Characterization of Efficient Simple Liability Rules with Multiple Tortfeasors

Satish K. Jain*

Rajendra P. Kundu**

Centre for Economic Studies and Planning
School of Social Sciences
Jawaharlal Nehru University
New Delhi 110 067
India

Abstract

The paper considers the efficiency of simple liability rules when there are multiple injurers. It is shown that a necessary and sufficient condition for any simple liability rule to be efficient is that it satisfies the condition of collective negligence liability. The condition of collective negligence liabilty, introduced in this paper, requires that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident.

Keywords: Simple Liability Rules, Efficient Simple Liability Rules, Multiple Tortfeasors, Rule of Negligence, Strict Liability, Condition of Collective Negligence Liability, Nash Equilibria.

JEL Classification: K13

* skjain@mail.jnu.ac.in

** rajendrakundu@sancharnet.in

Characterization of Efficient Simple Liability Rules with Multiple Tortfeasors

Satish K. Jain
Rajendra P. Kundu

Most of the results of the law and economics literature relating to the question of efficiency of liability rules have been obtained in the context of two-party interactions involving one victim and one tortfeasor. The main conclusion that has emerged is that while various negligence rules as well as the rule of strict liability with the defense of contributory negligence are efficient, the rules of no liability and strict liability are not.[1] The rules of negligence and strict liability with the defense of contributory negligence, however, have been analyzed in the context of one victim and multiple injurers as well.[2] It has been shown that while negligence is efficient in the context of multiple injurers, the rule of strict liability with the defense of contributory negligence is not.[3] An important implication of this is that the efficiency of a rule is not independent of the number of tortfeasors. In this paper we consider the entire class of simple liability rules, when there are multiple tortfeasors, and get a complete characterization of efficient simple liability rules by obtaining a necessary and sufficient condition for efficiency of any simple liability rule. A liability rule determines the proportions in which the parties involved in the interaction bear the loss in case of occurrence of accident. A simple liability rule determines the proportions in which various parties bear the loss in case of accident as a function of whether and which

[1]There is an extensive literature on the efficiency of liability rules. Pioneering contributions by Calabresi (1961, 1965, 1970) dealt with the effect of liability rules on parties' behaviour. The efficiency of the rule of negligence was analyzed by Posner (1972). The formal analysis of some of the most important liability rules was first put forward by Brown (1973). He showed that the rule of negligence as well as the rule of strict liability with the defense of contributory negligence induce the victim and the injurer alike to take optimal levels of care. Systematic and detailed treatment of liability rules is contained in Shavell (1987) and Landes and Posner (1987). A complete characterization of efficient liability rules has been obtained in Jain and Singh (2002).

[2]The basic result showing the efficiency of the rule of negligence with one victim and multiple injurers is due to Landes and Posner (1980). Multiple-tortfeasor context was also analyzed in Tietenberg (1989), Kornhauser and Revsez (1989) and Miceli and Segerson (1991). Landes and Posner (1987), Shavell (1987) and Miceli (1997) provide formal treatment of the topic.

[3]To be precise, one should speak of the class of negligence rules and the class of strict liability with the defense of contributory negligence rules rather than of the rule of negligence and the rule of strict liability with the defense of contributory negligence.

parties are negligent in the sense of having levels of care below the due care levels. Most liability rules used in practice, including the rules of negligence and strict liability with the defense of contributory negligence, are simple liability rules. An important exception is the rule of comparative negligence, under which the liability may depend on the extent of negligence as well in case both the victim and the injurer are negligent.

A liability rule is efficient if it induces all parties to take care levels which are total social cost minimizing. Total social costs are defined to be the sum of costs of care taken by all the parties and expected accident loss. In order to show that a liability rule is efficient one has to show that (i) all Nash equilibria are total social cost minimizing and (ii) that at least one Nash equilibrium exists.[4] In the presence of the assumption that there is a unique configuration of care levels which is total social cost minimizing, an assumption usually made in the literature, the question of efficiency of a liability rule reduces to the question of whether the configuration of total social cost minimizing care levels of the parties constitutes a unique Nash equilibrium or not. In this paper, while retaining most of the assumptions of the standard framework within which the question of efficiency of liability rules has been discussed in the literature, the problem is considered in a somewhat more general context. No assumptions are made on the costs of care and expected loss functions apart from postulating that they are such that the minimum of total social costs exists, and that a higher level of care never results in greater expected loss. The possibility that there could be more than one configuration of care levels at which total social costs are minimized is not ruled out.

The main result of the paper shows that a necessary and sufficient condition for efficiency of any simple liability rule is that it satisfy the condition of collective negligence liability. The condition of collective negligence liability, introduced in this paper, requires that if at least one party involved in the interaction is negligent then no party which is nonnegligent bears any liability in case of occurrence of accident. An immediate corollary of the above general characterization theorem is that when there are multiple tortfeasors every variant of the rule of negligence is efficient. Interestingly, it also follows that not all versions of strict liability with the defense of contributory negligence in a multi-tortfeasor context are inefficient.[5]

[4]Throughout this paper we consider only pure-strategy Nash equilibria.

[5]The observation that not all variants of the rule of strict liability with the defense of contributory negligence in the multi-tortfeasor context are inefficient was first made by Kornhauser and Revsez (1989). The general characterization theorem proved in this paper enables one to demarcate the efficient variants of the rule of strict liability with the defense of contributory negligence from those variants which are inefficient.

The paper is divided into four sections. Section 1 sets out the framework within which the efficiency problem is analyzed. Section 2 states and proves the general characterization theorem. The next section contains a brief discussion of the rules of negligence and strict liability with the defense of contributory negligence in the light of the general characterization theorem. The discussion pertains to the reasons because of which while all variants of negligence rule in a multi-tortfeasor context are efficient, all variants of strict liability with the defense of contributory negligence are not. We conclude in section 4 with some remarks on the efficiency question for the class of all liability rules in the multi-tortfeasor context.

1 Definitions and Assumptions

We consider accidents involving one victim (individual 1) and n injurers (individuals $2, \ldots, n+1$); where $n \geq 2$. It would be assumed that the entire loss, to begin with, falls on the victim. We denote by $a_i, i = 1, \ldots, n+1$, the index of the level of care taken by individual i. Let $N = \{1, \ldots, n+1\}$. For each $i \in N$, let $A_i = \{a_i \mid a_i$ is the index of some feasible level of care which can be taken by individual $i\}$. We assume:

Assumption A1 $(\forall i \in N)[(\forall a_i \in A_i)(a_i \geq 0) \ \wedge \ 0 \in A_i]$.

For each $i \in N$, we denote by $c_i(a_i)$ the cost of individual i's care level a_i. Let $C_i = \{c_i(a_i) \mid a_i \in A_i\}$, $i \in N$. We assume:

Assumption A2 $(\forall i \in N)[(\forall c_i \in C_i)(c_i \geq 0) \ \wedge \ c_i(0) = 0]$.

Furthermore, it would be assumed that:

Assumption A3 $(\forall i \in N)[(\forall a_i, a_i' \in A_i)[a_i > a_i' \ \rightarrow \ c_i(a_i) > c_i(a_i')]]$.

In other words, $c_i, i \in N$, is assumed to be a strictly increasing function of a_i . In view of this assumption, for each i, c_i itself can be taken to be an index of level of care taken by individual i. Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of $c_1, \ldots, c_{n+1}$; $\pi = \pi(c_1, \ldots, c_{n+1})$, $H = H(c_1, \ldots, c_{n+1})$. Let $L = \pi H$. L is thus a function of $c_1, \ldots, c_{n+1}$, $L = L(c_1, \ldots, c_{n+1})$; and denotes the expected loss due to accident. We assume:

Assumption A4 $(\forall (c_1, \ldots, c_{n+1}), (c_1', \ldots, c_{n+1}') \in C_1 \times \ldots \times C_{n+1})(\forall j \in N)[(\forall i \in N)(i \neq j \ \rightarrow \ c_i = c_i') \ \wedge \ c_j > c_j' \ \rightarrow \ L(c_1, \ldots, c_{n+1}) \leq L(c_1', \ldots, c_{n+1}')]$.

