# Econometrics I, quiz 17

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1. This is a 20-minute quiz
2. **NO** point in the exam can you discuss the questions/answers with any of your colleagues.
3. Good luck. :-)

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- **Preamble**: **Testing non-linear restrictions**

  In the previous quiz, the problem was of testing a null of the form:

  $$H_0 : \alpha + \beta = 1, H_a : \beta_1 + \beta_2 \neq 1$$

  What if we want to test a non-linear restriction on $\vec{\beta}$? Example, $\beta_1/\beta_2 = 1$, or $\beta_1\beta_2 = 1$?

  **The approach**: We know that the restriction on the coefficients can generally be written as $H_0 : R(\beta) = r$. $R()$ is nonlinear,and the test statistic remains:

  $$\frac{R(\hat{\beta}) - r}{\text{var}(R(\hat{\beta}))}$$

  It is easy to evaluate $R(\hat{\beta}) - r$ with the estimated coefficients. But what is the estimated variance of $R(\hat{\beta})$?

  Use a linear Taylor expansion of $c(\beta)$ to find out.

  $$\begin{aligned} R(\hat{\beta}) &= R(\beta) + \frac{\partial R(\beta)'}{\partial \beta}(\hat{\beta} - \beta) \\ \Rightarrow \text{var}(R(\hat{\beta})) &= \left(\frac{\partial R(\beta)}{\partial \beta}\right)' var(\hat{\beta}) \left(\frac{\partial R(\beta)}{\partial \beta}\right) \end{aligned}$$

  **Example:** $\beta_1/\beta_2 = 1$. $R(\beta)$ is non-linear. Then,

  $$\begin{aligned} dR_{\beta_1} = \partial R(\hat{\beta})/\partial\hat{\beta}_1 &= 1/\beta_2 \\ dR_{\beta_2} = \partial R(\hat{\beta})/\partial\hat{\beta}_2 &= -\beta_1/\beta_2^2 \\ \Rightarrow \text{var}(R(\hat{\beta})) &= dR_{\beta_1}^2\hat{\sigma}_{\hat{\beta}_1}^2 + dR_{\beta_2}^2\hat{\sigma}_{\hat{\beta}_2}^2 + 2dR_{\beta_1}dR_{\beta_2}c\hat{o}v_{(\hat{\beta}_1,\hat{\beta}_2)} \end{aligned}$$

**Q1**: Testing the long-run marginal propensity to consume (MPC).

A consumption equation differentiating between short-run and long run MPC is:

$$C_t = \alpha + \beta Y_t + \gamma C_{t-1} + \epsilon_t$$

where $\beta$ captures short-run MPC and $\delta = \beta/(1-\gamma)$ captures the long-run MPC.

If you are given the following estimation information on the above consumption equation,

| Variable | Estimate | Std. Err. |
|---|---|---|
| Intercept | 10.0130 | 9.451 |
| $Y_t$ | 0.6460 | 0.1041 |
| $C_{t-1}$ | 0.2884 | 0.1186 |
| N: 36 | | |
| $R^2$: 0.9987 | | |
| Estimated estimator covariance matrix | | |
| | $Y_t$ | $C_{t-1}$ |
| $Y_t$ | | -0.01234 |
| $C_{t-1}$ | | |

1. Write out the consumption equation with the estimated parameters. **(1 point)**

2. What is the test statistic to test $H_0 : \beta/(1-\gamma) = 1, H_A : \beta/(1-\gamma) \neq 1$? **(8 points)**

3. What will you use as the distribution to determine the critical value for this statistic? **(1 point)**