Bargaining Order and Delays in Multilateral Bargaining with Asymmetric Sellers

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We examine whether the hold-up problem is mitigated or exacerbated in a multilateral bargaining problem with two heterogeneous sellers who own units which are perfect complements. Using a complete information model we also analyze the optimal negotiation sequence of the buyer. We find that the buyer prefers to negotiate with the lower-valuation seller first, except in an equilibrium where both the lower-valuation seller and the buyer play "hold-out" strategies and the higher-valuation seller plays an "accommodative" strategy. For high discount factors we find that while there exists a unique efficient outcome when the buyer negotiates with the lower-valuation seller first and the sellers are sufficiently heterogeneous, significant delay in reaching agreements may arise when they are not. In case the buyer bargains with the higher-valuation seller first, an inefficient outcome is shown to exist even when players are extremely impatient.

Keywords: Multilateral Bargaining, Heterogeneous Sellers, Holdout, Negotiation Sequence.

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1 Introduction

It has been well-documented that multilateral bargaining games comprising patient homogeneous sellers selling perfect complementary units, suffer from a hold-up problem, when each seller endeavors to reach agreements later, with the hope of securing a larger share of the surplus. This leads to inefficient delays. The purpose of this paper is to analyze the sequence in which a buyer prefers to negotiate with sellers with different valuations for their objects and to solve for the conditions under which (in)efficient outcomes exist in such bargaining games.

There are several examples of multilateral negotiations where a single buyer has to negotiate with multiple heterogeneous sellers. These include an industrialist bargaining with several farmers in order to assemble plots of land for a project; a manufacturer negotiating with a group of upstream suppliers; and a manager bargaining with two different unions in order to end a strike. In each of these examples, it is possible that the sellers have different valuations for their objects. In the land assembly problem for example, sellers could be expected to have different valuations for their land, even if the plots are contiguous and similar in size, when they have different endowments of skill and capital or have varying access to alternative methods of earning a livelihood (Ghatak and Ghosh (2011)).

To examine whether the hold-up problem is mitigated or exacerbated in the presence of heterogeneous sellers, we consider a multilateral bargaining problem with one buyer and two sellers. We assume that each seller owns a single object, and that the objects are *perfect complementary* to the buyer, such that she realizes the value of a project (M) only when she reaches an agreement with both the sellers. The two sellers have different valuations for their objects. For analytical tractability, we normalize the value of the lower-valuation seller (V_1) to zero and assume the value of the higher-valuation seller (V_2) to be strictly positive. We assume that bargaining proceeds through an exogenously determined sequence, with the buyer negotiating with each of the sellers in alternate rounds. Each round of bargaining potentially consists of two periods. In the first period the buyer makes an offer to the seller. If the offer is rejected, the seller makes a counter-offer to the buyer in the second period, which the buyer then accepts or rejects. Both the offer and the counter-offer specifies the compensation (price) to be paid to the corresponding seller. If either the offer or the counter-offer is accepted, the buyer pays the seller the negotiated price and the seller leaves the game forever. Once an agreement is reached, the buyer proceeds to negotiate with the remaining seller through an infinite horizon, alternate offer bargaining game (à la Rubinstein (1982)). If on the other hand, both the offer and counter-offer are rejected, the buyer moves on to the next round to bargain with the other seller through an identical sequence of offers and counter-offers. Clearly there are two possible bargaining orders: in the first, the buyer negotiates with the lower-valuation seller first, and in the second, with the higher-valuation seller.

The key issue in such problems lies in the circumstances which lead the different players to hold-up, which when combined with a bargaining order, results in inefficient delays. Following the literature on homogeneous sellers, the incentive to hold-up negotiations for all the agents can be predicted to grow as the discount factor approaches one. For sufficiently patient homogeneous players, it has been shown that two of the three players choose to play "hold-up" strategies, and place the remaining player into a disadvantageous bargaining position. However with heterogeneous sellers, as the valuation of the higher-valuation seller rises, it is not immediately obvious when this seller would choose to play such "hold-up" strategies. If the higher-valuation seller is the first to reach an agreement, he would seek a larger compensation for giving up a land of higher value; if on the other hand, the seller chooses to hold out, the available compensation is rising in his valuation. We are therefore interested in answering the following questions:

(a) under what conditions does the buyer prefer to bargain with the lower-valuation (higher-valuation) seller first?

(b) given a bargaining order, what are the conditions which lead to efficient (inefficient) outcomes?

We find that there are two significant equilibria, which are mirror images of each other. In the first equilibrium (E3), negotiations between the buyer and the higher-valuation seller hold out, while in the second (E5), the buyer is able to successfully negotiate the first deal with the higher-valuation seller only. We find that the buyer prefers to negotiate with the lower-valuation seller first, except in an equilibrium where both the lower-valuation seller and the buyer play "hold-out" strategies and the higher-valuation player plays an "accommodative" strategy (i.e. E5). The project is completed with delay whenever the buyer and the first seller in a bargaining order choose to play aggressively. For sufficiently high values of the discount factor, the equilibrium E5 exists provided the ratio of $V_2/M = K$ is below a threshold[1], and corresponds to an inefficient outcome for the first bargaining order. However for the same order, similar values of the discount factor and with K larger than the same threshold we find that there exists a *unique, efficient* equilibrium (E3). Therefore, while the hold-up problem is mitigated in the first bargaining order when the sellers are "sufficiently heterogeneous", signifi-

[1]The threshold is given by $K = \delta/(1 + \delta)$, where δ is the discount factor.

cant delay in reaching agreements may emerge when they are not. In the second bargaining order, an inefficient outcome is shown to exist even if players are *extremely impatient*. Such an outcome corresponds to the equilibrium E3, such that no agreement can be reached in the first round of negotiations between the buyer and the higher-valuation seller. Given that there are instances where negotiations have failed when the participants have deemed the outcome to be unfair, we calculate the Gini coefficient for the two equilibria. We find that in the first equilibrium it is a constant, and that it increases with K in the second equilibrium.

Equilibrium	Bargaining Order 1	Bargaining Order 2
E3; Buyer and higher-valuation seller hold out	efficient	inefficient
E5; Buyer and lower-valuation seller hold out	inefficient	efficient

We also find that when the valuation of the higher-valuation seller is close to zero and when the discount factor is relatively low, none of the players play strategies that hold up negotiations. As the valuation of the higher-valuation seller increases, he initially demands a higher compensation for his land through higher counter-offers, which are then deemed to be unacceptable by the buyer. This leads to seller 2's counter-offer getting rejected. Eventually the higher-valuation seller sets cutoffs for accepting offers from the buyer, such that the buyer is better off offering an amount lower than that cutoff. This leads to both the offer to and the counter-offer from the higher-valuation seller to get rejected.

Our model is an extension of Cai (2000), who studies a multilateral bargaining model of complete information, in which one buyer negotiates with two *homogeneous sellers* selling perfect complementary units. He shows that when players are sufficiently patient, there exist multiple equilibria, of which one corresponds to an inefficient outcome. We build on this result by showing that for high values of the discount factor, there exists a unique equilibrium when the degree of heterogeneity between the sellers is high enough[2]. Cai (2000) also shows that the delay can become longer as the number of sellers increases and perpetual disagreement can occur in equilibrium for a large number of sellers. He finds that when the number of sellers is larger than or equal to three and when the discount factor is sufficiently close to one, there exists an equilibrium in which the buyer gets zero.

[2]This equilibrium leads to an efficient (inefficient) outcome, when the buyer bargains with lower-valuation (higher-valuation) seller first.

In our model with two heterogeneous sellers, we get a similar result in several equilibria when K reaches the threshold value mentioned above. Holdout-related inefficient delays are also salient in Cai (2003) and Menezes and Pitchford (2004). While the former studies a model similar to Cai (2000) with *contingent* contracts and shows that there are multiple Markov equilibria, the latter uses a two-seller framework with cash-offer contracts, and allows the buyer to negotiate with both the sellers at any given date.

While Coase (1960) provides the most famous example of the holdout problem where he describes a railroad trying to acquire plots of land from several farmers, Eckart (1985) and Asami (1988) were the first to offer game theoretic arguments to study such problems. In a recent paper Chowdhury and Sengupta (2012) broadly analyze the conditions which lead to a (in)significant holdout problem and find that the problem is largely resolved when either the bargaining protocol is transparent and the buyer has a positive outside option or the marginal contribution of the last seller is not too large. However, it continues to be severe whenever the buyer has no outside option or when the bargaining protocol is secret. Though the bargaining protocol is transparent in our setup, the buyer does not have access to an outside option. Strategic holdout has also been shown to pose a serious problem to R&D development, when licenses have to be obtained from multiple patentees (Shapiro (2001)).

In addition to the seller holdout problem our paper is related to the literature on optimal negotiation sequence. Li (2010) studies an infinite-horizon random-offer model with complementary goods, in which the bargaining strength of a seller is given by the probability with which he gets to make a take-it-or-leave it offer. The key finding of this paper is that any sequencing can be sustainable in equilibrium. A closely related paper to ours is Xiao (2010), who examines the preference over bargaining order in an infinite-horizon complete information multilateral bargaining game with asymmetric sellers. In his model, bargaining strength of a seller is measured by the size of the land he owns, with the seller of a larger plot having higher strength. The bargaining order is endogenously determined by the buyer. Xiao shows that there exists an efficient subgame perfect equilibrium, in which the buyer chooses to negotiate in order of increasing size. The buyer negotiates by choosing a seller with whom she continues to bargain through an alternating sequence of offers and counter-offers, until an agreement is reached. The first seller then leaves the game while the buyer moves on to the next seller. The multilateral bargaining game thus effectively becomes a sequence of Rubinstein bargaining games with a unique equilibrium. Allowing the buyer to negotiate with the same seller in consecutive rounds bestows her with an advantage, as the option of holding out is taken away from the seller. In contrast in our model, once negotiations fail in a round, the buyer is constrained to negotiate with the other seller in the following round; this offers each of the sellers a provision to hold-out.

Krasteva and Yildirim (2012) study strategic sequencing by a buyer with two sellers in an incomplete information framework, which is different from ours in several dimensions. While the buyer's valuation from both the units is commonly known, her stand-alone valuations are private information. The sellers are heterogeneous with respect to their bargaining strength, which is represented by the probability of making an offer in a one-shot random proposer bargaining game. Further, the buyer bargains with each seller individually and sequentially, and decides whether or not to buy the products after observing their respective posted prices and her own valuations. They show that the buyer cares about the sequencing order only when equilibrium trade is inefficient[3]; the buyer then begins with the weaker seller if the sellers have diverse bargaining strengths, and with the stronger of the two if both the sellers are strong bargainers. For the case where the stand-alone valuations are common knowledge, they find that the buyer is indifferent between the two sequences. Noe and Wang (2000) similarly show that a financially distressed firm is neutral to the debt negotiation sequence if each creditor's claim is higher than the firm's value. Chatterjee and Kim (2005) consider a multilateral bargaining game with an identical sequence of offers and counter-offers as ours. They assume that the sellers have the same valuation (zero) for their objects, and that while the buyer's valuation is private information, it is commonly known that one of the items is worth twice as much to the buyer as the other. They assume that the buyer can choose either to bargain simultaneously or sequentially and find that it is optimal for the buyer to either bargain simultaneously or to bargain first with the less important seller.

2 Model

We use a non-cooperative game theoretic model to solve for a bargaining problem, where a buyer (industrialist) bargains with two sellers (farmers), over an infinite time horizon. Each seller owns an object (a plot of land) and is represented by an index $i \in \{1, 2\}$. The buyer negotiates with one seller at a time, in order to purchase the land from him. These plots are perfectly complementary for the

[3] In their model trade is deemed to be socially efficient if the buyer acquires both goods with probability 1.

buyer, in the sense that the buyer must purchase both the plots of land before she can proceed with the construction of her plant and realize the value of the project.

