A Bootstrapped Historical Simulation Value at Risk Approach to S & P CNX Nifty

Debashis Dutta

&

Basabi Bhattacharya

* This is a preliminary version.

* Research Scholar, Department of Economics, Jadavpur University, Kolkata, Telephone: (033)-65885601(R), 9920096400 (M). Corresponding Author. E-mail: debashis_dutta001@yahoo.com, dutt.debashis@gmail.com

* Professor, Department of Economics, Jadavpur University, Kolkata–700032, India, Telephone: (033)-2422-4877 (R), (033)-2414-6328 (O), (033)-2414-6328 (Fax –O), 9830337210(M), E-mail: basabi54@yahoo.com, basabi54@gmail.com
A Bootstrapped Historical Simulation Value at Risk Approach to S & P CNX Nifty

Abstract

This paper proposes to evaluate the predictive performance of Value at Risk (VaR) methods, empirically applied to S&P CNX NIFTY, Index of National Stock Exchange of India. The traditional Value at Risk method assumes linearity as a distributional assumption, which entails high amount of model risk. The Value at Risk model based on historical simulation is a good candidate for the financial returns series, as it does not take any distributional assumption. The Bootstrapped Historical simulation VaR is found to be a better choice as it keeps the true distributional properties along with tackling the scarcity of adequate data point by bootstrapping, which is a necessity for historical simulation.

JEL classification: C51, G28

Keywords: Value at Risk, VaR, Bootstrapped Historical Simulation, Historical Simulation, S & P CNX Nifty.
I. Motivation:

The selection of appropriate Value at Risk methodology is a challenge for both researchers as well as practitioners. This has grown in its importance in wake of necessity for market risk capital charge computation for Internal Model approach of Basel II accord as well as the gradual global migration to Value at Risk based limit management system for prudent risk management purpose. The challenges take formidable shapes due to non-linearity of the data set and also non-availability of larger data set. The Historical Simulation Value at Risk can take care of the non-linearity of the data set, as it does not take any distribution assumption like Variance Covariance method where underlying assumptions is that the data set follows a linear distributions. But conventional Historical Simulation method requires 3 to 5 years of data or more. The Bootstrapped Historical Simulation takes care of the non-availability of large data set, by bootstrapping, while retains the main characteristics of the true distribution. As most of the portfolio managers benchmark their portfolio with a reference index like S & P CNX Nifty, our study takes S & P CNX Nifty as data set. We applied various types of Value at Risk techniques along with conventional Historical Simulation Value at Risk method and also Bootstrapped Historical Value at Risk method. Our study focuses largely on their relative performance.
II. Literature Survey:

As Value at Risk has long been a central focus of risk measurement and management, there has been a huge array of literature. We have referred a few major studies with its appropriateness with our present study. Amongst earlier studies, Crnkovic and Drachman\(^1\) (1995) developed a metric and compared relative performance comparison between standard variance-covariance method and historical simulation approach. Studies by Schinassi\(^2\) (1999) dwell on dependency of VaR models on historical relationships between price movements in different markets and their trend to break down during times of stress and turbulence in event of structural breaks in relationships across markets.

In the Indian context, some remarkable researches have been carried out on VaR. Srinivasan, Shah, Ganti and Shah\(^3\) (2000) pointed that the computational cost involved as one of the drawbacks of the method and proposed the computational geometry techniques. Sarma, Thomas and Shah\(^4\) (2000) evaluated performance of a few alternative VaR models, using India’s Nifty stock market index as a case study and adopted a bi-direction approach i.e., statistical model selection and model selection based on a loss function. Dharba\(^5\) (1999) presented a new method for computing the VaR for a set of fixed-income securities based on extreme value theory that models the tail

---


probabilities directly without making any assumption about the distribution of entire return process. Nath & Reddy\(^6\) (2003) worked on foreign exchange market in India and studied various VaR methods using the Rupee-Dollar exchange rate data to understand which method is best suited for Indian system. Varma\(^7\) (1999) empirically tested of different risk management models in the Value at Risk (VaR) framework in the Indian stock market with special emphasis on EWMA model and GARCH-GED specification. Samanta & Nath\(^8\) (2003) studied three categories of VaR methods, viz., Variance-Covariance (Normal) methods including Risk-Metric, Historical Simulation (HS) and Tail-Index Based approach. Raina & Mukhopadhyay\(^9\) (2004) found out optimal allocation of a unit capital between the portfolio elements so as to maximize VaR. The algorithm has been validated using a three-asset portfolio example. Samanta G.P. and Thakur, S.K.\(^10\) (2006) assess the accuracy of VaR estimates obtained through the application of tail-index. The database consists of daily observations on two stock price indices. BSE Sensex and BSE 100 from 1999 to 2005. Results show that tail index based methods provide relatively more conservative VaR estimates and have greater chances of passing through the regulatory backtesting. Among


a plethora of studies only broad contours of related literature are presented here.

