Risky asset valuation and the efficient market hypothesis

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Pricing risky assets

Principle of asset pricing: Net Present Value

- Every asset is a set of cashflow, maturity (C_i, T_i) pairs. There can be fixed/variable cashflows at fixed/variable times. (Eg. Bonds, options; insurance, equity.)
- Price of the asset is the price of all expected cashflows $E(C)$, at dates T.
- What is a cashflow $E(C)$ at T worth today?

$$\mathrm{NPV} = \frac{E(C)}{(1+r)^T}$$

Compound interest version:

$$\mathrm{NPV} = e^{-sT} E(C)$$

where we use $s = \log(1+r)$ and r is the discount rate.
- Valuation is all about getting the correct $E(C)$, T, and r.
- The work we did to understand risk with Markowitz optimisation and CAPM comes in handy here to define r for any asset with risk.

Recap: Value of a security with risky cashflows

- A security produces cashflows $E(C_t)$ from $t = 1$ till ∞.
- The security is worth P:

$$P = \sum_{t=1}^{N} \frac{E(C_t)}{(1 + r_f + \Delta)^t}$$

- Operationalising this requires:
  1. The distribution of all $E(C_t)$ in the future
  2. The risk premium, Δ, for the discount rate

  Which is hard!

Implementation of NPV, risky bond

- Let's assume that estimates of $E(C_i)$ are available.
- If (say) there is a risky bond with "N" cashflows, and we use risk neutrality, price P is:

$$P = \sum_{t=1}^{N} \frac{E(C_t)}{(1 + r_f)^t}$$

  where r_f is the risk free rate of return.
- If we know the credit premium Δ for the risk of cashflows, P becomes:

$$P = \sum_{t=1}^{N} \frac{E(C_t)}{(1 + r_f + \Delta)^t}$$

Implementation of NPV, equity

- Equity is harder than bonds in that future cashflows are even more uncertain. Equity promises a fraction of the profits of the company at some undefined future time, called “dividends”.
- The hard part is making estimates of $E(d_t)$ at future dates. Once we estimate of d_t, the pricing technology is the same.
- NPV the future values of $E(d_t)$,

$$P = \sum_{t=1}^{N} \frac{E(d_t)}{(1+r_f)^t} \text{ under risk-neutrality} \quad (1)$$

$$P = \sum_{t=1}^{N} \frac{E(d_t)}{(1+\Delta)^t} \quad (2)$$

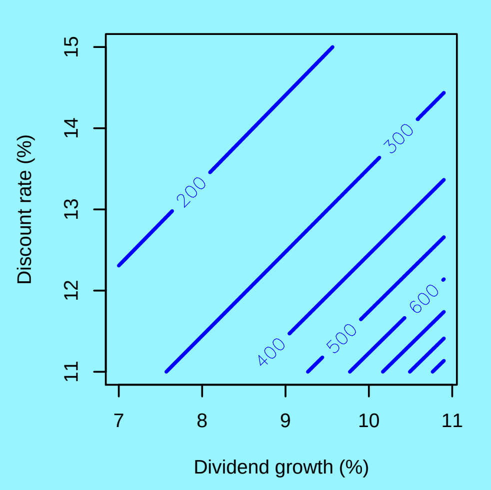
- **Note:** *Finance theory* focuses on modelling Δ. *Security analysis* focuses on models to forecast $E(d_t)$. This focuses on one security at a time – relatively little theory goes there.

Why price are hard to estimate, and volatile

- The NPV of the firm's share depends supremely on your views about
  1. future dividend growth, and
  2. the required risk premium.
- Slight changes to these views generate large changes in the price!

Sensitivity of stock prices: an example

Starting from $d_0 = 10$:



A huge range of stock prices associated with small changes in your view about future dividend growth and/or the risk

Summary

- In a risk neutral world, future $E(C)$ are discounted at r_f.
- However, in a risk-averse context, we need to incorporate a risk premium for risky cashflows Δ.
- Asset pricing theory is about looking at an asset and saying what the Δ should be for the risk characteristics of the firm.
- Even if Δ were known, valuation is hard!!
- It requires forecasting expected cashflows at future dates.
- Particularly for equity: NPV is very very sensitive to slight changes in either growth of dividends or risk premium.
- Every day, as views on these two numbers change, stock prices fluctuate.