That is to say, it is assumed that greater care by an individual, given the levels of care of all other individuals, results in lesser or equal expected accident loss. The decrease can take place because of decrease in the probability of occurrence of accident or because of decrease in loss in case of occurrence of accident or both. Total social costs (TSC) are defined to be the sum of costs of care of all the individuals and the expected loss due to accident; TSC = $[\Sigma_{i \in N} c_i] + L(c_1, \ldots, c_{n+1})$. Total social costs are thus a function of $c_1, \ldots, c_{n+1}$. Let $M = \{(c'_1, \ldots, c'_{n+1}) \mid [\Sigma_{i \in N} c'_i] + L(c'_1, \ldots, c'_{n+1})$ is minimum of $\{[\Sigma_{i \in N} c_i] + L(c_1, \ldots, c_{n+1}) \mid c_i \in C_i, i \in N\}\}$. Thus M is the set of all costs of care configurations $(c'_1, \ldots, c'_{n+1})$ which are total social cost minimizing. It will be assumed that:

Assumption A5 $C_1, \ldots, C_{n+1}$, *and* L *are such that* M *is nonempty.*

Let $(c^*_1, \ldots, c^*_{n+1}) \in M$. Given $c^*_1, \ldots, c^*_{n+1}$, we define for each $i \in N$, function p_i as follows:
$p_i : C_i \mapsto \{0, 1\}$
$p_i(c_i) = 1$ if $c_i \geq c^*_i$
$p_i(c_i) = 0$ if $c_i < c^*_i$.

Depending on the simple liability rule, there could be legally specified due care levels for all individuals, or for some of them or for none of them. If the simple liability rule specifies the due care level for individual $i, i \in N$, then c^*_i used in the definition of p_i would be taken to be identical with the legally specified due care level. If the liability rule does not specify the due care level for individual i then c^*_i used in the definition of p_i can be taken to be any $c^*_i \in C_i$ subject to the requirement that $(c^*_1, \ldots, c^*_{n+1}) \in M$. Thus in all cases, for each individual i, c^*_i would denote the legally binding due care level for individual i whenever the idea of legally binding due care level for individual i is applicable.[6]

$p_i(c_i) = 1$ would be interpreted as meaning that individual i is taking at least the due care and $p_i(c_i) = 0$ as meaning that individual i is taking less than due care. If $p_i(c_i) = 1$, individual i would be called nonnegligent; and if $p_i(c_i) = 0$, individual i would be called negligent.

Let I denote the closed interval $[0, 1]$.[7] A simple liability rule is a rule which specifies the proportions in which $n + 1$ individuals are to bear the loss in case of occurrence of

[6]Thus, implicitly it is being assumed that the legally specified due care levels are in all cases consistent with the objective of total social cost minimization. This standard assumption is crucial for results on the efficiency of liability rules.

[7]In addition to denoting the set $\{x \mid 0 \leq x \leq 1\}$ by $[0, 1]$, we use the following standard notation to denote:
by $[0, 1)$ the set $\{x \mid 0 \leq x < 1\}$,

accident as a function of whether and which individuals are negligent. Formally, a simple liability rule is a function f from $\{0,1\}^{n+1}$ to I^{n+1}, $f : \{0,1\}^{n+1} \mapsto I^{n+1}$, such that: $f(p_1,\ldots,p_{n+1}) = f[p_1(c_1),\ldots,p_{n+1}(c_{n+1})] = (x_1,\ldots,x_{n+1}) = [x_1(p_1(c_1),\ldots,p_{n+1}(c_{n+1})), \ldots, x_{n+1}(p_1(c_1),\ldots,p_{n+1}(c_{n+1}))]$, where $\Sigma_{i\in N}x_i = 1$. If accident takes place and loss of $H(c_1,\ldots,c_{n+1})$ materializes, then $x_i[p_1(c_1),\ldots,p_{n+1}(c_{n+1})]H(c_1,\ldots,c_{n+1})$ will be borne by individual i. As, to begin with, in case of occurrence of accident, the entire loss falls upon the victim (individual 1), $x_i[p_1(c_1),\ldots,p_{n+1}(c_{n+1})]H(c_1,\ldots,c_{n+1})$, $i \in \{2,\ldots,n+1\}$, represents the liability payment by injurer i to the victim. The victim's expected costs therefore are:
$c_1+L(c_1,\ldots,c_{n+1})-\Sigma_{i=2}^{n+1}x_i[p_1(c_1),\ldots,p_{n+1}(c_{n+1})]L(c_1,\ldots,c_{n+1}) = c_1+x_1[p_1(c_1),\ldots,p_{n+1}(c_{n+1})]L(c_1,\ldots,c_{n+1})$.

Every individual is assumed to regard an outcome O_1 to be at least as good as another outcome O_2 iff expected costs for the individual under O_1 are less than or equal to expected costs under O_2.

Given $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5), a simple liability rule f is efficient iff $(\forall(\overline{c}_1,\ldots,\overline{c}_{n+1}) \in C_1\times\ldots\times C_{n+1})[(\overline{c}_1,\ldots,\overline{c}_{n+1})$ is a Nash equilibrium $\rightarrow (\overline{c}_1,\ldots,\overline{c}_{n+1}) \in M] \wedge (\exists(\overline{c}_1,\ldots,\overline{c}_{n+1}) \in C_1\times\ldots\times C_{n+1})[(\overline{c}_1,\ldots,\overline{c}_{n+1})$ is a Nash equilibrium]. A simple liability rule f is defined to be efficient iff for every possible choice of $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5), f is efficient.

In other words, given $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5), a simple liability rule f is efficient iff (i) every Nash equilibrium is total social cost minimizing and (ii) there exists at least one Nash equilibrium. A simple liability rule is efficient iff it is efficient for every possible choice of $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5).

Remark 1 *It should be noted that if (A5) is not satisfied then no simple liability rule can be efficient.*

When one considers the class of all simple liability rules, with respect to Nash equilibria all possibilities are open as the following examples show:

Example 1 Let $N = \{1,2,3\}, C_1 = C_2 = C_3 = \{0,1\}$;
$f(0,0,0) = f(0,0,1) = f(0,1,0) = f(0,1,1) = (1,0,0), f(1,0,0) = f(1,0,1) = f(1,1,0) = f(1,1,1) = (0,\frac{1}{2},\frac{1}{2})$;

by $(0,1]$ the set $\{x \mid 0 < x \leq 1\}$, and
by $(0,1)$ the set $\{x \mid 0 < x < 1\}$.

$L(0,0,0) = 3.5, L(0,0,1) = L(0,1,0) = 2, L(1,0,0) = 3, L(0,1,1) = .5, L(1,0,1) = L(1,1,0) = 1.5, L(1,1,1) = 0.$

$(0,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (0,1,1)$.

Here $(0,0,0)$, which is not TSC-minimizing, is the only Nash equilibrium.

