Testing for efficient markets

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What is market efficiency?

- A market is efficient if prices *contain all information about the value of a stock*.
- An attempt at a more precise definition:
  an efficient market is defined with respect to an information set I_t if it is impossible to earn economic profits by trading on the basis of I_t.
  MICHAEL JENSEN. Some anomalous evidence regarding market efficiency. *Journal of Financial Economics*, **6**:pages 95–101 (1978)
- The efficient market hypothesis (**EMH**):
  There will be an absence of arbitrage opportunities in a market populated by rational, profit–maximising agents.
- EMH does not depend upon anything other than the rationality of agents.

The grand market efficiency debate

- A strong market efficiency position is: There is **zero** forecastability of returns.
- Some people get excited when a t stat of 2.5 turns up, they have "rejected the H_0 of market efficiency".
- A lot of talk about "inefficient markets" based on such rejections.
- But forecasting equation have no substantial power. When H_0 can be rejected only with a tiny R^2, the process is mostly white noise!
- One view is: Speculators are evil, the speculative process is gambling.
  Modern finance knows better.

EMH: Implications

- If the price is the correct discounted value of future cashflows, there are two sets of implications:
  1. There are no arbitrage opportunities: you only get returns if you take risk.
  2. There are implications on $E(r)$ of any asset: this ought to be a function only of the risk premium on equity.
     This means E(excess returns) across any pair of assets ought not to differ persistently.

  These ought to be true given a fixed information set.
- Research goal: Do these statements about no-arbitrage actually hold in a market?
- We need to test EMH for a given market.

Structure of tests of EMH

Tests of market efficiency are differentiated based on what is the I_t being used.

- Weak form or "returns predictability":
  I_t includes price information only.
- Semi-strong form or "event studies":
  I_t includes prices and information about firms and macroeconomic events.
- Strong form of "tests about insider trading":
  I_t allows for differences in information across different economic agents.

Tests of EMH

- **Weak form**: ACF, Variance Ratio analysis (Nelson and Plosser 1985, Summers 1988).
  Effects studied: serial correlation, seasonal effects (such as day of week, budget day, end of year effects).
- **Semi-strong form**: Event–study analysis (Brown and Warner 1980, 1985).
  Effects studied: corporate action (such as dividend announcements, bonus issues, rights issues, debt issues, defaults, etc), institutional changes (such as introduction of derivatives markets, changes in laws to shareholders/creditors, etc).
- **Strong form**:
  Effects studied: mutual fund/institutional fund performance wrt stock market index.

Interpreting tests of EMH

- All the above tests of EMH are joint tests of the market efficiency as well as an asset pricing model.
- For instance, all the first tests of EMH were based on the null of the random walk model of prices.
- The random walk assumes a normal distribution for the innovation series.
  However, stock prices were found to have several non-normalities in their returns behaviour: such as skewness, heteroscedasticity, etc.
- This shifted the behaviour of stock price under EMH from pure random walk to that of the more general martingale process.

$$E(P_{t+1}|P_t, P_{t-1}, \ldots) = P_t$$

Interpreting tests of EMH

- Rejection of the null hypothesis is a joint rejection of market efficiency *and* the asset pricing model.
- Standard literature is biased towards rejecting the asset pricing model rather than rejecting market efficiency. (Fama, 1970; Roll, 1977; Ball, 1978; Fama, 1991; **?**).
- There is a branch that builds models with inefficient markets built in explicitly with some success in explaining real-world price behaviour.
  (Grossman and Stiglitz, 1980; Summers, 1986; Poterba and Summers, 1988; Lo and MacKinlay, 1988).

Interpreting tests of EMH

- The earliest tests of EMH were independent of asset pricing theory: Serial correlation, runs tests, presence of day-of-week, month-of-year, size-of-firm, etc. effects.
  EUGENE FAMA. The behaviour of stock market prices. *JOB*, **38**:pages 34 – 105 (1965)
  These established some empirical characterstics of the data.
- Next, the tests based the behaviour of prices on specific asset pricing models.
  Then tests of EMH became *joint* tests of market efficiency and an asset pricing model: Tests of the random walk, event studies, performance of mutual fund managers, etc.

Statistical tests of the random walk behaviour of prices

Test of randomness #1: Runs test

- A returns sequence as follows – $+, +, +$ – is (a) a positive run and (b) a run of length 3.
- Runs can have different directions $(+, -, 0)$ and different lengths.
- Randomness of returns implies certain properties of runs.

