Risk measurement class 3
Value at risk

Susan Thomas

IGIDR, 25 January 2011

Risk measures

- For any position in finance, we have an *ex ante* return X which is a random variable
- To the best of our ability, we can try to work out $X \sim f(x;\theta)$
- If we are right, then all fore-knowledge about X is contained in our $f(x)$
- In general, X is multivariate so $f()$ is complicated
- Humans want a simple scalar measure of risk
- One proposal: Value at Risk

Value at risk

You choose an α, and then VaR is the number v such that

$$\int_{\infty}^{v} f(x)dx = \alpha$$

Note the full armament required:

- You have to choose a time horizon
- Over that time horizon you have to have a credible estimate of $f(x)$ for your (multivariate!) position
- Then choose an α
- Then solve out for v.

Interpretation

"The value at risk of this position, on a one-day horizon, at a 99% level, is Rs.1.34 million"

- This means that on 1% of the days, the loss will *exceed* Rs.1.34 million
- This *does not mean* that on the worst 1% of the days, the loss will be roughly Rs.1.34 million.
- In a year of 250 trading days, 1% of the days occur 2-3 days of the time
- So this means: "In the worst 2-3 days of the year, the loss on this position will exceed Rs.1.34 million".

A first warning

- Suppose someone says: "Let's work out the VaR of monthly return at a 99% level"
- This will be exceeded in roughly 1 month of 100
- I.e. the VaR will be exceeded once in 8.33 years
- When the geek says "In the worst 2-3 **days** of the year, the loss on this position will exceed Rs.1.34 million", this is in touch with practical physical intuition.
- When the geek says "In the worst **month** of a 8.33 year period, the loss on this position will exceed Rs.1.34 million", this runs the risk of getting disconnected with the practical physical intuition.

Why is VaR so useful?

- The mathematician says: "You have a whole rich $f(x;\theta)$. A vast amount of information is stripped away when we go from that to the VaR".
- The practical man says: "I cannot understand $f(x;\theta)$, but I think I can use the VaR".
- VaR is a tool to facilitate conversations between geeks and squares. The geeks know how to make it, the squares know how to use it.

Desirable properties for risk measures

Purpose: to summarise the entire distribution of value returns X by a single measure. Features:

- Monotonicity: if $X_1 \leq X_2$, then $RM(X_1) \leq RM(X_2)$
- Translation invariance: $RM(X + C) = RM(X) + C$ where C is cash.
  Higher levels of cash means lower the risk of the portfolio.
- Homogeniety: $V(mX) = mV(X)$
  A portfolio with the same constituents that is larger by a multiplier m will have a higher risk value, scaled up by multiplier m.
- Subadditivity: $V(X_1 + X_2) \leq V(X_1) + V(X_2)$
  Combining portfolios cannot increase risk.

Does VaR fit?

- VaR fits the criteria of monotonicity and translation invariance.
- On homogeniety, the standard VaR is calculated on returns distribution of the portfolio.
  However, larger portfolios have a larger market impact of trade – ie, liquidity risk.
  If this is not taken into account, VaR does not satisfy the homogeniety property.
- Subadditivity of credit portfolios may report VaR as having a higher VaR than an investment in any single credit instrument.

VaR can be dangerous is the simplicity: a single measure that captures *the edge* of how bad things can get. But does not tell us *everything* about how bad things can get.

Some alternative measures

- *The whole distribution*, $f(x)$: reporting a range of VaR numbers for higher confidence levels.
- *Conditional VaR*: the expectation of all losses that exceed the reported VaR.

$$E[X|X < v] = \frac{\int_{\infty}^{v} xf(x)dx}{\int_{\infty}^{v} f(x)dx}$$

- *Standard Deviation*, σ_x: VaR becomes a multiplier of this. It works well if $f(x)$ is well known.
- *Semi-standard deviation*, $\sigma_{V(x)}$: the dispersion of all points that represent a loss (or exceeds a VaR).
- *Drawdown*, $DD(x)_T$: drop wrt the maximum over a fixed time period, t. Calculated as:

$$DD(x)_T = \frac{x^{max} - x_T}{x^{max}}$$

It is useful if returns are *not* independent across time.

Cashflow at risk

- VaR applied to non-financial firms becomes about one-period earnings.
- *Cashflow at risk* (CFAR) is the worst case shortfall in cashflows to the firm.
- $CFAR \sim E(R - C)$ where $E(R)$ is the expected revenues, and $E(C)$ is the expected costs.

Part I

The most trivial case for a VaR calculation

Returns are i.i.d. normal

- X is i.i.d.
- $X \sim N(0, \sigma^2)$
- In this case, we have to just look up normal tables.