In our model time is discrete, with the game starting in period zero. The buyer bargains with the sellers in a fixed order, which is given exogenously. The buyer bargains with the first seller i over the price of his land in a round of bargaining. Each round starts with the buyer making an offer to the seller, which the seller either accepts or rejects. If he rejects, the seller makes a counter-offer, which the buyer then accepts or rejects. If either the offer or the counter-offer is accepted, the negotiation comes to an end with the buyer paying the seller the agreed price immediately and the seller leaving the game permanently. The buyer then participates in an infinite horizon alternate-offer bargaining game with the remaining seller j, which is identical to a Rubinstein game[4]. If the counter-offer is also rejected, the game moves to the next round where the buyer negotiates in a similar manner with seller $j \neq i$. Our model thus differs from that of Xiao (2012), who allows the buyer to negotiate with the same seller in consecutive rounds before the first agreement is reached. Hence, each round of negotiation comprises at most two periods. Offers are made and responded to in the first period, while counter-offers are made and either accepted or rejected in the second.

Once the buyer reaches an agreement with both the sellers, the project is completed immediately, and the benefit from completion to the seller is assumed to be M. We assume that the sellers are asymmetric, in the sense that while the valuation of land for seller 1 is V_1, that of seller 2 is V_2, with $V_2 > V_1 = 0$. Let $K = V_2/M$ such that $K \in (0,1)$. All players are assumed to be risk-neutral and have the same discount factor $\delta \in (0,1)$.

There are two possible bargaining orders in this framework. We define $\Gamma(1,2)$ as the infinite period game (or subgame) where the buyer bargains first with seller $i = 1$, and in the event both the offer and counter-offer are rejected, he moves on to bargain with the other seller in the same fashion. Similarly, we define $\Gamma(2,1)$ as the game (or subgame) where the buyer negotiates first with seller $i = 2$, followed by seller 1. Using this notation, we can denote the infinite horizon alternate-offer bargaining game with seller i, after the buyer successfully negotiates with seller j, as $\Gamma(i,i)$ with $i = 1,2$. We denote offers made to seller i as o_i and counter-offers made by seller i as co_i. Both the offer and the counter-offer denotes the price offered to the seller involved in that bargaining round. For example, if the first agreement involves seller i accepting an offer o_i from the buyer, this implies that $p_i = o_i$, such that the net payoff to the seller is $p_i - V_i$. Figure 1 summarizes

[4] Rubinstein game in this paper refers to the bargaining game analyzed in Rubinstein(1982)

$\Gamma(1,2)$ and $\Gamma(2,1)$.

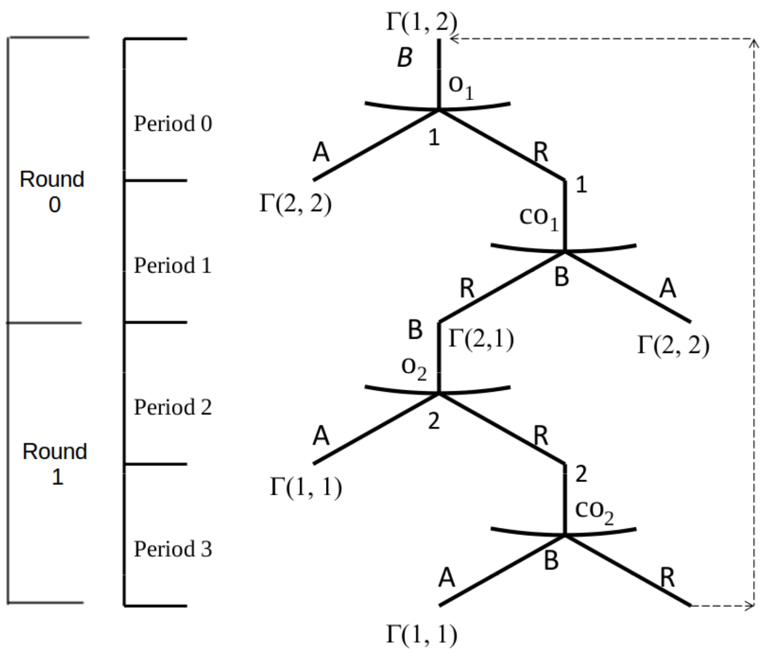

Figure 1: Extensive form representation of bargaining game.

We assume that the model is one of complete and perfect information and use the concept of Subgame Perfect Equilibrium (SPE). Also, we assume that players follow stationary strategies, which means in any two identical subgames players follow identical strategies. Henceforth the term *equilibrium* means a SPE where players follow stationary strategies. We describe an equilibrium by the strategy profile (s_1, s_2, b) where s_i denotes strategy of seller i and b denotes strategy of the buyer. The buyer's strategy specifies both an o_1 and an o_2, and uses cutoff rules to accept or to reject counter-offers. The sellers on the other hand announce their respective counter-offers and use a cutoff rule to reply to offers made by the buyer. These strategies describe offers, counter-offers and rules for accepting or rejecting the same *before the first successful negotiation has taken place*. Since binding cash-offer contracts are used to compensate sellers, the payment made to the first seller is a sunk cost for the buyer.

We represent the equilibrium outcome of the game by $\{y_1, y_2, x, t\}$ where y_i denotes the payoff of seller i, x denotes the payoff of the buyer and t denotes the final period in which the buyer reaches an agreement with both the sellers. We follow a convention similar to the one used by Cai (2000) by evaluating payoffs at the date when all negotiations are completed, while strategies being reported in current value terms. For instance, if the buyer agrees to pay the first seller an amount p_1 in period t_1, and agrees to a price p_2 in period t_2, the equilibrium outcome in that case, is denoted by $\left\{\frac{p_1 - V_1}{\delta^{t_2 - t_1}}, p_2 - V_2, M - p_2 - \frac{p_1}{\delta^{t_2 - t_1}}, t_2\right\}$ where $\sum_i y_i + x = M - V_2$. Similarly if the buyer succeeds in negotiating with seller 2 first by accepting a counter-offer $co_2 = p_2$ in period t_1, and agrees to pay seller 1 p_1 in period $t_2 > t_1$, the equilibrium outcome is denoted by $\left\{p_1 - V_1, \frac{1}{\delta^{t_2 - t_1}}(p_2 - V_2), M - p_1 - \frac{p_2}{\delta^{t_2 - t_1}}, t_2\right\}$ where $\sum_i y_i + x = M - \frac{V_2}{\delta^{t_2 - t_1}}$. If $t \geq 2$, we deem the bargaining outcome as inefficient.

3 Equilibria

We begin our analysis of the game where the seller has successfully negotiated with seller i in period t. The subgame following this transaction is denoted by $\Gamma(j, j)$ where the buyer negotiates with the remaining seller j in an alternate-offer infinite horizon bargaining game. The equilibrium outcome of the subgame is delineated by the following lemma.

Lemma 1 *In the subgame $\Gamma(j, j)$ the buyer reaches an agreement with the remaining seller immediately, with the buyer offering $o_j = p_j$, such that*

$$p_j - V_j = \frac{\delta}{1+\delta}(M - V_j),$$

which the seller accepts. The buyer's payoff in the subgame is therefore $M - p_j = \frac{1}{1+\delta}(M - V_j)$.

The first successful negotiation does not fetch any payoff to the buyer such that the payment made in the first deal is a sunk cost to the buyer. The buyer and the remaining seller therefore split the surplus $M - V_j$ as in the Rubinstein game. This result is similar to ones obtained by both Cai (2000) and Xiao (2012) whereby, the seller and the buyer get equal share of the surplus in the subgame $\Gamma(j, j)$ as δ approaches 1. Since the buyer makes the first payment out of this surplus, the first seller gets a lower share of the surplus than the second. This leads to a "last-mover advantage", and provides an incentive to hold-out, to both the sellers. Using the above lemma, we can previse the equilibrium outcome in the case where seller 1 accepts an offer o_1 from the buyer in period zero. The equilibrium outcome for $\Gamma(1,2)$ in this case will be $\left\{o_1/\delta, \frac{\delta}{1+\delta}(M-V_2), \frac{1}{1+\delta}(M-V_2)-(o_1/\delta), 1\right\}$. Similarly, if the counter-offer co_2 is accepted by the buyer in period 3, the equilibrium outcome will be $\left\{\frac{\delta}{1+\delta}M, \frac{co_2}{\delta}-\frac{V_2}{\delta}, \frac{1}{1+\delta}M-\frac{co_2}{\delta}, 4\right\}$.

3.1 Buyer Bargains with Seller 1 first

In this subsection we solve for the equilibrium of the game $\Gamma(1,2)$, where the buyer bargains first with seller 1 starting in period 0. We construct equilibria where offers and counter-offers get accepted or rejected and solve for conditions under which none of the players could profitably deviate from the corresponding prescribed strategy profile. In each case, the strategy profile (s_1, s_2, b) is said to be *subgame perfect*, if it satisfies the *one-stage deviation property*. For some combinations of parameter values K and δ, we get a unique equilibrium, while for others we get multiple equilibria. To build some intuition behind our results, we first deconstruct a strategy profile which can be used to support an equilibrium where both the counter-offers are rejected, when $M = 1$ and $V_1 = V_2 = 0$. These parameter values correspond to the model used in Cai(2000).

The strategy profile involves the use of symmetric strategies. The sellers reject any counter-offer smaller than $\delta^4/(1+\delta)$ and counter-offer $\delta^3/(1+\delta)$. The buyer on the other hand, offers $\delta^4/(1+\delta)$ and rejects any counter-offer greater than $\delta(1-\delta+\delta^4)/(1+\delta)$. For $\delta_0 \leq \delta < \delta_1$, these strategies constitute a unique equilibrium. The best outcome that a seller can get by rejecting the buyer's offer in period t is $\delta/(1+\delta)$, which is available in period $t+3$, the present value of which is $\delta^4/(1+\delta)$. The seller thus accepts any offer o_i such that $o_i - V_i \geq \delta^4/(1+\delta)$, i.e. $o_i \geq \delta^4/(1+\delta)$ $\forall i$. For the same reason, when the seller makes a counter-offer in period t, he tries to ensure the present value of the payoff $\delta/(1+\delta)$ which is available in period $t+2$, i.e. $\delta^3/(1+\delta)$.

The buyer has a maximum counter-offer that she is willing to accept. To solve for the maximum counter-offer that the buyer is willing to accept in any period t, she uses the equation

$$1 - \frac{\widehat{co}}{\delta} - \frac{\delta}{1+\delta} = \delta\left(1 - \frac{\delta}{1+\delta} - \frac{\delta^3}{1+\delta}\right) \tag{1}$$

The expression on the left denotes the buyer's payoff by accepting the counter-offer (from seller i) in present value terms of period $t+1$. If she rejects the counter-offer, she pays $\delta^4/(1+\delta)$ to the other seller in period $t+1$ and $\delta/(1+\delta)$ to the same seller i in period $t+2$. The payoff to the buyer in present value terms of period $t+1$ is denoted by the expression on the right. From this equation, the buyer gets $\widehat{co} = \delta(1-\delta+\delta^4)/(1+\delta)$ such that for counter-offers greater than $\widehat{co}$, she rejects. For $\delta > \delta_0$, $\delta^3/(1+\delta) > \delta(1-\delta+\delta^4)/(1+\delta)$ such that *counter-offers are rejected* while offers $\delta^4/(1+\delta)$ are accepted. The equilibrium outcome for this case is $\{\delta^3/(1+\delta), \delta/(1+\delta), (1-\delta^3)/(1+\delta), 1\}$. In case $\delta = \delta_0$, $\delta^3/(1+\delta) = \delta(1-\delta+\delta^4)/(1+\delta)$ such that while the equilibrium outcome remains the same, both *offers and counter-offers are accepted*.

