Returns, prices and volatility

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22 September, 2017

Goals

- From expected returns to prices
- Price volatility
- Variance tests of pricing models

Pricing models

Expected returns to valuation

- Expected return, $E(R_{t+1})$

$$E(R_{t+1}) = \frac{E(V_{t+1}) - V_t + E(D_{t+1})}{V_t}$$

- Assume constant expected returns, k

$$\begin{aligned} E_t(R_{t+1}) &= k, \quad k > 0 \\ V_t &= \frac{E_t V_{t+1} + E_t D_{t+1}}{(1+k)} = \delta(E_t V_{t+1} + E_t D_{t+1}) \end{aligned}$$

Iterated expectations:

$$V_t = E_t(\delta D_{t+1} + \delta^2 D_{t+2} + \ldots + \delta^N (D_{t+N} + V_{t+N}))$$

Terminal condition

$$\lim_{N\to\infty} \quad E_t \delta^N (D_{t+N} + V_{t+N}) \to 0$$

$$\text{Then } V_t = E_t \sum_{i=0}^{\infty} \delta^i D_{t+i}$$

Valuation to pricing

- If a market has investors with homogenous expectations, and
- who trade to remove arbitrage, then

$$P_t = V_t = E_t \sum_{i=0}^{\infty} \delta^i D_{t+i}$$

Valuation special cases

- Constant expected dividends, $D_{t+1} = D_t + w_t$, $w_t \sim$ white noise$(0, \sigma_w^2)$.

$$
\begin{aligned}
P_t &= \delta(1 + \delta + \delta^2 + \ldots)D_t = \frac{\delta}{1-\delta} D_t \\
P_{t+1} - P_t &= \frac{\delta}{1-\delta} w_{t+1} \\
var(P_{t+1} - P_t) &= \left(\frac{\sigma_w}{k}\right)^2
\end{aligned}
$$

- Dividends grow at a constant rate, $D_{t+1} = (1+g)D_t + w_{t+1}$

$$
\begin{aligned}
P_t &= \delta(1+g)D_t + \delta^2(1+g)^2 D_{t+1} + \ldots \\
&= \sum_{i=1}^{\infty} \delta^i (1+g)^i D_{t+i} \\
\rightarrow P_t &= \frac{1+g}{k-g} D_t, \text{ where } (k-g) > 0
\end{aligned}
$$

- **Q:** What is the $var(P_{t+1} - P_t)$ in the constant growth dividend model?

Time varying expected returns

- What if investors need returns to vary with time: $E(R_{t+1}) = k_{t+1}$?

$$
\begin{aligned}
P_t &= E_t\left(\delta_{t+1} D_{t+1} + \delta_{t+1}\delta_{t+2} D_{t+1} + \ldots + \delta_{t+1}\ldots\delta t + N D_{t+N}\right) \\
&= E_t\left(\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\delta_{t+i}\right) D_{t+j}\right)
\end{aligned}
$$

- Need both expected returns and expected dividends.
- Expected dividends – past data?
  Expected returns – time series models? CAPM?

CAPM as the model for time varying expected returns

- In CAPM, market returns are proportional to systematic risk, $E_t R_{m,t+1} = r_{f,t} + \lambda E_t \sigma^2_{m,t+1} = k_t$
- Merton (1973): portfolios are a combination of r_f, R_m.
  If investors take no systematic risk, $k_t = r_{f,t}$
- Expected returns for a single security, i:

  $$E_t R_{i,t+1} = r_{f,t} + \beta_{i,t} \lambda E_t \sigma^2_{m,t+1} = r_{f,t} + \lambda \sigma_{im,t+1} = k_{t+1}$$

  Where $\beta_{i,t} = E_t(\sigma_{im,t+1}/\sigma^2_{m,t+1})$
- Linear price-dividends ratio:

  $$\log P_t/D_t = p_t - d_t = \kappa + E_t \left(\sum_{j=1}^{\infty} \rho^{j-1} (\Delta d_{t+j} - h_{t+j}) \right) + \lim_{j \to \infty} \rho^j (p_{t+j} - d_{t+j})$$

  ρ is a linearising constant $(= \bar{P}/(\bar{P} + \bar{D}))$,
  h_{t+j} is one period log returns, and
  Δd_{t+j} are changes in log dividends.

Volatility of prices

Gaps between model and market

1. Price/dividend ratio is high.
2. Price volatility is much higher than what is predicted by the model.

- Dividend changes are not high enough; neither are changes in r_f.

  Changes in other pricing model factors: risk aversion; expected market risk.

- These puzzles have shaped the pricing literature.
  - Explaining changes in risk aversion: behavioural models
  - Theoretical models for changes in expected market risk: stochastic discount factor models.
  - Long run deviation between price and dividends: rational bubbles.
  - Empirical links between changes in volatility and changes in return: ARCH/GARCH-in-mean models.

Volatility and forecastability

- Alternative form of the linearised valuation formula:

$$h_t - E_{t-1}h_t = E_t - E_{t-1}\left[\sum_{j=0}^{\infty}\rho^j(\Delta d_{t+j} - \Delta h_{t+j+1})\right]$$

  LHS is *unexepected shock*, which comes from a revision in expectations.

- Changes in returns is driven by revisions to expected dividends and to expected rates of returns.
- Campbell (1991) identifies revisions to expected returns as the significant factor.
- Small changes in expected returns are not inconsistent with volatile prices, if returns have time dependence.

Variance tests of pricing models

Variance of prices

- Valuation model: $P_t = \sum_{j=1}^{\infty} \delta^i E_t D_{t+j}$.
- Test: is $var(P_{market,t}) = var(P_t)$?
- Inputs: market prices – easy; models of expected dividends – difficult.
- Shiller (1981), LeRoy and Porte (1981) – found a gap between the two variances.
- Tests of EMH: some tests are clear (variance ratio)

  For tests based on changes in news about fundamentals, the benchmark is unclear.
- Tests of variance bounds: model-free and model-based.
  - Model free tests don't have a benchmark or statistics for inference.
  - Model-based use a defined stochastic process for dividends. These are a joint test of efficiency and the defined model.
  - Central question: are dividends stationary?

Shiller's volatility test

- The approach:
  - For a given period of data, t $= 1 \ldots T$,
  - Pick a window of data from the past, and
  - Calculate $E(P_t^*) = \sum_{j=1}^{n} \delta^i D_{t+j} + \delta^n P_{t+n}$
  - Compare with actual P_t to calculate the ϵ_t.
  - Var(P_t^*) $=$Var(P_t)$+$Var(ϵ) or Var(P_t^*) $>$Var(P_t).
- Shiller (1981) showed a wide gap between model and market.

  LeRoy-Porte (1981) showed a borderline significant gap.

Statistical and modelling problems

- Testing with time varying expected dividends, risk-free rates and risk premium.

  The gaps persisted, even if they are smaller than Shiller (1981).
- Sensitivity to terminal price value: assume a moving terminal price. (Mankiw, Romer and Shapiro, 1991)
- Problems with assumptions of:
  - stationarity DGP vs.
  - stationarity with high persistence, OR non-stationarity, OR regime shifts (the peso problem).
- Even to differentiate across these four types require very long time series.
- Build tests around models of persistence or non-stationarity of prices and dividends.

  De-trending the series becomes vulnerable to estimation error. (Shiller, 1981)

  Better: apply Monte-Carlo Simulations for inference. (Kleidon, 1986)

  Recent: Test in a Vector AutoRegressive framework.

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Thank you