A course on applied statistics for finance practitioners

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May 2003

1 Pre-requisites

- Fluency with algebra and analytic geometry
- You can differentiate $f(x) = xsin(x)$ and integrate $f(x) = x * x$.
- Notions of maximisation using calculus.
- You know what is an option: American versus European, Call versus Put.
- You know what an option delta means.
- You what is put-call parity and why it works.
- In short, you must have good 12^{th} standard mathematics, good practical knowledge of finance.

Books used in the course of the lectures: *Introduction to probability models*, by Sheldon Ross. *Investment Science*, by David Leunberger.

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2 The Syllabus

1. Axioms of probability (Ross chapter 2)
2. Conditional probability and independence (Ross chapter 3)
3. Random variables RVs (Ross chapter 4)
4. Continuous RVs (Ross chapter 5)
5. Introduction to returns
   - Defining returns.
   - Normal versus lognormal.
   - Concept of *Value at Risk* VaR - a percentile point off a PDF of ex–ante MTM loss.
6. Jointly distributed RVs (Ross chapter 6)
   - The multivariate normal distribution.
   - The concept of 'correlation'.
7. Portfolio VaR versus security VaR
   - Basic concept in portfolio theory - securities intermingle in complex and non-obvious ways.
   - Linear combination of security VaR $\neq$ portfolio VaR.
   - Examples –
     - Diversification with two uncorrelated securities,
     - Diversification with two correlated securities
     - Nonlinear securities: Nifty spot + Nifty put
     - A ten-stock portfolio - individual stocks versus portfolio
     - portfolio of a few government bonds
   - Core message: Portfolio analysis $\neq$ security analysis.
8. Expectation (Ross chapter 7)
9. The microeconomics of risk and risk aversion
   - Expected utility hypothesis
   - Risk neutral valuation
   - Construct pretty diagrams highlighting how U(certainty eqvt) is better.
10. Portfolio theory in an MVN world

    Assets are MVN.
    - Markowitz analysis.
    - Portfolio optimisation in a Markowitz world.
11. Principles of asset pricing
    - State space
    - Complete markets
    - Arrow-Debreu securities

- Risk-neutral measure

12. The method of Monte Carlo
13. How to compute VaR (simple MVN+linear case)
    - The simplest case: multivariate normal returns, linear products $\text{Var}(r_p) = w'Sw$, read off a percentile from the normal distribution.
    - Do a few examples, questions in a problemset.
    - Evaluation - this is great *but* –
        - Needs normality,
        - Need linear products, *and*
        - Need to know the true S.
14. Limit theorems (Ross chapter 8)
15. Introduction to statistics
    - Notion of population and sample
    - Concept of estimator
    - Desirable properties of estimators: *efficiency, bias, consistency*
    - analysis of sample mean
    - Use of the *central limit theorem* CLT to get distribution.
    - Mean versus median.
16. OLS

    Example of OLS - *market model*:
    - Using the market model with daily data
    - Benefits of going intra-day
    - Using market model for variance decomposition
    - Testing value added of a fund manager ($H_0 : \alpha = 0$)
    - Single index market model (SIMM)
    - Using OLS to lead on to a SIMM covariance matrix estimator.
17. Computation-intensive statistics
    - Difficulties with CLT approach
    - Simulations when the true distribution is known
    - Concept of bootstrap
18. The methodology used to calculate the NSE MIBOR
    - Illustrates issues like mean versus median
    - Illustrates use of simulation-based estimators.
    - Teaches you how MIBOR works.
19. Introduction to time-series
    - Autocorrelation function (ACF)

- AR(1)
- AR(k)
- MA(1)
- MA(k)
- ARMA(1,1)
- ARMA(p,q)

Warning - all assume homoscedastic innovations.

20. VaR by historical simulation (HS)
    - What is HS?
    - Evaluation: very nice for i.i.d. problems, breaks for non-i.i.d. problems.
21. Volatility clustering
    - Most real-world financial returns exhibit clustering
    - Examples: Nifty, INR-USD.
    - Consequences of volatility clustering for VaR problems: long-run average volatility is too-high or too-low specifically, if recent days were volatile, we would expect high VaR.
22. Heteroskedasticity models
    - ARCH(1)
    - ARCH(q)
    - GARCH
    - IGARCH
23. Implementing VaR in an ARCH world.
    - ARCH(1)
    - Riskmetrics
24. Modelling the Nifty series
25. Modelling the ten-year interest rate
26. Testing VaR implementations
    - Simple notions
    - Christoffersen's test
    - Examples with Indian data
    - Difficulties with power:
      - Difficulties in disambiguating closely related models.
      - Worst case-situations to watch out for.
27. Introduction to continuous time
    - Weiner processes
    - Models using the Weiner process
      - Arithmetic brownian motion (ABM)

- Geometric brownian motion (GBM)
- Mean-reversion model (O-U process)

- Simulating from the GBM

28. Ito's lemma

    Application: an interest rate follows O-U, what is the process that the bond price follows?

29. The Black/Scholes analysis
    - Black-Scholes (Sharpe, Alexander, Bailey proof, binomial approximation)
    - Getting to it using Ito's lemma
    - Derivations of all greeks
    - Delta neutral hedging

30. VaR for nonlinear products
    - Failure of simple approaches for portfolios with nonlinear products
    - CME's SPAN
    - Option greeks as sensitivity measures - but they don't give VaR.

31. VaR by Monte Carlo
    - Very general - $r \sim f(\theta)$, without restrictions.
    - Any products based on this (pricing formulas should be easy)
    - As long as we can simulate from f, we can do VaR
    - But this is costly.

32. Models of interest rates
    - Simple mean-reverting models in continuous time.
    - Matching the term structure to calibrate them.
    - Using these to price interest rate derivatives.

NOTE: In all these topics, we will be dealing explicitly with Indian financial markets and financial time series.