Example 2 Let $N = \{1,2,3\}, C_1 = C_2 = C_3 = \{0,1\}$;

$f(0,0,0) = f(0,0,1) = f(0,1,0) = f(0,1,1) = (1,0,0), f(1,0,0) = (0, \frac{1}{2}, \frac{1}{2}), f(1,0,1) = (0,1,0), f(1,1,0) = (0,0,1), f(1,1,1) = (0, \frac{1}{2}, \frac{1}{2})$;

$L(0,0,0) = 3.5, L(0,0,1) = L(0,1,0) = 2, L(1,0,0) = 3, L(0,1,1) = .5, L(1,0,1) = L(1,1,0) = 1.5, L(1,1,1) = 0.$

$(0,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (0,1,1)$.

Here $(0,1,1)$, which is TSC-minimizing, is the only Nash equilibrium.

Example 3 Let $N = \{1,2,3\}, C_1 = C_2 = C_3 = \{0,1\}$;

$f(0,0,0) = f(0,0,1) = f(0,1,0) = f(0,1,1) = (1,0,0), f(1,0,0) = f(1,0,1) = f(1,1,0) = f(1,1,1) = (0, \frac{1}{2}, \frac{1}{2})$;

$L(0,0,0) = 8, L(0,0,1) = L(0,1,0) = 6.2, L(0,1,1) = 3, L(1,0,0) = 5, L(1,0,1) = L(1,1,0) = 3.2, L(1,1,1) = 0.$

$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (1,1,1)$.

Here both $(1,0,0)$, which is not TSC-minimizing, and $(1,1,1)$, which is TSC-minimizing, are Nash equilibria.

Example 4 Let $N = \{1,2,3\}, C_1 = C_2 = C_3 = \{0,1\}$;

$f(0,0,0) = f(0,0,1) = f(0,1,0) = f(0,1,1) = (1,0,0), f(1,0,0) = (0, \frac{1}{2}, \frac{1}{2}), f(1,0,1) = (0, \frac{3}{17}, \frac{14}{17}), f(1,1,0) = (0, \frac{2}{17}, \frac{15}{17}), f(1,1,1) = (0, \frac{2}{7}, \frac{5}{7})$;

$L(0,0,0) = 13, L(0,0,1) = L(0,1,0) = 11.5, L(0,1,1) = L(1,0,0) = 10, L(1,0,1) = L(1,1,0) = 8.5, L(1,1,1) = 7.$

$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (1,1,1)$.

Here there is no Nash equilibrium.

2 Characterization of Efficient Simple Liability Rules

Condition of Collective Negligence Liability (CNL): A simple liability rule f satisfies the

condition of collective negligence liability iff $(\forall(p_1,\ldots,p_{n+1}) \in \{0,1\}^{n+1})[(p_1,\ldots,p_{n+1}) \neq (1,\ldots,1) \;\rightarrow\; (\forall i \in N)(p_i = 1 \;\rightarrow\; x_i(p_1,\ldots,p_{n+1}) = 0)]$.
In other words, a simple liability rule satisfies the condition of collective negligence liability iff its structure is such that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident.

Lemma 1 *If a simple liability rule satisfies condition CNL then for any arbitrary choice of $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5), $(c_1^*,\ldots,c_{n+1}^*)$ is a Nash equilibrium.*

Proof: Let simple liability rule f satisfy condition CNL. Take any $C_1,\ldots,C_{n+1},L$ and $(c_1^*,\ldots,c_{n+1}^*) \in M$ satisfying (A1) - (A5). Denote $f(1,\ldots,1)$ by $(x_1^1,\ldots,x_{n+1}^1)$. Suppose $(c_1^*,\ldots,c_{n+1}^*)$ is not a Nash equilibrium. Then, for some $k \in N$ there is some $c_k' \in C_k$ which is a better strategy for individual k than c_k^*, given that every other individual j uses $c_j^*, j \in N, j \neq k$. That is to say:
$(\exists k \in N)(\exists c_k' \in C_k)[c_k' + x_k[p_1(c_1^*),\ldots,p_k(c_k'),\ldots,p_{n+1}(c_{n+1}^*)]L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*) < c_k^* + x_k[p_1(c_1^*),\ldots,p_k(c_k^*),\ldots,p_{n+1}(c_{n+1}^*)]L(c_1^*,\ldots,c_{n+1}^*)]$. (1.1)

First consider the case: $c_k' < c_k^*$.
If $c_k' < c_k^*$ then: $x_k[p_1(c_1^*),\ldots,p_k(c_k'),\ldots,p_{n+1}(c_{n+1}^*)] = 1$, by condition CNL. Therefore:
(1.1) $\rightarrow$ $c_k' + L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*) < c_k^* + x_k^1 L(c_1^*,\ldots,c_k^*,\ldots,c_{n+1}^*)$.
As $0 \leq x_k^1 \leq 1$, we obtain:
$c_k' + L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*) < c_k^* + L(c_1^*,\ldots,c_{n+1}^*)$.
Adding $\Sigma_{j \in N-\{k\}} c_j^*$ to both sides of the above inequality one obtains:
$[\Sigma_{j \in N-\{k\}} c_j^*] + c_k' + L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*) < [\Sigma_{j \in N} c_j^*] + L(c_1^*,\ldots,c_{n+1}^*)$. (1.2)
Inequality (1.2) says that total social costs at $(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*)$ are less than total social costs at $(c_1^*,\ldots,c_{n+1}^*)$. But total social costs attain their minimum at $(c_1^*,\ldots,c_{n+1}^*)$.
This contradiction establishes that if $c_k' < c_k^*$ then (1.1) cannot hold. (1.3)

Next consider the case: $c_k' > c_k^*$.
If $c_k' > c_k^*$ then:
(1.1) $\rightarrow$ $c_k' + x_k^1 L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*) < c_k^* + x_k^1 L(c_1^*,\ldots,c_{n+1}^*)$,
as $x_k[p_1(c_1^*),\ldots,p_k(c_k'),\ldots,p_{n+1}(c_{n+1}^*)] = x_k^1$.
Adding $x_k^1 \Sigma_{j \in N-\{k\}} c_j^*$ to both sides of the above inequality one obtains:
$(1 - x_k^1)c_k' + x_k^1[[\Sigma_{j \in N-\{k\}} c_j^*] + c_k' + L(c_1^*,\ldots,c_k',\ldots,c_{n+1}^*)] < (1 - x_k^1)c_k^* + x_k^1[[\Sigma_{j \in N} c_j^*] + L(c_1^*,\ldots,c_{n+1}^*)]$.

As the minimum of total social costs = $[\Sigma_{j\in N}c_j^*] + L(c_1^*, \ldots, c_{n+1}^*)$, it must be the case that:

$[\Sigma_{j\in N-\{k\}}c_j^*] + c_k' + L(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*) \geq [\Sigma_{j\in N}c_j^*] + L(c_1^*, \ldots, c_{n+1}^*)$;

and consequently:

$x_k^1[[\Sigma_{j\in N-\{k\}}c_j^*] + c_k' + L(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*)] \geq x_k^1[[\Sigma_{j\in N}c_j^*] + L(c_1^*, \ldots, c_{n+1}^*)]$.

Therefore we conclude:

$(1 - x_k^1)c_k' < (1 - x_k^1)c_k^*$.

If $(1 - x_k^1) > 0$, then $c_k' < c_k^*$, contradicting the hypothesis that $c_k' > c_k^*$. If $(1 - x_k^1) = 0$, then we obtain $0 < 0$, a contradiction.

This establishes that if $c_k' > c_k^*$ then (1.1) cannot hold. (1.4)

(1.3) and (1.4) establish the lemma.

