,000.00)	(188,000.00)	(178,000.00)	(168,000.00)
		106,000,000.00							(192,000.00)	(182,000.00)	(172,000.00)
		108,000,000.00							(196,000.00)	(186,000.00)	(176,000.00)
		110,000,000.00							(200,000.00)	(190,000.00)	(180,000.00)
		112,000,000.00								(194,000.00)	(184,000.00)
		114,000,000.00								(198,000.00)	(188,000.00)
		116,000,000.00									(192,000.00)

Figure: Summarizing from Above- Vanna Limits within the Y shaped area



§ 3.3 Greek Limits

The Greek Limits are as under:

1. **Delta: +/- 625,000**

Assuming Delta Limit is set at **+/-625,000** and the daily Loss limit is **–Rs.5, 00,000/-** we shall detail the limits for other Greeks:

2. **If Delta limit is +/-625,000; Gamma limit is from –17.5 Million to +40 Million.**
3. Vega depends on Delta and Gamma position of the portfolio. If Gamma is positive then the **Limit for Vega is +/-35 Million. If Gamma is negative then the Vega Limit progressively reduces.** For a **Gamma** of **– 17.5 Million,** the **Vega limit is 0.** For a **Gamma** of **–15 Million,** the **Vega limit** is **+/-5 Million.** For a **Gamma of –10 Million, the Vega limit is +/-15 Million.** For a **Gamma of –5 Million, the Vega limit is +/-25 Million.** For a **Gamma of –2 Million, the Vega limit is +/-31 Million**
4. Vanna also depends on the Delta, Vega & Gamma of the portfolio. For a Negative Gamma of -7 Million the Vanna limit is +/-4 Million. For all other Negative Values of Gamma greater than –7 Million the Vanna limit is +/- 6 Million. For positive Gamma Values: +/-Gamma x 10 is the Vanna limit.

§ 4. Management of Option Portfolio under Greek limits

We have undertaken a rigorous exercise of managing a dummy portfolio of options since July 2005.

The following Greeks were monitored:

1. Delta & Net Delta
2. Gamma
3. Speed($\partial^3 P/\partial S^3 = \partial \text{Gamma}/\partial S$)
4. Vanna ($\partial \text{Delta}/\partial \sigma = \partial \text{Vega}/\partial S$)
5. Vega
6. Theta
7. Rho
8. Phi
9. Fwd Delta

§ 4.1 Managing Delta

Delta is the simplest Greek to manage, since we have to take a position in Spot or Forward market to offset the impact of Delta. If we take a position in Spot, it would have an impact on Delta only. However, if we take a position in forwards, it would have an impact on Delta, Rho and Phi. Thus, to cover an option position, it is better to take a position in forwards, since it would cover the Delta, Rho and Phi risk. However, considering that this would require two interventions in the market (Spot and Buy-sell or Sell-buy swap) and also that forward quotes are not as liquid as quotes in Spot, traders often take spot positions to square off option delta.

I. Hedging delta for a new option

Let us consider the following example. If a Client buys a 6-month ATM Call for USD 1 million, the Delta would be 50%, i.e. USD 0.50 million. Thus, to hedge the transaction, we would have to either:

a. Buy USD 0.50 mio Forward [i.e Buy Spot 0.5 million, and Sell-Buy 6m Forward 0.5 million]

b. Buy USD 0.50*$\exp^{-rf*t}$ = 0.5 $e^{-(4.35\%*0.5)}$ =0.489 million Forward. (approx, assuming 6 mth r_f at 4.35%)

Assuming that we buy USD 0.5 million forward, our position would be squared on the Delta, Rho and Phi. However, the option would have a Gamma of USD 0.40 Mio (negative) and Vega of USD 0.125 mio (negative), which are unhedged.

II. Managing delta for the option portfolio

Let us assume that the delta limits for the option portfolio is USD 1 million. Since the portfolio has a certain gamma, the portfolio delta would keep on changing during the day with movement in Spot, forwards and volatilities. Thus, the portfolio delta is to be monitored and when it reaches the delta limit, a position is to be taken to square off the delta. For example, when the portfolio delta exceeds USD 1 million and we have to sell USD 1 million in spot to square off the delta. Thus,

Delta of the portfolio = USD 1 million

Delta of USD 1 million sell position = - USD 1 million

Net delta of the book = 0

Similarly, when the portfolio delta exceeds – USD 1 million, we may have to buy USD 1 million in spot to square off the delta. Thus,

Delta of the portfolio = - USD 1 million

Delta of USD 1 million buy position = USD 1 million

Net delta of the book = 0

Thus, we can manage the delta of a portfolio of options. Please note that the delta limits are to be monitored on an end-of-the-day basis, meaning that the net delta of the portfolio is to be within the prescribed limits at the end of the trading session.

