Varying volatility in parameteric linear VaR

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Recap

- So far, if the portfolio has a mean daily return of $E(r_p) = w'E(r_h)$ and V_h is the h-day covariance matrix over a h-day horizon, then

$$\mathrm{VaR}_{h,\alpha} = \Phi^{-1}(1-\alpha)\sqrt{w'V_h w} - w'E(r_h)$$

- So far, volatility estimates have been historical:

$$\hat{\sigma}_t^2 = T^{-1}\sum_{k=1}^{T} r_{t-k}^2$$

- Similarly, covariances are historically equally weighted estimates.
- What if the variance-covariances input could be improved over the historical estimate of V_h?
  Ie, short-term returns are more representative of short-term volatility compared to long-run historical averages.

Conditional volatility in estimating portfolio VaR

Exponentially weighted moving averages (EWMA)

- EWMA for the variance estimate is summarised as:

$$\hat{\sigma}_t^2 = (1 - \lambda) r_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2$$

  Assume r_t are independent.
- λ is called a *smoothing constant* and follows $0 < \lambda < 1$.
- The latter condition is because the alternative representation of EWMA volatility is:

$$\hat{\sigma}_t^2 = (1 - \lambda)(r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \lambda^3 r_{t-4}^2 + \ldots)$$

  As $k \to \infty, \lambda^k \to 0$
- This captures the decaying impact of old information well.

EWMA covariances

- A similar approach to calculate the covariance between i, j securities:

$$\hat{\sigma}^2_{ij,t} = (1-\lambda)(r_{i,t-1}r_{j,t-1}+\lambda r_{i,t-2}r_{j,t-2}+\lambda^2 r_{i,t-3}r_{j,t-3}+\lambda^3 r_{i,t-4}r_{j,t-4}+\ldots)$$

- Or it is written as:

$$\hat{\sigma}^2_{ij,t} = (1-\lambda)r_{i,t-1}r_{j,t-1} + \lambda\hat{\sigma}^2_{ij,t-1}$$

- The EWMA correlation, $\hat{\rho}_{ij}$ is calculated using the EWMA covariance and the two EWMA variances.
- Covariances based on EWMA must have the same λ for the variance and covariance estimates, to *ensure* a positive semi-definite covariance matrix.

Operationalising EWMA covariances

- Given a time series of returns and covariances, $\hat{\sigma}^2_{ij,t+1}$ is a weighted average of $r_{i,t}$, $r_{j,t}$ and $\sigma^2_{ij,t}$.
- If you don't have the time series of covariances, use the historical covariance $\hat{\sigma}_{ij}$ as the initial value.

$$\sigma^2_{ij,1} = (1-\lambda) r_{i,0} r_{j,0} + \lambda \hat{\sigma}^2_{ij}$$

- This is part of the input to $\hat{\Omega}_{t+1}$ used for the VaR calculation of the portfolio from constituents as:

$$\text{EWMA VAR}_{\alpha,t} = \Phi^{-1}(1-\alpha)\sqrt{w'\hat{\Omega}_{t+1} w}$$

- Same approach applies to calculating VaR using *risk factors*, where $\hat{\Omega}_{t+t}$ is the variance-covariance matrix for the time series of risk factors.

Example of EWMA variances and different λ

- EWMA VaR estimes for mid-June 2008, FTSE-100.
- Calculated for different $\lambda = 0.9, 0.95, 0.99$

α	λ		
	0.90	0.95	0.99
5%	7.42%	8.08%	7.81%
1%	10.49%	11.43%	11.05%
0.1%	13.93%	15.19%	14.86%

- Higher the α, higher the volatility estimates.
- However, higher λ does not automatically generate higher values of σ.
  λ only captures how much weight is given to the recent observations.

EWMA in regulation: RiskMetrics VaR

- Several regulators adopted RiskMetrics as a easy-to-operationalise model for covariance matrix forecasts.
- The approach can be applied to a large set of underlying securities. This framework includes:
  1. *Regulatory matrix:* Matrix calculated as *equally* weighted averages based on the last 250 observations.
  2. *Daily matrix*: EWMA with $\lambda = 0.94$ for all elements.
  3. *Monthly matrix*: EWMA with $\lambda = 0.97$ for all elements, multiplied by 25 to shift from daily to monthly.