If the seller makes a counter-offer greater than $\delta^3/(1+\delta)$ and all other parts of strategies remain the same as those in the equilibrium constructed above, the modified strategy profile is also subgame perfect for $\delta \in [\delta_0, \delta_1)$ and has the same equilibrium outcome as the one described above. However, in this case the counter-offers are always rejected. For the sake of consistency, we use strategies which are similar in spirit to the latter, to describe an equilibrium where counter-offers are rejected. The next lemma rules out indefinite delay as an outcome of the bargaining process.

Lemma 2 *For the two-seller game, perpetual disagreement cannot be an equilibrium outcome.*

Proof. Assume that there exists an equilibrium with perpetual disagreement, such that all players get zero payoff. If the buyer deviates and offers $\varepsilon > 0$ to seller 1 in period zero, the seller should accept. If the seller refuses, the sellers and the buyer proceed to make the same offers and counter-offers in the following rounds, such that he gets zero. The buyer would then offer $\frac{\delta(M-V_2)}{1+\delta}$ to seller 2 in period 1, and get the payoff $\frac{M-V_2}{1+\delta} - \frac{\varepsilon}{\delta} > 0$, a contradiction. ■

We now proceed to describe the equilibrium for different values of the parameters K and δ. There are seven possible equilibria, labeled E1 to E7, which are enumerated in a way such that an increasing number of offers and counter-offers are rejected. Using one step deviation rule it can be checked that each of following strategy profiles constitute equilibrium.

3.1.1 Equilibrium 1: All offers and counter-offers are accepted.

The players' strategies which support this equilibrium are as follows.

$$b^1 = \begin{cases} o_1 = \frac{\delta^2(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] \\ \text{reject } co_1 > \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] = \widehat{co}_1 \\ o_2 = \frac{\delta^2}{(1+\delta)^2}M + \frac{1+\delta+\delta^3}{(1+\delta)^2(1+\delta^2)}V_2 \\ \text{reject } co_2 > \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M + \delta(1+\delta+\delta^2)V_2] = \widehat{co}_2 \end{cases},$$

$$s_1^1 = \begin{cases} co_1 = \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] \\ \text{reject } o_1 < \frac{\delta^2(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] = \widehat{o}_1 \end{cases},$$

$$s_2^1 = \begin{cases} co_2 = \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M + \delta(1+\delta+\delta^2)V_2] \\ \text{reject } o_2 < \frac{\delta^2}{(1+\delta)^2}M + \frac{1+\delta+\delta^3}{(1+\delta)^2(1+\delta^2)}V_2 = \widehat{o}_2. \end{cases}$$

We now provide a brief description of how such strategies were constructed. To do so, we assume that a_1 and a_2 represents the offers made by the buyer to sellers 1 and 2 respectively. The maximum co_1 and co_2 that is acceptable to the buyer is denoted by $\widehat{Y}_1$ and $\widehat{Y}_2$ respectively. Given that all offers and counter-offers are accepted, these four unknowns can be solved by the following four equations:

$$a_1 - V_1 = \delta(\widehat{Y}_1 - V_1) \tag{2}$$

$$\frac{M - V_2}{1+\delta} - \frac{\widehat{Y}_1}{\delta} = \frac{\delta}{1+\delta}(M - V_1) - a_2 \tag{3}$$

$$a_2 - V_2 = \delta(\widehat{Y}_2 - V_2) \tag{4}$$

$$\frac{M - V_1}{1+\delta} - \frac{\widehat{Y}_2}{\delta} = \frac{\delta}{1+\delta}(M - V_2) - a_1 \tag{5}$$

The buyer offers seller 1 a_1 such that the seller is indifferent between accepting a_1 or rejecting it, in which case he gets $\widehat{Y}_1$ in the next period. The second equation solves for the maximum co_1 that is acceptable to the buyer, by making the buyer indifferent between accepting or rejecting $\widehat{Y}_1$ in period t. By rejecting $\widehat{Y}_1$, the buyer would have to make an offer a_2 to seller 2 in period $t+1$, which is accepted. The project would then be completed in period $t+2$, with the buyer getting the discounted payoff $\frac{\delta}{1+\delta}(M - V_1)$ in period $t+1$. Similar equations are derived for the negotiation between the buyer and the second seller. If $a_1^*, a_2^*, \widehat{Y}_1^*$ and $\widehat{Y}_2^*$ represents the solution to the above system of equations, we get $\widehat{o}_i = a_i^*$ and $\widehat{co}_i = \widehat{Y}_i^*$ for $i = 1, 2$. We now state our first proposition, which states the necessary condition for the first equilibrium.

Proposition 1 *If* $K \leq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4}$ *then the strategies* (s_1^1, s_2^1, b^1) *constitute an SPE of the game* $\Gamma(1,2)$*. The equilibrium outcome is* $\{X, \frac{\delta}{1+\delta}(M - V_2), \frac{1}{1+\delta}(M - V_2) - X, 1\}$ *where,* $X = \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2]$.

Proof. See Appendix A. ■

In the above proposition, to ensure that $\frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4} \geq 0$ we must have $0 < \delta \leq 0.755$. For these parameter values, neither farmer is sufficiently patient to hold up the negotiation process and is amenable to (i) accepting the relevant offer prices of the buyer and (ii) making reasonable counter-offers which are accepted by the buyer. However, the minimum offer that seller 2 is willing to accept, as well as the highest counter-offer that the buyer is willing to accept from seller 2, are higher than that of seller 1 ($i.e.$ $\widehat{o}_1 < \widehat{o}_2$ and $\widehat{co}_1 < \widehat{co}_2$). Since offers made to the sellers are equal to the respective threshold levels, the second seller therefore demands, and gets a higher compensation for foregoing a land of higher valuation.

3.1.2 Equilibrium 2: Only co_2 is rejected.

We now look for an equilibrium or equilibria where at most one offer or counter-offer is rejected. We find that an equilibrium comprising the counter-offer made by seller 2 being rejected is the only such equilibrium and that the strategies which constitute it are as follows.

$$b^2 = \begin{cases} o_1 = \frac{\delta^2}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \\ \text{reject } co_1 > \frac{\delta}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \\ o_2 = \frac{\delta^4}{1+\delta}(M - V_2) + V_2 \\ \text{reject } co_2 > \frac{\delta}{(1+\delta)}[(1-\delta+\delta^2-\delta^3+\delta^6)M + (\delta+\delta^3-\delta^6)V_2] \end{cases}$$

$$s_1^2 = \begin{cases} co_1 = \frac{\delta}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \\ \text{reject } o_1 < \frac{\delta^2}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \end{cases}$$

$$s_2^2 = \begin{cases} co_2 > \frac{\delta^3}{1+\delta}(M - V_2) + V_2 \\ \text{reject } o_2 < \frac{\delta^4}{1+\delta}(M - V_2) + V_2 \end{cases}$$

The offers and maximum acceptable counter-offers in these strategies were solved through a system of equations which are similar to those used in the first equilibrium. Equations (2), (3) and (5) remain the same, while equation (4) is replaced by

$$a_2 - V_2 = \frac{\delta^4}{1+\delta}(M - V_2).$$

When the buyer makes an offer to the second seller in any period t, she knows that the best payoff that he can get by rejecting the offer is $\frac{\delta}{1+\delta}(M - V_2)$ in period $t+3$. Thus, the seller will accept any offer $a_2 \geq \frac{\delta^4}{1+\delta}(M - V_2) + V_2$. Similarly, when seller 2 makes a counter-offer, the best possible payoff that he can get in case that counter-offer is rejected is $\frac{\delta^3}{1+\delta}(M - V_2)$. With $co_2 > \frac{\delta^3}{1+\delta}(M - V_2) + V_2 \geq \widehat{Y}_2$,[5] the counter-offer is rejected.

Proposition 2 *The strategies* (s_1^2, s_2^2, b^2) *constitute an SPE of the game* $\Gamma(1,2)$ *if the following conditions hold:*

$$\begin{aligned} K &\geq \frac{\delta+\delta^2-\delta^4-1}{\delta-\delta^4} \\ K &\geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4} \\ \textit{and } K &\leq \frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5}. \end{aligned}$$

The equilibrium outcome in this case is given by $\{X, \frac{\delta}{1+\delta}(M - V_2), \frac{1}{1+\delta}(M - V_2) - X, 1\}$ *where,* $X = \frac{\delta}{(1+\delta)}[(1 - \delta + \delta^4)M + (\delta - \delta^4)V_2]$.

Proof. Similar to that of Proposition 1. ■

To ensure that there exists values of K which satisfy all the conditions mentioned above, we must have $\delta \in (0, 0.838]$. As is evident from the necessary conditions under which this equilibrium can be sustained, for the same level of δ, K needs to be higher than that in $E1$. The only necessary condition in the first equilibrium is derived from the inequality

$$\widehat{co}_2 - V_2 \geq \delta^2 \left(\frac{\delta}{1+\delta}(M - V_2) \right)$$

[5] $\frac{\delta^3}{1+\delta}(M - V_2) + V_2 \geq \widehat{Y}_2$ iff $K \geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4}$.

which ensures that seller 2 is better off counter-offering $\widehat{co}_2$ than by offering a higher amount. At $K = \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4}$, the net payoff that seller 2 gets by counter-offering $\widehat{co}_2$ in $E2$ is equal to $\frac{\delta^3}{1+\delta}(M-V_2)$. However, since

$$\frac{\partial}{\partial K}\left(\frac{\delta^3}{1+\delta}(M-V_2)-(\widehat{co}_2-V_2)\right) \geq 0 \; \forall \delta$$

such that while both $\frac{\delta^3}{1+\delta}(M-V_2)+V_2$ and $\widehat{co}_2$ are increasing in K, the former increases faster than the latter as K increases. This implies that in $E2$

$$\frac{\delta^3}{1+\delta}(M-V_2) > \widehat{co}_2 - V_2,$$

which implies that the second seller can do better by ensuring that his counter-offer gets rejected[6]. He does so by asking for $co_2 > \frac{\delta^3}{1+\delta}(M-V_2)+V_2$. As K increases, the maximum compensation that the buyer is willing to offer to seller 2 is therefore unable to keep up with the payoff available to seller 2 in the event negotiations fail.

The first necessary condition ensures that seller 1 is better off counteroffering $\widehat{co}_1$, than by offering a higher amount, while the last condition ensures that the buyer cannot do better by offering seller 2 an amount lower than $\widehat{o}_2$. As in the first equilibrium, the seller with the higher valuation demands a higher compensation for his land than the other seller (i.e. $\widehat{o}_2 > \widehat{o}_1$) when an offer is made to him. The buyer is also willing to provide a higher price to seller 2 than the first seller through a higher maximum acceptable counter-offer (i.e. $\widehat{co}_1 < \widehat{co}_2$). For the limiting case where the players are extremely impatient, the strategies described above do not constitute an SPE. This can be corroborated by the necessary condition which ensures that the buyer offers $o_2 = \widehat{o}_2$, i.e.

$$\frac{M-V_1}{1+\delta} - \frac{\widehat{o}_2}{\delta} \geq \delta^2\left(\frac{M-V_2}{1+\delta}\right) - \delta\widehat{o}_1.$$

In this case, $\widehat{o}_1 = \delta\widehat{co}_1$ and $\widehat{o}_2 = \frac{\delta^4}{1+\delta}(M-V_2)+V_2$. Substituting, we get

$$\frac{M}{1+\delta} + \delta^2\widehat{co}_1 \geq \frac{\delta^3}{1+\delta}(M-V_2) + \frac{V_2}{\delta} + \frac{\delta^2}{1+\delta}(M-V_2),$$

such that the above condition fails to hold as $\delta \to 0$, given $V_2 > 0$.

