Estimating Volatility in the Indian Stock Markets: Some explorations

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Abstract

This paper explores to develop alternative models from the Autoregressive Conditional Heteroskedasticity (ARCH) or its generalisation, the Generalised ARCH (GARCH) family, to estimate volatility in the Indian equity market return. For this purpose, we have selected two indices each from the two widely traded stock exchanges in India – the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). For empirical analysis, the 30-script and 100-script Stock Indices from the BSE, and the S&P CNX-500 and S&P CNX-Nifty from NSE, have been selected. The sample covers daily observations from the beginning of January 2000 till the end of October 2007. The stock returns are found to possess the asymmetrical property. Apart from using own past information, we have explored two additional indicators - total Foreign Institutional Investors (FII) transactions and overnight changes in prices to explain the return prices. Empirically, it has been found that these indicators contain information in explaining the stock returns. The Threshold GARCH (TARCH) models are found to have explained the volatilities better for both the BSE Indices and S&P-CNX 500, while Exponential GARCH (EGARCH) models for the S&P CNX-Nifty. Evidence of increase in volatility due to certain negative factors has been found in all the equity markets. The estimates of the volatilities for all the indices are found to move in tandem through the application of spectral analysis.

Introduction:

Estimation of volatility in the equity market has got important implications for many issues in economics and finance. High volatility in the stock prices has many adverse effects in an economy. The investment decisions by investors may undergo changes due to high volatility, which may lead to a fall in the long-term capital flows from foreign as well as domestic investors.

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The views expressed in this paper are solely of the authors and not of the institution to which they belong.

This paper explores to develop alternative models from the Autoregressive Conditional Heteroskedasticity (ARCH) family to explain the Indian equity markets. For this purpose, we have selected two indices from each of the two main stock exchanges in India – the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). For empirical analysis, the 30-script and 100-script Stock Indices from the BSE and the S&P CNX-100 and S&P CNX-Nifty have been selected. In the literature of finance, alternative measures of volatility based on both models and estimators, have been proposed. Models and estimators, which assume constant volatility, are known as *unconditional volatility*. Afterwards it was recognized that volatility in the financial market occurs in clusters in time and is time varying. Models based on time varying volatility are known as *conditional volatility*.

After the seminal work of Engle (1982), and later on as generalized by Bollerselv (1986), the Autoregressive Conditional Heteroskedasticity (ARCH) models become quite popular to estimate the volatility, especially in the equity markets. Generally, it is observed that upward movements in the equity markets are followed by lower volatilities than the downward movements of the same magnitude. This asymmetric effect is referred as the *leverage effect.* In such cases the ARCH/ GARCH models, which are symmetrical in nature, would be inadequate to model the volatility. To capture the asymmetrical impact of volatility, Glosten *et al.* (1993) and Zakonian (1994) proposed the Threshold GARCH (TARCH) and Nelson (1991) proposed the Exponential GARCH (EGARCH) models.

In the literature, a vast amount of research effort has been done to estimate the volatility in the equity markets through the application of the family of ARCH models. In the context of the Indian economy, Karmakar (2006) through the application of an TARCH(1,1) model found existence of asymmetry in the daily returns in the Indian stock market. Evidence of contemporaneous transmission effects were also found across volatilities of the stocks and Index futures market using a TARCH model. Chen and Lian (2005) found existence of asymmetry in the equity markets of five ASEAN countries, *viz.* Malaysia, Singapore, Thailand, Indonesia and the Philippines, and found that the TARCH and EGARCH models performed better in forecasting the equity markets post Asian financial countries. Sollis (2005) found that macroeconomic variables contain valuable information to forecast stock returns and volatility in the S&P Composite Index during the 1970's, but not during the 1990's, under the GARCH framework.

The remainder of the paper is organized into six sections. Section 2 describes various models and estimators to measures volatility, both conditional and unconditional. Section 3 describes some basic statistical properties of the daily returns of the four selected stock prices. Results based on the empirical analysis are discussed in Section 4. Section 5 explores the relationship between the volatilities in different stock indices through the application of spectral analysis. Finally, Section 6 concludes.

2. Review of Volatility models and estimators:

2.1. Unconditional Volatility estimators and models:

2.1.1. Traditional estimators:

During the initial stages, volatility of asset return was estimated as the square of the asset returns, estimated based on the closing prices, and was defined by the estimator –

$$\sigma_t^2 = R_t^2$$
$$where\ R_t = \log(C_t) - \log(C_{t-1})$$
$$where\ C_t = \text{closing price of day t}$$

2.1.2. Simple Variance:

Another commonly used measure used to estimate the volatility is the simple variance, defined as,

$$\sigma_t^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} R_{t-i}^2 - n\overline{R}^2\right)$$
$$where\ \overline{R} = \text{simple average return}$$

One major limitation of this measure of volatility is the selection of the appropriate period to use. A long period may smooth out a lot of information and a short period will lead to very noisy estimate.

2.1.3. Extreme-Value estimators:

Parkinson (1980) proposed an extreme-value volatility estimator for an asset following driftless Geometric Brownian motion (GBM) and is defined as,

$$\sigma_{P.t}^2 = \frac{1}{4\ln(2)}\{\ln(H_t) - \ln(L_t)\}^2$$
$$where$$
$$H_t = \text{Highest price observed on day t}$$
$$L_t = \text{Lowest price observed on day t}$$

Later on, German and Class (1980) proposed an alternative extreme-value estimator based on the opening, closing, high and low prices. The estimator is defined as,

$$\sigma^2_{GC,t} = 0.5\{\ln(\frac{H_t}{C_t})\}^2 - \{2\ln(2)-1\}\{\ln(\frac{C_t}{O_t})\}^2$$

where

C_t = closing price

O_t = opening price

Though both the Parkinson and German-Class estimators are theoretically efficient, one major disadvantage of these estimators is that they are based on the assumption of driftless GBM process. Roger and Satchell (1991) proposed an estimator, which relaxes this assumption and proposed an alternative estimator. The Roger-Satchell estimator is defined as,

$$\sigma^2_{RS.t} = \ln(H_t / C_t)\ln(H_t / O_t) + \ln(L_t / C_t)\ln(L_t / O_t)$$

2.2. Conditional Volatility Models:

2.2.1. Symmetrical Models:

The Conditional volatility models incorporate time varying second order moments, where the series $\{y_t\}$ is decomposed into its conditional mean $\underset{\sim}{\phi}' \underset{\sim}{X}_t$ and conditional variance σ_t^2.

Both $\underset{\sim}{\phi}' \underset{\sim}{X}_t$ and σ_t^2 depends on all past information available upto period (t-1). $\underset{\sim}{\phi}'$ is the value of the co-efficient and $\underset{\sim}{X}_t$ is the vector of independent variables.

