# Portfolio measurement, CAPM and APT

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# Goals

- Portfolio performance measures
- Factor models

# Portfolio performance measurement

- A portfolio is defined as:
  - The total value of the portfolio.
  - The assets held in the portfolio.
  - The weight of each asset in the portfolio, $w_i$.
- There are infinite $\vec{w}$ – how is performance measured?

# Elements of performance evaluation framework

- The weights are optimised to deliver minimum $\sigma_r$ for a given E($r$). (Markowitz)
  Insight: performance must include both E($r_p$) and $\sigma_p$.
- If E($r_p$) is the reward for holding only systematic risk, not unsystematic risk, then
  Insight: Use $\beta_p * \sigma_M$ as the risk measure.
- The one-fund separation theory suggests that efficient portfolios are a weighted average of E($r_f$) and E($r_M$).
  Insight: performance of your choice of portfolio must benchmark against these.
- Returns on all alternative portfolios must be calculated to be directly comparable.
  Insight: annualise returns, use the same method of compounding, the same currency units of value, include dividend payouts.

# Questions of performance evaluation

1. What is a single measure that we use to compare the performance of the portfolio?
2. What is the benchmark portfolio?
3. What is the measure that captures whether the selected portfolio performed “well” relevant to “the relevant benchmark”?
4. Is the measure sufficient to differentiate between “actual” ability and “luck”?

# Portfolio performance measures

- Sharpe's ratio.
- Treynor's measure.
- Jenson's Alpha
- Information Ratio
- Tracking Error
- $M^2$ measure

# Mean-variance measures of performance, Sharpe's measure

- Sharpe's measure:
$$\frac{(\bar{r}_p - \bar{r}_f)}{\sigma_p}$$
- Returns is adjusted for the risk-free rate.
- Risk is total risk of the portfolio returns. (Question: is this $\sigma$ of $r_p$ or $(r_p - r_f)$?)
- It is an ordinal measure: it ranks different portfolios by their return-risk performance.
- The higher the Sharpe's measure, the higher it's rank in performance.

# Treynor's measure

- Treynor's measure:

$$\frac{(\bar{r}_p - \bar{r}_f)}{\beta_p}$$

- Returns is adjusted for the risk-free rate.
- Risk is systemic risk of the portfolio returns. (Question: do we have to worry about the $r_p$ vs. $(r_p - r_f)$ issue here?)
- Higher Treynor's measure $\rightarrow$ higher the portfolio performance ranking.

# Jensen's alpha

- Jensen's measure:

$$\alpha_p = \bar{r}_p - \bar{r}_f - \beta_p * (\bar{r}_p - \bar{r}_f))$$

- Here, the focus is on "excess returns".
  Net of the returns predicted by the systemic risk of the portfolio, does this portfolio have more than zero returns?
- The larger the Jensen's measure (also called the Alpha of the portfolio), the higher the rank in the portfolio performance.

# Information Ratio

- Information Ratio:

$$\frac{\alpha_p}{\sigma_{(e_p)}}$$

- $\alpha_p$ is Jensen's measure for the portfolio.
- $\sigma_{e_p}$ is called the "tracking error" of the portfolio.
- The larger the Information Ratio, the higher the rank in the portfolio performance.

# Tracking error

- Tracking Error (TE) is a measure of how well the portfolio adheres to it's stated scheme of investment.
- TE measures how much the returns of the managed portfolio "tracks" the returns of the stated benchmark portfolio.
- Typically, all managed portfolios attract an investor set by stating the target portfolio allocation across different assets: this is called the "stated scheme".
- Example, "liquid funds" would be invested in short-term fixed income securities/fixed deposits.
- Example, "growth funds" would be invested in equity with high capital appreciation, rather than steady dividend payouts.
- Example, "index funds" invest only in the stocks of the index, and in the correct proportion.
- For a "well-managed portfolio", TE is very small.
- For any portfolio, TE can never be zero.

# $M^2$ measure

- The $M^2$ measure helps to more directly compare the difference between portfolio performance than the ranking.
- $M^2$ is measured in two steps:
  1. Calculate $\sigma_p$, and adjust $r_p$ by the ratio:

     $$A = \sigma_{\text{benchmark}} / \sigma_p$$

     Example: if $\sigma_p = 3 * \sigma_{\text{benchmark}}$, then $\hat{r}_{p,t} = r_{p,t}/3$
  2. Then, the performance measure is:

     $$M^2 = \hat{r}_p - r_{\text{benchmark}}$$

# HW: calculate performance measures

| | Portfolio, P | Market, M |
|---|---|---|
| $\bar{r}$ | 35% | 28% |
| $\beta$ | 1.2 | 1.0 |
| $\sigma$ | 42% | 30% |
| $\sigma_{e_p}$ | 18% | 0% |

- What is the Sharpe's, Treynor, Jenson, Information Ratio measures for Portfolio $P$?
- Does $P$ outperform the market using any of these measures – if so, by which measure does $P$ outperform?