Test of randomness #2: autocorrelation coefficients

- If a series of data is "random", then it will have no significant autocorrelation coefficients.
- $H_0 : \rho = 0$
- The standard deviation for the autocorrelation coefficient approximated by

$$\sigma_\rho = 1/\sqrt{N}$$

Variance Ratios as a test of EMH

Tests of randomness #3: Variance ratio

- If innovations are independent, *and* the distribution has constant variance, *then* σ_K^2, the variance of returns over k periods is $K\sigma_1^2$.
- Variance Ratio at lag K is defined as $VR(K)$ where

$$VR(K) = \frac{V(K)}{V(1)}\frac{1}{K}$$

- Under the null of iid returns, $VR(K) = 1$ for any K.

Does the world work like this?

- There are 52 weeks in 12 months, i.e. 4.333 weeks a month.

Product	VR
S&P 500	4.21
Nifty	5.06
USD/EUR exchange rate	3.83

Where else in economics do you get a numerical formula that works like this?

From the idea of $\sqrt{T}$ scaling to a test

- Okay, so we believe that in a fairly efficient, homoscedastic market, we will get $\sqrt{T}$ scaling of volatility.
- But how can we look at data from the realworld and reject the null?
- This need a test.
  E.g: is 4.21 far enough from 4.33 to reject? What about 5.06? 3.83?

Calculating $VR(k)$

- To calculate $V(k)$, daily returns are aggregated over k periods.
- **Cochrane 1988** showed that $VR(k)$ can be approximated by:

$$VR(k) \sim 1 + 2\sum_{j=1}^{k-1} \frac{k-j}{k} \hat{\rho}_j$$

where $\hat{\rho}_j$ is the estimated autocorrelation coefficient at lag j.
- **Fama and French 1988, 1989** formulated another form based on OLS estimates of an autoregressive equation as:

$$\begin{aligned} r_{t,t+k} &= \alpha_k + \beta_k r_{t-k,t} + \epsilon_{t,t+k}, \text{ and} \\ \beta_k &\sim \frac{\hat{\rho}_1 + 2\hat{\rho}_2 + \ldots + (k+1)\hat{\rho}_{k+1} + \ldots + \hat{\rho}_{2k-1}}{k + 2[(k-1)\hat{\rho}_1 + \ldots + \hat{\rho}_{k-1}]} \end{aligned}$$

where β_k is distributed around 0, and negative values indicate mean reversion.

Inference for $VR(k)$

- The test statistic has to be adjusted for the heteroskedasticity.
- **Lo, Mackinlay 1988** have a heteroskedasticity consistent estimator for $VR(k)$:

$$\sqrt{T}(VR(k) - 1) \sim N(0, \theta_k)$$

where

$$\begin{aligned} \theta_k &= 4 \sum_{i=1}^{T/k-1} \left(1 - \frac{i}{k}\right)^2 \hat{\delta}_i \\ \hat{\delta}_i &= T \sum_{j=i+1}^{T} \frac{\sigma_j^2 \sigma_{j-i}^2}{\sigma_j^4} \end{aligned}$$

- **Kim, Nelson, Startz 1988** propose using bootstrap and randomisation to infer the VR distribution when returns have an unknown distribution.

Using the *bootstrap* for VR inference

- For sample size of T data, VR at any lag K is:

$$\hat{VR}(K)$$

- Question: how do we know that $\hat{VR}(K)$ is significantly different from 1?
- We create the empirical distribution of $\hat{VR}(K)$ by bootstrapping.
  - Boostrap:sample from the T data with replication.
  - Create N datasets from the original sample. Each dataset has to be of size T
  - Calculate $VR(k)$ for each "bootstrap datasets".
- References for bootstrap:
  1. Google for *Bradly Efron*, *R. Tibshirani*
  2. The**wikipedia** entry on "Bootstrap (Statistics)" is very good.

VR inference using the *bootstrap* distribution

- In the end, we get N values of $VR(K)$.
  The empirical distribution of these $VR(k)$ is the benchmark distribution for VR(K).
- If the original data is *iid*, the bootstrap distribution of $\hat{VR}(K)$ will be centered around 1.
- The value of the estimated $\hat{VR}(K)$ will be within the 95% bounds of this distribution.