§ 4.1.1 Managing Volatility Smile effect on Delta Hedging of Currency Options book

According to Thomas F. Coleman,Yohan Kim,Yuying Li and Arun Verma(2001) the volatility smile is derived out of the implied volatility. Though this ensures integrity of valuation, it distorts the delta and other Greeks because using the implied volatility cannot ensure integrity of Greeks. Thus there is a vol smile adjustment to be made to the Greeks, especially Delta, because we use this Greek to spot hedge. The following analysis shows how we make the adjustment:

If $\mathbf{C}$ = Market price of Call/Put option

$\mathbf{C_{BS}}$ = Black-Scholes/Garman-Kohlhagen price of Call/Put option

$\mathbf{S_0}$ = Spot

$\boldsymbol{\sigma^c_{imp}}$ = Implied Volatility

True Delta = B-S Delta + Adjustment

$$\frac{\partial C}{\partial S_0} = \frac{\partial C_{BS}}{\partial S_0} + \frac{\partial C_{BS}}{\partial \sigma^c_{imp}} \frac{\partial \sigma^c_{imp}}{\partial S_0}$$

$$\frac{\partial C_{BS}}{\partial S_0} = \text{B-S Delta}$$

$$\frac{\partial C_{BS}}{\partial \sigma^c_{imp}} = \text{B-S Vega}$$

$$\frac{\partial \sigma^c_{imp}}{\partial S_0} = \frac{\partial \sigma^c_{imp}}{\partial \Delta_{B-S}} \times \frac{\partial \Delta_{B-S}}{\partial S_0} = \text{Smile Slope x B-S Gamma}$$

Hence

True Delta = B-S Delta + Adjustment = B-S Delta + B-S Vega x Smile Slope x B-S Gamma

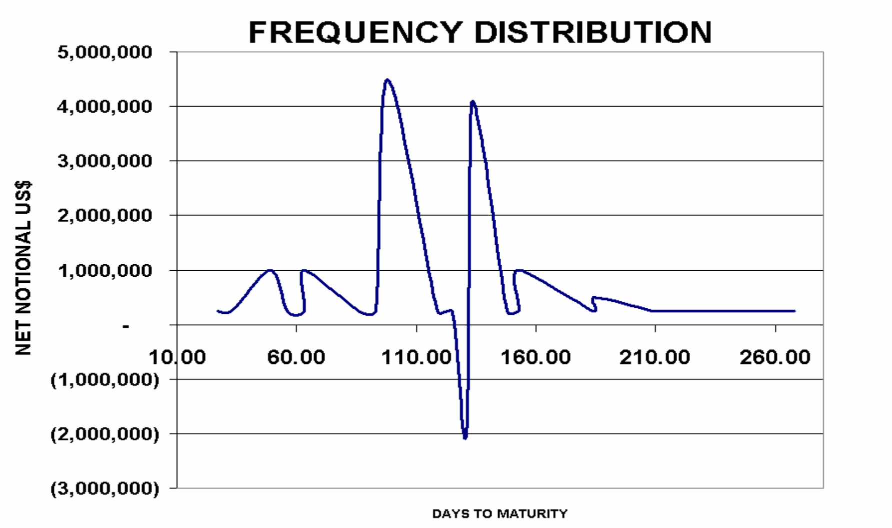
In practice one must take this adjustment into account while booking spot hedges using the delta.

§ 4.2 Managing risk arising out of Gamma

Spot & Forward hedging cannot change the Gamma because these do not have any second order effects. Gamma can be kept under control by practicing the following:

(For the forthcoming discussion let us define the maturity bucket wise distribution of Net Option Bought & Sold (Bought Notional – Sold Notional) as the Net Option Notional Maturity Profile.)

