Testing for structural changes in the Market Model

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Recap: framework for estimating β_i

- We want to estimate the following linear relationship:

$$r_i - r_f = \beta_i(r_M - r_f)$$

- Form of the econometric model:

$$Y_i = \alpha + \beta_i X_i + \epsilon_i$$

- For the market model, the null hypotheses:

$$\begin{aligned} H_0 : \beta_i &= 1; H_a : \beta \neq 1 \\ H_0 : \alpha &= 0; H_a : \alpha \neq 0 \end{aligned}$$

- Model assumption: β_i is constant, α is constant in the sample.

Testing for change in parameters in the sample

- Defining structural change in the data.
- Example, the data is generated out of the "real" data as follows.
  - There are two sections of the data $N = n_1 + n_2$ such that:

$$
\begin{aligned}
Y_i &= \alpha_1 + \beta_1 X_i + e_i \text{ for period 1} \\
Y_i &= \alpha_2 + \beta_2 X_i + e_i \text{ for period 2}
\end{aligned}
$$

  - Opposed to the model: $Y_i = \alpha + \beta X_i + e_i$ for the whole period.
  - The first set of two equations become the "unrestricted" model, M_U and the second becomes the "restricted" model, M_R.
- If the "restriction" does not hold, it is equivalent to saying that there is structural change in the DGP.

Operationalising the test for structural change in the model

- Test: compare model variance $\sigma^2_{M_u}$ and $\sigma^2_{M_R}$
  - If $\sigma^2_{M_U} < \sigma^2_{M_R}$ then there is structural change in the sample.
- $\sigma^2_{M_R} = \sum_{i=1}^{N} (Y_i - \alpha - \beta X_i)^2$
- $\sigma^2_{M_U} = \sum_{i=1}^{n_1} (Y_i - \alpha_1 - \beta_1 X_i)^2 + \sum_{i=n_1}^{N} (Y_i - \alpha_2 - \beta_2 X_i)^2$
- Test: F-test

$$\frac{(\sigma^2_{M_R} - \sigma^2_{M_U})/k}{\sigma^2_{M_U}/(n_1 + n_2 - 2K)} \sim F(k, N - 2k)$$

where number of restrictions, $m = k$.

Case I: allowing only the intercept to change

- Model: $Y_i = \alpha + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \epsilon_i$
- Unrestricted model: for samples n_1, n_2

$$\begin{aligned} Y_i &= \alpha_1 + \beta_{11} X_{1,i} + \beta_{21} X_{2,i} + \beta_{31} X_{3,i} + \epsilon_{i1} \\ Y_i &= \alpha_2 + \beta_{12} X_{1,i} + \beta_{22} X_{2,i} + \beta_{32} X_{3,i} + \epsilon_{i2} \end{aligned}$$

- Restricted model: n_1, n_2

$$\begin{aligned} Y_i &= \alpha_1 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + e_{i1} \\ Y_i &= \alpha_2 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + e_{i2} \end{aligned}$$

Case I: allowing only the intercept to change

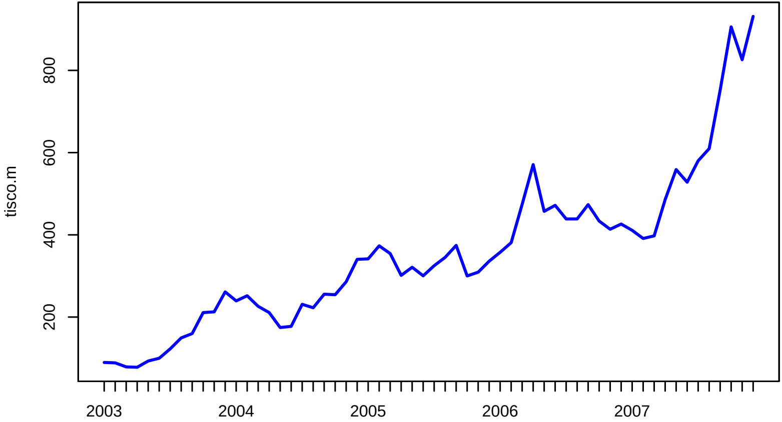
- What is the test statistic?

$$
\begin{aligned}
M_R &= \sum_{i=1}^{n_1} e_{i1}^2 + \sum_{i=1}^{n_1} e_{i2}^2 \\
M_u &= \sum_{i=1}^{n_1} \epsilon_{i1}^2 + \sum_{i=1}^{n_1} \epsilon_{i2}^2 \\
F &= \frac{(M_R - M_U)/(2(k-1))}{M_U/(n_1 + n_2 - 2k)}
\end{aligned}
$$