III. Methodology and Data sources:

Methodology

In our study we have chosen the requisite confidence level, forecast horizon and historical observation period, which are enumerated below.

Confidence Level

The confidence level is \( p = (1- \alpha) \), which defines the probability of the expected maximum loss. The market risk surface can be analyzed by varying the level of confidence. The most common confidence levels are between 95 \% and 99 \%, although they can vary between 90 \% and 99.9\% (Hendricks\(^\text{11}\), 1996). The Basel Committee requires the use of 99 \% confidence level in official reporting (Basel Committee, 2006)\(^\text{12}\), as it must be high enough for capital requirement calculations, but a lower level of confidence (e.g. 95 \%) can be used for internal reporting. In our study, we have selected 95\% level of confidence in order to find out VaR for internal reporting purpose.


**Forecast Horizon**

The length of the period, for which the expected maximum loss is forecasted, is known as forecast horizon or holding period. Large deviations in the portfolio value are more probable over a long period than a short one, and VaR is usually greater for a holding period of one month than for a day, for instance. The portfolio composition is assumed to remain static for VaR over the holding period. The adequate length of the holding period depends on whether the risk is measured from a private or a regulatory perspective (Christoffersen et al.\textsuperscript{13}, 1998). Trading activity and the liquidity of the assets (i.e. the time and ability to convert a position to cash) has also an impact on the adequate length of the holding period (Khindanova and Rachev\textsuperscript{14}, 2000). In practice, the holding period can vary from one trading day to some years, but the Basel Committee requires the use of 10-day holding period for official reporting. They still permit the use of a shorter holding period and scaling of VaR to correspond 10-day holding period\textsuperscript{1} (Basel Committee\textsuperscript{15}, 2006). Khindanova and Rachev\textsuperscript{16}(2000) suggest that a 10-day holding period is inadequate for frequently traded assets and restrictive for illiquid assets. As such we have taken 5-

\begin{footnotes}
\end{footnotes}
days horizon for computing VaR i.e. the reference data remains static for 5-day period.

**Historical Observation Period**

The length of the data sample in VaR calculation is known as the historical observation period. This observation period connects VaR to the history of the market risk factors, as the volatility of the risk factors is determined based on the length of the historical observation period. In practice the observation period may vary from a month to several years. The regulatory standard sets a minimum length of one year for the historical observation period (Basel Committee\(^\text{17}\), 2005), while the period may vary from a month to several years in practice. A one-period VaR can be scaled to a long horizon VaR by multiplying by the square root of the length of the horizon. For instance, a one-day VaR may be scaled to ten-day VaR by multiplying it by 10. However, this is permitted only if short horizon returns are i.i.d., which is not always the case (Christoffersen et al.\(^\text{18}\), 1998). The regulatory requirement of 250 trading days produces rather accurate VaR forecasts when used with the most common volatility models and Historical

---


Simulation VaR (Hendricks\textsuperscript{19}, 1996). Longer historical observation periods provide the most accurate forecasts (Khindanova and Rachev\textsuperscript{20}, 2000). Hendricks\textsuperscript{21} (1996) reports the superiority of 1,250-day historical observation period on the basis of an analysis of several VaR models with 95\% and 99\% levels of confidence. He finds the stability of unconditional distribution of changes in portfolio value to support the use of long periods. Hendricks\textsuperscript{22}'s (1996) results highlight the Basel Committee requirement for a minimum historical observation period of 250 days, as he finds shorter periods to produce inaccurate VaR measures. We have taken considerable long period from 1\textsuperscript{st} April 2000 to 31\textsuperscript{st} March 2007 having 1755 data points.

\textit{Data sources}

The data set used is S\& P CNX Nifty as available from National Stock Exchange website for the period from 1\textsuperscript{st} April 2000 to 31\textsuperscript{st} March 2007 as for Historical Simulation Value at risk, time horizon should be 3 to 5 years at least.