Traditional accounting methods

- For equity holders, the cashflows that are relevant are the free cashflow available to equity.
- These have been proxied by (a) the dividends paid out and (b) income net of the cashflows to debt holders, net of repayments, including new debt issued etc.
- These led to traditional approaches such as the **free cashflow** and the **dividend discount** models.
- Such models assume there is:
  1. Consensus on the future free cashflows to equity, D_i.
  2. Consensus on the discount rate, r_i.
  3. Equity is infinitely lived.
- Established markets have financial analysts that forecast future cashflows from the balance sheets/P&Ls of companies.

How on earth does any finance get done?

The best economist would face a huge struggle to get a fix on P in any precise sense.

The revolutionary idea of finance

"Speculative trading by atomic traders on organised financial markets does a pretty good job of getting the correct P".

Markets as a valuation methodology

Markets: The wisdom of crowds

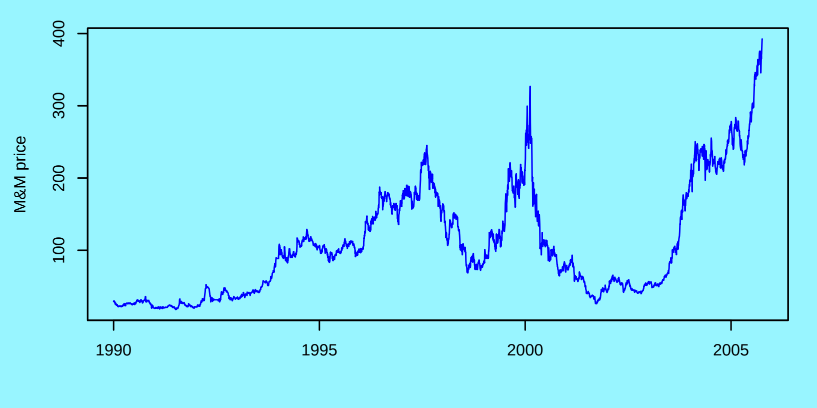
- There are millions of speculators on the market.
- If a security is “too cheap”, speculators buy.
- If a security is “too costly”, speculators sell.
- No **one** speculator has market power.
  Each speculator is well incentivised: he makes huge profits if he’s right, and huge losses if he’s wrong.
- The equilibrium price works out to be remarkably smart.
  “Market efficiency”: The proposition that the price discovered by a speculative market does a pretty good job of embedding forecasts of future d_t and a sensible risk premium Δ.
- We shift gears from modelling equity prices, and try the understand the behaviour of prices from speculative markets.

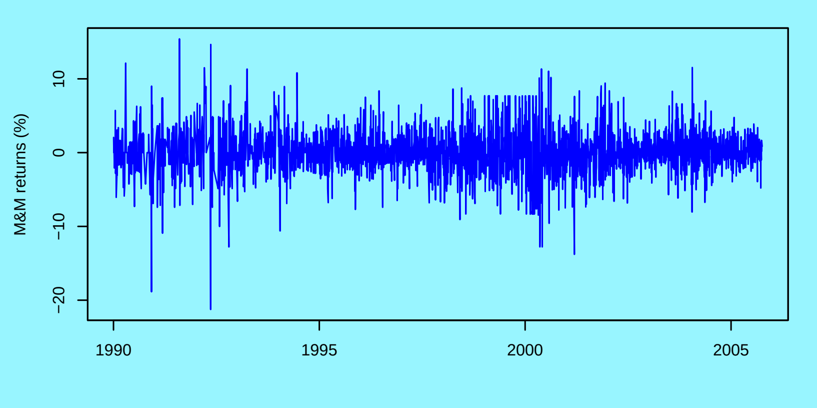
Understanding prices and returns

The notion of returns

- The market produces a time-series $P_1, P_2, \ldots$
- We like to focus on the percentage change in prices, the "returns".
- Prowess jargon: "Adjusted Closing Price" (ACP).