The ARCH(q) model is defined as,

$$y_t = \underset{\sim}{\phi}' \underset{\sim}{X}_t + \varepsilon_t$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

where,

$w, \alpha_1, \alpha_2, ---, \alpha_q$ = parameters to be estimated

ε_t = innovations at time t

One major problem with the application of ARCH model is that it requires long lag length and consequently a large number of parameters need to be estimated. Subsequently, Bollerslev (1986) extended the Engel's ARCH model by incorporating the autoregressive terms of the conditional variance. The specification of a GARCH(p,q) model is,

$$y_t = \underset{\sim}{\phi}' \underset{\sim}{\mathrm{X}}_{\mathrm{t}} + \varepsilon_t$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where,

$w, \alpha_1, \text{--}, \alpha_q, \beta_1, \text{--}, \beta_p =$ parameters to be estimated

$\varepsilon_{\mathrm{t}} =$ innovations at time t

If the empirical estimates of the coefficients $\{\alpha_i\}$ and $\{\beta_j\}$ are such that their sum adds upto one or more in a statistically significant sense, any shocks to the variance will be persistent in the sense that the conditional variance tends to explode as t increases.

Engle *et al.* (1987) proposed the ARCH-in-Mean (ARCH-M) model by introducing the conditional standard deviation into the mean equation. This model is commonly used where the expected returns on an asset is related to expected asset risk. The conditional standard deviation is used as a proxy for the risk and the estimated coefficient on the expected risk is considered as a measure of the risk-returns tradeoff. The ARCH-M(p,q) model is defined as,

$$y_t = \underset{\sim}{\phi}' \underset{\sim}{\mathrm{X}}_{\mathrm{t}} + \gamma\sigma_t + \varepsilon_t$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

2.2.2. Asymmetrical Models:

The TARCH (p,q) model is defined as,

$$y_t = \underset{\sim}{\phi}' \underset{\sim}{\mathrm{X}}_{\mathrm{t}} + + \varepsilon_t$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 \xi_{t-i}$$

where

$\xi_{t-i} = 1 \text{ if } \varepsilon_{t-i} < 0$

and $\xi_{t-i} = 0 \text{ if } \varepsilon_{t-i} > 0$

In particular for a TARCH(1,1) model, the specification for the conditional variance is defined as,

$$\sigma_t^2 = \varpi + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 \xi_{t-1}$$

The TARCH model is formulated based on the assumption that unexpected changes in the market return measured by $\{\varepsilon_t\}$ have different impact on the conditional variance of the returns. In this specification, good news $(\varepsilon_t > 0)$ and bad news $(\varepsilon_t < 0)$ have different impact on the conditional variance. Good news, *i.e.* during the period of upward movement in the equity market, the variance will increase through the coefficient α_1. On the other hand, bad news, *i.e.* during the period of downward movement in the equity market, the variance will increase through the coefficient $(\alpha_1 + \gamma_1)$. A non-zero value of the coefficient γ_1 implies the asymmetrical nature of the return with a positive value of γ_1 indicates the presence of leverage effect.

The EGARCH (p,q) model is defined as,

$$y_t = \underset{\sim}{\phi}' \underset{\sim}{\mathrm{X}_t} + + \varepsilon_t$$

$$\ln(\sigma_t^2) = \varpi + \sum_{i=1}^{q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^{q} \gamma_i (\frac{\varepsilon_{t-i}}{\sigma_{t-i}})$$

In particular for a EGARCH(1,1) model, the specification for the conditional variance is defined as,

$$\ln(\sigma_t^2) = \varpi + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

In the EGARCH model, where the conditional variance is characterized by exponential nature assumes that the external unexpected shocks will exert a stronger influence on the variance than in the TARCH model. A non-zero value of γ_1 indicates the existence of asymmetrical effect in the returns and a negative value indicates the presence of leverage effect.

3. Basic Statistical Properties of the Stock Return:

In India, the National Stock Exchange (NSE) and Bombay Stock Exchange (BSE) are the two major stock exchanges involved in the equity market. For our empirical analysis, we have considered two stock indices each from both the stock exchanges. The BSE-30 and BSE-100, which are based on 30-script and 100-script, have been taken from the BSE for the analysis. On the other hand the two indices, *viz.,* the NSE-S&P CNX-500 and NSE-S&P Nifty, which are based on 500-script and 50-script, respectively, have been selected from the NSE. The nominal stock return is estimated as:

The daily data for the indices are taken from the website of the two indices. Also the total value of the purchase and sales by the foreign investors during a day in the equity market may also influence on the equity prices. Daily data on the purchase of sales by foreign investors in the equity market has been collected from the website of Securities and Exchange Bank of India (SEBI). The data series covers the period from 3-January-2000 to 31-October-2007 having 1954 daily observations. Figure-1 presents the daily returns of the four indices. By visual inspection, the possibility of existence of heteroskedasticity cannot be ruled out, as the amplitude of changes varies over time.

Table-1 presents the descriptive statistics for all the four indices. The average daily returns are found to be approximately equal for all the four indices. The average daily return for BSE-100 is found to be slightly lower at 0.06%, while for the other three indices it is found to be 0.07%.

The standard deviation of the return series is 1.5% daily for both BSE-30 and S&P-CNX Nifty or 24.3% annually, assuming 252 working days per year. For both BSE-100 and S&P-CNX 500, the standard deviation of the return series is found to be 1.7% or 27.0% annually.

The coefficients of the skewness are found to be significant and negative for all the returns. The negative values indicate that the average investor in the equity market prefers negative asymmetry as compared to positive asymmetry. This indicates that a rational investor prefers portfolios with lower probability of large payoffs.

Similarly, the coefficients of kurtosis are found to be positive and are significantly higher than 3, indicating highly leptokurtic distribution compared to the normal distribution for all the returns. The investor's preferences for higher moments are important for security valuation and thus such preference take positive values.

The Jarque-Bera statistic indicates lack of normal distribution in the equity returns, suggesting lack of symmetric nature in the equity returns. Figure-2 presents the Karnel density function of the all the return series, which confirms our finding that the series does not follow normal distribution.

Both the Augmented Dickey-Fuller (ADF) and Philips-Perron test statistic rejects the null hypothesis of presence of unit roots suggest that all the return series are stationary in nature. Thus, in the long-run, the return series reverts back to its mean level and the unconditional variances of all the series are constant in nature. However, there may be periods in which the variance may be relatively high.

Table-2 presents the Ljung-Box (LB) Q-statistic for high-order serial correlation for all the four return series up to lag 24. The Q-statistic rejects the null hypothesis of independence, suggesting that equity returns exhibit dependencies on its past behavior Table-3 presents the LB statistic of the squared series (also termed as a measure of volatility). The squared series indicates significant second-order dependencies in all the return series, suggesting the possibility of conditional variance heteroskedasticity effect. Thus the above results indicate that the return series are not independently and identically distributed, even though they follow a stationary process.