Lemma 2 *If a simple liability rule satisfies condition CNL then for every possible choice of $C_1, \ldots, C_{n+1}, L$ and $(c_1^*, \ldots, c_{n+1}^*) \in M$ satisfying (A1) - (A5): $(\forall(\overline{c}_1, \ldots, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_{n+1})[(\overline{c}_1, \ldots, \overline{c}_{n+1})$ is a Nash equilibrium $\rightarrow (\overline{c}_1, \ldots, \overline{c}_{n+1}) \in M]$.*

Proof: Let simple liability rule f satisfy condition CNL. Take any $C_1, \ldots, C_{n+1}, L$ and $(c_1^*, \ldots, c_{n+1}^*) \in M$ satisfying (A1) - (A5). Let $(\overline{c}_1, \ldots, \overline{c}_{n+1})$ be a Nash equilibrium.

$(\overline{c}_1, \ldots, \overline{c}_{n+1})$ being a Nash equilibrium implies:

$(\forall i \in N)(\forall c_i \in C_i)[\overline{c}_i + x_i[p_1(\overline{c}_1), \ldots, p_{n+1}(\overline{c}_{n+1})]L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq c_i + x_i[p_1(\overline{c}_1), \ldots, p_i(c_i), \ldots, p_{n+1}(\overline{c}_{n+1})]L(\overline{c}_1, \ldots, c_i, \ldots, \overline{c}_{n+1})]$. (2.1)

(2.1) $\rightarrow$ $(\forall i \in N)[\overline{c}_i + x_i[p_1(\overline{c}_1), \ldots, p_{n+1}(\overline{c}_{n+1})]L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq c_i^* + x_i[p_1(\overline{c}_1), \ldots, p_i(c_i^*), \ldots, p_{n+1}(\overline{c}_{n+1})]L(\overline{c}_1, \ldots, c_i^*, \ldots, \overline{c}_{n+1})]$. (2.2)

(2.2) $\rightarrow$ $[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*] + \Sigma_{i\in N}x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L[(\forall j \in N - \{i\})(c_j = \overline{c}_j) \wedge c_i = c_i^*]$, (2.3)

as $\Sigma_{i\in N}x_i[p_1(\overline{c}_1), \ldots, p_{n+1}(\overline{c}_{n+1})] = 1$.

Designate $f(1, \ldots, 1)$ by $(x_1^1, \ldots, x_{n+1}^1)$. Let $\{i \in N \mid \overline{c}_i < c_i^*\}$ be designated by N_0.

First consider the case when $N_0 = \phi$.

$N_0 = \phi \rightarrow (\forall i \in N)[\overline{c}_i \geq c_i^* \wedge x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)] = x_i^1]$.

(2.3), therefore, reduces to:

$[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*] + \Sigma_{i\in N}x_i^1 L[(\forall j \in N - \{i\})(c_j = \overline{c}_j) \wedge c_i = c_i^*]$. (2.4)

As $(\forall i \in N)[\overline{c}_i \geq c_i^*]$, we have by (A4):

$(\forall i \in N)[L[(\forall j \in N - \{i\})(c_j = \overline{c}_j) \wedge c_i = c_i^*] \leq L(c_1^*, \ldots, c_{n+1}^*)]$. (2.5)

(2.4) and (2.5) $\rightarrow$ $[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1,\ldots,\overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*] + \Sigma_{i\in N}x_i^1 L(c_1^*,\ldots,c_{n+1}^*) = [\Sigma_{i\in N}c_i^*]+L(c_1^*,\ldots,c_{n+1}^*)$. (2.6)

(2.6) says that total social costs at $(\overline{c}_1,\ldots,\overline{c}_{n+1})$ are less than or equal to total social costs at $(c_1^*,\ldots,c_{n+1}^*)$. As total social costs at $(c_1^*,\ldots,c_{n+1}^*)$ are minimum, it must be the case that total social costs at $(\overline{c}_1,\ldots,\overline{c}_{n+1})$ are equal to total social costs at $(c_1^*,\ldots,c_{n+1}^*)$. Therefore we conclude:

$(\overline{c}_1,\ldots,\overline{c}_{n+1})$ is a Nash equilibrium $\wedge\ N_0 = \phi\ \rightarrow\ (\overline{c}_1,\ldots,\overline{c}_{n+1}) \in M$. (2.7)

Next consider the case $\#N_0 = 1$. Let $N_0 = \{k\}$.

$N_0 = \{k\}\ \rightarrow\ (\forall i \in N - \{k\})[x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j))\ \wedge\ p_i = p_i(c_i^*)] = 0]$, (2.8)

by condition CNL.

In view of (2.8), (2.3) reduces to:

$[\Sigma_{i\in N}\overline{c}_i]+L(\overline{c}_1,\ldots,\overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*]+x_k^1 L(\overline{c}_1,\ldots,c_k^*,\ldots,\overline{c}_{n+1})$, (2.9)

as $x_k[p_1(\overline{c}_1),\ldots,p_k(c_k^*),\ldots,p_{n+1}(\overline{c}_{n+1})] = x_k^1$.

Now, $(\forall i \in N - \{k\})[\overline{c}_i \geq c_i^*]\ \rightarrow\ L(\overline{c}_1,\ldots,c_k^*,\ldots,\overline{c}_{n+1}) \leq L(c_1^*,\ldots,c_{n+1}^*)$, by (A4).

Consequently, (2.9) implies:

$[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1,\ldots,\overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*] + x_k^1 L(c_1^*,\ldots,c_{n+1}^*) \leq [\Sigma_{i\in N}c_i^*] + L(c_1^*,\ldots,c_{n+1}^*)$

(2.10)

(2.10) establishes that:

$(\overline{c}_1,\ldots,\overline{c}_{n+1})$ is a Nash equilibrium and $\#N_0 = 1\ \rightarrow\ (\overline{c}_1,\ldots,\overline{c}_{n+1}) \in M$. (2.11)

Finally consider the case when $\#N_0 > 1$.

$\#N_0 > 1\ \rightarrow\ (\forall i \in N)[x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j))\ \wedge\ p_i = p_i(c_i^*)] = 0]$, (2.12)

by condition CNL.

In view of (2.12), (2.3) reduces to:

$[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1,\ldots,\overline{c}_{n+1}) \leq \Sigma_{i\in N}c_i^*$. (2.13)

(2.13) in turn implies: $[\Sigma_{i\in N}\overline{c}_i] + L(\overline{c}_1,\ldots,\overline{c}_{n+1}) \leq [\Sigma_{i\in N}c_i^*] + L(c_1^*,\ldots,c_{n+1}^*)$. (2.14)

(2.14) establishes that:

$(\overline{c}_1,\ldots,\overline{c}_{n+1})$ is a Nash equilibrium and $\#N_0 > 1\ \rightarrow\ (\overline{c}_1,\ldots,\overline{c}_{n+1}) \in M$. (2.15)

(2.7), (2.11) and (2.15) establish the lemma.

Lemma 3 *If a simple liability rule is efficient for every possible choice of* $C_1,\ldots,C_{n+1},L$ *and* $(c_1^*,\ldots,c_{n+1}^*) \in M$ *satisfying (A1) - (A5), then it satisfies condition CNL.*

Proof: Let f be a simple liability rule.
Suppose condition CNL is violated. Then:
$(\exists(p'_1,\ldots,p'_{n+1}) \in \{0,1\}^{n+1})[\phi \neq \{i \in N \mid p'_i = 0\} = N' \ \wedge \ \Sigma_{i\in N'}x_i(p'_1,\ldots,p'_{n+1}) \neq 1]$.
Among all $(p'_1,\ldots,p'_{n+1}) \in \{0,1\}^{n+1}$ for which $[\phi \neq \{i \in N \mid p'_i = 0\} = N' \ \wedge \ \Sigma_{i\in N'}x_i(p'_1,\ldots,p'_{n+1}) \neq 1]$, choose anyone for which $\#N'$ is the smallest. Designate the selected $(p'_1,\ldots,p'_{n+1})$ by $(p^0_1,\ldots,p^0_{n+1})$.
Let:
$N_0 = \{i \in N \mid p^0_i = 0\}, N_1 = N - N_0 = \{i \in N \mid p^0_i = 1\}, N_{00} = \{i \in N_0 \mid x_i(p^0_1,\ldots,p^0_{n+1}) = 0\}, N_{01} = \{i \in N_0 \mid x_i(p^0_1,\ldots,p^0_{n+1}) > 0\},\ \#N_0 = n_0,\ \#N_1 = n_1,\ \#N_{00} = n_{00},\ \#N_{01} = n_{01}$.
Designate $x_i(p^0_1,\ldots,p^0_{n+1})$ by $x^0_i, i \in N$; and $x_i(1,\ldots,1)$ by $x^1_i, i \in N$.
We consider the following three mutually exclusive and exhaustive cases separately:

(i) $N_{00} = \phi, N_{01} \neq \phi$

(ii) $N_{00} \neq \phi, N_{01} = \phi$

(iii) $N_{00} \neq \phi, N_{01} \neq \phi$.