[6]For $K \geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4}$, the $\widehat{co}_2$ in E2 is larger than or equal to that in E1.

3.1.3 Equilibrium 3: Both o_2 and co_2 are rejected.

Following the intuition developed from the analysis of the previous equilibrium, as the parameter K further increases, we would expect both o_2 and co_2 to get rejected. The equilibrium is sustained by the strategies (s_1^3, s_2^3, b^3) where

$$b^3 = \begin{cases} o_1 = \frac{\delta^2(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2) \\ \text{reject } co_1 > \frac{\delta(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2) \\ o_2 < \frac{\delta^4}{1+\delta}(M - V_2) + V_2 \\ \text{reject } co_2 > \frac{\delta}{(1+\delta)(1-\delta^4)}[(1 - \delta + \delta^2 - \delta^4)M + (\delta - \delta^2)V_2] \end{cases}$$

$$s_1^3 = \begin{cases} co_1 = \frac{\delta(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2) \\ \text{reject } o_1 < \frac{\delta^2(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2) \end{cases}$$

$$\text{and } s_2^3 = \begin{cases} co_2 > \frac{\delta^3}{1+\delta}(M - V_2) + V_2 \\ \text{reject } o_2 < \frac{\delta^4}{1+\delta}(M - V_2) + V_2 \end{cases} .$$

The o_1 and co_1 are solved from the equations

$$a_1 - V_1 = \delta(\widehat{Y}_1 - V_1)$$
$$\frac{M-V_2}{1+\delta} - \frac{\widehat{Y}_1}{\delta} = \frac{\delta^3}{1+\delta}(M - V_2) - \delta^2 a_1$$

such that if the buyer rejects $\widehat{Y}_1$ in period t, she gets $\frac{M-V_2}{1+\delta}$ in period $t+4$, after paying a_1 to seller 1 in $t+3$. The second equation therefore, solves for the maximum acceptable counter-offer for the buyer from seller 1. A similar equation is used to solve for $\widehat{Y}_2$. The only necessary condition for this equilibrium is given by the following proposition.

Proposition 3 *If* $K \geq \frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5}$ *then the strategies* (s_1^3, s_2^3, b^3) *constitute an SPE of the game* $\Gamma(1,2)$. *The equilibrium outcome is* $\{X, \frac{\delta}{1+\delta}(M - V_2), \frac{1}{1+\delta}(M - V_2) - X, 1\}$
where $X = \frac{\delta(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2)$.

Proof. Similar to that of Proposition 1. ■

While making an offer to seller 2, the buyer uses the same intuition used in E2 to figure out that seller 2 accepts offers $a_2 \geq \frac{\delta^4}{1+\delta}(M - V_2) + V_2$. However, if the buyer offers a smaller amount, it gets rejected. In that case, the buyer gets $\frac{M-V_2}{1+\delta}$ in period $t+3$ after paying o_1 to seller 1 in period $t+2$. For the buyer to offer a smaller amount, the necessary condition is

$$\frac{M-V_1}{1+\delta} - \frac{1}{\delta}\left(\frac{\delta^4}{1+\delta}(M-V_2)+V_2\right) \leq \frac{\delta^2}{1+\delta}(M-V_2) - \delta o_1$$
$$\iff K \geq \frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5}$$

At $K = \frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5}$, the net payoff to the buyer by getting o_2 rejected is equal to that by offering $o_2 = \widehat{o}_2 = \frac{\delta^4}{1+\delta}(M-V_2)+V_2$. However, since

$$\frac{\partial}{\partial K}\left[\frac{\delta^2}{1+\delta}(M-V_2) - \delta o_1 - \left(\frac{M}{1+\delta} - \frac{\delta^3}{1+\delta}(M-V_2) - \frac{V_2}{\delta}\right)\right] > 0 \; \forall \delta,$$

the difference of the net payoff that the buyer gets by getting o_2 rejected and that by getting it accepted, increases with K. This ensures that the buyer offers $o_2 < \frac{\delta^4}{1+\delta}(M-V_2)+V_2$ in the relevant parameter space. Further since $\frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5} \geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4} \; \forall \delta$, the condition $K \geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4}$ is satisfied whenever $K \geq \frac{\delta+\delta^8-\delta^3-\delta^5}{1+\delta+\delta^8-\delta^3-\delta^4-\delta^5}$, where the former ensures that seller 2 counter-offers $co_2 > \frac{\delta^3}{1+\delta}(M-V_2)+V_2 \geq \widehat{Y}_2$ such that it is rejected.

The strategies followed by the buyer and seller 2 ensure that seller 2 is never the first seller to reach an agreement, which implies that the first seller faces the same situation in periods 4 and 5, as he did in periods 0 and 1. Seller 1 therefore, cannot become the last seller to sign a contract, and he thus accepts $o_1 = \frac{\delta^2(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M-V_2)$.

3.1.4 Equilibrium 4: Both co_1 and co_2 are rejected.

We show that the following strategies constitute an SPE for a particular range of parameter values K and δ.

$$b^4 = \begin{cases} o_1 = \frac{\delta^4}{1+\delta}M \\ \text{reject } co_1 > \frac{\delta}{1+\delta}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \\ o_2 = \frac{\delta^4}{1+\delta}(M-V_2)+V_2 \\ \text{reject } co_2 > \frac{\delta}{1+\delta}[(1-\delta+\delta^4)M + \delta V_2] \end{cases},$$

$$s_1^4 = \begin{cases} co_1 > \frac{\delta^3}{1+\delta}M \\ \text{reject } o_1 < \frac{\delta^4}{1+\delta}M \end{cases},$$

$$\text{and } s_2^4 = \begin{cases} co_2 > \frac{\delta^3}{1+\delta}(M - V_2) + V_2 \\ \text{reject } o_2 < \frac{\delta^4}{1+\delta}(M - V_2) + V_2 \end{cases} .$$

We use the equations

$$a_i - V_i = \frac{\delta^4}{1+\delta}(M - V_i)$$
$$\text{and } \frac{M - V_j}{1+\delta} - \frac{\widehat{Y}_i}{\delta} = \frac{\delta(M - V_i)}{1+\delta} - a_j$$

to solve for $\widehat{Y}_i^* = \widehat{co}_i$ and $a_i^* = \widehat{o}_i$ for $i = 1, 2$ with $i \neq j$. From the buyer's strategy, it is easy to verify that $\widehat{co}_2 > \widehat{co}_1$ and that while $o_i = \frac{\delta^4}{1+\delta}(M - V_i) + V_i$ for $i = 1, 2$, $\widehat{o}_2 > \widehat{o}_1 \because V_2 > 0$.

Proposition 4 *If* $K \leq \frac{\delta+\delta^2-\delta^4-1}{\delta-\delta^4}$ *and* $K \leq \frac{\delta+\delta^6-\delta^3-\delta^4}{1+\delta-\delta^3-\delta^4}$ *then the strategies* (s_1^4, s_2^4, b^4) *constitute an SPE of the game* $\Gamma(1, 2)$ *and the equilibrium outcome is* $\{X, \frac{\delta}{1+\delta}(M - V_2), \frac{1}{1+\delta}(M - V_2) - X, 1\}$ *where,* $X = \frac{\delta^3}{1+\delta}M$.

Proof. Similar to that of Proposition 1. ■

To ensure that there exists values of K which satisfy both the conditions in proposition 4, we must have $\delta \in [0.755, 1)$. The first necessary condition ensures that the first seller is better off counter-offering an amount higher than the maximum counter-offer than the buyer is willing to accept. At $K = \frac{\delta+\delta^2-\delta^4-1}{\delta-\delta^4}$, the net payoff seller 1 gets by counteroffering $\widehat{co}_1$ is equal to $\frac{\delta^3}{1+\delta}(M - V_1)$, which is the payoff that he gets in case the counter-offer is rejected. However, since $\frac{\partial}{\partial\delta}\left(\frac{\delta^3}{1+\delta}M - \widehat{co}_1\right) > 0$, seller 1 prefers to counter-offer $co_1 > \frac{\delta^3}{1+\delta}M \geq \widehat{Y}_1$ as he becomes more patient, such that it gets rejected. The second necessary condition ensures that the buyer cannot be better off by offering an amount smaller than o_2 to seller 2. Finally, the necessary condition which ensures that seller 2 is better off counter-offering $co_2 > \frac{\delta^3}{1+\delta}(M - V_2) + V_2$, is automatically satisfied when the two necessary conditions stated in the above proposition hold.

A necessary condition for the buyer to offer $o_i = \widehat{o}_i$ for $i = 1, 2$ is

$$\frac{M - V_j}{1+\delta} - \frac{\widehat{o}_i}{\delta} \geq \frac{\delta^2}{1+\delta}(M - V_j) - \delta\widehat{o}_j$$
$$\implies (M - V_j)\delta(1 - \delta) + \delta^2\widehat{o}_j \geq \widehat{o}_i.$$

Since the counter-offer is rejected in the following period, $\widehat{o}_i = \frac{\delta^4}{1+\delta}(M - V_i) + V_i$ for $i = 1, 2$. Substituting, we get the necessary condition

$$(M - V_j)\delta(1-\delta) + \delta^2 \left(\tfrac{\delta^4}{1+\delta}(M - V_j) + V_j \right) \geq \tfrac{\delta^4}{1+\delta}(M - V_i) + V_i.$$

Assuming that $V_j = V_1 = 0$ and $V_i = V_2 > 0$ and $\delta \to 1$, the above condition does not hold. Therefore, we can conclude that the above strategies cannot constitute an equilibrium for $V_2 > 0$, $\delta \to 1$.

3.1.5 Equilibrium 5: Both o_1 and co_1 are rejected.

The first of the equilibria which is inefficient is the mirror equilibrium of E3, where the roles of sellers 1 and 2 are reversed, such that in this case it is the buyer and the first seller who adopt a "hold-out" strategies. The following strategies support an SPE for the relevant range of parameter values K and δ.

$$b^5 = \begin{cases} o_1 < \frac{\delta^4}{1+\delta} M \\ \text{reject } co_1 > \frac{\delta}{1+\delta}\left(\frac{1-\delta+\delta^2-\delta^4}{1-\delta^4} M - \frac{\delta^2}{1+\delta^2} V_2\right) \\ o_2 = \frac{\delta^2(1-\delta^3)}{(1-\delta^4)(1+\delta)} M + \frac{1-\delta}{1-\delta^4} V_2 \\ \text{reject } co_2 > \frac{\delta}{(1-\delta^4)}\left(\frac{1-\delta^3}{1+\delta} M + (\delta^2 - \delta^3) V_2\right) \end{cases},$$

$$s_1^5 = \begin{cases} co_1 > \frac{\delta^3}{1+\delta} M \\ \text{reject } o_1 < \frac{\delta^4}{1+\delta} M \end{cases},$$

$$s_2^5 = \begin{cases} co_2 = \frac{\delta}{(1-\delta^4)}[\frac{1-\delta^3}{1+\delta}(M) + (\delta^2 - \delta^3) V_2] \\ \text{reject } o_2 < \frac{\delta^2(1-\delta^3)}{(1-\delta^4)(1+\delta)} M + \frac{1-\delta}{1-\delta^4} V_2. \end{cases}$$

As must be evident, the equations used to solve for o_2, $\widehat{co}_1$ and $\widehat{co}_2$ are similar to the ones used in E3. This equilibrium is similar to the one with equilibrium outcome $(s', 3)$ in Theorem $1(b)$ of Cai (2000). However, unlike Cai, the range of parameter values over which E3 and E5 coexist, *are not identical*.