Table-1: Descriptive Statistics of daily returns

	BSE-30	**BSE-100**	**S&P CNX-500**	**S&P CNX-Nifty**
Mean	+0.0007	+0.0006	0.0007	+0.0007
Std. Deviation	+0.0153	+0.0166	+0.0167	+0.0153
Maximum	+0.0793	+0.0875	+0.1635	+0.0797
Minimum	-0.1181	-0.1194	-0.1607	-0.1305
Skewness	-0.6122	-0.6397	-0.7561	-0.6794
Kurtosis	+7.1659	+7.5142	+15.7186	+8.2785
Jarque-Bera Statistics	1534.22 (0.000)	1786.84 (0.000)	13342.71 (0.000)	2417.54 (0.000)
Unit Root Test				
ADF test[3]	-31.57	-31.20	-12.35	-14.29
Philips-Perron test	-41.07	-39.97	-5.36	-40.50

*The Critical values for the ADF test and Phillips-Perron test are –3.968 and –3.415 at 1% and 5% level of significance, as provided by MacKinnon.

[3] The general form of the ADF test is estimated by, $\Delta y_t = a_0 + bt + a_1 y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t$

where a_0 is a constant, t represents deterministic trend and the lag length *'p'* of differences are incorporated to ensure that error term ε_t becomes white noise. The lag length *'p'* is determined based on the Akaike's Information Criterion (AIC). The null hypothesis to be tested is $H_0 : a_1 = 0$ against the alternative $H_1 : a_1 < 0$

Table-2: Autocorrelation and Ljung-Box Q-statistic for serial correlation

Lag	BSE-30			BSE-100			S&P CNX-500			S&P CNX-Nifty		
	AC	Q-stat	*p-val*	AC	Q-stat	*p-val*	AC	Q-stat	*p-val*	AC	Q-stat	*p-val*
1	0.076	11.239	0.001	0.102	20.475	0.000	0.085	14.094	0.000	0.088	15.187	0.000
2	-0.038	14.120	0.001	-0.038	23.286	0.000	-0.034	16.342	0.000	-0.077	26.701	0.000
3	0.005	14.168	0.003	0.027	24.762	0.000	0.018	16.979	0.001	0.018	27.327	0.000
4	0.056	20.217	0.000	0.070	34.260	0.000	0.072	27.113	0.000	0.059	34.196	0.000
5	-0.006	20.285	0.001	0.013	34.580	0.000	0.006	27.196	0.000	0.008	34.310	0.000
6	-0.034	22.536	0.001	-0.054	40.384	0.000	-0.023	28.223	0.000	-0.036	36.917	0.000
7	-0.017	23.115	0.002	0.013	40.728	0.000	-0.011	28.472	0.000	-0.022	37.907	0.000
8	0.003	23.137	0.003	0.017	41.261	0.000	0.028	30.018	0.000	-0.001	37.908	0.000
9	0.053	28.703	0.001	0.053	46.757	0.000	0.051	35.220	0.000	0.052	43.184	0.000
10	0.035	31.058	0.001	0.041	50.047	0.000	0.055	41.101	0.000	0.044	47.040	0.000
11	-0.023	32.083	0.001	-0.008	50.170	0.000	0.001	41.102	0.000	-0.013	47.385	0.000
12	-0.012	32.350	0.001	0.003	50.184	0.000	-0.014	41.509	0.000	-0.035	49.732	0.000
13	0.023	33.369	0.001	0.021	51.017	0.000	0.045	45.505	0.000	0.044	53.620	0.000
14	0.018	34.013	0.002	0.047	55.312	0.000	0.030	47.222	0.000	0.027	55.026	0.000
15	0.010	34.203	0.003	0.002	55.317	0.000	-0.005	47.272	0.000	-0.001	55.026	0.000
16	0.005	34.249	0.005	-0.001	55.318	0.000	-0.036	49.815	0.000	-0.011	55.257	0.000
17	0.023	35.335	0.006	-0.003	55.335	0.000	0.050	54.714	0.000	0.027	56.729	0.000
18	0.007	35.420	0.008	0.002	55.341	0.000	0.008	54.832	0.000	0.013	57.049	0.000
19	-0.045	39.398	0.004	-0.060	62.390	0.000	-0.037	57.602	0.000	-0.028	58.624	0.000
20	-0.061	46.693	0.001	-0.041	65.666	0.000	-0.055	63.656	0.000	-0.066	67.283	0.000
21	0.036	49.280	0.000	0.034	67.891	0.000	0.025	64.846	0.000	0.088	15.187	0.000
22	0.016	49.776	0.001	0.008	68.011	0.000	0.002	64.853	0.000	-0.077	26.701	0.000
23	0.020	50.571	0.001	0.051	73.197	0.000	0.027	66.276	0.000	0.018	27.327	0.000
24	0.031	52.511	0.001	0.002	73.205	0.000	0.004	66.301	0.000	0.059	34.196	0.000

4. Model to estimate the volatility:

As the return series are found to be asymmetrical as well as with leptokurtic in nature, the simple ARCH/ GARCH models, which are useful for symmetrical data series, will not be the appropriate models for analysis of the return series. This led to the adoption of the asymmetrical models, *viz.,* TARCH and EGARCH models to explore the return series. Initially, TARCH/ GARCH models are estimated based on its own past information only. Thus the conditional variances estimated based on these models ignore completely the possible impact of other related economic variables such as the foreign inflows and domestic investment in the equity market. The value of the total purchase and sales by the foreign investors, during a day in the equity market, is an important factor, which impacts on the equity prices which may further lead to volatility in the Indian equity market. Apart from the foreign investment factor, other unforeseen factors may also have impact on the equity prices. Any unforeseen factors that may occur after the closing of the business day may be reflected through the opening prices of the next day. Especially, the movement in the stock prices in the western countries of the previous day or of the eastern Asian countries on the same day may have impact on the Indian equity market. To capture the impact of these unforeseen factors on the equity prices, we have defined a technical variable *overnight_return,* to measure the overnight changes in the stock prices, as follows:

$$Overnight\_return_t = \log(\frac{O_t}{C_{t-1}})$$

All the models are estimated assuming Student t-distribution for the error terms to allow for kurtosis and TARCH/ GARCH to allow for skewness.