# Summarising performance measures

- Sharpe's measure: $\frac{(\bar{r}_p - \bar{r}_f)}{\sigma_p}$
- Treynor's measure: $\frac{(\bar{r}_p - \bar{r}_f)}{\beta_p}$
- Jenson's Alpha: $\alpha_p = \bar{r}_p - \bar{r}_f - \beta_p * (\bar{r}_p - \bar{r}_f))$
- Information Ratio: $\frac{\alpha_p}{\sigma_{(e_p)}}$
- $M^2$ measure: $M^2 = \hat{r}_p - r_{\text{benchmark}}$ where $\hat{r}_p$ is returns adjusted for the ratio $\sigma_{\text{benchmark}} / \sigma_p$
- A measure is used in different situations as portfolio performance.
  What to use, where and when?

# When performance is measured for the whole investment

- If the portfolio being evaluated is the whole investment, then Sharpe's measure is the best measure.
- Here, all that matters is whether the $E(r)$ is commensurate with the total expected risk of the investment portfolio.

# When performance is measured for a part of the investment portfolio

- Question: what is the correct allocation of fresh funds to the portfolio?
- Assumption: the full portfolio is well diversified, and has only systemic risk.
  In that case, the risk of the total investment is likely to be the market index.
- Here, the best measure is Treynor's measure.
  It identifies excess return for higher systematic risk.

# A portfolio held with a index fund

- An index fund (equivalent to the one-fund of the one-fund separation theorem) is optimally diversified.
- If fresh funds have to be allocated to any portfolio, then the new portfolio must provide $\alpha_p > 0$.
- This must be adjusted for the risk of the portfolio over the index risk.
- The measure to use here is the information ratio: $\alpha_p/\sigma_{e_p}$

# Example of the above case: partial indexation

- A fund broadly tracks the index fund, but outperforms the index. How can this happen?
- Two cases:
  - ‘Closet indexation’ - 80% in index, 20% is actively managed.
  - ‘Index+ funds’ - basically an index fund, some stocklending, some index arbitrage, to juice up returns.
- IR is a very useful measure here:
  How much do you outperform, per unit TE incurred?

# HW: Using measures

- Indian indexes: Nifty, Nifty Junior, CMIE Cospi
- Overseas: S&P 500, FTSE-100
- An internationally diversified portfolio: 20% in India, 60% in the US, 20% in the UK.

Compute and compare their Sharpe's ratio. Sounds easy?

# Implementation problems

- Dividend trouble - CMIE only reports it with dividends; other indexes (by default) are price indexes.
- Long time-series are essential. We do the latest 10 years.
- Measure all series over an identical span!
- Currency complexities: We convert everything into USD!
- Expected returns: 52 times the average weekly return.
- Standard deviation of returns: $\sqrt{52}$ times the $\sigma$ of weekly returns.

# Sharpe's ratio of some portfolios, 2008

| | Mean | SD | SR |
|---|---|---|---|
| Nifty | 14.69 | 26.30 | 0.56 |
| Junior | 19.55 | 31.83 | 0.61 |
| COSPI | 20.18 | 27.02 | 0.75 |
| S&P 500 | 5.35 | 16.87 | 0.32 |
| FTSE-100 | 4.66 | 16.33 | 0.29 |
| Intnl divn | 6.99 | 15.06 | 0.46 |

HW: What was the actual average performance of these indexes in the following five year period?

# Factor models

# Statistical vs. theory

- Factor models are statistical models.
- Problem: There exists $n$ assets, $i = 1, 2, \ldots, n$. Find a single factor, $f$, such that

$$
\begin{aligned}
r_i &= a_i + b_i f + \epsilon_i \\
E(\epsilon_i) &= 0 \\
E(\epsilon_i, f) &= 0
\end{aligned}
$$

- This factor is common to **all** the assets.
- The factor affects the price of one asset through its mean, variance and covariance with other assets:

$$
\begin{aligned}
E(r_i) &= a_i + b_i E(f) \\
\sigma_i^2 &= b_i^2 \sigma_f^2 + \sigma_{\epsilon_i}^2 \\
\sigma_{ij} &= b_i b_j \sigma_f^2 \text{ where} \\
b_i &= cov(r_i, f)/\sigma_f^2
\end{aligned}
$$

- The factor weight differs for different assets.