	99 pc	95 pc
0.5	1.16	0.82
1	2.33	1.64
2	4.65	3.29

There be dragons

- The ordinary spreadsheet user can compute out portfolio standard deviations
- Multiply by the quantile and you're done
- But this only works when portfolio returns are i.i.d. normal.
- This is better than knowing nothing
- But we know this is not the reality: there are fat tails, volatility clustering, etc. Nonlinear products generate non-normal distributions.

Part II

Historical simulation

The idea

1. You are presented with a complicated VaR situation
2. You ignore all the complexity and say: Show me the historical time-series of returns.
3. You now pretend this is i.i.d.
4. Work out the 1% quantile on this empirical distribution.
5. And say this is a VaR estimate.

Hysterical simulation

- Non-normality?
- Non-iid?
- Is the data span long enough?
- Are you even sure that the position that we have today was held intact through the entire history in which this data is generated?

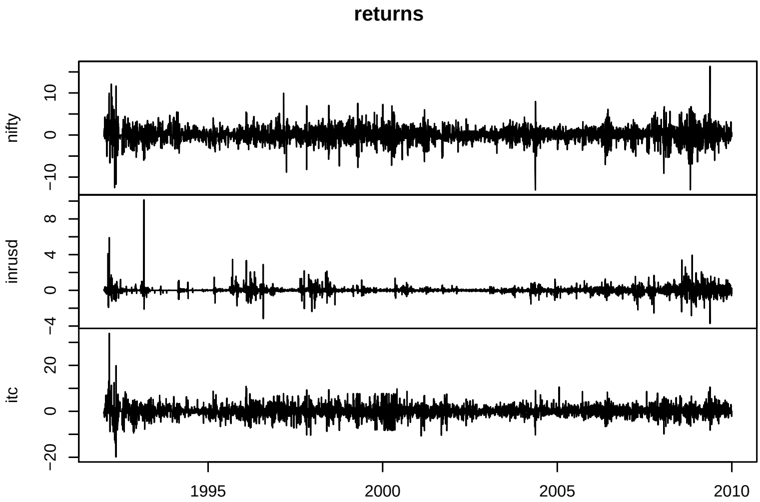
Bottom line: Better than knowing nothing, and can be done with a spreadsheet, but mostly not a good idea.

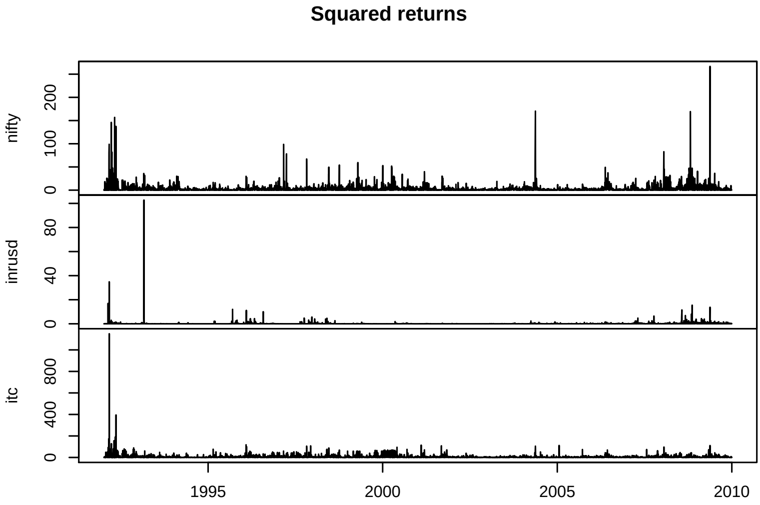
Part III

Let's absorb the facts

A little dataset

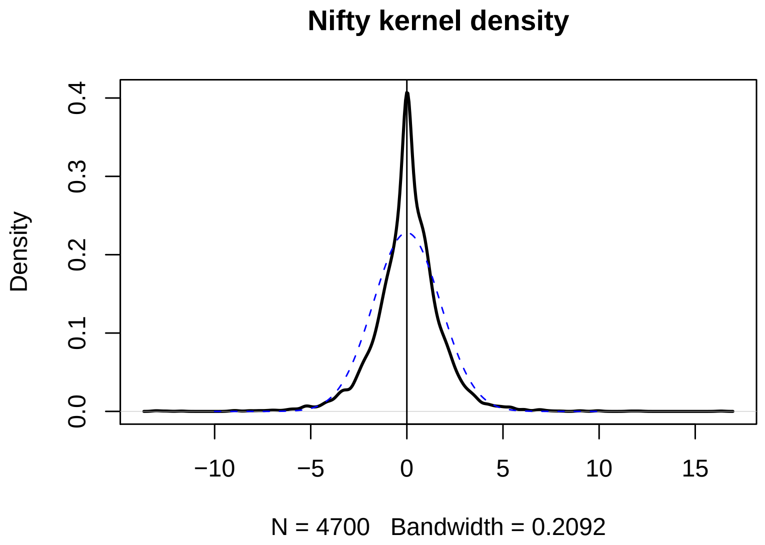
- Let's look at 3 interesting series: Nifty, INR/USD and one common stock - I T C LTD.
- Daily returns from 1992-01-07 to 2009-12-31, 4700 days of data.
- You can criticise this dataset saying that India changed dramatically over this period, but from the viewpoint of the statistical estimation, it does not get better than this.