Table 3: Alternative models for stock return (r_t)

Name	Model Type	Form of the Model
Model A	TARCH(1,1)	$r_t = \underset{\sim}{\phi}' \underset{\sim}{X_t} + +\varepsilon_t$ $\sigma_t^2 = \varpi + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2\xi_{t-1}$ *where* $\xi_{t-1} = 1 \text{ if } \varepsilon_{t-1} < 0$ $and\ \xi_{t-1} = 0 \text{ if } \varepsilon_{t-1} > 0$ $\underset{\sim}{X} = \{r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}\}'$
Model B	EGARCH(1,1)	$y_t = \underset{\sim}{\phi}' \underset{\sim}{X_t} + +\varepsilon_t$ $\ln(\sigma_t^2) = \varpi + \beta\ln(\sigma_{t-1}^2) + \alpha\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right\rvert + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ $\underset{\sim}{X} = \{r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}\}'$
Model C	Multivariate TARCH(1,1)	$r_t = \underset{\sim}{\phi}' \underset{\sim}{X_t} + +\varepsilon_t$ $\sigma_t^2 = \varpi + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2\xi_{t-1}$ *where* $\xi_{t-1} = 1 \text{ if } \varepsilon_{t-1} < 0$ $and\ \xi_{t-1} = 0 \text{ if } \varepsilon_{t-1} > 0$ $\underset{\sim}{X} = \{r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}, \log(FII\_total), overnight\_return\}'$
Model D	Multivariate EGARCH(1,1)	$y_t = \underset{\sim}{\phi}' \underset{\sim}{X_t} + +\varepsilon_t$ $\ln(\sigma_t^2) = \varpi + \beta\ln(\sigma_{t-1}^2) + \alpha\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right\rvert + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ $\underset{\sim}{X} = \{r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}, \log(FII\_total), overnight\_return\}'$

We have explored four alternative models to estimate the volatility present in the four return series. Table-3 presents the alternative form of the models. The estimate of the volatility for a particular series has been obtained based on the best-selected model. The best model has been selected based on two statistical criteria, *viz.,* with higher $\bar{R}^2$ and having minimum AIC. Also, we have compared the estimate of the volatility in the return series of stock indices with the estimates based on the Roger-Satchell estimator, though they are not strictly comparable.

4.1. Model for BSE-30:

The estimated four alternative models for BSE-30 are provided in Table-4. The Lagrange Multiplier (LM) test for the presence of residual serial correlation accepts the null hypothesis of lack of residual auto-correlation, as suggested by the *p-values.* Also, the ARCH-LM test for the presence of residual ARCH effect indicates lack of residual ARCH-effect. The LM tests suggest no significant specification error in the formulation of the models. The current return is found to be significantly influence by the previous-day return except as estimated under the Model C. None of the models could establish any significant impact on the current return by its second lag. However, the third and fourth lags are found to have significant impact in determining the current return. The total FII transaction's under Model D is found to have significant impact in determining the daily return. On the other hand, the overnight return is found to contain significant information about the daily return, as suggested by the *p-values*.

Under the two TARCH models (Model A and Model C), the coefficient of σ_{t-1}^2 are found to be significant, suggesting the salient features of the time-varying volatility of BSE-30 stock returns. The term ε_{t-1}^2 is found to have significant impact on the conditional volatility under Model C. at the conventional level. The $\varepsilon_{t-1}^2\xi_{t-1}$ terms are found to be significant and positive, thereby indicating evidence of asymmetrical impact of good/ bad news on the stock returns, *i.e.,* existence of leverage effect. As mentioned earlier, the coefficient of ε_{t-1}^2 measures the impact of good news, while, the sum of the two coefficients of the terms ε_{t-1}^2 and $\varepsilon_{t-1}^2\xi_{t-1}$ measures the impact of bad news. Thus, under Model A, bad news is expected to lead to an increase the volatility from 0.019 to 0.019+0.238 = 0.257. The volatility is expected to move up from 0.050 to 0.050+0.202=0.252 due to bad news under Model C.

The two terms $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ and $\ln(\sigma^2_{t-1})$ under the two EGARCH models, are found to be significant. Also the coefficients of the term $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ are found to be significant and negative under Model B and Model D, thus confirming the existence of leverage effect on the stock returns.

The $\bar{R}^2$ is found to be maximum for both Model C and Model D. However, the AIC of Model C is found to be lower than the Model D, suggesting the superiority of Model C. This led to the adoption of Model C (Multivariate TARCH) to estimate the conditional volatility of daily return of BSE-30.

Table-4: Estimated models for daily return – BSE 30

	Model A	Model B	Model C	Model D
Conditional Mean equation:				
C1	+0.001 (0.045)	+0.001 (0.623)	-0.003 (0.049)	-0.004 (0.002)
r_{t-1}	+0.108 (0.000)	+0.116 (0.000)	+0.042 (0.089)	+0.051 (0.034)
r_{t-3}	+0.055 (0.015)	+0.057 (0.009)		
r_{t-4}	+0.053 (0.030)	+0.046 (0.054)	+0.067 (0.005)	+0.070 (0.004)
log (FII_ total)			+0.0001 (0.053)	+0.001 (0.002)
Overnight_ return			+0.747 (0.000)	+0.753 (0.000)
Conditional Variance equation:				
C2	+1.54e-05 (0.000)	-0.989 (0.000)	+1.38 e-05 (0.000)	-0.988 (0.000)
ε^2_{t-1}	+0.019 (0.203)		+0.050 (0.003)	
σ^2_{t-1}	+0.784 (0.000)		+0.768 (0.000)	
$\varepsilon^2_{t-1}\xi_{t-1}$	+0.238 (0.000)		+0.202 (0.000)	
$\lvert \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \rvert$		+0.266 (0.000)		+0.276 (0.000)
$\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$		-0.155 (0.000)		-0.138 (0.000)
$\ln(\sigma^2_{t-1})$		+0.910 (0.000)		+0.912 (0.000)
$\bar{R}^2$	0.002	0.001	0.130	0.130
AIC	-5.808	-5.800	-5.936	-5.930
Serial Correlation - LM	6.481 (0.691)	7.477 (0.588)	10.171 (0.426)	10.714 (0.380)
ARCH-LM	0.677 (0.775)	0.892 (0.555)	1.236 (0.252)	1.224 (0.260)

4.2. Model for BSE-100:

Table-5 presents the estimates of the four alternative models for BSE-100. The Lagrange Multiplier (LM) test for residual serial correlation and ARCH-effects accepts the null hypothesis of lack of residual auto-correlation and ARCH-effect as suggested by the *p-values.* Thus there is no significant specification error in the formulation of the models. The current return is found to be significantly influence by the previous-day return except for all the models. Also, as in the case of BSE-30, we could not establish any significant impact of the second lag on the current return. However, the third and fourth lags are found to have significant impact in determining the current return. The total FII transactions and the overnight return are found to have significant impact about the daily return, as suggested by the *p-values* under the multivariate setup.

Under the two TARCH models (Model A and Model C), ε_{t-1}^2 and σ_{t-1}^2 are found to be significant, suggesting the salient features of the time-varying volatility of BSE-100 stock returns. The $\varepsilon_{t-1}^2 \xi_{t-1}$ terms are found to be significant and positive, thereby indicating evidence of asymmetrical impact of good/ bad news on the stock returns. Under Model A, bad news is expected to increases the volatility from 0.087 to 0.087+0.169 = 0.256. Under Model C, the bad news increases volatility from 0.057 to 0.057+0.233=0.290.