Case (i)

Let t be a positive number. For every $i \in N_{01}$, choose r_i such that:
$tx^0_i < r_i < t\frac{x^0_i}{\Sigma_{i\in N_{01}}x^0_i}$ (3.1.1)

(3.1.1) $\rightarrow$ $\Sigma_{i\in N_{01}}tx^0_i < \Sigma_{i\in N_{01}}r_i < t.$ (3.1.2)
Let $t - \Sigma_{i\in N_{01}}r_i = \mu$.
Choose a positive number α such that $\alpha < min\ \{r_i - tx^0_i \mid i \in N_{01}\}$. (3.1.3)
Let $\beta \in (0,1)$. (3.1.4)
Now, let $C_1,\ldots,C_{n+1}$ be specified as follows:
$(\forall i \in N_1)[C_i = \{0,c^0_i\} \ \wedge \ c^0_i = \frac{\alpha}{n_1}], (\forall i \in N_{01})[C_i = \{0,c^0_i\} \ \wedge \ c^0_i = r_i]$. (3.1.5)
Let (N'_1, N''_1) be a decomposition[8] of N_1, and (N'_{01}, N''_{01}) a decomposition of N_{01}.
Let L be specified by:
$(\forall i \in N'_1)(c_i = 0) \ \wedge \ (\forall i \in N''_1)(c_i = c^0_i) \ \wedge \ (\forall i \in N'_{01})(c_i = 0) \ \wedge \ (\forall i \in N''_{01})(c_i = c^0_i) \ \rightarrow L(c_1,\ldots,c_{n+1}) = \Sigma_{i\in N'_1}\frac{\alpha\beta}{n_1} + \Sigma_{i\in N'_{01}}(r_i + \frac{\mu}{n_{01}})$. (3.1.6)

[8]Let w be a positive integer; and let $W = \{1,2,\ldots,w\}$. $(S_1,\ldots,S_w)$ is a decomposition of set S iff (i) $\phi \subseteq S_i \subseteq S,\ i \in W$; (ii) $\cup_{i\in W}S_i = S$ and (iii) $S_i \cap S_j = \phi,\ i \neq j,\ i,j \in W$.

(3.1.6) $\rightarrow$ $(\forall i \in N_1')(c_i = 0) \wedge (\forall i \in N_1'')(c_i = c_i^0) \wedge (\forall i \in N_{01}')(c_i = 0) \wedge (\forall i \in N_{01}'')(c_i = c_i^0)$ $\rightarrow$ $\text{TSC}(c_1, \ldots, c_{n+1}) = \Sigma_{i \in N_1''} \frac{\alpha}{n_1} + \Sigma_{i \in N_{01}''} r_i + \Sigma_{i \in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i \in N_{01}'}(r_i + \frac{\mu}{n_{01}})$

(3.1.7)

As $\Sigma_{i \in N_1''} \frac{\alpha}{n_1} + \Sigma_{i \in N_{01}''} r_i + \Sigma_{i \in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i \in N_{01}'}(r_i + \frac{\mu}{n_{01}}) = \alpha\beta + \Sigma_{i \in N_1''}(1-\beta)\frac{\alpha}{n_1} + \Sigma_{i \in N_{01}} r_i + \Sigma_{i \in N_{01}'} \frac{\mu}{n_{01}}$, it follows that TSC is minimized when $N_1' = N_1$ and $N_{01}'' = N_{01}$; and the unique TSC minimizing $(c_1, \ldots, c_{n+1})$ is given by:

$[(\forall i \in N_1)(c_i = 0) \wedge (\forall i \in N_{01})(c_i = r_i)]$.

Now we show that $(\forall i \in N)(c_i = 0)$ is a Nash equilibrium.

It should be noted that we have:

$(p_1(0), \ldots, p_{n+1}(0)) = (p_1^0, \ldots, p_{n+1}^0)$.

Take any $j \in N_1$.

Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;

if j uses $c_j = 0$, then j's expected costs $[EC_j]$ are

$= 0 + x_j(p_1(0), \ldots, p_j(0), \ldots, p_{n+1}(0))L(0, \ldots, 0, \ldots, 0) = x_j^0 L(0, \ldots, 0, \ldots, 0) = x_j^0[\alpha\beta + \mu + \Sigma_{i \in N_{01}} r_i] = x_j^0[\alpha\beta + t]$;

if j uses $c_j = c_j^0$, then EC_j

$= \frac{\alpha}{n_1} + x_j(p_1(0), \ldots, p_j(\frac{\alpha}{n_1}), \ldots, p_{n+1}(0))L(0, \ldots, \frac{\alpha}{n_1}, \ldots, 0) = \frac{\alpha}{n_1} + x_j^0[\alpha\beta + t - \frac{\alpha\beta}{n_1}] = \frac{\alpha}{n_1}(1 - \beta x_j^0) + x_j^0[\alpha\beta + t]$;

as $(p_1(0), \ldots, p_j(0), \ldots, p_{n+1}(0)) = (p_1(0), \ldots, p_j(\frac{\alpha}{n_1}), \ldots, p_{n+1}(0)) = (p_1^0, \ldots, p_j^0, \ldots, p_{n+1}^0)$.

$\beta \in (0,1) \wedge x_j^0 \in [0,1] \rightarrow EC_j[c_j = 0] < EC_j[c_j = c_j^0]$

$\rightarrow$ for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.1.8)

Next consider any $j \in N_{01}$.

Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;

if j uses $c_j = 0$, then EC_j

$= 0 + x_j^0\, L(0, \ldots, 0, \ldots, 0) = x_j^0[\alpha\beta + t]$;

if j uses $c_j = r_j$, then EC_j

$= r_j + x_j(p_1(0), \ldots, p_j(r_j), \ldots, p_{n+1}(0))L(0, \ldots, r_j, \ldots, 0)$.

Therefore,

$EC_j = r_j$, if $\#N_{01} > 1$, as $x_j(p_1(0), \ldots, p_j(r_j), \ldots, p_{n+1}(0)) = 0$ by the choice of $(p_1^0, \ldots, p_{n+1}^0)$; and

$EC_j = r_j + x_j^1 \alpha\beta$, if $\#N_{01} = 1$, i.e., if $N_{01} = \{j\}$.

Thus, $c_j = r_j \rightarrow EC_j \geq r_j$.