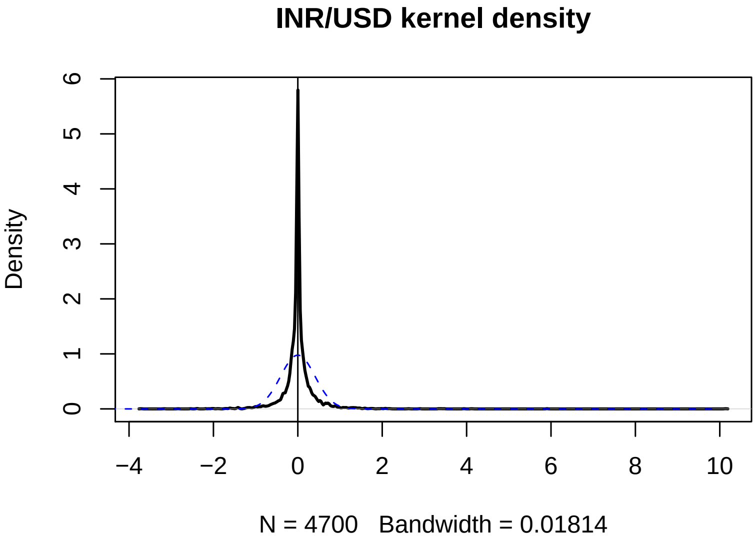

returns
nifty
inrusd
itc
10
0
−10
8
4
0
−4
20
0
−20
1995
2000
2005
2010

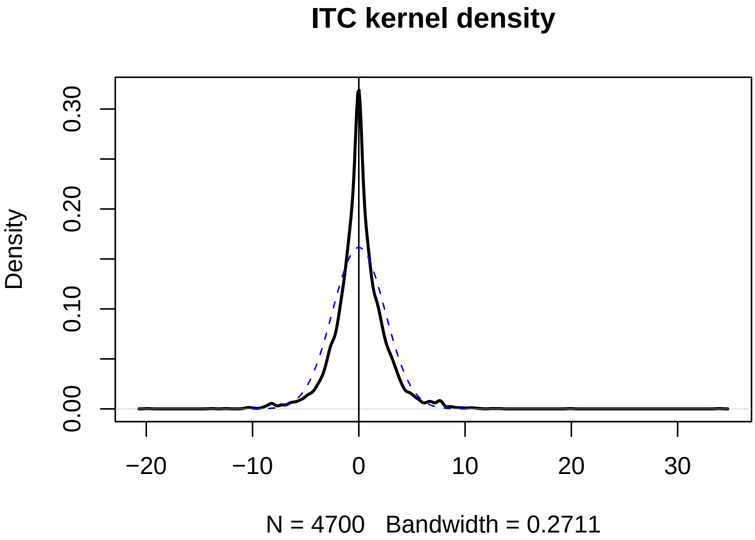

Squared returns
nifty
0
100
200
inrusd
0
40
80
itc
0
400
800
1995
2000
2005
2010

What do we see?

- Volatility clustering!
- Sleepy periods are followed by sleepy periods, and vice versa
- There is no "overall average volatility"
- Currency: Periods of fixing and then the fixing fails so there is an outburst of volatility. (But in 2003 and then in 2007 they changed things).
- Nifty is less volatile than I.T.C.


Nifty kernel density
Density
0.0
0.1
0.2
0.3
0.4
−10
−5
0
5
10
15
N = 4700   Bandwidth = 0.2092


INR/USD kernel density
Density
0
1
2
3
4
5
6
−4
−2
0
2
4
6
8
10
N = 4700   Bandwidth = 0.01814


ITC kernel density
Density
0.00
0.10
0.20
0.30
−20
−10
0
10
20
30
N = 4700   Bandwidth = 0.2711

Implications for risk measurement

- We must absolutely stay away from i.i.d. stories
- We must worry about non-normality (can partly come from sheer volatility clustering)
- We must always have physical intuition about reality, never let it become a game of numbers.