Proposition 5 *If $K \geq \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$ and $K \leq \frac{\delta}{1+\delta}$, then the strategies (s_1^5, s_2^5, b^5) constitute an SPE of the game $\Gamma(1,2)$. The equilibrium outcome is represented by $\{\frac{\delta}{1+\delta}M,\ X - \frac{V_2}{\delta},\ \frac{1}{1+\delta}M - X, 3\}$ where, $X = \frac{\delta(1-\delta^3)}{(1-\delta^4)(1+\delta)} M + \frac{1-\delta}{\delta-\delta^5} V_2$.*

Proof. Similar to that of Proposition 1. ■

In the proposition above, for such K to exist, we must have $\delta \in [0.819, 1)$. The first necessary condition is derived from the inequality

$$\frac{\delta^2}{1+\delta}(M - V_1) - \delta a_2 \geq \frac{M-V_2}{1+\delta} - \frac{o_1}{\delta}$$

which assures the buyer of a higher payoff by offering seller 1 an amount lower than $\frac{\delta^4}{1+\delta}M$, than by offering $o_1 = \frac{\delta^4}{1+\delta}M$. The second necessary condition sees to it that the buyer cannot do better by offering an amount smaller than o_2 to seller 2. For the first seller to counter-offer $co_1 > \frac{\delta^3}{1+\delta}M \geq \widehat{co}_1$ it must be the case that there's no profitable deviation. The condition which guarantees this, is $K \geq \frac{1+\delta^6-\delta-\delta^4}{\delta^2-\delta^4}$, which is automatically satisfied whenever the first necessary condition holds.

While $\frac{\partial y_2}{\partial V_2} < 0$, it can be verified that if $K = \frac{\delta}{1+\delta}$, $\widehat{co}_2 = V_2$ and therefore $\widehat{o}_2 - V_2 = \delta(\widehat{co}_2 - V_2) = 0$. It is evident that at $K = \frac{\delta}{1+\delta}$, the buyer bargains with the second seller aggressively enough to drive the maximum counter-offer that he is willing to accept to its lower bound, V_2. This implies that the payoff of the second seller $y_2 = 0$. It is also possible to check that while the necessary condition for the buyer to offer $o_2 = \widehat{o}_2$ is

$$\frac{M-V_1}{1+\delta} - \frac{\widehat{o}_2}{\delta} \geq \frac{\delta}{1+\delta}(M - V_1) - \widehat{co}_2,$$

the condition is satisfied with equality if $K = \frac{\delta}{1+\delta}$. The buyer is therefore indifferent between getting her offer accepted or rejected. Similarly, it can be shown that the seller is indifferent between getting his counter-offer accepted or rejected at the same value of K. This leads us to prognosticate that there will be equilibria in which both o_1 and co_1 will be rejected, and either co_2 or o_2 will not be accepted (equilibrium 6 and 7 respectively).

For the equilibria E1-E5, the sellers adopt one of two types of strategies:

(i) reject any offer $o_i < \frac{\delta^4}{1+\delta}(M - V_i) + V_i$ and counter-offer $co_i > \frac{\delta^3}{1+\delta}(M - V_i) + V_i$, and (ii) sellers choose to counter-offer $co_i = \widehat{co}_i$ and reject any $o_i < \delta(\widehat{co}_i - V_i) + V_i$.

While making the counter-offer, each seller compares the payoff from making a counter-offer $co_i = \widehat{co}_i$ with the payoff that he can get in case the counter-offer is rejected. The latter constitutes the holding-out payoff for the seller and is given by $\frac{\delta^3}{1+\delta}(M - V_i)$, except for sellers 1 and 2 in equilibria E3 and E5 respectively[7]. In case $\frac{\delta^3}{1+\delta}(M - V_i) > \widehat{co}_i - V_i$, the seller chooses to counter-offer $co_i > \frac{\delta^3}{1+\delta}(M - V_i) + V_i$, which ensures that the counter-offer is rejected. It can be easily verified that

$$\frac{\delta^3}{1+\delta}(M - V_i) + V_i > \widehat{co}_i$$
$$\Leftrightarrow \frac{\delta^4}{1+\delta}(M - V_i) + V_i > \delta(\widehat{co}_i - V_i) + V_i$$

which implies that if the seller prefers to have the counter-offer rejected (accepted), the minimum offer that he is willing to accept is higher in the first (second) type of strategy than the second (first). For the equilibria in which a counter-offer gets rejected, we solve for the condition which ensures that $\frac{\delta^3}{1+\delta}(M - V_i) + V_i > \widehat{co}_i$. This ensures that the counter-offer, as well as the minimum acceptable offer for the first type of strategy are higher than the second.

The buyer's strategy on the other hand, comprises an offer to and a maximum acceptable counter-offer for each seller. While making offers, we could assume that the buyer chooses one of two actions: she either offers $o_i = \frac{\delta^4}{1+\delta}(M - V_i) + V_i$ or $o_i = \delta(\widehat{co}_i - V_i) + V_i$. For the equilibria E1-E5, we find that the buyer offers $o_i = \delta(\widehat{co}_i - V_i) + V_i$ whenever she is bargaining with a seller with $\widehat{o}_i = \delta(\widehat{co}_i - V_i) + V_i$. For this to happen, it must be the case that $\frac{\delta^3}{1+\delta}(M - V_i) < \widehat{co}_i - V_i$. The buyer never offers $o_i = \frac{\delta^4}{1+\delta}(M - V_i) + V_i < \widehat{o}_i$, such that in the equilibria E1-E5, it is never the case that an offer is rejected in period t, and a counter-offer is accepted in the following period. However, while bargaining with a seller who sets $\widehat{o}_i = \frac{\delta^4}{1+\delta}(M - V_i) + V_i$, the buyer either offers $\widehat{o}_i$ or a smaller amount. For sellers to set such an $\widehat{o}_i$, it must be the case that $\frac{\delta^3}{1+\delta}(M - V_i) + V_i > \widehat{co}_i$. In such an eventuality, if the buyer offers $o_i = \delta(\widehat{co}_i - V_i) + V_i$ it will get rejected, as $\delta(\widehat{co}_i - V_i) + V_i < \widehat{o}_i$. While bargaining with sellers 1 and 2 in the equilibria E3 and E5 respectively, the maximum counter-offer that the buyer is willing to accept is given by $\widehat{co}_i$, where $\widehat{co}_i$ solves

$$\frac{M-V_i}{1+\delta} - \frac{\widehat{co}_i}{\delta} = \frac{\delta^3}{1+\delta}(M - V_j) - \delta^2\widehat{o}_i,\ i = 1, 2. \tag{6}$$

In these cases, if the counter-offer is rejected, the buyer returns to the same seller and makes a payment $\widehat{o}_i = \delta(\widehat{co}_i - V_i) + V_i$ before moving on to the other seller.

[7]The payoff from holding out in this case is given by $\delta^3(o_i - V_i)$. However, $\because\ o_i = \widehat{o}_i = \delta(\widehat{co}_i - V_i) + V_i$, the payoff from holding out becomes $\delta^4(\widehat{co}_i - V_i)$. The sellers in these cases prefer to counter-offer $\widehat{co}_i$ as $\widehat{co}_i - V_i \geq \delta^4(\widehat{co}_i - V_i)$.

Substituting for $\widehat{o}_i$ in the equation (6) we get,

$$\widehat{co}_i = \frac{\delta}{1-\delta^4}\left(\frac{1-\delta^3}{1+\delta}(M - V_j) + \delta^2(1-\delta)V_i\right)$$

In all the other cases, $\widehat{co}_i$ solves

$$\frac{M-V_j}{1+\delta} - \frac{\widehat{co}_i}{\delta} = \frac{\delta}{1+\delta}(M - V_i) - \widehat{o}_j \quad (7)$$

$$\implies \frac{\widehat{co}_i}{\delta} = \frac{M-V_j}{1+\delta} - \frac{\delta}{1+\delta}(M - V_i) + \widehat{o}_j \quad (8)$$

where $\widehat{o}_j = \delta(\widehat{co}_j - V_j) + V_j$ or $\widehat{o}_j = \frac{\delta^4}{1+\delta}(M - V_j) + V_j$, depending on whether the other player gets his counter-offer accepted or rejected. If $\widehat{o}_j = \frac{\delta^4}{1+\delta}(M - V_j) + V_j$, using (7) we get

$$\widehat{co}_i = \frac{\delta}{1+\delta}\left[(1-\delta+\delta^4)M + V_j(\delta-\delta^4) + \delta V_i\right]. \quad (9)$$

Similarly, if $\widehat{o}_j = \delta(\widehat{co}_j - V_j) + V_j$, the corresponding

$$\widehat{co}_i = \delta\left[\frac{M-V_j}{1+\delta} - \frac{\delta}{1+\delta}(M - V_i) + \delta(\widehat{co}_j - V_j) + V_j\right]. \quad (10)$$

While solving for $\widehat{co}_i$, the buyer calculates the payoff that she will get if the counter-offer is rejected. If the buyer successfully negotiates with the other seller in period $t+1$, the $\widehat{co}_i$ is higher in the case where the counter-offer from seller j in period $t+2$ is rejected, than where it is accepted.

3.1.6 Equilibrium 6: Only o_2 is accepted.

The equilibrium in which only o_2 is accepted is supported by the following strategies:

$$b^6 = \begin{cases} o_1 < \frac{\delta^4}{1+\delta}M \\ \text{reject } co_1 > \frac{\delta}{1+\delta}[(1-\delta)M + \delta V_2] \\ o_2 = V_2 \\ \text{reject } co_2 > \frac{\delta(1-\delta^3)}{(1+\delta)}M + \delta^3 V_2 \end{cases}$$

$$s_1^6 = co_1 > \frac{\delta^3}{1+\delta}M \text{ and reject } o_1 < \frac{\delta^4}{1+\delta}M$$

$$s_2^6 = co_2 > V_2 \text{ and reject } o_2 < V_2.$$

In this case, seller 2 accepts any offer $o_2 \geq V_2$, and the buyer completes the first negotiation by offering the minimum amount possible. The intuition follows from the equation

$$o_2 - V_2 \geq \delta^4 (o_2 - V_2) \geq 0,$$

which implies that if the second seller rejects the offer a_2 in period t, the same offer is made to him in period $t + 4$. The buyer then immediately proceeds to negotiate with the first seller, from which she gets $\frac{1}{1+\delta}(M - V_1)$. It must therefore be the case that $\frac{1}{1+\delta}(M - V_1) - \frac{V_2}{\delta} \geq 0 \iff K \leq \frac{\delta}{1+\delta}$. For the second seller to counter-offer an amount higher than $\widehat{co}_2$, it must be the case that

$$\widehat{co}_2 - V_2 \leq \delta^3 (o_2 - V_2) = 0,$$

which holds iff $K \geq \frac{\delta}{1+\delta}$. These two conditions then imply that $K = \frac{\delta}{1+\delta}$ is a necessary condition for the equilibrium.