As, $x_j^0[\alpha\beta + t] < x_j^0\alpha + x_j^0 t < \alpha + x_j^0 t < (r_j - x_j^0 t) + x_j^0 t = r_j$, it follows that:

for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.1.9)

(3.1.8) and (3.1.9) establish that $(\forall i \in N)(c_i = 0)$, which is not TSC-minimizing, is a Nash equilibrium. (3.1.10)

Case (ii):

Let t and v be positive numbers such that $v < t$.
Let $\alpha > 0$ and $\beta \in (0, 1)$.
Let $C_1, \ldots, C_{n+1}$ be specified as follows:
$(\forall i \in N_1)[C_i = \{0, c_i^0\} \ \wedge \ c_i^0 = \frac{\alpha}{n_1}]$,
$(\forall i \in N_{00})[C_i = \{0, c_i^0\} \wedge c_i^0 = \frac{v}{n_{00}}]$. (3.2.1)
Let (N_1', N_1'') be a decomposition of N_1, and (N_{00}', N_{00}'') a decomposition of N_{00}.
Let L be specified by:
$(\forall i \in N_1')(c_i = 0) \ \wedge \ (\forall i \in N_1'')(c_i = c_i^0) \ \wedge \ (\forall i \in N_{00}')(c_i = 0) \ \wedge \ (\forall i \in N_{00}'')(c_i = c_i^0) \ \rightarrow$
$L(c_1, \ldots, c_{n+1}) = \Sigma_{i \in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i \in N_{00}'} \frac{t}{n_{00}}$ (3.2.2)

(3.2.2) $\rightarrow$ $(\forall i \in N_1')(c_i = 0) \ \wedge \ (\forall i \in N_1'')(c_i = c_i^0) \ \wedge \ (\forall i \in N_{00}')(c_i = 0) \ \wedge \ (\forall i \in N_{00}'')(c_i = c_i^0) \ \rightarrow \ \text{TSC}(c_1, \ldots, c_{n+1}) = \Sigma_{i \in N_1''} \frac{\alpha}{n_1} + \Sigma_{i \in N_{00}''} \frac{v}{n_{00}} + \Sigma_{i \in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i \in N_{00}'} \frac{t}{n_{00}}$

(3.2.3)

As $\Sigma_{i \in N_1''} \frac{\alpha}{n_1} + \Sigma_{i \in N_{00}''} \frac{v}{n_{00}} + \Sigma_{i \in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i \in N_{00}'} \frac{t}{n_{00}} = \alpha\beta + v + \Sigma_{i \in N_1''}(1 - \beta)\frac{\alpha}{n_1} + \Sigma_{i \in N_{00}'} \frac{t-v}{n_{00}}$, it follows that TSC is minimized when $N_1' = N_1$ and $N_{00}'' = N_{00}$; and the unique TSC-minimizing $(c_1, \ldots, c_{n+1})$ is given by:
$[(\forall i \in N_1)(c_i = 0) \ \wedge \ (\forall i \in N_{00})(c_i = \frac{v}{n_{00}})]$.
Now we show that $(\forall i \in N)(c_i = 0)$ is a Nash equilibrium.
Take any $j \in N_1$.
Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;
if j uses $c_j = 0$, then $EC_j = 0 + x_j(p_1(0), \ldots, p_j(0), \ldots, p_{n+1}(0)) \ L(0, \ldots, 0, \ldots, 0) = x_j^0 \ L(0, \ldots, 0, \ldots, 0) = x_j^0[\alpha\beta + t]$;
if j uses $c_j = c_j^0$, then $EC_j = \frac{\alpha}{n_1} + x_j(p_1(0), \ldots, p_j(\frac{\alpha}{n_1}), \ldots, p_{n+1}(0))L(0, \ldots, \frac{\alpha}{n_1}, \ldots, 0) = \frac{\alpha}{n_1} + x_j^0[\alpha\beta + t - \frac{\alpha\beta}{n_1}] = \frac{\alpha}{n_1}(1 - \beta x_j^0) + x_j^0[\alpha\beta + t]$.
$\beta \in (0, 1) \ \wedge \ x_j^0 \in [0, 1] \ \rightarrow \ EC_j[c_j = 0] < EC_j[c_j = c_j^0]$
$\rightarrow$ for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.2.4)

Next consider any $j \in N_{00}$.
Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;
if j uses $c_j = 0$, then $EC_j = 0 + x_j^0 L(0, \ldots, 0, \ldots, 0) = 0$, as $x_j^0 = 0$;
if j uses $c_j = \frac{v}{n_{00}}$, then $EC_j = \frac{v}{n_{00}} + x_j(p_1(0), \ldots, p_j(\frac{v}{n_{00}}), \ldots, p_{n+1}(0))L(0, \ldots, \frac{v}{n_{00}}, \ldots, 0)$.

Therefore,

$EC_j = \frac{v}{n_{00}}$ if $\#N_{00} > 1$, as $x_j(p_1(0), \ldots, p_j(\frac{v}{n_{00}}), \ldots, p_{n+1}(0)) = 0$ by the choice of $(p_1^0, \ldots, p_{n+1}^0)$; and

$EC_j = \frac{v}{n_{00}} + x_j^1\alpha\beta$, if $\#N_{00} = 1$ i.e., if $N_{00} = \{j\}$.

Thus, $c_j = \frac{v}{n_{00}} \;\; \rightarrow \;\; EC_j \geq \frac{v}{n_{00}}$.

Consequently, for $j, c_j = 0$ is better than $c_j = c_j^0$. (3.2.5)

(3.2.4) and (3.2.5) establish that $(\forall i \in N)(c_i = 0)$, which is not TSC-minimizing, is a Nash equilibrium. (3.2.6)

Case (iii):

Let t be a positive number. For every $i \in N_{01}$, choose r_i such that:

$tx_i^0 < r_i < t\frac{x_i^0}{\Sigma_{i\in N_{01}} x_i^0}$ (3.3.1)

(3.3.1) $\rightarrow$ $\Sigma_{i\in N_{01}} tx_i^0 < \Sigma_{i\in N_{01}} r_i < t$. (3.3.2)

As $0 < \Sigma_{i\in N_{01}} r_i < t$, one can choose a positive number v such that $\Sigma_{i\in N_{01}} r_i < v < t$.

Let $v - \Sigma_{i\in N_{01}} r_i = \epsilon$, and $t - v = \delta$.

Choose a positive number α such that $\alpha < min\ \{r_i - tx_i^0 \mid i \in N_{01}\}$; and let $\beta \in (0, 1)$.

(3.3.3)

Now, let $C_1, \ldots, C_{n+1}$ be specified as follows:

$(\forall i \in N_1)[C_i = \{0, c_i^0\} \;\wedge\; c_i^0 = \frac{\alpha}{n_1}]$,

$(\forall i \in N_{00})[C_i = \{0, c_i^0\} \;\wedge\; c_i^0 = \frac{\epsilon}{n_{00}}]$,

$(\forall i \in N_{01})[C_i = \{0, c_i^0\} \;\wedge\; c_i^0 = r_i]$. (3.3.4)

Let (N_1', N_1'') be a decomposition of N_1, (N_{00}', N_{00}'') a decomposition of N_{00}, and (N_{01}', N_{01}'') a decomposition of N_{01}.