The terms $|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}|$ and $\ln(\sigma_{t-1}^2)$ in both the EGARCH models are found to be significant. Also the $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ is found to be significant and negative, suggesting the existence of leverage effect on the stock returns.

The $\bar{R}^2$ both the two Model C and Model D are found to be maximum at 0.173. However, the AIC of Model C is found to be –5.854, which is lower than the AIC of the Model D, suggesting the superiority of Model C in explaining the return in BSE-100. This led to the adoption of Model C (Multivariate TARCH) to estimate the conditional volatility of daily return of BSE-100.

Table-5: Estimated models for daily return – BSE 100

	Model A	Model B	Model C	Model D
Conditional Mean equation				
C1	-0.001 (0.013)	+0.001 (0.030)	-0.004 (0.029)	-0.005 (0.003)
r_{t-1}	+0.130 (0.000)	+0.141 (0.000)	+0.086 (0.000)	+0.096 (0.000)
r_{t-3}	+0.076 (0.001)	+0.073 (0.001)	+0.061 (0.005)	+0.066 (0.002)
r_{t-4}	+0.044 (0.080)	+0.031 (0.201)	+0.079 (0.001)	+0.085 (0.000)
log (FII_ total)			+0.001 (0.038)	+0.001 (0.006)
Overnight_ return			+0.720 (0.000)	+0.731 (0.000)
Conditional Variance equation				
C2	+1.29e-05 (0.000)	-0.858 (0.000)	+1.78 e-05 (0.000)	-1.101 (0.000)
ε_{t-1}^2	+0.087 (0.000)		+0.057 (0.006)	
σ_{t-1}^2	+0.776 (0.000)		+0.734 (0.000)	
$\varepsilon_{t-1}^2 \xi_{t-1}$	+0.169 (0.000)		+0.233 (0.000)	
$\mid \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \mid$		+0.334 (0.000)		+0.302 (0.000)
$\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$		-0.106 (0.000)		-0.148 (0.000)
$\ln(\sigma_{t-1}^2)$		+0.930 (0.000)		+0.900 (0.000)
$\overline{R}^2$	0.009	0.001	0.173	0.173
AIC	-5.716	-5.712	-5.854	-5.850
Serial Correlation - LM	9.002 (0.437)	11.055 (0.272)	6.057 (0.734)	7.512 (0.584)
ARCH-LM	0.683 (0.769)	0.602 (0.842)	1.102 (0.354)	1.230 (0.256)

4.3. Model for S&P CNX-500:

Table-6 presents the estimates of the four alternative models for S&P CNX-500. The Lagrange Multiplier (LM) test for residual serial correlation and ARCH-effects accepts the null hypothesis of lack of residual auto-correlation and ARCH-effect as suggested by the *p-values,* signifying the lack of specification error in the models formulation. The current return is found to be significantly influence by the previous-day return and also with a lag of three days. The overnight return is found to have significant impact on the daily return, as suggested by the significant by the *p-values* under the multivariate setup. However, we could not found any significant impact of the total FII transactions on the daily return of S&P CNX-500.

Under the two TARCH models (Model A and Model C), ε_{t-1}^2 and σ_{t-1}^2 are found to be significant, suggesting the salient features of the time-varying volatility of S&P CNX-500 stock returns. The $\varepsilon_{t-1}^2\xi_{t-1}$ terms are found to be significant and positive, thereby indicating evidence of asymmetrical impact of good/ bad news on the stock returns. The $|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}|$ and $\ln(\sigma_{t-i}^2)$ terms in both the EGARCH models are found to be significant. Also the $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ is found to be significant and negative, suggesting the existence of leverage effect on the stock returns. Under Model A, bad news is expected to push up the volatility from 0.075 to 0.075+0.301 = 0.376.

The AIC of Model C is found to be –5.918, which is lower than the AIC of the Model D, suggesting the superiority of Model C in explaining the return in S&P CNX-500. This led to the adoption of Model C (Multivariate TARCH) to estimate the conditional volatility of daily return of S&P CNX-500.

Table-6: Estimated models for daily return – S&P CNX 500

	Model A	Model B	Model C	Model D
Conditional Mean equation				
C1	+0.001 (0.019)	+0.001 (0.001)	-0.0004 (0.142)	-0.001 (0.038)
r_{t-1}	+0.143 (0.000)	+0.154 (0.000)	+0.896 (0.000)	+0.080 (0.001)
r_{t-2}			+0.092 (0.000)	
r_{t-3}	+0.055 (0.019)	+0.039 (0.094)	+0.052 (0.033)	+0.048 (0.037)
r_{t-4}	+0.076 (0.002)	+0.067 (0.009)		+0.054 (0.030)
Overnight_ return			+0.896 (0.000)	+0.881 (0.000)
Conditional Variance equation				
C2	+4.68e-05 (0.000)	-1.757 (0.000)	+1.25 e-05 (0.000)	-0.907 (0.000)
ε_{t-1}^2	+0.075 (0.000)		+0.118 (0.006)	
σ_{t-1}^2	+0.582 (0.000)		+0.761 (0.000)	
$\varepsilon_{t-1}^2 \xi_{t-1}$	+0.301 (0.000)		+0.128 (0.000)	
$\left\| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right\|$		+0.360 (0.000)		+0.326 (0.000)
$\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$		-0.139 (0.000)		-0.097 (0.000)
$\ln(\sigma_{t-1}^2)$		+0.824 (0.000)		+0.925 (0.000)
$\overline{R}^2$	0.004	0.003	0.221	0.221
AIC	-5.645	-5.637	-5.918	-5.911
Serial Correlation - LM	3.831 (0.872)	4.860 (0.770)	8.349 (0.499)	9.283 (0.324)
ARCH-LM	0.67 (0.78)	0.34 (0.98)	1.064 (0.387)	1.235 (0.250)

4.4. Model for S&P CNX-Nifty:

The estimated four alternative models for S&P CNX-Nifty are provided in Table-7. The Lagrange Multiplier (LM) test for suggests lack of residual serial correlation and ARCH-effect. The current return is found to be significantly influence by the previous four days return and overnight return. The total FII transaction's under Model D is found to have significant impact in determining the daily return.

Under the two TARCH models (Model A and Model C), σ_{t-1}^2 are found to be significant, suggesting the salient features of the time-varying volatility of S&P CNX-Nifty stock returns. The ε_{t-1}^2 term is found to be significant under Model C. The term $\varepsilon_{t-1}^2 \xi_{t-1}$ in the two TARCH models are found to be significant and positive, thereby indicating evidence of asymmetrical impact of good/ bad news on the stock returns. Under Model A, bad news is expected to lead to an increase the volatility from 0.020 to 0.020+0.290 = 0.310. The volatility is expected to move up from 0.027 to 0.027+0.238=0.265 due to bad news under Model C. Similarly, under the two EGARCH models (Model B and Model D), the term $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ is found to be significant and negative, suggesting the existence of leverage effect on the stock returns.