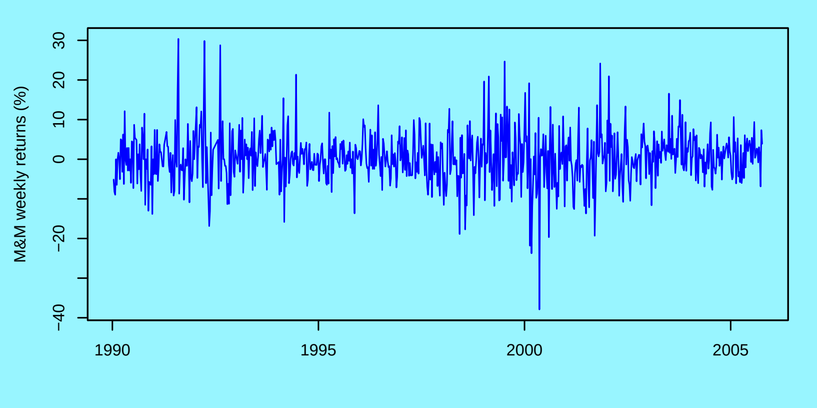
Example: Mahindra & Mahindra




M&M returns (%)
10
0
−10
−20
1990
1995
2000
2005

Returns can be computed over any frequency

- Daily returns is common
- Weekly returns is useful
- You can go intra-day! Returns over five-minute intervals is precious.

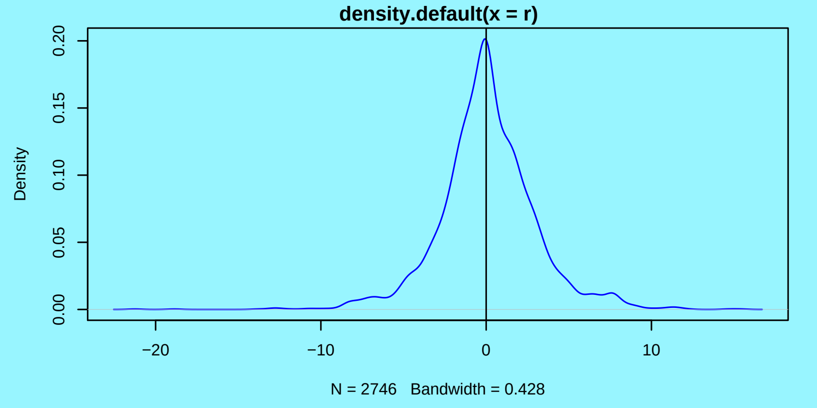

M&M weekly returns (%)
30
20
10
0
−20
−40
1990
1995
2000
2005

Numerical example

```
> load("mnm.rda")
> tail(p)
2005-09-27 2005-09-28 2005-09-29 2005-09-30 200
364.95     371.10     371.85     377.50     389
> prices2returns(tail(p))
2005-09-28 2005-09-29 2005-09-30 2005-10-04
1.6711210  0.2018979  1.5080022  0.8442107
```

Summary statistics about returns

```
> load("mnm.rda")
> r <- prices2returns(p)
> summary(r)
     Index                    r
 Min.   :1990-01-02   Min.   :-21.25614
 1st Qu.:1995-02-08   1st Qu.: -1.46368
 Median :1998-09-06   Median :  0.00000
 Mean   :1998-07-13   Mean   :  0.08008
 3rd Qu.:2002-03-20   3rd Qu.:  1.64140
 Max.   :2005-10-04   Max.   : 15.41507
> sd(r)
[1] 2.950983
```


density.default(x = r)
Density
0.00
0.05
0.10
0.15
0.20
−20
−10
0
10
N = 2746 Bandwidth = 0.428

Market efficiency

- In an efficient market, all speculators know the historical prices.
- Competition between them will eliminate opportunities for earning money “for free”.
- This is like the zero-profit condition under perfect competition.
- In the limit, when millions of smart speculators are in play, returns should become non-forecastable (i.e. random).
- *This is a testable statement*.
- Simplest model: Returns are homoscedastic normal. But reality doesn’t have to oblige.