Let L be specified by:

$(\forall i \in N_1')(c_i = 0) \;\wedge\; (\forall i \in N_1'')(c_i = c_i^0) \;\wedge\; (\forall i \in N_{00}')(c_i = 0) \;\wedge\; (\forall i \in N_{00}'')(c_i = c_i^0) \;\wedge\; (\forall i \in N_{01}')(c_i = 0) \;\wedge\; (\forall i \in N_{01}'')(c_i = c_i^0) \;\rightarrow\; L(c_1, \ldots, c_{n+1}) = \Sigma_{i\in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i\in N_{00}'}(\frac{\epsilon}{n_{00}} + \frac{\delta}{n_0}) + \Sigma_{i\in N_{01}'}(r_i + \frac{\delta}{n_0})$. (3.3.5)

(3.3.5) $\rightarrow$ $(\forall i \in N_1')(c_i = 0) \;\wedge\; (\forall i \in N_1'')(c_i = c_i^0) \;\wedge\; (\forall i \in N_{00}')(c_i = 0) \;\wedge\; (\forall i \in N_{00}'')(c_i = c_i^0) \;\wedge\; (\forall i \in N_{01}')(c_i = 0) \;\wedge\; (\forall i \in N_{01}'')(c_i = c_i^0) \;\rightarrow\;$ TSC$(c_1, \ldots, c_{n+1}) = \Sigma_{i\in N_1''} \frac{\alpha}{n_1} + \Sigma_{i\in N_{00}''} \frac{\epsilon}{n_{00}} + \Sigma_{i\in N_{01}''} r_i + \Sigma_{i\in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i\in N_{00}'}(\frac{\epsilon}{n_{00}} + \frac{\delta}{n_0}) + \Sigma_{i\in N_{01}'}(r_i + \frac{\delta}{n_0})$.

(3.3.6)

As $\Sigma_{i\in N_1''} \frac{\alpha}{n_1} + \Sigma_{i\in N_{00}''} \frac{\epsilon}{n_{00}} + \Sigma_{i\in N_{01}''} r_i + \Sigma_{i\in N_1'} \frac{\alpha\beta}{n_1} + \Sigma_{i\in N_{00}'}(\frac{\epsilon}{n_{00}} + \frac{\delta}{n_0}) + \Sigma_{i\in N_{01}'}(r_i + \frac{\delta}{n_0}) = \alpha\beta + \Sigma_{i\in N_{00}} \frac{\epsilon}{n_{00}} + \Sigma_{i\in N_{01}} r_i + \Sigma_{i\in N_1''}(1-\beta)\frac{\alpha}{n_1} + \Sigma_{i\in N_{00}'\cup N_{01}'} \frac{\delta}{n_0} = v + \alpha\beta + \Sigma_{i\in N_1''}(1-$

$\beta)\frac{\alpha}{n_1} + \Sigma_{i \in N'_{00} \cup N'_{01}} \frac{\delta}{n_0}$, it follows that TSC is minimized when $N'_1 = N_1, N''_{00} = N_{00}$ and $N''_{01} = N_{01}$; and the unique TSC-minimizing $(c_1, \ldots, c_{n+1})$ is given by:
$[(\forall i \in N_1)(c_i = 0) \ \wedge \ (\forall i \in N_{00})(c_i = \frac{\epsilon}{n_{00}}) \ \wedge \ (\forall i \in N_{01})(c_i = r_i)]$.
Now we show that $(\forall i \in N)(c_i = 0)$ is a Nash equilibrium.
Take any $j \in N_1$.
Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;
if j uses $c_j = 0$, then $EC_j = 0 + x_j(p_1(0), \ldots, p_j(0), \ldots, p_{n+1}(0))L(0, \ldots, 0, \ldots, 0) = x_j^0 L(0, \ldots, 0, \ldots, 0) = x_j^0[\alpha\beta + \epsilon + \delta + \Sigma_{i \in N_{01}} r_i] = x_j^0[\alpha\beta + t]$;
if j uses $c_j = c_j^0$, then $EC_j = \frac{\alpha}{n_1} + x_j(p_1(0), \ldots, p_j(\frac{\alpha}{n_1}), \ldots, p_{n+1}(0))L(0, \ldots, \frac{\alpha}{n_1}, \ldots, 0) = \frac{\alpha}{n_1} + x_j^0[\alpha\beta + t - \frac{\alpha\beta}{n_1}] = \frac{\alpha}{n_1}(1 - \beta x_j^0) + x_j^0[\alpha\beta + t]$.
$\beta \in (0,1) \ \wedge \ x_j^0 \in [0,1] \ \rightarrow \ EC_j[c_j = 0] < EC_j[c_j = c_j^0]$
$\rightarrow$ for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.3.7)

Next consider any $j \in N_{00}$.
Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;
if j uses $c_j = 0$, then $EC_j = 0 + x_j^0 L(0, \ldots, 0, \ldots, 0) = 0$;
if j uses $c_j = \frac{\epsilon}{n_{00}}$, then $EC_j = \frac{\epsilon}{n_{00}} + x_j(p_1(0), \ldots, p_j(\frac{\epsilon}{n_{00}}), \ldots, p_{n+1}(0))L(0, \ldots, \frac{\epsilon}{n_{00}}, \ldots, 0) = \frac{\epsilon}{n_{00}}$, as $\#N_0 > 1$ and consequently $x_j(p_1(0), \ldots, p_j(\frac{\epsilon}{n_{00}}), \ldots, p_{n+1}(0)) = 0$ by the choice of $(p_1^0, \ldots, p_{n+1}^0)$.
Therefore for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.3.8)

Finally, consider any $j \in N_{01}$.
Given that every other individual $i, i \neq j$, is using strategy $c_i = 0$;
if j uses $c_j = 0$, then $EC_j = 0 + x_j^0 L(0, \ldots, 0, \ldots, 0) = x_j^0[\alpha\beta + t]$;
if j uses $c_j = r_j$, then $EC_j = r_j + x_j(p_1(0), \ldots, p_j(r_j), \ldots, p_{n+1}(0))L(0, \ldots, r_j, \ldots, 0) = r_j$, as $\#N_0 > 1$ and therefore $x_j(p_1(0), \ldots, p_j(r_j), \ldots, p_{n+1}(0)) = 0$ by the choice of $(p_1^0, \ldots, p_{n+1}^0)$.
As, $x_j^0[\alpha\beta + t] < x_j^0\alpha + x_j^0 t < \alpha + x_j^0 t < (r_j - x_j^0 t) + x_j^0 t = r_j$, it follows that:
for j, $c_j = 0$ is better than $c_j = c_j^0$. (3.3.9)
(3.3.7) - (3.3.9) establish that $(\forall i \in N)(c_i = 0)$, which is not TSC-minimizing, is a Nash equilibrium. (3.3.10)

(3.1.10), (3.2.6) and (3.3.10) establish the lemma.

Theorem 1 *A simple liability rule is efficient for every possible choice of* $C_1, \ldots, C_{n+1}, L$ *and* $(c_1^*, \ldots, c_{n+1}^*) \in M$ *satisfying (A1) - (A5) iff it satisfies the condition of collective* *negligence liability.*

Proof: If simple liability rule f satisfies the condition of collective negligence liability then by Lemmas 1 and 2 it is efficient for every possible choice of $C_1, \ldots, C_{n+1}, L$ and $(c_1^*, \ldots, c_{n+1}^*) \in M$ satisfying (A1) - (A5). Lemma 3 establishes that if f is efficient for every possible choice of $C_1, \ldots, C_{n+1}, L$ and $(c_1^*, \ldots, c_{n+1}^*) \in M$ satisfying (A1) - (A5) then it satisfies the condition of collective negligence liability.

3 The Rule of Negligence and Strict Liability with the Defense of Contributory Negligence

In the one-victim one-injurer context the rule of negligence is defined by the condition that the injurer is fully liable iff he is negligent; and that the injurer is not at all liable iff he is nonnegligent. The rule of strict liability with the defense of contributory negligence is defined by the condition that the victim is fully liable iff he is negligent and that he is not at all liable iff he is nonnegligent. The two rules are mirror images of each other with the interchange of the two parties and their liability assignments.

Consider the class of simple liability rules defined for multiple injurers by:

(a) If an injurer is nonnegligent then he is not at all liable; and

(b) If at least one injurer is negligent then the set of all injurers taken together is fully liable.