Empirical evidence about VR

- Cochrane (1988), Poterba and Summers (1988), Lo and Mackinlay (1988) – all found evidence that $VR(K)$ for US stock market prices show a pattern of
  - Positive deviations from 1 over the short horizon, and
  - Negative deviations from 1 over the longer horizon

Economic interpretation of the VR observations

- When prices show positive deviations from 1 in the short term, followed by negative deviation in the longer term, it is referred to as the “mean-reversion” property of prices.
  - Prices over-react and overshoot the “mean-level” prices initially (VR > 1).
  - Prices then “revert” to the mean over a longer period.
- The earlier literature also identified varying **magnitudes** of mean-reversion in different periods.
  For example, mean-reversion was much stronger in the pre-WWII period as compared to in the post-WWII period.

Causes for mean-reversion

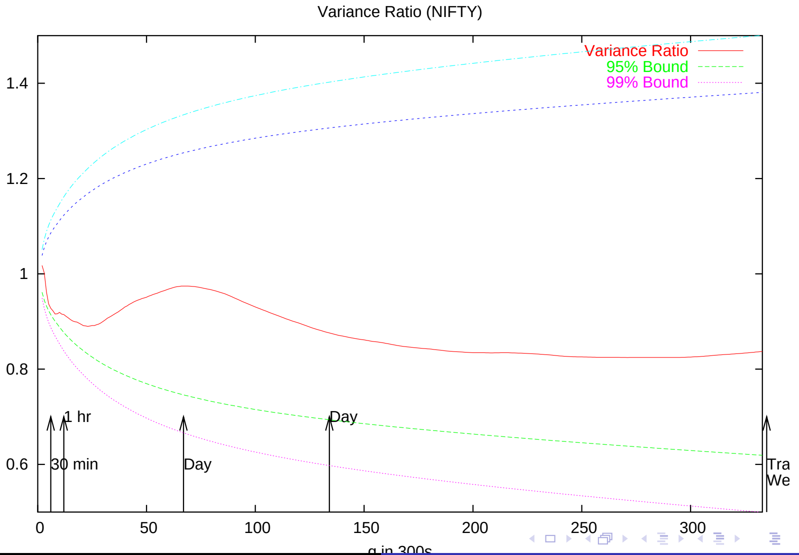
- On the short-run, bid-ask spread causes a negative serial correlation: Roll (1984).
- Across stocks of different liquidity, those with higher liquidity will have smaller serial correlation: Hasbrouck (1991).
- For a portfolio containing stocks of different liquidity, the same information will get absorbed sooner by some stocks, a little later by others.
  This ought to cause positive serial correlation in an index: Lo and Muthuswamy (1996).

Causes for mean-reversion

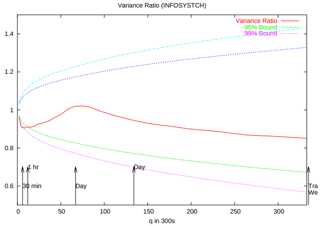
- HF Finance: These deviations are even more pronounced when the horizon reduces to within the day – to hour/minutes/seconds.
- The behaviour of the VR using extremely high frequency data becomes a story of how information transmits into prices.
  This can be studied at the level of individual stocks, pairs of stocks and the entire market.
- HF data helps trace out the path of market efficiency.

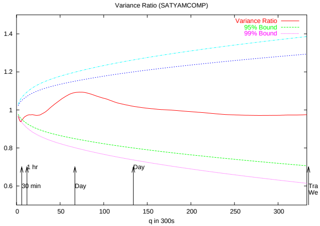
Serial correlation in Indian stock market data

Serial correlation in Nifty, March 1999 to February 2001

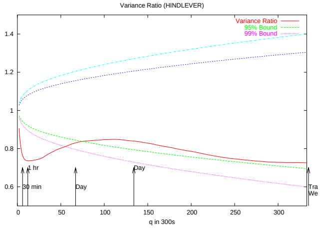


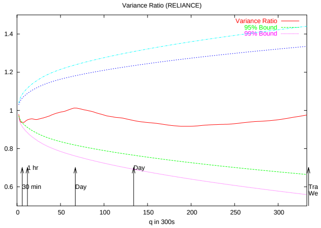
Serial correlation in IT stocks, March 1999 to February 2001





Serial correlation in manufacturing stocks, March 1999 to February 2001





Recapitulation

Core idea of variance ratio: Uncertainty goes up as $\sqrt{T}$ • Approximation of VR using ACF • Test statistic and inference based on overlapping samples • Nelson-Kim-Startz strategy of scrambling • Tests which address heteroscedasticity • Standard explanations for serial correlations in returns data – nonsynchronous trading and indexes, and bid-ask bounce.