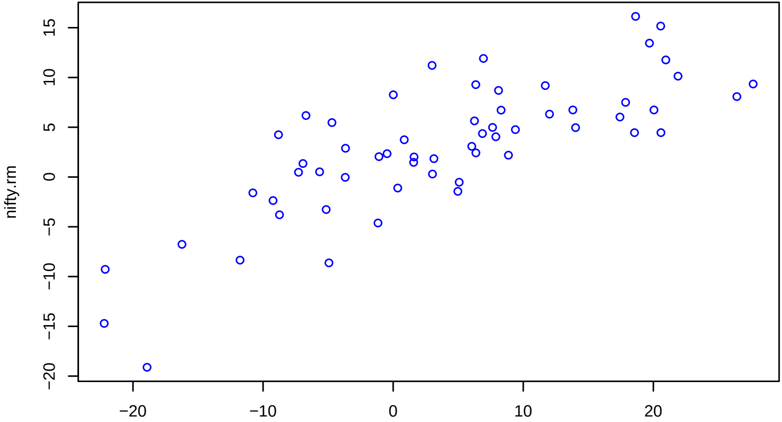
- Number of parameters in unrestricted model: 8
- Number of parameters in restricted model: 5
- Number of restrictions = 3
- Critical value: F(3, N - 8)

Structural breaks in the market model for Tata Steel

Tata Steel prices



Tata Steel vs. Nifty returns



Statistics on Tata Steel

	Tata Steel
Mean	3.968
Std. Dev.	11.898
Skewness	-0.105
	(0.108)
Kurtosis	2.432
	(0.793)
N = 59	

- $\chi^2(1)$, 0.05% level of significance = 3.84, 0.01% = 6.63
- $\chi^2(2)$, 0.05% level of significance = 5.99, 0.01% = 9.21
- Not significantly different from a normal.

Tata Steel regression on Nifty

- Regression:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.0192	1.0377	1.95	0.0566
$r_M - r_f$	1.3926	0.1433	9.72	0.0000

F-stat(1, 57) = 94.39
prob value = 1.072e-13
R-squared = 0.6235
Adjusted R-squared: 0.6169

- Model: $E(r_{\text{tata-steel}} - r_f) = 1.393 E(r_{\text{Nifty}} - r_f)$

Testing for a structural break in Tata steel beta

- Economic hypothesis: The Tata Steel–Corus deal which was signed in January 2007 caused a shift in the beta of Tata Steel.
- Operationalising the test: there is a “structural break” in the market model for returns on Tata Steel before and after March 2006.
- Unrestricted model:

$$\begin{aligned}(r_i - r_f) &= \alpha^* + \beta_i^* E(r_M - r_f) + e_i \text{ period: 2003-2006} \\ (r_i - r_f) &= \alpha^{**} + \beta_i^{**} E(r_M - r_f) + e_i \text{ period: 2007-}\end{aligned}$$

- Restricted model:

$$(r_i - r_f) = \alpha + \beta_i E(r_M - r_f) + \epsilon_i \text{ whole period}$$

- Number of parameters in unrestricted model: 4
- Number of parameters in restricted model: 2
- Number of restrictions = 2
- Critical value: F(2, N - 4)

Testing for a structural break in Tata steel beta

- Unrestricted model variance:

$$\begin{aligned} \text{Period 1 RSS}, \sigma_1^2 &= 1941.107 \\ \text{Period 2 RSS}, \sigma_2^2 &= 1172.618 \\ \sigma_{M_U}^2 &= 3113.725 \end{aligned}$$

- Restricted model variance:

$$\sigma_{M_R}^2 = 3120.476$$

- Test statistic:

