The implications of axioms of coherent risk measures to risk management

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Motivation: risk needs to be aggregated

- Risk management is about understanding and managing the potential losses in the total portfolio.

  Risk measurement is often done at the level of single assets, or sub-portfolios.

- For example: risk of an equity trading desk, risk at a bond trading desk.

  What risk measure should be used consistently over these two parts of the trading portfolio so that at the firm level, the management of the risk is “coherent”?

- Central to this problem: understanding *aggregate risk* because the ultimately liability is that of a *single entity* – the firm.

- For example, **VaR** is ultimately useful when it dictates what is the risk capital to be allocated.

  A single risk measure feeds into a single risk capital measure for the firm.

- Coherent risk measures are important when the risk comes from separate risk-taking economic agents within the same firm.

Where does coherence fit?

- Worry: if one trading desk has double the position of another, how should the risk measure treat the losses across the two?
- Worry: if risk capital needs to be set aside, then does setting (say):

$$\text{Capital} = \text{Total VaR} = \text{Var(equity)} + \text{Var(equity)}$$

  ensure that an *acceptable* level of loss the firm has covered for?
- If VaR is a "coherent risk measure", this should always be true.

Artzner et al, 1999

Step 0: Defining “acceptable loss”

- Sometimes, acceptable losses are set outside the firm: regulator, clearing corporation (for a trading firm).
- Often the acceptable losses are set within the firm: chief risk officer setting position limits across different levels of operations.
- The benefit of the definition is that it sets the benchmark level of risk.
- Artzner et al state that if the risk measure is a coherent measure, then it can be used to set the benchmark level of risk.
  This in turn becomes the risk capital for the firm.

Step 1: Measuring the risk of the portfolio

- There are several ways of measuring total risk of the portfolio.
- Three that Artzner et al 1999 consider (considering a portfolio of linear and non-linear securities):
  1. Regulatory "rules of thumb": SEC/NASD rules on final networth.
  2. Quantile-based measure of the portfolio returns: **VaR**
  3. Scenario based measure: Standard Portfolio Analysis of Risk, **SPAN**
- Question for the risk officer to consider: are these approaches coherent?

Describing SPAN

- SPAN was a good general risk measure operationalised on a portfolio with spot, futures and options assets.
- Key insight: for a given underlying, the movements of (non-linear) option prices track the spot price.
- SPAN implements a limited "scenario analysis" on a portfolio: where the scenarios are fixed at multiples of $0, \pm 1, \pm 2, \pm 3\sigma$, and two "extreme" values of underlying price movements.
- **Note:** This typically does not account for *basis risk*.
- More difficult to implement for general portfolios. Diversification is handled using (generally) rules of thumb similar to the regulatory "rules of thumb".

Describing Regulatory Rules of Thumb

- (Typically) Identify derivatives positions which exactly (*not including* basis risk) cancel out each other, and adjust the capital requirement downward to the extent of these positions.
- Eg: **Spread positions** – *Long* 10 units of near-month Nifty futures and *short* 10 units of next-month Nifty futures.
- Eg: **Long** 100 units of Axis Bank futures near-month and **Short** 100 units of SBIN futures.
- **Caveat**: India does not permit netting of positions taken in different underlyings.
- Rules of thumb can be very complicated – particularly when trying to adjust capital/margin requirements to account for
  1. imperfect correlation measures across different underlyings
  2. basis risk.

SEC/NASD rules

- Mark the full portfolio to market.
- Deduct market value of calls
- *Deduct* margins for spread positions
- Check that this is zero.
- **Note:** Typically, the margin paid corresponds to what the position may lose in the worst scenario.

Analysing for coherence

Recap on definition of coherent measures

1. Monotonicity: if $X_1 \leq X_2$, then $RM(X_1) \leq RM(X_2)$.
   Obvious: positions that lead to more losses require higher capital.
2. Translation invariance: $RM(X + C) = RM(X) + C$ where C is cash.
   Note: This leads to the "risk capital" interpretation of the RM. If $C = RM(X)$, then $RM(X) + C = 0$ (given $RM(X)$ is a loss).
3. Subadditivity: $RM(X_1 + X_2) \leq RM(X_1) + RM(X_2)$.
4. Homogeniety: $RM(mX) = mRM(X)$
   Addendum: Move to needing *Convexity* such that:

$$RM(mX) \geq mRM(X)$$

Why is subadditivity so important?

1. Reflects the idea that risk can be reduced by diversification.
   Eg., non-subadditive measures of risk in portfolio optimisation may create portfolios with high concentration risk.
2. If risk capital comes from non-subadditive risk measures, firms have an incentive to create subsidiaries to reduce regulatory capital.
3. With subadditivity, decentralisation of risk management is possible.
   Eg., CRO can set position limits on different traders M_1, M_2 such that firm's capital $M \geq M_1 + M_2$ always, where $M = RM(L_{\text{firm}}), M_1 = RM(L_1), M_2 = RM(L_2)$.

VaR can violate sub-additivity

- For a full-equity portfolio, sub-addivitity holds in the case of VaR.
  Eg., for a 2-stock portfolio, S_1, S_2, with variances σ_1^2, σ_2^2 and correlation $-1 < \rho_{1,2} < 1$, we know the portfolio variance is lower than individual stock variance.
  VaR works with linear combinations of underlying "elliptically distributed risk factors".
- VaR is not sub-additive in:
  1. Very skewed loss distributions: with portfolios of defaultable bonds, options.
  2. Dependence structures is a special, highly asymmetric form.
  3. Underlying assets are independent but very heavy-tailed (high persistence in volatility dependence, prone to jumps in volatility).

Scenario analysis is coherent

- Strength of the approach lies in defining losses upfront for the entire portfolio.
- If the underlying distribution is correct, then the scenario analysis does present a coherent measure.
- Worldwide, exchanges are moving to a more continuous set of scenarios.
- Question: is it optimal?

Next

How to measure efficacy of the risk measure?