Proposition 6 *If* $K = \frac{\delta}{1+\delta}$ *and* $\delta^3(1 + \delta) \geq 1$, *then the strategies* (s_1^6, s_2^6, b^6) *constitute an SPE of* $\Gamma(1,2)$ *with the equilibrium outcome* $\{\frac{\delta}{1+\delta}M, 0, 0, 3\}$.

Proof. Similar to that of Proposition 1. ■

Since $K = \frac{\delta}{1+\delta}$, the total surplus from trade becomes $M - \frac{V_2}{\delta} = \frac{\delta}{1+\delta}M$. The buyer offers seller 2 $o_2 = V_2$ in period 2, the current value of which in period 3 is $\frac{V_2}{\delta} = \frac{1}{1+\delta}M$, such that the entire payoff from the Rubinstein bargaining game is offset by the current value of the payment made in the previous round. The equilibrium outcome therefore comprises both the buyer and seller 2 getting zero payoff, and with seller 1 getting the entire surplus. This equilibrium exists whenever $\delta \in [0.819, 1)$.

3.1.7 Equilibrium 7: Only co_2 is accepted.

In the equilibrium where only co_2 is accepted, the intuition behind the results is similar to that of the previous equilibrium. We show that the following strategies support an SPE:

$$b^7 = \begin{cases} o_1 < \frac{\delta^5}{1+\delta}M \\ \text{reject } co_1 > \frac{\delta}{(1+\delta)}[M - V_2] \\ o_2 < \frac{\delta^2}{1+\delta}M + (1-\delta)V_2 \\ \text{reject } co_2 > \frac{\delta}{1+\delta}M. \end{cases},$$

$$s_1^7 = co_1 > \frac{\delta^4}{1+\delta}M \text{ and reject } o_1 < \frac{\delta^5}{1+\delta}M,$$

$$\text{and } s_2^7 = \begin{cases} co_2 = \frac{\delta}{1+\delta}M \\ \text{reject } o_2 < \frac{\delta^2}{1+\delta}M + (1-\delta)V_2 \end{cases}.$$

With all other offers and counter-offers being rejected, the maximum counter-offer that the buyer can accept from seller 2 is given by the equation

$$\frac{M-V_1}{1+\delta} - \frac{\widehat{co}_2}{\delta} = 0 \implies \widehat{co}_2 = \frac{\delta}{1+\delta}M.$$

In this case, the necessary conditions which ensure that o_2 and co_2 are rejected and accepted respectively, entail that $K = \frac{\delta}{1+\delta}$.

Proposition 7 *If* $K = \frac{\delta}{1+\delta}$ *and* $\delta^4(1+\delta) \geq 1$, *then the strategies* (s_1^7, s_2^7, b^7) *constitute an SPE of* $\Gamma(1,2)$ *with the corresponding equilibrium outcome* $\{\frac{\delta}{1+\delta}M, 0, 0, 4\}$.

Proof. Similar to that of Proposition 1. ■

With $K = \frac{\delta}{1+\delta}$, it can be easily verified that the buyer's payoff $\frac{1}{1+\delta}M - \frac{\widehat{co}_2}{\delta} = 0$ and that the seller 2 gets $\widehat{co}_2 - V_2 = 0$. As in E6, the first seller gets the entire surplus $\frac{\delta}{1+\delta}M$. Our final proposition rules out any other equilibria. If we define $\overline{\delta}$ as the solution to $\delta^4(1+\delta) = 1$ we get $\overline{\delta} = 0.857$.

3.1.8 Discussion

In the first part of our analysis, we chose to solve for the game where the buyer bargains with seller 1 first. The parameter values which support the different equilibria are shown in figure 2.

Since Cai's (2000) model is a special case of our model with $V_1 = V_2 = 0$, the results from his model coincide with ours along the horizontal axis. Thus, if players are impatient, the unique equilibrium is E1 for $\delta < \delta_0$ where δ_0 solves $\delta^2(1+\delta) = 1$, i.e. $\delta_0 = 0.755$. As δ increases, such that $\delta_0 \leq \delta < \delta_1$[8], the unique equilibrium is E4. Finally for $\delta \geq \delta_1$, we have a region with multiple equilibria where E3, E4 and E5 coexist.

We use an argument similar to Cai's (2000) to build some intuition around the existence of multiple equilibria in the region where V_2 is close to zero and $\delta \geq \delta_1$.

[8] $\delta_1 \in (0,1)$ is the solution to $\delta^4(1 + \frac{\delta^2}{1+\delta}) = 1$, i.e. $\delta_1 = 0.913$.

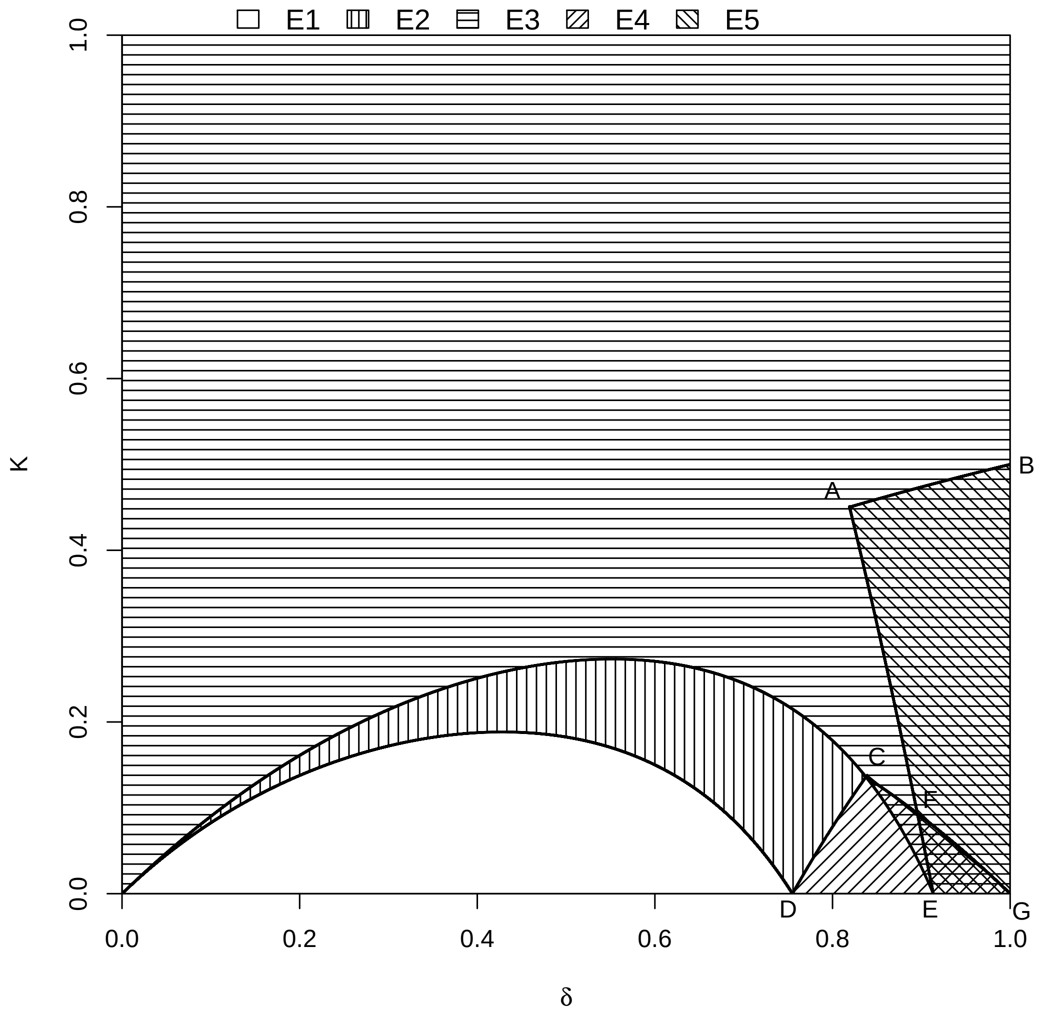

Figure 2: SPE of the bargaining game. The equilibria E6 and E7 lie on the curve AB.

We define $p = \frac{\delta^2(1-\delta^3)}{(1+\delta)(1-\delta^4)}$ and $a = \frac{\delta}{1+\delta} - p$. For such values, we can show that the sellers follow one of two types of strategies: in the first, they counter-offer p/δ and reject any offer less than p. In the second type, they reject any offer less than $\delta^4/(1+\delta)$ and counter-offer more than $\delta^3/(1+\delta)$. While seller 1 follows the first type of strategy in the equilibrium E3, he follows the second type in the equilibria E4 and E5. The higher valuation seller on the other hand, follows the first type of strategy in the equilibrium E5, and the second in the equilibria E3 and E4. In response, the buyer adopts one of these strategies: in the first, she offers $\frac{\delta^4}{1+\delta}$, and rejects any counter-offer greater than $\frac{\delta(1-\delta+\delta^4)}{1+\delta}$. In the second strategy, while she offers p, she uses a different cutoff for the two sellers to reject counter-offers; she rejects counter-offers greater than p/δ and $\delta\left(\frac{1}{1+\delta}-a\right)$ while bargaining with sellers who adopt the first and second strategy respectively. Since

$$\delta\left(\tfrac{1}{1+\delta}-a\right) < \tfrac{p}{\delta} \leq \tfrac{\delta^3}{1+\delta} \text{ and } \tfrac{p}{\delta} \leq \tfrac{\delta(1-\delta+\delta^4)}{1+\delta} \text{ when } \delta \geq \delta_1, \tag{11}$$

this suggests that two of the three players choose to play aggressively, leading to the three equilibria in this region.

	Type I	Type II
Buyer's strategy	$o = \frac{\delta^4}{1+\delta}$	$o = p$
	reject $co > \frac{\delta(1-\delta+\delta^4)}{1+\delta}$	reject $co > p/\delta$
		reject $co > \frac{\delta}{1+\delta} - a\delta$
	E4	E3, E5
Seller's strategy	$co = p/\delta$	$co = \frac{\delta^3}{1+\delta}$
	reject $o < p$	reject $o < \frac{\delta^4}{1+\delta}$
	S1: E3; S2: E5	S1: E4, E5; S2: E3, E4

While it is difficult to show that the inequalities given by (11) hold for all parameter values for which the equilibria E3, E4 and E5 coexist, we can presume that a similar intuition applies for low values of V_2 and high values of δ. This leads negotiations to hold-out between the buyer and sellers 1 and 2 in the equilibria E5 and E3 respectively. For the case where both V_2 and δ are relatively small, none of the players have any incentive to hold up the negotiation process, such that all offers and counter-offers are accepted. As V_2 increases, both $\widehat{co}_2$ and $\frac{\delta^3}{1+\delta}(M-V_2)+V_2$ rise, with the latter increasing at a faster rate than the former. This initially leads to co_2 getting rejected in E2, as the Rubinstein game payoff available to the higher valuation seller is higher than the payoff that the seller can get by counter-offering $\widehat{co}_2$. As V_2 increases further, the strategies used by the buyer and seller 2 lead to both o_2 and co_2 getting rejected in E3. This result is once again driven by the increasing value of the holding out payoff to the higher-valuation seller, who now keeps setting a higher minimum acceptable offer from the buyer. The buyer then offers $o_2 < \widehat{o}_2 = \frac{\delta^4}{1+\delta}M + (1-\frac{\delta^4}{1+\delta})V_2$.