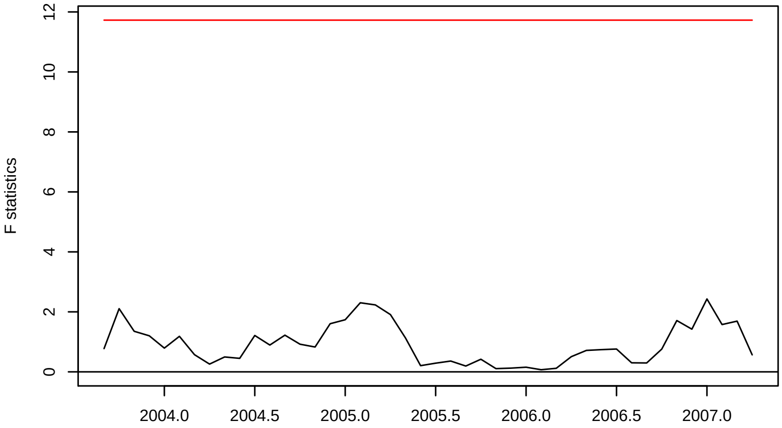
$$\frac{(3120.476 - 3113.725)/2}{3113.725/(59-4)} = 0.0596$$

- Critical value:

$$\begin{aligned} F(2,55) &= 3.168, 5\%\text{level of significance.} \\ F(2,55) &= 5.021, 1\%\text{level of significance.} \end{aligned}$$

- Reject a structural break happened in March 2006.

F-stats for a series of possible structural breaks in the Tata Steel market model



A multi-factor model for Infosys Technologies

Model estimation for Infy beta

- $r_{\text{infy}} - r_f = \alpha_0 + \beta_{\text{infy}}(r_{\text{Nifty}} - r_f) + \epsilon$
- Regression Results:

	Estimate	Std. Error	t-value	Prob value
(Intercept)	-1.6413	1.0974	-1.496	0.140
Market returns	0.7474	0.1516	4.930	7.45e-06
	F-stat(1, 57) = 24.31			
	prob value = 7.5e-6			
	R-squared = 0.2989			
	Adjusted R-squared: 0.2866			

- Model: $E(r_{\text{infy}} - r_f) = 0.747E(r_{\text{Nifty}} - r_f)$

Multiple variables regression for Infosys returns

- Alternative model with multiple variables explaining returns: APT
- Two variables deemed important in explaining security returns:
  - interest rate spread – difference between short and long term rates
  - Foreign exchange fluctuations.
- Proposed model:
  $(r_{\text{infosys},t} - r_f) = \alpha + \beta_{\text{infosys}}(r_{M,t} - r_f) + \beta_{\text{fx}} F_t + \beta_{\text{spread}} S_t + e_t$
- Null hypotheses:
  - β_{fx} is significant.
    $H_0 : \beta_{\text{fx}} = 0, H_A : \beta_{\text{fx}} \neq 0$
  - β_{spread} is insignificant. $H_0 : \beta_{\text{spread}} = 0, H_A : \beta_{\text{spread}} \neq 0$

Statistical analysis of the data

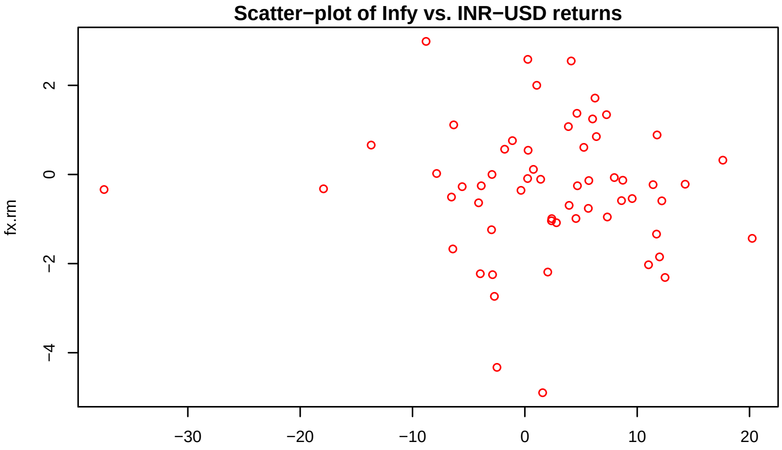
- Summary statistics for monthly data

	Nifty	Infy	FX-ret	Spread
Mean	3.0061	2.0449	-0.327	1.421
Std. Dev.	6.7305	9.0119	1.489	1.339
Skewness	-0.8148	-1.3728	-0.431	0.266
	(6.70)	(18.85)	(1.829)	(0.696)
Kurtosis	4.1742	7.6354	3.961	1.539
	(3.45)	(53.72)	(2.274)	(5.244)