The random walk of speculative market prices

A model for speculative market prices

- We know that many rational speculators in a market ought to eliminate any arbitrage.
  Ie, similar assets will be similarly priced.
- Speculative markets ought to have prices with no forecastability – no predictable runs, no autocorrelations in returns.
- **Samuelson 1965** was the first paper to put a model to prices in such a speculative market.
  The model: perfectly competitive markets with rational agents have prices which are a “random walk”.
  This became the first widely accepted “quantitative model” for the DGP of speculative market prices.

The random walk

- If x_t is a random walk variable, then

$$x_t = x_{t-1} + \epsilon_t$$

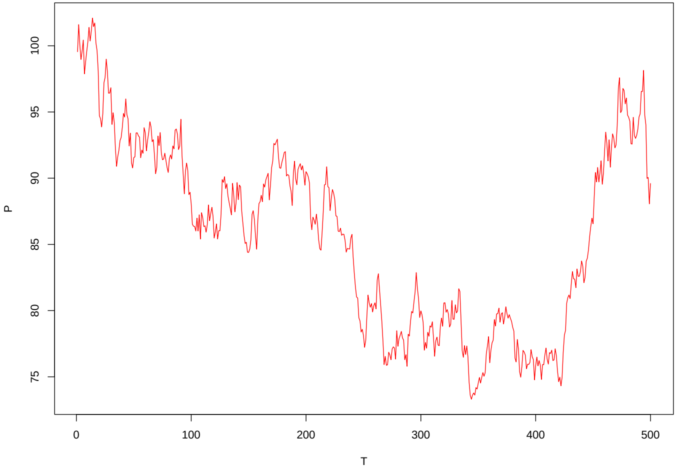
  where ϵ_t is iid.
- Prices are log-normally distributed. Then, prices being a random walk means:

$$\log p_t = \log p_{t-1} + \epsilon_t$$

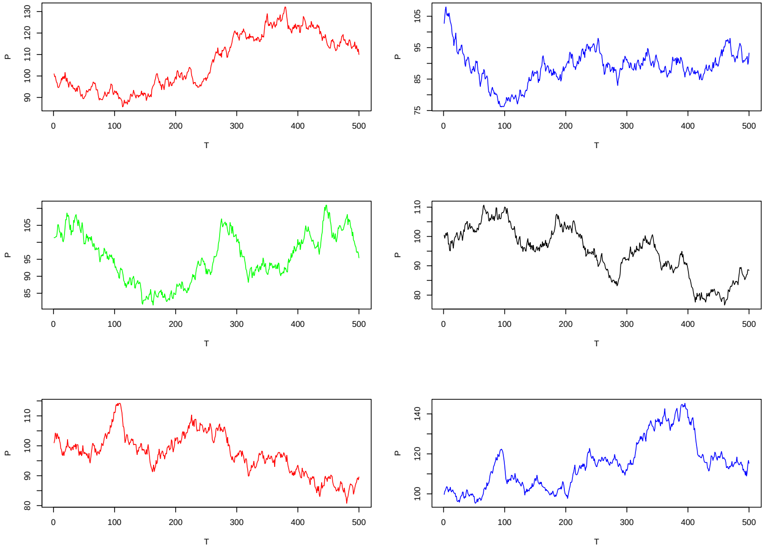
  where ϵ_t is iid as $N(0, \sigma^2)$.

Example: plotting a simulated random walk

```
> P = 100  # In Rs.
> N = 500
> m = 0.01 # In percent
> sg = 1.2 # In percent
>
> plot(P*cumprod(1+(rnorm(N,m,sg)/100)),type="l",
>                             ylab="P",xlab="T",
>                             col="red")
```


100
95
90
85
80
75
P
0
100
200
300
400
500
T

Simulations off the same DGP



Properties of a random walk

- The innovation at time t is ϵ_t.
- ϵ_t is **i.i.d.** drawn from $N(0, \sigma^2)$.
- All innovations to the DGP are permanent.

$$\begin{aligned} \log P_{t+1} &= \log P_t + \epsilon_t \\ \text{And, } \log P_{t+1} &= \log P_{t-k} + \sum_{i=0}^{k} \epsilon_{t-i} \end{aligned}$$

- The best estimate of the forecasted price P_{t+1} is P_t. This is true for forecasts at all horizons, h, in the future. Ie,

$$E(P_{t+h}) = P_t$$

- These are also properties of a time series with a unit root.