This class of rules can be thought of as embodying the essential features of the negligence rule in the multi-injurer context in a natural way. From the definition of the negligence rule it follows that in the two-party context it is characterized by the following four implications: (i) If the injurer is negligent then he is fully liable, (ii) If the injurer is fully liable then he is negligent, (iii) If the injurer is nonnegligent then he is not at all liable, and (iv) If the injurer is not at all liable then he is nonnegligent. It is clear that in a multi-injurer context (i) cannot be satisfied. It is equally clear that one cannot abandon (i) without giving up the very idea behind the negligence rule. (b), however, seems a natural way to retain the idea of (i) in the multi-injurer context. (ii) and (iii) also must be retained if the essential idea behind the negligence rule is to remain intact. (a) requires for every injurer what (iii) requires for the single injurer in the two-party context. It should be noted that (a) implies (ii) for every injurer. In the multi-injurer context, with respect to (iv) there is some leeway; nothing essential seems to hinge on whether it holds or not. Thus it seems appropriate to term the class of simple liability rules defined by (a) and (b) as the class of negligence rules. It is obvious that (a) and (b) together imply satisfaction of collective negligence liability. Thus as a corollary of Theorem 1 it follows that every variant of negligence rule in a multi-injurer context is efficient.

Unlike the rule of negligence, in the case of strict liability with the defense of contributory negligence, all the four implications characterizing the rule in the two-party context can continue to hold in the multi-injurer context. Indeed, the implications: (1) If the victim is negligent then he is fully liable, (2) If the victim is fully liable then he is negligent, (3) If the victim is nonnegligent then he is not at all liable, and (4) If the victim is not at all liable then he is nonnegligent; must be satisfied if the essential features of strict liability with the defense of contributory negligence are to be retained. Now, every rule in the class of simple liability rules satisfying (1) - (4), which we can term as the class of strict liability with the defense of contributory negligence rules, does not satisfy the condition of collective negligence liability. On the other hand, it is clear that there is a subclass of the class of simple liability rules satisfying (1) - (4) which does satisfy the condition of collective negligence liability. Thus, in the multi-injurer context, while every variant of the negligence rule is efficient, only some variants of strict liability with the defense of contributory negligence are efficient.

4 Concluding Remarks

Under a simple liability rule the proportions in which the loss is apportioned among the parties in case of occurrence of accident is determined on the basis of whether and which parties involved in the interaction are negligent. A more general notion than that of a simple liability rule is that of a liability rule. A liability rule determines the proportions in which the loss is apportioned among the parties in case of occurrence of accident on the basis of not only whether and which parties are negligent but also on the basis of proportions of negligence or nonnegligence of the parties. The condition of collective negligence liability has been shown to be both necessary and sufficient for efficiency for the entire class of simple liability rules satisfying assumptions (A1) - (A5). The class of simple liability rules is a subclass of the class of liability rules. An obvious question that arises is as to whether the condition of collective negligence liability is a characterizing condition for efficiency also for the class of all liability rules satisfying assumptions (A1) - (A5). It is quite straightforward to show that the condition of collective negligence liability indeed is a sufficient condition for any liability rule satisfying assumptions (A1) - (A5) to be efficient. Because of the generality of the notion of a liability rule, the necessity question, however, appears to pose seemingly intractable difficulties.

In view of the facts (i) that a necessary condition for any simple liability rule satisfying assumptions (A1) - (A5) to be efficient is that it satisfy the condition of collective negligence liability, (ii) that a sufficient condition for any liability rule satisfying assumptions (A1) - (A5) to be efficient is that the condition of collective negligence liability holds, and (iii) that the class of simple liability rules satisfying assumptions (A1) - (A5) is a proper subclass of the class of liability rules satisfying assumptions (A1) - (A5); it follows that logically there are only two possibilities regarding efficiency conditions for the class of liability rules satisfying assumptions (A1) - (A5). These possibilities are: (i) the condition of collective negligence liability is a necessary and sufficient condition for any liability rule satisfying assumptions (A1) - (A5), (ii) for the class of all liability rules satisfying assumptions (A1) - (A5) there does not exist any condition which is both necessary and sufficient for efficiency. That is to say, there is no condition which is both necessary and sufficient for any liability rule satisfying assumptions (A1) - (A5) to be efficient. It is an open question as to which of these two possibilities in fact holds.

References

Barnes, David W. and Stout, Lynn A. (1992), *The Economic Analysis of Tort Law*, St. Paul, West Publishing Company.

Brown, John Prather (1973),'Toward an Economic Theory of Liability', 2 *Journal of Legal Studies*, 323-350.

Calabresi, Guido (1961), 'Some Thoughts on Risk Distribution and the Law of Torts', 70 *Yale Law Journal*, 499-553.

Calabresi, Guido (1965), 'The Decision for Accidents: An Approach to Non-Fault Allocation of Costs', 78 *Harvard Law Review*, 713-745.

Calabresi, Guido (1970), *The Costs of Accidents: A Legal and Economic Analysis*, New Haven, Yale University Press.

Cooter, Robert D. (1985), 'Unity in Torts, Contracts and Property: The Model of Precaution', 73 *California Law Review*, 1-51.

Cooter, Robert D. (1991), 'Economic Theories of Legal Liability', 5 *Journal of Economic Perspectives*, 11-30.

Cooter, Robert D. and Ulen, Thomas S. (1999), *Law and Economics*, 3rd ed., New York, Addison-Wesley.

Eisenberg, Melvin Aron (1988), *The Nature of the Common Law*, Cambridge (MA), Harvard University Press.

Jain, Satish K. and Singh, Ram (2002), 'Efficient Liability Rules: Complete Characterization', 75 *Journal of Economics* (*Zeitschrift für Nationalökonomie*), 105-124.

Keeton, W. Page, Dobbs, Dan B., Keeton, Robert E., and Owen, David G. (1984), *Prosser and Keeton on the Law of Torts*, 5th ed., St. Paul, West Publishing Company.

Kornhauser, Lewis A. and Revesz, Richard L. (1989), 'Sharing Damages Among Multiple Tortfeasors', 98 *Yale Law Journal*, 831-884.

Landes, William M. and Posner, Richard A. (1980), 'Joint and Multiple Tortfeasors: An Economic Analysis', 9 *Journal of Legal Studies*, 517-556.

Landes, William M. and Posner, Richard A. (1987), *The Economic Structure of Tort Law*, Cambridge (MA), Harvard University Press.

Levmore, Saul (1994), *Foundations of Tort Law*, Oxford, Oxford University Press.

Miceli, Thomas J. (1997), *Economics of the Law: Torts, Contracts, Property, Litigation*, Oxford, Oxford University Press.

Miceli, Thomas J. and Segerson, Kathleen (1991), 'Joint Liability in Torts: Marginal and Infra-Marginal Efficiency', 11 *International Review of Law and Economics*, 235-249.

Polinsky, Mitchell A. (1989), *An Introduction to Law and Economics*, 2nd ed., Boston, Little, Brown and Company.

Posner, Richard A. (1972), 'A Theory of Negligence', 1 *Journal of Legal Studies*, 28-96.

Posner, Richard A. (1973), 'Strict Liability: A Comment', 2 *Journal of Legal Studies*, 205-221.

Posner, Richard A. (1992), *Economic Analysis of Law*, 4th ed., Boston, Little, Brown and Company.

Shavell, Steven (1980), 'Strict Liability versus Negligence', 9 *Journal of Legal Studies*, 1-25.

Shavell, Steven (1987), *Economic Analysis of Accident Law*, Cambridge (MA), Harvard University Press.

Tietenberg, Tom H. (1989), 'Indivisible Toxic Torts: The Economics of Joint and Several Liability', 65 *Land Economics*, 305-319.