# Example: CAPM as the first single factor model

- CAPM is a single factor model with excess returns on the *market portfolio* as the factor:

$$E(r_i - r_f) = \beta_i E(r_M - r_f)$$

- Here, the single factor is identified as the **market** portfolio. The above formula is sometimes called the **single index market model** (SIMM) or the market model.
- The single factor is supposed to be the SML market portfolio $m$.
  Implementation: use the market index as the market portfolio.
- Roll, 1977: criticism of small changes in the market returns

# From CAPM to factor models

- CAPM has a single factor – for “systematic risk” – common across all stocks.
  Everything else is stock specific or unsystematic.

# From CAPM to factor models

- CAPM has a single factor – for “systematic risk” – common across all stocks.
  Everything else is stock specific or unsystematic.
- Empirical observation: single factors capture a relatively small fraction of $\sigma_r$.
  Example: in the best case, market returns captures upto 35% of $\sigma_p$.
- Observation 1: covariance between assets tend to be higher.
  Covariance between portfolios are much higher.
- Observation 2: covariance tends to be clustered.
  For example, returns of stocks from a given industry have a greater covariance than with stocks from other industries.
- Can we exploit these observations to get a better pricing model?
  Solution: Multi-factor models.

# Multi-factor models

- Given our observations at the start of the lecture, returns on an asset could perhaps be better explained by a larger number of factors:

$$E(r_i) = a_i + b_{i,1} f_1 + b_{i,2} f_2 + \ldots + b_{i,n} f_n + \epsilon_i$$

- The factors are $f_1, f_2, \ldots, f_n$.
- The weight of each factor on the returns of $i$ is $b_{i,1}, b_{i,2}, \ldots, b_{i,n}$.
  These are called "factor loadings".
  When factor $f_1$ goes up, $b_{i,1}$ predicts what happens to the expected returns and variance of $i$: what direction and by how much?

# The econometric approach to factor models

- The statistics tries to isolate a set of common factors that can be used to model a set of random numbers.
- In our problem of asset pricing, the random numbers are the returns of the traded assets.
- Statistical method 1: if the factors are identified and available exogenously, the factor loadings can be estimated using linear regressions of the time series of returns on that of the factors.

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- Statistical method 1: if the factors are identified and available exogenously, the factor loadings can be estimated using linear regressions of the time series of returns on that of the factors.
- Statistical method 2: even without knowledge of the factors, they can be estimated from the data using the *Principal Components Analysis* methodology (PCA).

# The PCA approach

- In a system which are highly correlated, there is likely to be a small set of independant sources of variation, which can be explained by a few principal components.
- PCA is based on the analysis of the eigenvalues and eigenvectors of the variance–covariance matrix of returns, where
  1. The first PC explains the greatest amount of the variation, the second explains the next greatest amount, etc.
  2. Each PC is independent of each other.

# Arbitrage Pricing Theory (APT) – an operational multi-factor model

- APT was developed by Stephen Ross, 1976:

$$r_i = a_i + b_{i,1}f_1 + b_{i,2}f_2 + b_{i,3}f_3 + \ldots + b_{i,n}f_n$$

- The factors $f_1, f_2, \ldots, f_n$ described the asset returns perfectly.
  The uncertainty in $E(r_i)$ is only due to the uncertainty in $Ef_1, f_2, \ldots, f_n$.
- APT states that if there are $k$ assets, and that $k > n$, there are constants $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n$ such that

$$\bar{r}_i = \lambda_0 + b_{i,1}\lambda_1 + b_{i,2}\lambda_2 + b_{i,3}\lambda_3 + \ldots + b_{i,n}\lambda_n$$

# APT assumptions

- The assumptions of APT are that investors prefer larger returns to less, given certain returns. It does not require the risk-preference assumptions of the CAPM.
- It assumes that a very large set of assets, $k$ is very large, and that every asset $i$ is different from another.
- It should be possible to construct portfolios of any set of assets such that the portfolio has
  1. zero risk, and
  2. zero net investment

  In this case, the return on this portfolio should be the risk-free rate, $r_f$.
- When you solve for the asset returns using this framework, you arrive at the APT.

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- The first implementation of the APT was a model where the factors are "derived" from the data directly. It was a model with five selected factors. The factor with the largest "importance" was identified to be the market portfolio.
- The typical implementation of APT has between three to 15 factors.

# What factors are to be used?

- Exogenous factors: typically macro–economic variables like interest rates, exchange rates, GDP growth, etc.
- Factors specific to the sample set: industry factors, financial and accounting data
- Factors estimated from the sample set itself: There are techniques like factor analysis, principle component analysis that derive factors that are weighted averages of the data itself (ie, returns and/or linear/non–linear functions of returns).
  Problem: These factors are typically treated as black–boxes, and cannot be linked back to an economic variable without effort. This becomes a problem when these models are to be used for prediction.

# Homework: Data constraints on estimating asset pricing models

- Leunberger, pages 212–222.

# References


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