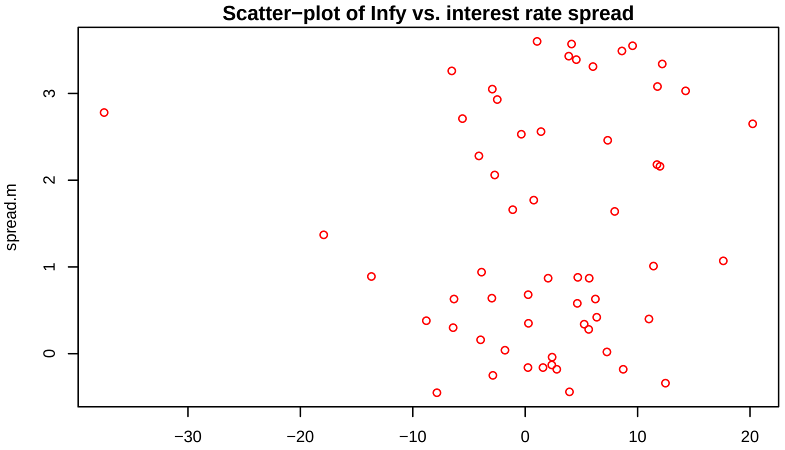
Number of observations = 60 observations

- No serious problems of non-normality in the newly added variables.
- Note: the spread between the long and short rates are a "rate". This tends to be a non-negative number.

Plot of Infy returns vs. fx returns



Plot of Infy returns vs. spread



Checking for multicollinearity

- The correlation matrix for the dataset is:

	Infy	Nifty	FX-ret	Spread
Infy	1.00	0.52	-0.05	0.07
Nifty.rm	0.52	1.00	-0.39	-0.04
FX-ret	-0.05	-0.39	1.00	0.11
Spread.m	0.07	-0.04	0.11	1.00

Regression results on the multifactor model no.1 for Infosys returns

- Regression results, M#1:

	Estimate	Std. Error	t-value	Prob value
(Intercept)	-1.6413	1.0974	-1.496	0.140
Market returns	0.826	0.1616	5.110	4.18e-06
FX fluctuations	1.1418	0.7373	1.549	0.127
Spread	0.6027	0.7697	0.783	0.437
	F-stat(3, 55) = 9.46			
	prob value = 3.879e-05			
	R-squared = 0.3404			
	Adjusted R-squared: 0.3044			
	Residual std.err.: 7.727 on 55 dof			

Regression results on the multifactor model for Infosys returns

- Neither the INR-USD rate changes nor the US interest rate spreads are influential in explaining the monthly returns on Infosys.
- The R^2 has gone from 0.2989 (SIMM) to 0.3404.
  But the "adjusted R^2" has gone from 0.2866 to 0.3044 - a much smaller difference.
- The adjusted R^2 is calculated as:

$$\bar{R}^2 = 1 - \frac{(N-1)}{(N-K)}(1 - R^2)$$

  The larger the "K", the greater has to be the increase in R^2.
- The F-statistic is still significant – but the base model is the pure intercept model.

Regression results on the multifactor model#2 for Infosys returns

- Regression results, M#2:

	Estimate	Std. Error	t-value	Prob value
(Intercept)	-0.9813	1.1483	-0.85	0.3964
Market returns	0.8432	0.1596	5.28	0.0000
FX fluctuations	1.2287	0.7264	1.69	0.0963
	F-stat(3, 56) = 13.98 prob value = 1.189e-05 R-squared = 0.3330 Adjusted R-squared: 0.3092 Residual Std.Err: 7.7 on 56 dof			

Regression results on the multifactor model#3 for Infosys returns

- Regression results, M#3:

	Estimate	Std. Error	t-value	Prol
(Intercept)	-1.6756	2.4235	-0.69	
Market returns	0.7327	0.1530	4.79	
FX forward rate changes	-0.4755	0.7824	-0.61	
Spread	0.6006	0.8303	0.72	
	F-stat(3, 55) = 8.478			
	prob value = 0.00019			
	R-squared = 0.3160			
	Adjusted R-squared: 0.2789			

- Here, the adjusted R^2 is *lower* than the adjusted R^2 of the SIMM.

Checking for multicollinearity

- The correlation matrix for the dataset is:

	Infy	Nifty	FX premium	Spread
Infy	1.00	0.52	-0.07	0.07
Nifty	0.52	1.00	0.04	-0.04
FX premium	-0.07	0.04	1.00	-0.36
Spread	0.07	-0.04	-0.36	1.00

HW: Predicting Infy returns

- If Nifty returns in January 2008 is -17.803, INR-USD was -0.5% and $r_f = 7.085$, what is the forecast for returns on Infy?
- What is the 95% confidence interval for this?