However, the AIC of Model D is found to be lowest than the other models, suggesting the superiority of Model D in explaining the return in the S&P CNX- Nifty prices. This led to the adoption of Model D (Multivariate EGARCH) to estimate the conditional volatility of daily return of S&P CNX-Nifty.

Table-7: Estimated models for daily return – S&P CNX Nifty

	Model A	Model B	Model C	Model D
Conditional Mean equation				
C1	+0.001 (0.052)	+0.001 (0.084)	+0.001 (0.086)	-0.004 (0.028)
r_{t-1}	+0.130 (0.000)	+0.135 (0.000)	+0.121 (0.000)	+0.123 (0.000)
r_{t-2}	-0.056 (0.018)	-0.055 (0.022)	-0.058 (0.015)	-0.059 (0.014)
r_{t-3}	+0.091 (0.000)	+0.095 (0.000)	+0.072 (0.002)	+0.074 (0.001)
r_{t-4}	+0.059 (0.011)	+0.056 (0.016)	+0.056 (0.020)	+0.052 (0.032)
Log (FII_ total)				+0.001 (0.012)
Overnight_ return			+0.710 (0.000)	+0.696 (0.000)
Conditional Variance equation				
C2	+1.82e-05 (0.000)	-1.161 (0.000)	+1.57 e-06 (0.000)	-1.016 (0.000)
ε_{t-1}^2	+0.020 (0.215)		+0.027 (0.093)	
σ_{t-1}^2	+0.748 (0.000)		+0.772 (0.000)	
$\varepsilon_{t-1}^2 \xi_{t-1}$	+0.290 (0.000)		+0.238 (0.000)	
$\left\| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right\|$		+0.295 (0.000)		+0.267 (0.000)
$\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$		-0.187 (0.000)		-0.166 (0.000)
$\ln(\sigma_{t-1}^2)$		+0.892 (0.000)		+0.907 (0.000)
$\bar{R}^2$	0.009	0.008	0.035	0.0352
AIC	-5.812	-5.810	-5.8316	-5.8318
Serial Correlation - LM	4.605 (0.799)	5.702 (0.681)	4.010 (0.856)	4.106 (0.847)
ARCH-LM	0.405 (0.962)	0.572 (0.866)	0.668 (0.783)	0.627 (0.821)

5. Some exploration of the conditional volatility:

Generally, it is expected that the occurrence of volatilities in the equity market of a country should be reflected in all the indices in tandem. To test this hypothesis, whether the estimate of the volatilities in the indices as estimated by the TARCH/ EGARCH models move in tandem or not, one can perform statistical test either in the time-domain or frequency-domain. In such situation, results based on the frequency domain are expected to be more powerful than those based on the time domain. For our empirical analysis, we considered the estimated volatilities of the BSE-30 index and compare these with the volatilities of the two selected indices of NSE.

To test this hypothesis, the spectral analysis has been performed. For the application of a spectral analysis it is desirable that all the series should possess the stationary property. For a formal determination, whether volatilities of the three series BSE-30, S&P CNX-500 and S&P CNX-Nifty possess the stationary property or not, the unit root test are conducted applying the ADF test. The empirical results of the ADF test are reported in Table-8. The ADF- test rejects the null hypothesis of presence of unit-roots, *i.e.* the estimates of volatilities are stationarity in nature.

Table 8: Unit Root Test of the estimate of volatilities

Series	ADF-test statistic	Critical Value	
		5%	1%
BSE-30	-10.596	-2.864	-3.437
S&P CNX-500	-6.224	-2.864	-3.437
S&P CNX-Nifty	-6.940	-2.864	-3.437

The spectral analysis between two variables can be analysed using squared coherence[4] and phase[5]. The cross-spectra between two variables reflect the importance of the relationship between the variables across all frequency[6].

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[4] The squared coherence between two variables measures the degree to which the variables move together. It is analogous to the square of the correlation co-efficient at each frequency.

[5] The phase statistic measures the lead- lag relationship between two variables at each frequency. The phase conception is similar to the concept of Granger causality used in the time series analysis.

[6] Here the frequency 'w' is measured in radian, with $0 \le w \le \pi$ and each frequency corresponds to a periodicity of $period = \frac{2\pi}{w}$. Thus for quarterly data, the frequency w = 0.20 corresponds to a cycle of period 32-quarters, while the frequency w = 1.26 corresponds to a cycle of period 5- quarters. Thus if one assumes the periodicity of the business cycle to be in the range of 5 to 32- quarters, the corresponding frequency band for this will be between 0.20 and 1.26.

Figure-5.1 and Figure-5.2 presents the squared coherence between the volatilities of BSE-30 with S&P CNX-500 and S&P CNX-Nifty respectively. The squared coherence of volatilities of BSE-30 with both the volatilities of the other two indices are found to be quite high at all frequencies, suggesting that the volatilities move together at all points of time.

Figure-5.3 and Figure-5.4 presents the phase-statistic of the cross- spectrum between volatilities of BSE-30 with S&P CNX-500 and S&P CNX-Nifty respectively. The phase values between the volatilities of BSE-30 and S&P CNX-Nifty are found to be approximately zero at all frequency, implying that the volatilities of both the series move in tandem. However, some positive slopes were found between the volatilities of BSE-30 and S&P CNX-500 at the high frequency range, suggesting the possibility of leading behavior of volatility of BSE-30 on S&P CNX-500.

Thus both squared coherence and phase-statistic indicates the possession of similar characteristics between the volatilities of BSE-30 with S&P CNX-500 and S&P CNX-Nifty, at all points of time. Thus the estimates of the volatilities for all the indices move in tandem.

6. Conclusions:

This paper explores to develop alternative models from the ARCH/ GARCH family to model the Indian equity markets. The equity market has been represented by the two widely traded stock exchanges in India – the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). Two stock indices, from each of the exchanges are selected for empirical analysis. The widely quoted 30-script BSE Index (known as the SENSEX) and the BSE-100 script have been selected from the BSE. The two indices, S&P CNX-500 and S&P CNX-Nifty have been selected from the NSE. The sample covers daily observations from the beginning of January 2000 till the end of October 2007. The stock returns are found to have possessed the asymmetrical property.

