Empirical evidence on CAPM and APT

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22 September, 2017

Goals

- Testing CAPM
- Testing APT

Testing CAPM

Testing CAPM using the time-series approach

- CAPM: $\rightarrow$ E$(R_{it}) - r_f = \alpha_i + \beta_i \mathrm{E}(R_m - r_f)$ where CAPM says: $\alpha_i = 0$.
- Testing approach:
  - $\epsilon_{it}, \epsilon_{jt}$ are individually iid.
  - Assume $\vec{r}$ is jointly normal, use MLE.
    Data is panel with $i = 1 \dots N$, $t = 1 \dots T$.
  - But E$(R_m - r_f)$ is the same, $\forall i$.
    Use OLS for each $i, t = 1 \dots T$ to estimate α_i.
- 1970's: Papers find that $\alpha_i = 0$.
  Later research find opposite results.

Testing CAPM using the cross-sectional approach

- The Securities Markets Line: linear relationship between E(r) and β.
- Then, average returns across stocks must vary *only* on their β.
- Testing approach: $\bar{R}_i = \lambda_0 + \lambda_i \hat{\beta}_i + \nu_i$
  where CAPM says: $\lambda_0 = r_f, \lambda_1 = \bar{R}_m - r_f > 0$
- Implementation: a two-stage procedure.
  - Assume β_i is constant over the sample
  - Step 1: Market model estimation for $\hat{\beta}_i$ using

    $$R_{it} - r_f = \alpha_i + \beta_i (ER_m - r_f) + \epsilon_{it}$$

  - Step 2: Cross-sectional model estimation using sample average returns $\forall i = 1 \ldots N$ in:

    $$\bar{R}_i = \lambda_0 + \lambda_1 \hat{\beta}_i + \nu_i$$

    where $\bar{R}_i$ is the sample average.
- Only the β_i should influence $\bar{R}_i$.

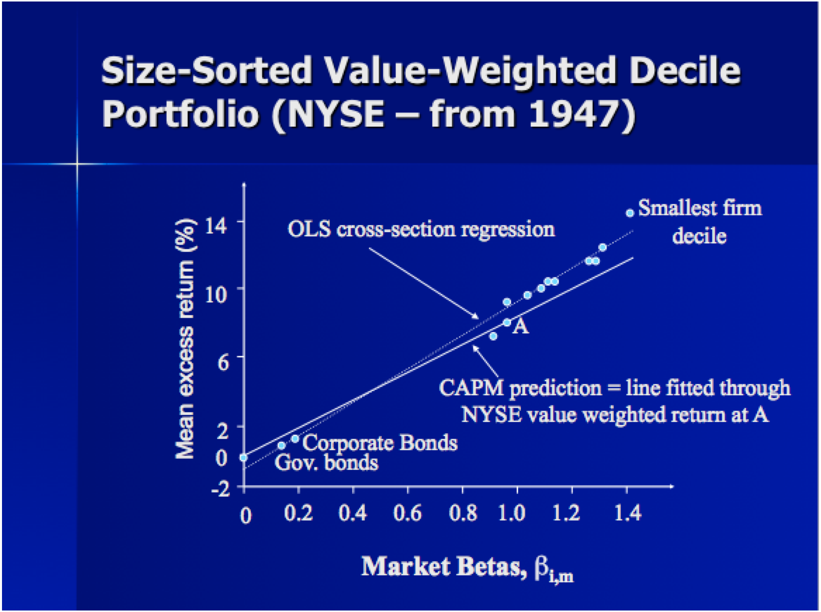
Problems in the estimations

- Single stock returns are very volatile.
  $\Longrightarrow$ Very difficult to differentiate cross-sectional variation in stock returns.
- Market model β is estimated with error.
- Normality assumption need not hold for ϵ_{it}, ν_i.
  Example, skew in ν_i can appear as linking residual risk and return.