Figure: An Actual Portfolio's Net Option Notional Maturity Profile



(Assuming we run each and every option on the portfolio to maturity (and do not offload very near expiry options into the market by netting/terminating) then we need to work out a systematic method as follows)

§ 4.2.1 AVOIDANCE OF GAMMA CONCENTRATION IN MATURITY-STRIKE TOPOGRAPHY

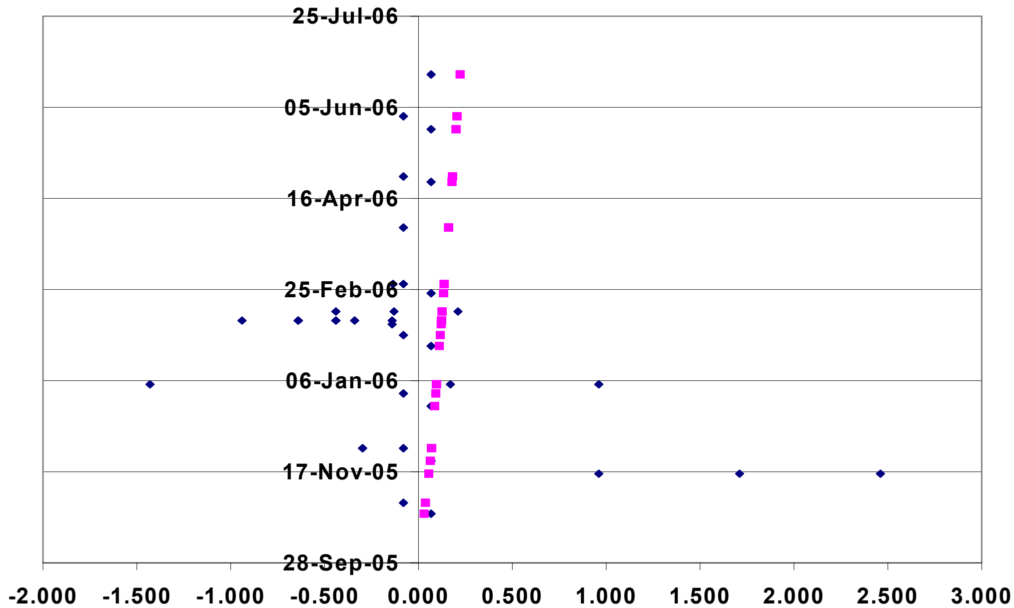
Uniform Net Option Notional Maturity Profile

This is necessary so that as time advances we should not be left with any excess net option notional principal, which is expiring within1 week.

Uniform distribution of Strikes on any given Day of Maturity

Systematically ensuring that on no single maturity day there are unidirectional bought/sold options that are outstanding on a single strike. The Maturity Strike topography with bought/sold indicators gives a fair understanding of the Gamma risks that are likely to emerge as expiry dates near. In the given topography we are not depicting the notional net/sold but the forward levels are depicted to show whether the strikes are ATMF or not.

MATURITY-STRIKE TOPOGRAPHY



§ 4.2.1 Managing Volatility Smile effect on Gamma Hedging of Currency Options book

The volatility smile is derived out of the implied volatility. Though this ensures integrity of valuation, it distorts the delta and other Greeks because using the implied volatility cannot ensure integrity of Greeks. Thus there is a vol smile adjustment to be made to the Gamma. The following analysis shows how we make the adjustment:

If C = Market price of Call/Put option

C_{BS} = Black-Scholes/Garman-Kohlhagen price of Call/Put option

S_0 = Spot

σ^c_{imp} = Implied Volatility

True Gamma = B-S Gamma + Adjustment.

Following a similar procedure as in Section §4.1.1.

True Gamma = B-S Gamma + B-S Vanna x Smile Slope x B-S Gamma

+B-S Vega x (Smile Slope x Speed + Smile Convexity x Gamma^2)

In practice one must take this adjustment into account while calculating Gamma.

§4.3 Managing risk arising out of Vega

Vega is also a Greek that can be managed only by taking a position in an offsetting option. The impact of Vega is potent for longer dated options. Vega for short dated options drops and for very short dated options it is close to zero. ATM & long dated options have the maximum Vega[1].

In practice the volatility levels in the USD/INR pair is very low at 3-6% levels. Moreover the risk arising out of Vega is very minimal due to the fact that on any given day the change in vol levels is not more than 1-1.25%. On an daily basis vols have never changed more than +/-1%. Hence even if the Vega is about 50 Million, the change by 1% will generate a worst case loss of Rs. 5 Lakhs. Based on this we could have a considerably smaller Vega limit for our portfolio (Please see Section § 3.3 Greek Limits) after Extreme Scenario Loss analysis in Section § 3.2 Greeks - The Trade-Off. If the Vega limits are broken or there is a need to position the book to take advantage of probable market views then it may necessitate a non-linear hedge (i.e. hedge using options).

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[1] Please see §3.1.3

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