Apart from using own past information, we have explored two indicators - total FII transactions and overnight changes in stock prices, to explain the return prices. Empirically, it has been found that these indicators contain information in explaining the return prices. The Threshold GARCH (TARCH) models are found to have explained the volatilities better for both the BSE Indices and S&P-CNX 500, while the Exponential GARCH (EGARCH) model is found to be superior for the S&P CNX-Nifty. Empirically, bad news, which can also be termed as contribution of certain negative factors, has also been found to have lead to an increase in the equity market. To test whether the volatilities for all the indices move in tandem or not, we performed the statistical tests in the frequency domain and found that the volatilities for all the indices move in tandem.

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Figure-1: Daily return of stock prices

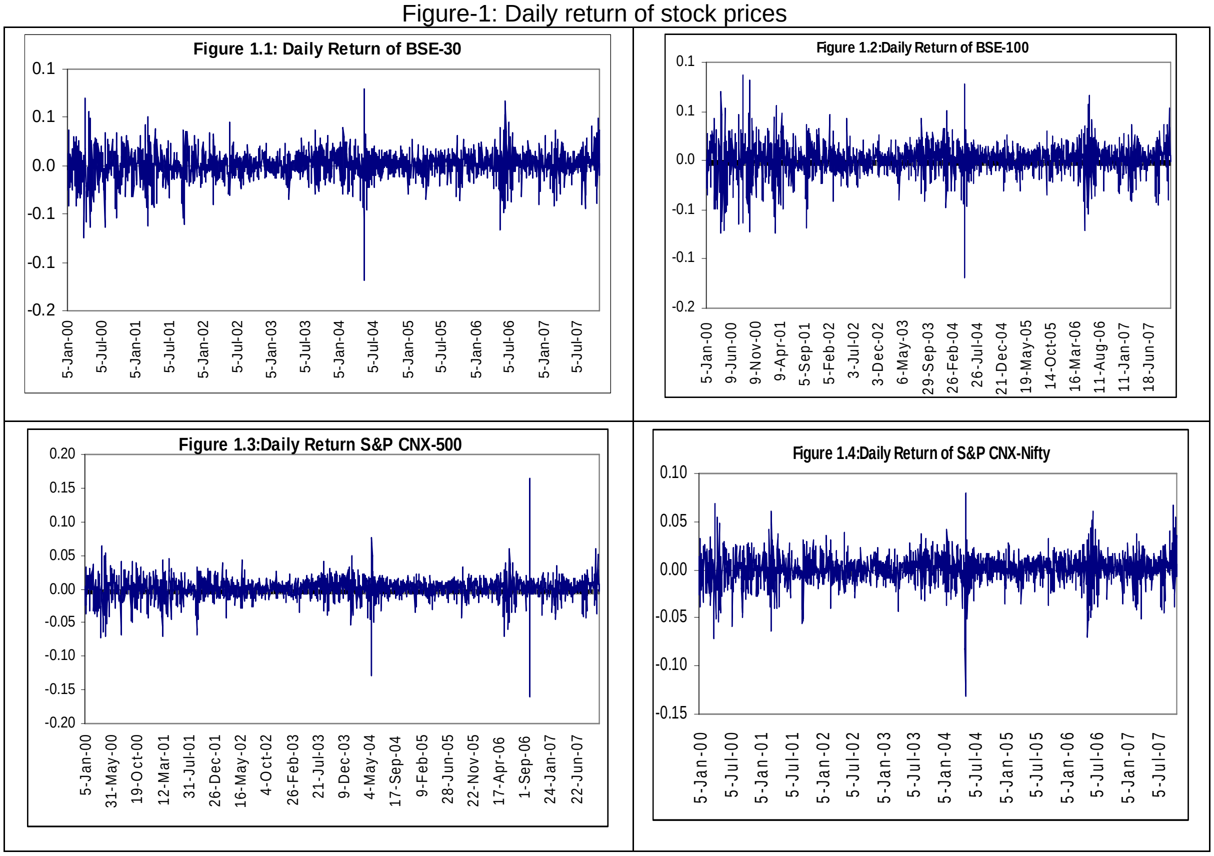

Figure-2: Kernel Density of the return series

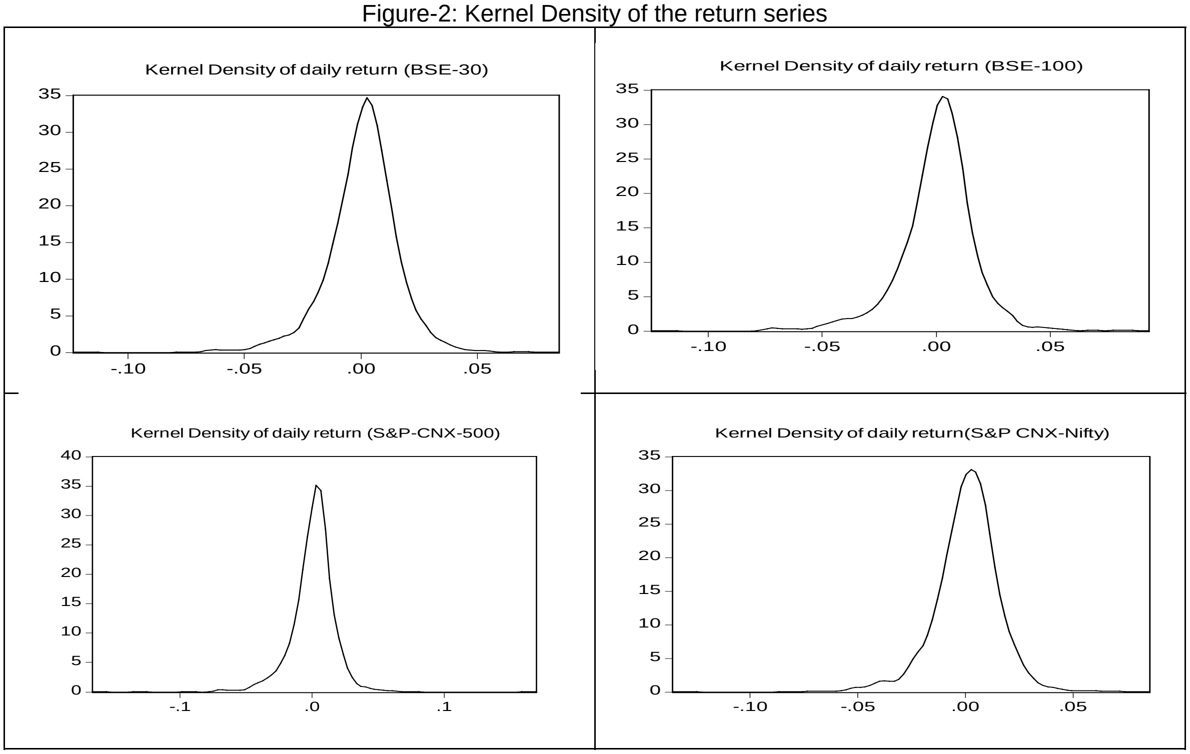

Figure-3: Estimates of Volatility of stock returns – TARCH/ EGARCH models

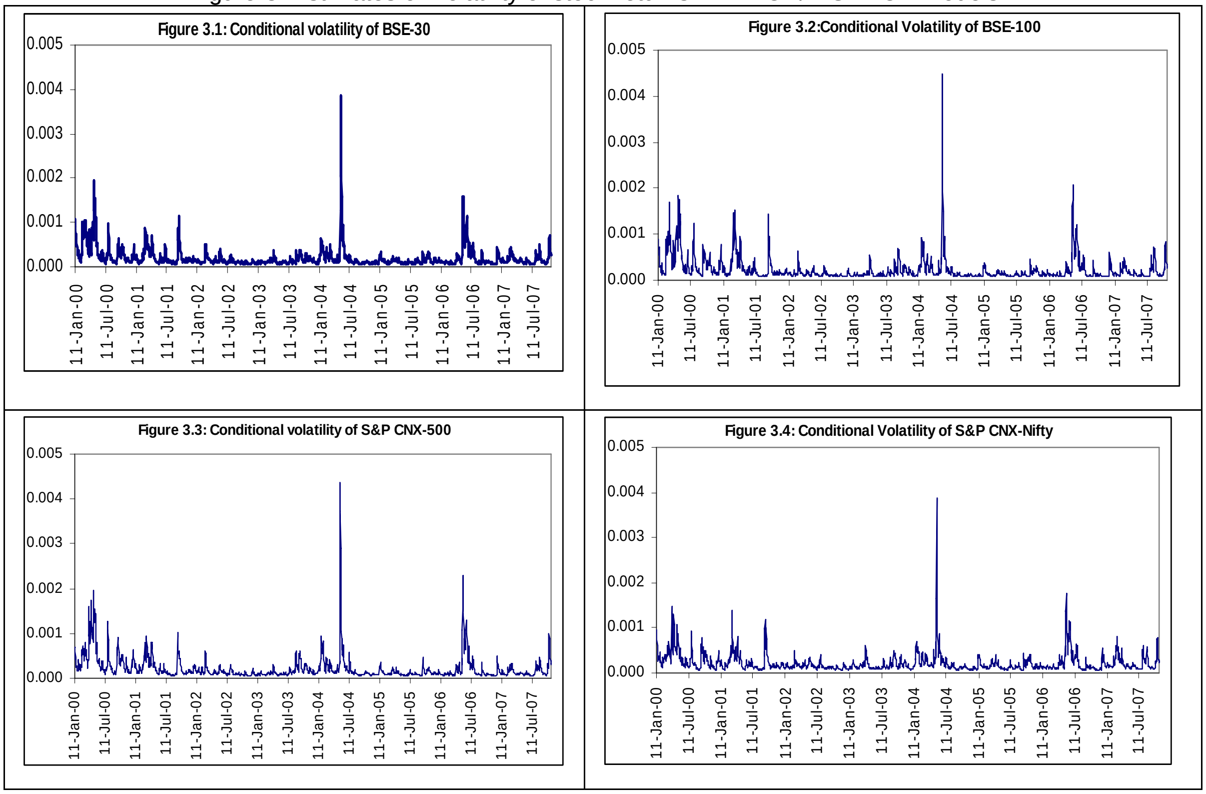

Figure-4: Estimates of Volatility of stock returns – Rogers- Satchell estimator

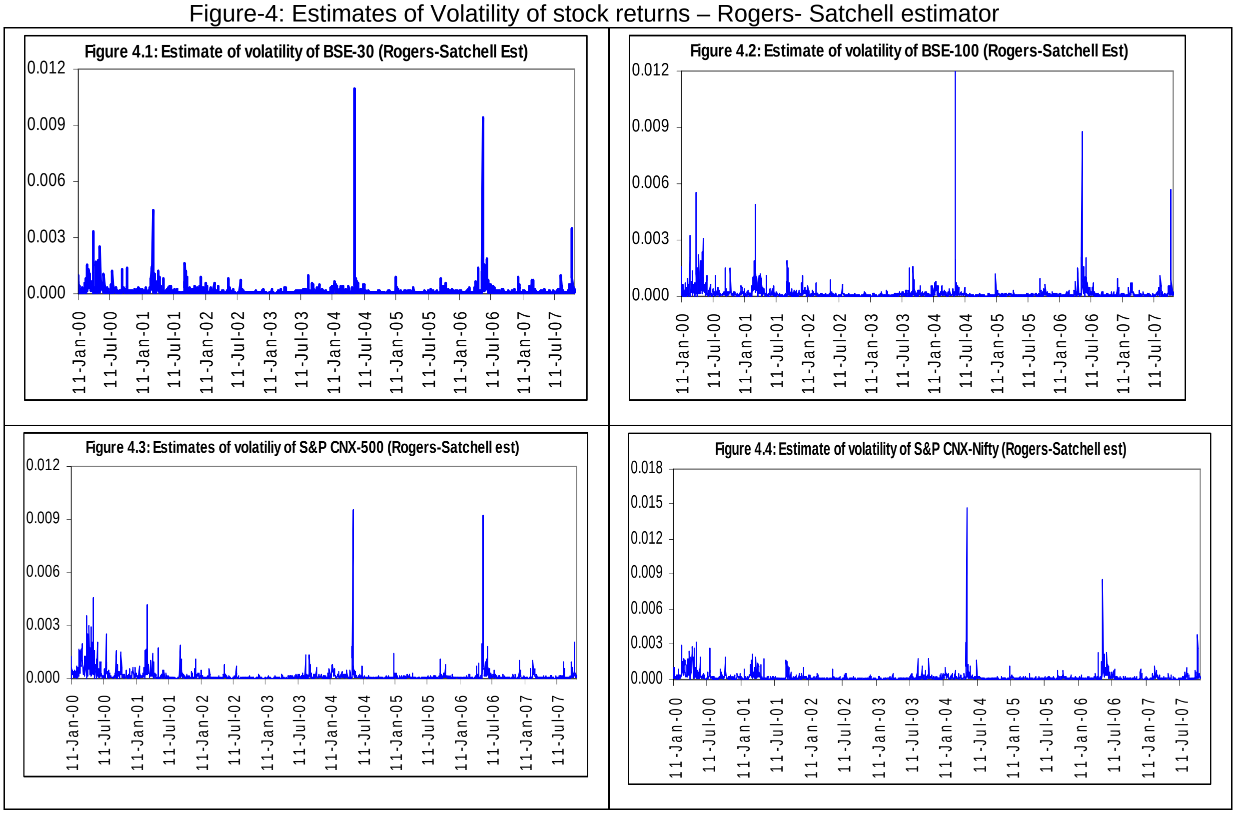

Figure 5: Squared Coherence and Phase-Statistic between estimated volatilities