Example of RiskMetrics vs. EWMA VaR

- *Problem:* Portfolio of two sets of U.S. stocks with
  1. Set 1: $\beta = 1.1$ with S&P500
  2. Set 2: $\beta = 0.85$ with NASDAQ-100
- 3 million invested in Set 1
- 1 million invested in Set 2
- Want to calculate the 1% 10-day linear normal VaR based on the RiskMetrics and disaggregate the VaR into standalone S&P 500 VaR and NASDAQ-100 VaR.

Example of RiskMetrics vs. EWMA VaR

- RiskMetrics and *regulatory* volatilities of S&P500 and NASDAQ-100 are as follows:

	Volatility		Correlation
	S&P	NASDAQ-100	
EWMA	22.81%	28.03%	94.91%
Regulatory	19.63%	22.89%	89.88%

- Annualised covariances:

	S&P500	NASDAQ-100
EWMA		
S&P500	0.052	0.061
NASDAQ-100	0.061	0.079
Regulatory		
S&P500	0.039	0.040
NASDAQ-100	0.040	0.052

Example of using RiskMetrics for portfolio VaR

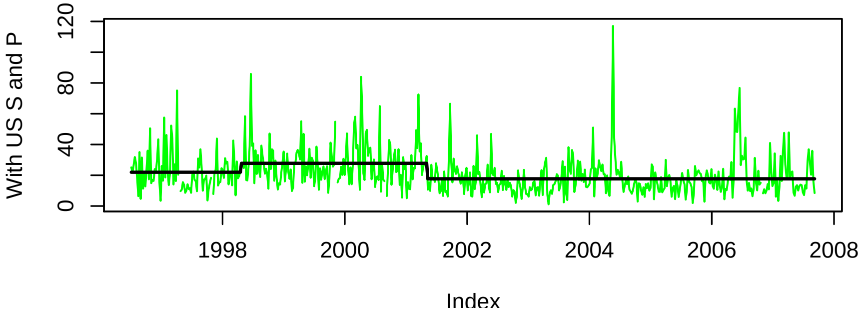
- The answer depends upon calculating (a) stand-alone VaR (using $\hat{\Omega}$ above, β wrt the two indexes and the amount invested in each Set and (b) summing it to get the Systematic VaR.
- This works out to be:

	Standalone VaR		Systematic VaR (In USD)
	S&P500	NASDAQ-100	Total
EWMA	350,284	110,852	456,833
Regulatory	301,377	90,522	384,789

Evidence against constant λ

- λ measures the *persistence* of shocks to the volatility of returns.
- Persistence changes.

Structural changes in Indian index volatility



Structural changes in equity index volatility

	Weekly realised vol	
	10-year average	Latest
Sensex	21.27	18
Japan Nikkei	20.22	16
US S and P	15.48	11
UK FTSE	15.36	11
S.Korea Kospi	28.43	18
Singapore STI	17.45	12

Structural changes in persistence of equity volatility

- Data – daily frequency
- GARCH(1,1) model estimates

	Full period, 1998-2007				Present, 2001-2007			
	γ_1	γ_2	Sum	Half-life	γ_1	γ_2	Sum	Half-life
Sensex	0.04	0.95	0.99	62.20	0.17	0.75	0.92	8.68
Nifty	0.03	0.95	0.98	35.68	0.17	0.75	0.92	8.68

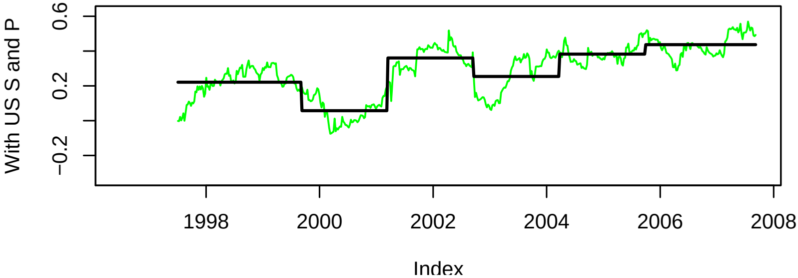
Structural changes in persistence of equity volatility

- Global phenomenon.
- GARCH(1,1) model estimates

	Full period, 1998-2007				Present, 2001-2007			
	γ_1	γ_2	Sum	Half-life	γ_1	γ_2	Sum	Half-life
Sensex	0.04	0.95	0.99	62.20	0.17	0.75	0.91	8.68
S&P	0.04	0.95	1.00	139.77	0.03	0.70	0.73	3.19
FTSE	0.14	0.82	0.96	18.62	0.16	0.56	0.72	3.11
Nikkei	0.01	0.97	0.99	62.95	0.10	0.68	0.78	3.81
Kospi	0.11	0.89	0.99	127.96	0.16	0.78	0.94	11.57
STI	0.10	0.90	1.00	−4167.23	0.14	0.74	0.88	6.55