For $\delta \to 1$ and $V_2 > 0$ we find that there are two equilibria, E3 and E5, which coexist for the range $K \in (0, \frac{\delta}{1+\delta}]$, and that there exists a unique equilibrium E3 for $K > \frac{\delta}{1+\delta}$. Therefore for sufficiently high values of K and for $\delta \to 1$ there exists a unique efficient equilibrium. For the equilibrium E5, we find that at $K = \frac{\delta}{1+\delta}$, $\widehat{co}_2 = V_2$, and that while the buyer is indifferent between offering $\widehat{o}_2$ or a smaller amount, the seller is indifferent between counter-offering $\widehat{co}_2$ or a larger amount. This indifference leads to two additional equilibria E6 and E7, in which in addition to o_1 and co_1, co_2 and o_2 get rejected respectively. Outcomes with delay are observed in equilibria E5, E6 and E7, which are supported by high values of δ. When $K = \frac{\delta}{1+\delta}$, the net payoff to seller 2 equals zero in all the three equilibria.

3.2 Buyer Bargains with Seller 2 First

We now proceed to solve for the equilibria of the game where the buyer bargains first with the higher-valuation seller. It is evident from figure 1, that the game $\Gamma(2,1)$ is a subgame of $\Gamma(1,2)$. Following the definition of SPE, any strategy profile which constitutes an SPE in a game, induces a Nash equilibrium in all its subgames. Thus, by construction, the strategy profiles which supported SPE of $\Gamma(1,2)$, do so in the game $\Gamma(2,1)$ as well. The subgame perfect outcomes, however, will be different. We use the notation EP_i, $i = 1, 2, ..., 7$ to denote the subgame perfect outcomes of the equilibria E1 to E7 respectively.

$$EP_1 \equiv \{\tfrac{\delta}{1+\delta}M, X - \tfrac{V_2}{\delta}, \tfrac{1}{1+\delta}M - X, 1\};\ X = \tfrac{\delta}{(1+\delta)^2}M + \tfrac{1+\delta+\delta^3}{\delta(1+\delta)^2(1+\delta^2)}V_2$$

$$EP_2 \equiv \{\tfrac{\delta}{1+\delta}M, X - \tfrac{V_2}{\delta}, \tfrac{1}{1+\delta}M - X, 1\};\ X = \tfrac{\delta^3}{1+\delta}(M - V_2) + \tfrac{V_2}{\delta}$$

$$EP_4 \equiv \{\tfrac{\delta}{1+\delta}M, X - \tfrac{V_2}{\delta}, \tfrac{1}{1+\delta}M - X, 1\};\ X = \tfrac{\delta^3}{1+\delta}(M - V_2) + \tfrac{V_2}{\delta}$$

$$EP_5 \equiv \{\tfrac{\delta}{1+\delta}M, X - \tfrac{V_2}{\delta}, \tfrac{1}{1+\delta}M - X, 1\};\ X = \tfrac{\delta(1-\delta^3)}{(1-\delta^4)(1+\delta)}M + \tfrac{1-\delta}{\delta-\delta^5}V_2$$

$$EP_6 \equiv \{\tfrac{\delta}{1+\delta}M, 0, \tfrac{1}{1+\delta}M - \tfrac{V_2}{\delta}, 1\}$$

$$EP_3 \equiv \{X, \tfrac{\delta}{1+\delta}(M - V_2), \tfrac{1}{1+\delta}(M - V_2) - X, 3\};\ X = \tfrac{\delta(1-\delta^3)}{(1+\delta)(1-\delta^4)}(M - V_2)$$

$$EP_7 \equiv \{\tfrac{\delta}{1+\delta}M, \tfrac{1}{1+\delta}M - \tfrac{V_2}{\delta}, 0, 2\}$$

Thus while outcomes for the first five equilibria are efficient, those related to E3 and E7 turn out to be inefficient. This leads us to the following corollary.

Corollary 1 *There exists inefficient outcomes even with extremely impatient players in the game* $\Gamma(2,1)$.

This corollary follows directly from proposition 3. The result is interesting because it goes against the general intuition that impatient players are unwilling to wait and are thus willing to make a deal as early as possible. If the buyer starts bargaining with seller 2 first, there will be two-period delay before the first deal is made. In this case, seller 2 demands the appropriately discounted Rubinstein payoff, which he gets if he sells second. However, it is not profitable for the buyer to give in to his demands, such that she would prefer to wait and to make the first deal with the seller with lower valuation, *i.e.* $\delta^2(\frac{M-V_2}{1+\delta} - \frac{o_1}{\delta}) \geq \frac{M}{1+\delta} - \frac{\delta^3}{1+\delta}(M - V_2) - \frac{V_2}{\delta}$. When it is seller 2's turn to counteroffer, the discounted Rubinstein payoff is better than what the buyer is willing to accept, *i.e.* $\frac{\delta^3}{1+\delta}(M - V_2) \geq \widehat{co_2} - V_2$, where $\widehat{co}_2$ is solved using the equation $\delta(\frac{M-V_2}{1+\delta} - \frac{o_1}{\delta}) = \frac{M}{1+\delta} - \frac{\widehat{co}_2}{\delta}$. As both the buyer and seller 2 are unwilling to make the first deal with each other, seller 1 is left with little choice but to relent and to follow an accommodative strategy.

One of the main questions that we attempt to answer is whether there exists a range of parameter values for which the buyer prefers to bargain first with the lower-valuation (higher-valuation) seller.

Corollary 2 *For* $K < \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$ *or* $K > \frac{\delta}{1+\delta}$, *the buyer prefers to negotiate with the lower-valuation seller first.*

Proof. See Appendix B. ■

In the parameter space defined by the above two conditions, we have regions with either a unique equilibrium or with multiple equilibria. These equilibria are E1, E2, E3 and E4. In each of these equilibria, it is beneficial for the buyer to begin the bargaining process by negotiating with the lower-valuation seller. In E1, E2 and E4 the buyer offers $o_1 < o_2$ such that the smaller sunk payment made to seller 1 in the first round, compensates the buyer for the smaller payoff obtained through Rubinstein bargaining with the higher-valuation seller in the second round. However, in the event where both the lower-valuation seller and the buyer choose to play strategies which lead to both o_1 and co_1 to get rejected, the buyer prefers to negotiate first with the higher-valuation seller. This corresponds to the equilibrium E5.

Corollary 3 *For* $K \geq \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$ *and* $K \leq \frac{\delta}{1+\delta}$*, it is possible to have an equilibrium in which the buyer prefers to bargain with the higher-valuation seller first.*

The region described above corresponds to the equilibrium E5. The payoffs of the players in E5 remain the same in current value terms of the period in which the project is completed, as the bargaining order is changed. However, since the outcome corresponding to E5 is inefficient in BO_1, the buyer is able to get the payoff sooner in the second order than in the first. There also exists other equilibria in the region described by the conditions above, in which the buyer prefers to bargain with the lower-valuation seller first. Hence the result follows.

In order to identify the regions which are associated with equitable (inequitable) payoffs, we begin by looking at values of δ close to 1, where the relevant equilibria are E3 to E7. The equilibria E6 and E7 result in highly unequal payoffs, with the first seller cornering the entire surplus, while the outcome corresponding to both the bargaining orders for E4 is $\{\frac{1}{2}M, \frac{1}{2}M, 0, 1\}$. The outcomes relating to E3 and E5 for the first bargaining order are $\{\frac{3}{8}(M-V_2), \frac{1}{2}(M-V_2), \frac{1}{8}(M-V_2), 1\}$ and $\{\frac{1}{2}M, \frac{3}{8}M - \frac{3}{4}V_2, \frac{1}{8}M - \frac{1}{4}V_2, 3\}$ respectively. When the order is reversed, these payoffs are available in periods 3 and 1 respectively. Thus, the payoff of the buyer is lower than the minimum of the sellers' payoff. If the buyer was allowed to choose the bargaining order (as in Xiao (2012)), she would have chosen to bargain with the lower-valuation seller first until an agreement was reached, before moving on to seller 2. In such an event, the buyer would have earned the payoff $\frac{1}{4}(M-V_2)$, while sellers 1 and 2 would have got $\frac{1}{4}(M-V_2)$ and $\frac{1}{2}(M-V_2)$ respectively. The Gini coefficient for the equilibrium E3 was found to be a constant (0.26) while that for the equilibrium E5 was found to increase monotonically from 0.26 to 0.67 as K increased from zero to 1/2 (i.e. $\delta/(1+\delta)$).

For $\delta \in (0,1)$ we computed the share of the players' payoffs in total payoffs for the equilibria E3 and E5, and found that the shares of the two sellers (buyer) to be rising (falling) in V_2 for the equilibrium E3 and that $\frac{\partial(f_1/\sum f_i+h)}{\partial V_2} > 0$, $\frac{\partial(f_2/\sum f_i+h)}{\partial V_2} < 0$, $\frac{\partial(h/\sum f_i+h)}{\partial V_2} < 0$ in E5. Using the Gini coefficient, the regions with maximum and minimum inequity for the two bargaining orders are shown in figure 3.

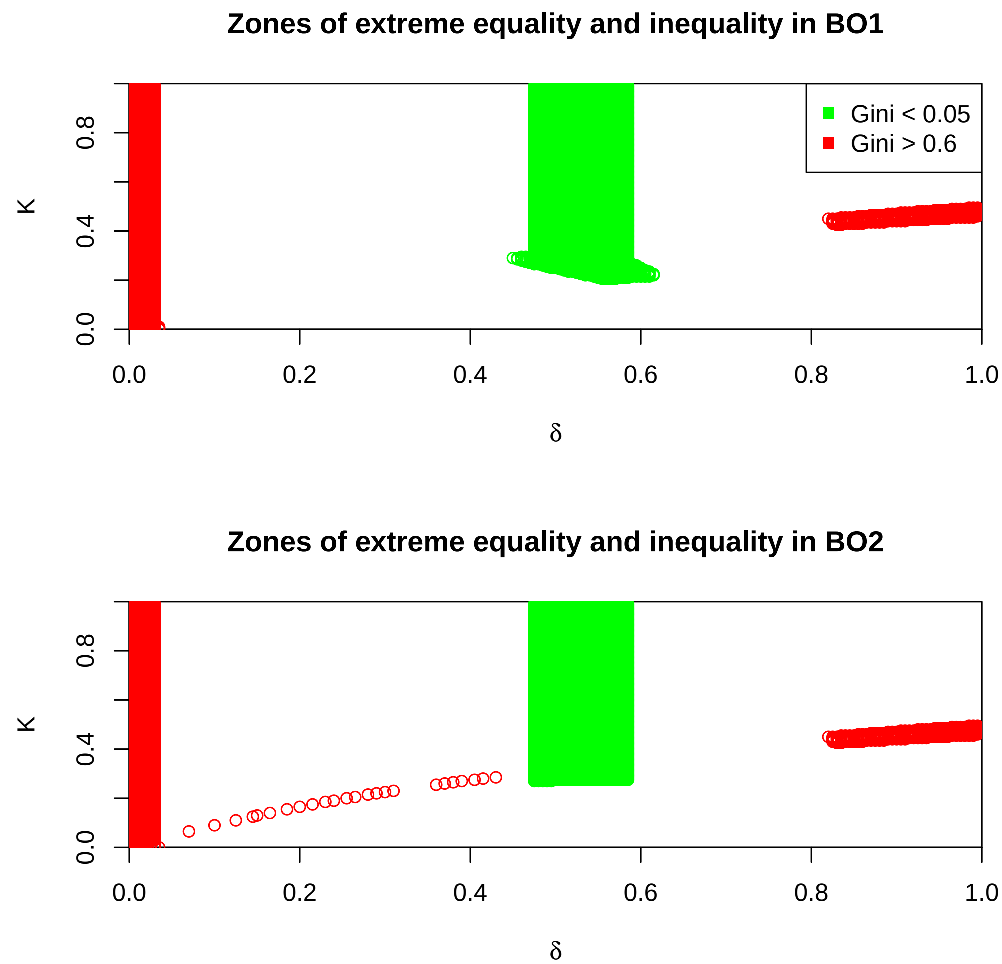

Figure 3:

4 Main Results and Conclusion

It has been well-established that there exists a hold-up problem in non-cooperative, multilateral bargaining games of complete information, when negotiations take place between sufficiently patient players. In a setting where a buyer negotiates with a single seller in each round of the bargaining process, sellers who reach agreements later, get a larger share of the surplus than others. This provides an incentive to each seller to stall the process. Cai (2000) showed that when a buyer negotiates with symmetric sellers having the same zero valuation for their land, delay could arise when players are sufficiently patient, and that as the number of bargainers increased, perpetual disagreement could be an equilibrium outcome. Another result which is common to such bargaining models is that different orders of making offers and counter-offers results in different distributions of the total surplus.