A random walk is non-forecastable

- $P_{t+1} = P_t + \epsilon_{t+1}$
- Forecastability is focussed on any new information/pattern, ϵ_{t+1} over P_t. This is a problem because:
  1. ϵ_{t+1} tends to be a small change over P_t.
  2. ϵ_{t+1} is a random number.
- The focus of speculators tend to be on picking patterns in the data, either in the short run or the long run.
  Most appear to forget that random draws from a normal distribution have some **non-zero** probability of (a) runs and (b) temporal serial correlation.

Random walk prices, white noise returns

- If prices follow a random walk, then

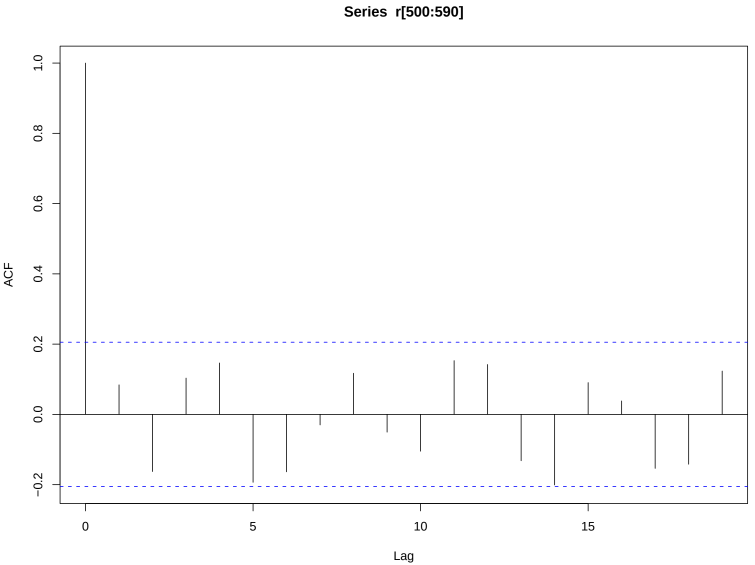
$$\log p_{t+1} = \log p_t + \epsilon_{t+1}$$

  where ϵ_{t+1} is iid as $N(0, \sigma^2)$.
- Quantitatively, this implies that
  - $E(r_t) = E(\epsilon_t) = E(\epsilon) = 0$
  - $E(r_t r_t)^2 = \sigma_\epsilon^2$
    This should hold irrespective of what point t in the time series is observed.
  - $E(r_{t+1} r_t) = 0$; there is no autocorrelation in the series.
    This should hold for autocorrelations at **all** lags.
    Eg., $E(r_{t+k} r_t) = 0, \forall k$

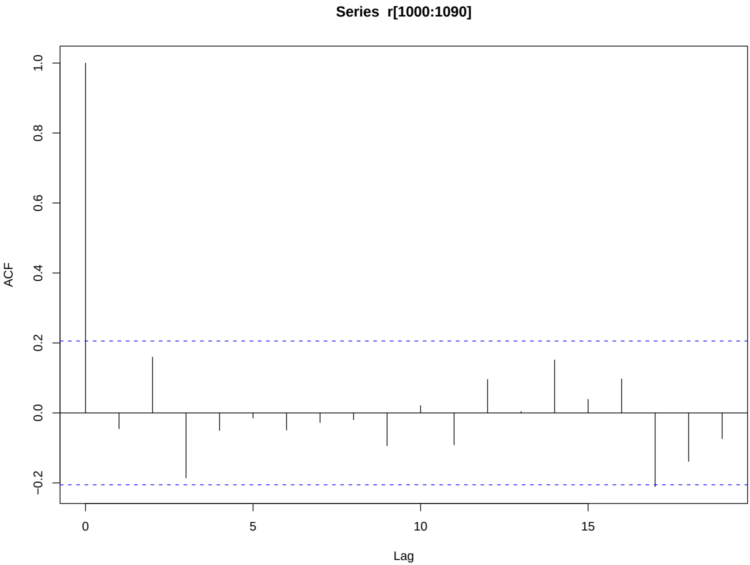
Autocorrelations in white noise series

```
> library(tseries)
> load("6_5.rda")
>
> acf(r[1000:1090])
```