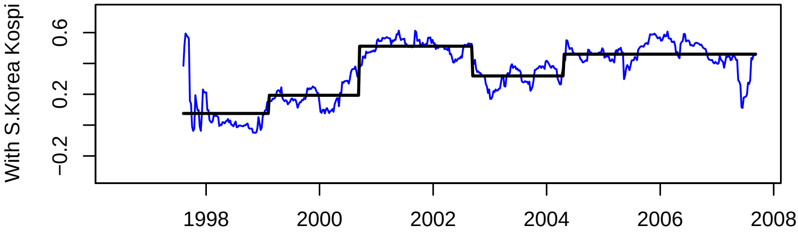
Structural changes in persistence of equity correlation, OECD

- Correlations change.
- Correlation of Sensex with S&P500



Structural changes in persistence of equity correlations, S. Asia

- Correlation of Sensex with S. Korea KOSPI index



Structural changes in persistence of equity correlations

Correlation of Sensex with:	Correlation in first period (%)	Date of last change	Correlation in recent period (%)
S&P	0.22	23/09/2005	0.44
FTSE	0.17	16/09/2005	0.52
Nikkei	-0.03	17/06/2005	0.42
Kospi	0.08	16/04/2004	0.46
STI	0.24	30/04/2004	0.53

Varying λ in covariance-variance

- Persistence changes. Why?
- Multiple reasons:
  - Macro (international) : Cross-country fund flows, changes in captail account convertibility, changes in differential risk factors across countries.
  - Macro (domestic) : Institutional changes – improvements in trading, clearing, settlement systems; introduction of new products; changes in interest rate environment – monetary policy.
  - Micro : changes in firm level features – maturing firms; changes in operations/management of firms.
- Constant λ models don't fit.

Multivariate GARCH

- Alternative is to constantly re-estimate covariances and variances for all constituents/factors.
- y_{1t} and y_{2t} can be modelled as:

$$\begin{aligned} y_{1t} &= \mu_{1t} + \epsilon_{1t} \\ y_{2t} &= \mu_{2t} + \epsilon_{2t} \end{aligned}$$

- ϵ_{1t} and ϵ_{2t} have unconditional distributions

$$\begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right]$$

- and conditional distributions:

$$\begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} | \mathcal{I}_{t-1} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} h_{1t} & h_{12t} \\ h_{12t} & h_2 \end{pmatrix} \right]$$

- Parameterisation of the above matrices are the heart of the MVGARCH models.

General MV-GARCH

- MV-GARCH(p,q) model for a set of N time series vectors is:

$$vech(H_t) = vech(\Sigma) + \sum_{i=1}^{q} A_i \;\; vech(\epsilon_{t-i}\epsilon'_{t-i}) + \sum_{j=1}^{p} G_j \;\; vech(H_{t-j})$$

  where Σ is an $(N \times N)$ matrix and A_i and G_i are $(N(N+1)/2 \times N(N+1)/2)$ matrices.

General MV-GARCH

- When $N = 2$ and $p, q = 1$, H_t is:

$$\begin{bmatrix} h_{1t} \\ h_{12t} \\ h_{2t} \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} \\ \epsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{12,t-1} \\ h_{2,t-1} \end{bmatrix}$$

- There are 21 parameters to be estimated.
- This makes $\frac{5N^2}{2} + \frac{N}{2}$ parameters to be estimated for a model with
  - N time series and
  - GARCH(1,1) for the terms of the variance covariance matrix.

Choices in MV-GARCH

- Research involves imposing meaningful economic restrictions on the A_i and G_i while simultaneously reducing the parameter space.
- Engle, Granger, Kraft (1986):
  - bivariate GARCH, normal errors.
  - Restrictions on diagonal matrix of A, G.
- Baba, Engle, Kraft, Kroner (1988) modified the above to include a quadratic restriction on A, G which further reduced the parameters to be estimated. This is the BEKK model.
- Bollerslev (1990): Constant correlations matrix.
- Most popularly used today: DCC (Dyanmic Conditional Correlations) MVGARCH.
- Key objective of this alphabet soup: attain positive definite variances and covariances, minimise estimation costs.

Task for a risk manager

- Out of all these choices, choose one.
- Focus on creating a framework that
  - Can be operationalised, to
  - Constantly evaluate risk as accurately as possible.
  - Ensure that the risk management strategies are in line with the current risk position.