Evaluating risk measures using portfolio optimization

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Overview

- Measures of correlations
- Testing Strategy: Uniform weights
- Testing Strategy: Portfolio optimization

The portfolio optimisation framework

- Given N stocks, how do we create an "efficient" portfolio?
- Markowitz approach:
  - Get the correct values of $E(\vec{r}_a)$, and
  - the correct estimate of $\vec{\Sigma}$.
- Find $\vec{w}$ such that:

$$\begin{array}{lll} \text{minimise} & \frac{1}{2}\sum_{i,j=1}^{n} w_i w_j \sigma_{ij} & \\ \text{subject to} & \sum_{i=1}^{n} w_i E(r_i) & = E(r_p) \\ & \sum_{i=1}^{n} w_i & = 1 \end{array}$$

- Resultant portfolio should have the best portfolio performance in the investment period.

Using portfolio optimisation to pick the correct risk measure

- Risk comparison framework: Given risk candidates $\sigma_1, \sigma_2, \ldots, \sigma_n$ and a T sized data set of returns:
  - Select K days to estimate $\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_n$ for the portfolio. This is the "in-sample" data.
  - Use these to calculate $\vec{w}_1, \vec{w}_2, \ldots, \hat{\vec{w}}_n$.
  - For the next $K + m$ days, observe $\hat{r}_1, \hat{\sigma}_1, \hat{r}_2, \hat{\sigma}_2, \ldots, \hat{r}_n, \hat{\sigma}_n$. Use this to calculate (say) SR_1, $\mathrm{SR}_2, \ldots,$ SR_n. This is the "out of sample" data, and typically $m = K$.
  - Repeat this for the full sample T, moving the in-sample period up by K days.
- σ_i that generated $\vec{w}_i$ with the lowest value of $\hat{\sigma}_p$ or highest value of SR is the "best" measure of risk.

Example of operationalising the portfolio optimisation framework

Candidates for covariance matrix estimation

- Historical covariance matrix (HC)
- Single index-market model (SIMM)
- Vasicek beta correction (VB) – modification of SIMM

Sample covariance matrix and SIMM

- Sample covariance: traditional method, estimation error-prone.
- SIMM: Assumes that returns of any asset Y_i are correlated with returns on the market index.
- Covariance matrix:

$$C = s_{00}^2 bb' + D$$

- s_{00}^2 = sample variance of market returns, b_i = slope estimate, D = diagonal matrix containing residual variance estimates d_{ii}
- Use package `stockPortfolio`

Vasicek beta

- Adjusts past β's towards the average β by modifying each β using the sampling error around it
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$$b'' = \frac{\frac{b'}{s_b'^2} + \frac{b'}{s_b^2}}{\frac{1}{s_b'^2} + \frac{1}{s_b^2}}$$
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$$s_b''^2 = \frac{1}{\frac{1}{s_b'^2} + \frac{1}{s_b^2}}$$
- s_b^2 is the variance of beta; b' and s_b' are parameters from the prior distribution.
- Use the modified β to obtain the covariance matrix
- Use package `stockPortfolio`

Testing framework: Intuition

- Models must generate *optimal* portfolios
- One measure of optimal: Least variance
- Other measures: Tracking error, Sharpe Ratio

Comparing alternative estimators #1

- Compare the portfolio variance predicted by the various covariance matrices to the actual out-of-sample portfolio variance
- The weights are generated from a uniform distribution
    1. Calculate the mean, median and standard-deviation of the difference
- Rolling-window estimates of the same

Predetermined weights

623 days in-sample, from 24/08/07 to 09/03/10
100 days out-of-sample, from 10/03/10 to 30/07/10
10,000 replications, True SD - Predicted SD

	In-sample			Out-of-sample		
Covariance matrix	Mean	Median	SD	Mean	Median	SI
HC	-9.23e-18	0.00	3.77e-16	-1.4640	-1.5130	0.07
SIMM	0.078	0.078	0.015	-1.386	-1.384	0.07
VB	0.0887	0.08797	0.017	-1.377	-1.376	0.06

Comparing alternative estimators for portfolio size 5

- Randomly choose a portfolio of 5 stocks
- Create 100 such portfolios
- Compare the three estimators for each portfolio

Results: 5 stock portfolio for fixed N

623 days in-sample, from 24/08/07 to 09/03/10
100 days out-of-sample, from 10/03/10 to 30/07/10

	Mean of average difference	Mean of average SD
HC	-1.6127	0.5414
SIMM	-1.5773	0.5391
VB	**-1.5717**	**0.5354**

Results: 5 stock portfolio for varying N

100 days out-of-sample, from 10/03/10 to 30/07/10
10,000 replications, True SD - Predicted SD

Days	HC	SIMM	VB
805	-1.4359	-1.3498	**-1.3461**
	-2.3212	-2.2404	-2.2590
	-1.1585	-1.1936	-1.1903
	-1.7095	**-1.6032**	-1.6074
1028	-1.9357	-1.8959	**-1.8955**
	-1.6973	-1.6630	**-1.6472**
	-0.6975	-0.6695	**-0.6674**
	-1.4616	-1.4640	-1.4496

Comparing alternative estimators #2

- Models forecasting performance compared using the *variance* of the optimized portfolio's returns.
- Carry out the portfolio optimization

$$min w' \Sigma w$$

$$w' r = E(r)$$

- Obtain the weights w
- Use the weights and the out of sample returns to compute the σ_p
- Compare the σ_p
- Use package `portfolio.optim` in `tseries`

Portfolio optimization

Portfolio variance using weights from the portfolio optimization exercise.
623 days in-sample, from 24/08/07 to 09/03/10
100 days out-of-sample, from 10/03/10 to 30/07/10

	Out-of-sample SD
HC	0.707
SIMM	0.682
VB	**0.683**

Thank you.