# Comparing risk measures using VaR forecasts

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# Recap

- Multiple measures of “risk” (volatility) of asset/portfolio returns: $\sigma$, range-based, systematic risk, etc.
- How do we choose the “best” from the viewpoint of a volatility forecast?
- Step 1: Select a financial context – VaR/Optimal Portfolio.
- Step 2: Apply the different candidates to the selected context.
- Step 3: Measure the actual VaR/Sharpe’s Ratio of the portfolio observed forward in time.
- Step 4: Use a statistical test to answer the question: How do our different candidates behave relative to the observed?
- Step 5: Select a candidate based on the test.

# Testing the performance of VaR forecasts

# Defining the variable of focus

- Define VaR at $p$ level of confidence and $t$ interval: $v_t$

$$\int_{-\infty}^{v_t} f_t(r)dr = p$$

- Define "failure" of the model as $r_t < v_t$.
- Define a good model (ie, a forecast of risk), $f(r)$ such that $\Pr(r_t < v_t) == p$.

# Simplest statistical tests

- A good statistical test is one which focusses on $r_t, v_{1,t}$ as outcomes to be compared without any dependence on the model that generated $v_{1,t}$.
- For example, $r_t^2, \beta^2 r_{m,t}^2$ as the estimate of $\sigma^2$ in a normal distribution generating two possible values of VaR ($V_1$, $V_2$).
- Compare ($V_1$, $V_2$) against actual return, $r_{t+1}$.
- If $V_1 < r_{t+1}$ and $V_2 > r_{t+1}$, then $V_1$ is *better* than $V_2$.
- Statistical test: Repeat this many times, and see which of $\Pr(V_1, V_2 > r_{t+1})$ is closer to $p$.

# Real world complications

- Typically, both $V_1$, $V_2$ show similar performance on the $\Pr(V_i < r_{t+1})$ wrt $p$.
  Which do you choose?
- The test itself does not recognise *heteroskedasticity*: ie, it wants the unconditional probability of failure to match, $p$.
- But what if:
  - $\Pr(V_1 > r_{t+1})$ = $\Pr(V_2 > r_{t+1}) = p$, but:
  - $\Pr(V_1 > r_{t+2} | V_1 > r_{t+1}) \neq 0$, and
  - $\Pr(V_2 > r_{t+2} | V_2 > r_{t+1}) = 0$?
- Solution: Christoffersen, 1998 and the test of interval forecast evaulation.

# Christoffersen's tests

- Transform the data on $r_t, v_t$ into $I_t$, where:
  1. $I_t = 1$ if $v_t > r_t$, and
  2. $I_t = 0$ if not.
- The forecasts are efficient if they show both correct unconditional coverage and no independence. Called "correct conditional coverage".
- Three steps to a definitive test of "coverage":
  1. Necessary: test of *unconditional coverage* – $\Pr(v_i > r_{t+1}) = p$.
  2. Test of *independence* – test $I_t$ against an alternative of first order markov process.
  3. Test of *correct conditional coverage* – $H_0$ : independent process with unconditional coverage of $p$. $H_1$ : first order markov process with unconditional coverage different from $p$.

# Limitations to Christoffersen

- *Problem #1* The alternative is limited – we only test for first order markov process.
  What if there is a higher order of dependence?
- *Solution* Christoffersen and Diebold 2000, Clements and Taylor 2000 do have a more general test.
  This includes cyclical dependencies, as well as higher order lags. But specific dependencies have to be tested for.
- *Problem #2* Too many forecast models are admitted in by these tests as well.

# Lopez's loss function approach

- Critical observation: The indicator variable does not care about the *magnitude* of the error.
  In the real world we do.
- Critical observation: Some users of volatility forecasts care only about *loss* (for example, regulators).
  Others care about both (for example, business cares about capital efficiency, rather than just a loss).
- Lopez incorporates the user's utility function into the calculation of $I_t$ as follows:
  - Example 1: $I_t = (r_t - v_t)^2$ if $v_t > r_{t+1}$; 0 if not.
  - Example 2: $I_t = (r_t - v_t)^2$ if $v_t > r_{t+1}$; $-\alpha v_t$ if not.
- In the second example, the magnitude of the error is part of the test. The larger the error, the greater the penalty to the model.

# Lopez' tests

- Define $z_t = l_{1,t} - l_{2,t}$. This will have some distribution.
- Define $\nu_t = 1$ if $z_t \geq 0$; $\nu_t = 0$ if $z_t < 0$.
- If $z_t$ is iid, then $\sum_T \nu_t$ is binomial$(T, 0.5)$.

# SarmaShahThomas 2003

- Competing forecasts of risk: Equally Weighted Moving Average (EWMA) model, RiskMetrics (RM), GARCH(1,1), Historical Simulation.
- Variable: Daily Nifty, 1990 to 2000.
- Results:
  - At 95% daily VaR: Christoffersen does not reject GARCH(1,1) and RM. ChristoffersenDiebold does not reject GARCH(1,1).
  - At 99% daily VaR: Christoffersen does not reject GARCH(1,1) and RM. Lopez finds that the regulatory loss function does not differentiate between RM and GARCH(1,1).