\textsuperscript{22} Ibid.
IV. Empirical Results

We have generated the profit and loss from the index returns, which replicate that of a portfolio. The profit and loss generated by an asset (or portfolio) over the period $t$, $P/L_t$, can be defined as the value of the asset (or portfolio) at the end of $t$ minus the asset value at the end of $t-1$:

$$ P/L_t = P_t - P_{t-1} $$

Wherein the positive value indicates profit, the negative value indicates loss.

**Figure 1: Histogram of absolute Profit and Loss**

Then we have attempted a distributional fitting to capture the nature of the
distribution, as the distributional assumption is one of the key points for Value at Risk computation.

**Figure 2: Fitting of Profit and Loss distribution**

In the above diagram, fit 1 is normal and fit 2 is non-normal. The fitting is attempted to find the nature of the distribution as methods of value at risk computation that we shall approach is dependent on distributional assumptions. The fitting says that the distribution in our study is a good candidate for non-normal one. Therefore, it is not a good candidate for general parametric value at risk measure, where underlying assumption is returns are normally distributed.

Then we have applied Historical Simulation method, which does not take any distributional assumption and as such a good candidate for Value
at Risk. In the historical simulation, the distribution of the future shifts in the risk factors of a portfolio is treated as the same way as the prior period distribution of shifts to simulate the value at risk. The most advantage is that it is non-parametric and as such does not assume any distributional assumption as to normality. We computed Value at Risk (called as VaR) at risk and subsequent expected shortfall (called ES) as per historical simulation. Expected Shortfall is a coherent risk measure which considers risk beyond VaR level.

**Figure 3: Historical Simulation Approach to VaR and Expected Shortfall**

The empirical result for historical simulation VaR is 49.935 and Expected Shortfall is 92.5742. The major advantage of this method is that it neither assumes returns are normally distributed nor it assumes returns.
are identically distributed over time. As a result; historical simulation model can well accommodate the fat tail for VaR computation unlike other simple approaches. This model does not bear much model risk for incorrect estimation of parameters as there is no necessity to estimate any parameter like volatilities, correlations or others.

Further, an attempt has been made to identify the behaviour of the tail as it is an important tool for risk measurement. We have constructed exploratory tool like QQ plot to get a view of the heaviness of the tail.

**Figure 4: Application of Graphical Exploratory Tool: QQ Plot**

As the QQ plot has steeper slopes at the tails and the tails have the slope different from the central mass, are suggestive of the empirical
distribution have heavier, or thinner, tails than the reference distribution. This QQ plot is a good tool for identifying outliers (e.g. observations contaminated by large errors).

The improvement over the conventional Historical Simulation Approach is Bootstrap Approach. In bootstrap method the samples are drawn from same historical data with replacement. The benefit of the method is that it implicitly takes the volatilities and correlations present in the historical data. The major advantage of this bootstrap method is that we can draw any amount of large data which is essential for model validation that may be not be case in historical simulation with less historical data. Then we computed bootstrapped VaR as below:
The result of the bootstrapped historical VaR is 51.2312 with 10000 resample from the historical data set.

**V. Concluding Observations**

Result of our study for historical bootstrapped VaR is 51.2312, which is a little higher than historical VaR i.e. 49.935. The main purpose of using bootstrapped historical VaR is that it takes care of the necessity of large data for model validation even the sample size is not adequate. As we have shown above that historical simulation is used for non-linearity present in the data set as historical simulation method takes care of
volatilities and correlations present in the referenced historical data. As such, in present context of non-linear data set which may not have all stressed scenarios, the use of bootstrapped historical VaR method is a better choice. It also be noted that the main motivation for this comparative study is that a well-defined optimization process of VaR accuracy would be a valuable asset to risk managers, though analytical derivation of such optimization process can be difficult, as the portfolio composition is not often static and the market risk factors change randomly. Accordingly, the statistical properties of VaR can vary. However, our present study leaves scope for further research as bootstrapped historical simulation, although does take care of the true distribution along with the stressed period present in the data set; it is unable to explain in full the true movement in full, where Extreme Value Theory based VaR is able to address the same.

References:


Dowd, K., 2005a., Measuring Market Risk, John Wiley & Sons Ltd,