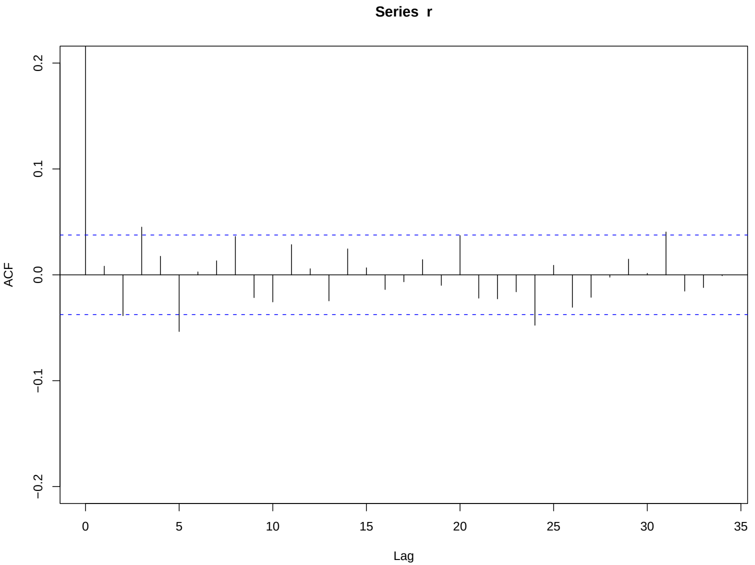
Example: Nifty, 90 days 1



Example: Nifty, 90 days 2



Example: Nifty, 2000 days



Efficient Market Hypothesis (EMH)

The grand market efficiency debate

- A strong market efficiency position is: There is **zero** forecastability of returns.
- Some people get excited when a t stat of 2.5 turns up, they have "rejected the H_0 of market efficiency".
- There is a lot of talk about "inefficient markets" based on such rejections.
- But no forecasting equations have substantial power.
- H_0 can be rejected, but with a tiny R^2, the process is mostly white noise! What is remarkable is not that there are small chinks: what is remarkable is how the broad idea works rather well.
- The socialist view is: Speculators are evil, the speculative process is gambling. Modern finance knows better.

EMH: Definition

EMH claims that investment in an asset priced in a speculative market is done at the “fair value” of the asset.

- "Asset prices fully and instantaneously rationally reflect all available relevant information." (Fama 1969,1971)
- "Asset prices reflect information to the point where the marginal benefits of acting on information (the profits to be made) do not exceed the marginal costs."

Good textbook reference: John Y Campbell, Andrew W. Lo, Craig A. MacKinlay, 1995, “The econometrics of financial markets”, published by Princeton University Press.

EMH: Implications

- If the price is the correct discounted value of future cashflows, there are two sets of implications:
  1. There are no arbitrage opportunities: you only get returns if you take risk.
  2. There are implications on $E(r)$ of any asset: this ought to be a function only of the risk premium on equity.
     This means E(excess returns) across any pair of assets ought not to differ persistently.

  These ought to be true given a fixed information set.
- Research goal: Do these statements about no-arbitrage actually hold in a market?
- We need to test EMH for a given market.

Tests of EMH

- Tests of EMH are categorised depending upon the information captured by market prices.
- The test categories are:
  1. Weak form: tests based on publicly observed information.
  2. Semi-strong form: based on information that is originally observed by a few, and then becomes publicly disclosed.
  3. Strong form: based on information that only a small set of investors could be privy to.
- For example, testing for autocorrelation in a price series is a **weak form** test of EMH.
  The tests are based on prices, which are publicly observed.

References

- ANDREW LO, (editor) *Market Efficiency: Stock Market Behaviour in Theory and Practice (International Library of Critical Writings in Economics)*. Edward Elgar Publishing, 1997