Solutions to statistical problems

- Step 1 (β estimation): based on groups / portfolios of stocks, not single stocks.
  Reasoning: portfolio returns are more stable.
- Examples:
  - Black, Jensen, Scholes (1972) group stocks into 10 portfolios based on β_i. Top portfolio has the highest *beta* stocks, bottom portfolio has lowest β stocks.
  - Cochrane (2001) group stocks into 10 portfolios based on size (measured by market capitalisation).
- Step 2 (cross-sectional): sample average returns regressed against the portfolio betas.
- Cochrane (2001) finds that the estimated β does capture average returns, but not completely.

Cochrane 2001: testing CAPM



Fama Macbeth (1973)

- Two 'innovations'
  1. Include additional cross-sectional variables.
  2. Estimate 'rolling window' cross-sectional regressions across each month.

$$R_{it} = \alpha_t + \beta_i \gamma_t + \delta_t Z_i + \epsilon_{it}$$

- Generates a T $\times$1 vector of α, γ, δ.
- If CAPM holds, then $\alpha_t = \delta_t = 0; \gamma_t > 0$.
- Test statistic: If returns are iid(N), then

$$t(\bar{\vec{\gamma}}_i) = \frac{\bar{\vec{\gamma}}}{\frac{s(\hat{\gamma}_j)}{\sqrt{T}}}$$

  which is t-distributed.
- $s(\hat{\gamma}_i)$ is standard deviation, and T is the number of observations.

Testing CAPM, Fama Macbeth (1973, 1974)

- Fama Macbeth (1973): 100 portfolios across 2000 stocks.
- Fama Macbeth (1974),
  - Monthly returns for the cross-sectional regressions.
  - Period: 1935 to 1968
  - $\bar{R}_{it}^{p} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 \beta_i^2 + \lambda_3 \sigma_{\epsilon_i^2} + \nu_i$

  $H_0 : \lambda_1 > 0, \lambda_2, \lambda_3 = 0$
- Conclude that CAPM holds.

Testing multi-factor models

Fama-French 3-factor model (1993)

- 25 portfolios grouped on additional factors to explain monthly stock excess returns:
  - Size: Small, Medium, Big (SMB)
    SMB_t = difference between return on small and big stock portfolio.
  - Value: High, Medium, Low (HML)
    SML_t = return on high (Book to market value) stocks versus low (Book to market value) stocks. Picks on distressed stocks.
- New 2-step estimation:
  - Step1: $R_{it} = \beta_{1i} R_{mt} + \beta_{2i} SMB_t + \beta_{3i} HML_t$
  - Step 2: $\bar{R}_i = \lambda_m \beta_{1i} + \lambda_{smb} \beta_{2i} + \lambda_{hml} \beta_{3i}$
- Period: 1963 to 1991.

Results of Fama-French 3-factor model (1993)

- 25 portfolios sorted by (1) size, (2) book to market and (3) book to market and size.
  - Market β clustered between 0.8 and 1.5
  - Average monthly returns between 0.25 and 1.

  CAPM hypothesis: positive correlation.
- For (2), CAPM is rejected.
  $\rightarrow$ arbitrage opportunity in buying low book to market value stocks and selling high book to market value stocks.
- If R^2 of the cross-section estimation is 1,
  $\rightarrow$ the 3-factors perfectly capture the portfolio average returns.
- The range of R^2 is $0.83 - 0.97$

Interpreting the multi-factor models

- Section 8.3, Cuthbertson-Nietsche.
- Each of the additional factors is interpreted as an alternative risk factor.
- Holds across countries: Size, value (distress), momentum
- Additional risk factors: macro-economic variables like inflation, labour income, investment growth.
  These work – significant coefficients in the cross-sectional regression.
  Not so well in the spread of returns on β as does size and value.
- Lettau Ludvigson (2001) also identified being in a recession as important.

Research on factor models in India

- *Four factor model in Indian equities market*, Sobhesh K. Agarwalla, Joshy Jacob and Jayanth R. Varma, IIM Ahmedabad working paper, W.P. No. 2013-09-05, September 2013
- Updated estimated monthly factors at `http://www.iimahd.ernet.in/~iffm/Indian-Fama-French-Momentum/`
- Additional factor is 'momentum': difference in returns from the previous month's return.

Thank you.