To investigate, among other things, whether the hold-up problem is exacerbated when the bargaining process involves one buyer and *heterogeneous sellers* with varying valuations for their land, we set up a model with the following features. We assumed that the buyer negotiated with two sellers, with seller 2 having a higher valuation for his land than the other seller. Negotiations in each round comprised a sequence of offers and counter-offers, with the buyer making an offer to the seller, followed by a counter-offer from the seller in case the offer was rejected. Two possible bargaining orders were considered, with the buyer negotiating first with the lower-valuation buyer in the first order, and with the higher-valuation buyer in the second. We then characterized the set of equilibria for each order.

In the equilibria E1, E2 and E4, the identity of the seller who reaches an agreement first, changes, when the bargaining order is changed. As a result, the payoffs earned by the players differ across the two bargaining orders, and the final period in which negotiations are completed remains unchanged. On the other hand, in the equilibria E3 and E5, changes in the bargaining order do not result in any change in the identity of the farmer who signs the first contract. In these equilibria players earn the same payoffs (in current value terms of the period in which the project is completed), while the period in which the final agreement is reached, changes from one sequence to the other. By comparing the buyer's payoff across the two bargaining orders, we show that she prefers to negotiate first with the lower-valuation seller for the equilibria E1, E2, E3 and E4, and with the higher-valuation seller in E5. This suggests that the buyer prefers to bargain first with the lower-valuation seller, except in the equilibrium E5, where both the buyer and the lower-valuation seller choose strategies which lead to o_1 and co_1 to get rejected. This is contrast to the result obtained by Xiao (2012), who constructs a model where the bargaining order is determined endogenously and shows that the buyer prefers to negotiate with the sellers in increasing size (valuations).

In the bargaining order where the buyer negotiates with the lower-valuation seller first, we find that an inefficient outcome may exist for sufficiently high values of δ, when $V_2/M = K$ is below a threshold. This finding is similar to a result obtained by Cai (2000), who shows that for $\delta \geq 0.913$ there exists multiple equilibria, of which one corresponds to an inefficient outcome. However, in the second bargaining order, an inefficient outcome corresponding to a unique equilibrium is shown to exist for $V_2 > 0$ and for very low values of δ. In this equilibrium, both the buyer and the seller with a higher-valuation choose strategies, which lead negotiations to reach a deadlock in the first round of the bargaining process. To the best of our knowledge, our paper is the first to show that such inefficient outcomes exist for extremely impatient players in such multilateral models of bargaining. Inefficiency in the form of delay, is thus exacerbated in the presence of heterogeneous sellers, when the buyer negotiates with the higher-valuation seller first.

For $V_2 > 0$ and $\delta \to 1$ we show that for sufficiently high K, there exists a unique equilibrium which is efficient (inefficent) when the bargaining order is the first (second). The hold-up problem is thus resolved in a setting with heterogeneous sellers when the buyer negotiates with the lower-valuation seller first and when the degree of heterogeneity between the sellers is sufficiently high. We find that the distribution of the surplus changes according to the equilibria which correspond to different values of K. The two significant equilibria E3 and E5 are supported by strategies which ensure that the identity of the seller reaching the first agreement does not change as the bargaining order changes. This results in payoffs which remain unchanged over the two bargaining orders. Given that there are instances where negotiations have failed when participants have deemed the process to be unfair, we use the Gini coefficient to check how equitable are the payoffs in these equilibria. The Gini coefficient[9] for E3 turns out to be 0.26, while that for E5 increases monotonically from 0.26 to 0.67 as K increases from zero to 1/2. We hypothesize that changing the sequence of offers and counter-offers, such that the sellers make the first offer in a round of bargaining will result in a different distribution of the surplus, and that randomizing over the identity of the player who makes the first offer might lead to more equitable distributions. This is left for future work.

[9] For a population with values y_i, $i = 1, 2, ..., n$ that are indexed in non-decreasing order ($y_i \leq y_{i+1}$) the Gini coefficient G is given by $2 \sum_{i=1}^{n} i y_i / (n \sum_{i=1}^{n} y_i) - \frac{n+1}{n}$, such that for the case where $y_i =$ a constant $\forall i$, $G = 0$. In our case the payoffs are given by y_1, y_2 and x for sellers 1, 2 and the buyer respectively.

5 Acknowledgements

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Appendix A

Proof of Proposition 8
Here we focus on the game before the first deal is made. Buyer negotiates with the sellers alternately. After every four offers and counteroffers or eight nodes (nodes are numbered N1 to N8 as shown in figure 4), the game repeats itself. We consider stationary strategies only. The bargaining game following the first deal is precisely a Rubinstein game (1982), which has a unique equilibrium strategy profile and outcome.

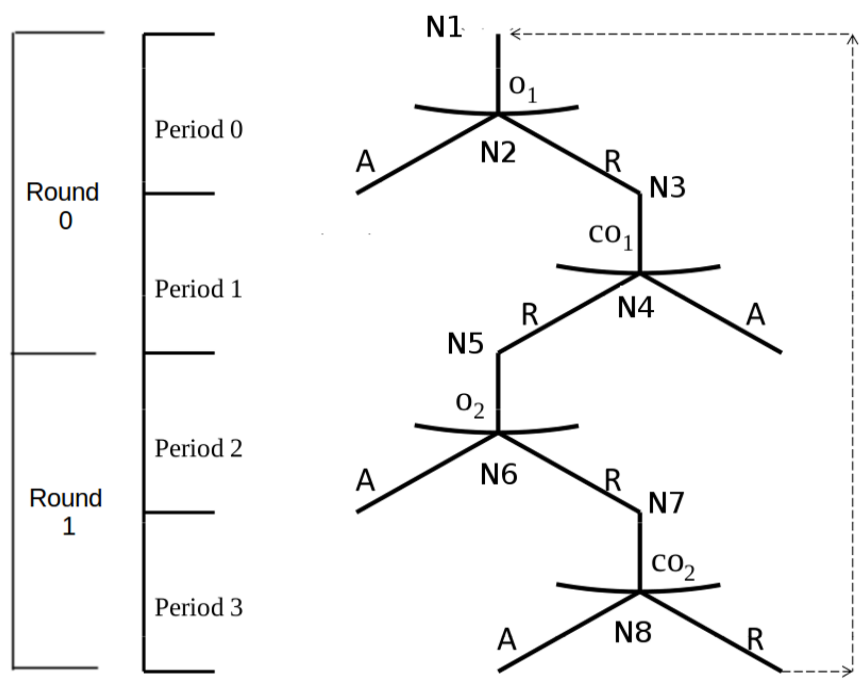

Figure 4: Extensive form representation of bargaining game.

In any equilibrium strategy profile, each of the four offers and counteroffers will either be accepted or rejected. Hence we have a total of 16 possible cases, depending upon various offers and counteroffers being accepted or rejected. We now try to construct equilibrium (or equilibria) in each of the 16 cases one by one. We expect some cases to yield an equilibrium while the remaining will not yield any. We use the *one step deviation rule* to check whether a strategy profile is subgame perfect.

Case 1: All offers and counteroffers, o_1, o_2, co_1 and co_2, are accepted.

This case leads to proposition 1.

Refer figure 4. We will start constructing equilibrium strategy profile at node 8 and move towards node 1.

Consider node $N8$.

Here buyer responds to seller 2's counteroffer. Let $\widehat{co_2}$ be the counteroffer which makes buyer indifferent between accepting and rejecting. If buyer accepts $\widehat{co_2}$, he gets $\frac{M-V_1}{1+\delta}$ in next period. In case he rejects it, he pays o_1 next period before receiving $\frac{M-V_2}{1+\delta}$ a period further later. Therefore,

$$\frac{\delta}{1+\delta}(M-V_2) - o_1 = \frac{M-V_1}{1+\delta} - \frac{\widehat{co_2}}{\delta} \tag{12}$$

Consider node $N7$.

Seller 2 makes a counteroffer at this node. Since in this case co_2 is being accepted, $co_2 = \widehat{co_2}$ must hold.

Also, the seller's payoff must not be less than the payoff he gets if counteroffer is rejected. Otherwise he will counteroffer himself high enough to get rejected. If co_2 is rejected, buyer makes deal with seller 1 in next period (as o_1 is accepted in this case) and seller 2 gets Rubinstein payoff a period further later. Therefore,

$$\widehat{co_2} - V_2 \geq \frac{\delta^3}{1+\delta}(M-V_2) \tag{13}$$

Consider node $N6$.

At this node, seller 2 responds to the buyer's offer. He accepts any payoff equal to or greater than the discounted value of the payoff he will receive next period. Hence, seller 2 accepts if $o_2 - V_2 \geq \delta(\widehat{co_2} - V_2)$.

Consider node $N5$.

Since o_2 is accepted, following must hold.

$$o_2 - V_2 = \delta(\widehat{co_2} - V_2) \tag{14}$$

If buyer offers less, his offer will be rejected(refer seller 2's strategy at $N6$), and he pays $\widehat{co_2}$ to seller 2 in next period before obtaining Rubinstein payoff a period further later. Following constraint ensures that offering less is not profitable for the buyer.

$$\frac{M - V_1}{1+\delta} - \frac{o_2}{\delta} \geq \frac{\delta}{1+\delta}(M - V_1) - \widehat{co_2} \tag{15}$$

Consider node $N4$ to node $N1$.

The game from node 4 to node 1 is symmetric to that from node 8 to node 5. Hence, we obtain following 4 equations by interchanging subscript 2 and 1 in above equations.

$$\frac{\delta}{1+\delta}(M - V_1) - o_2 = \frac{M - V_2}{1+\delta} - \frac{\widehat{co}_1}{\delta} \tag{16}$$

$$\widehat{co}_1 - V_1 \geq \frac{\delta^3}{1+\delta}(M - V_1) \tag{17}$$

$$o_1 - V_1 = \delta(\widehat{co}_1 - V_1) \tag{18}$$

$$\frac{M - V_2}{1+\delta} - \frac{o_1}{\delta} \geq \frac{\delta}{1+\delta}(M - V_2) - \widehat{co}_1 \tag{19}$$

Also, by symmetry we can say that $co_1 = \widehat{co}_1$.

Equations 11, 13, 15 and 17 have four unknowns. These, along with $V_1 = 0$, give us

$$\begin{aligned}
o_1 &= \tfrac{\delta^2(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] \\
\widehat{co}_1 &= \tfrac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] \\
o_2 &= \tfrac{\delta^2}{(1+\delta)^2}M + \tfrac{1+\delta+\delta^3}{(1+\delta)^2(1+\delta^2)}V_2 \\
\widehat{co}_2 &= \tfrac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M + \delta(1+\delta+\delta^2)V_2]
\end{aligned}$$

Substituting these values in equations 12, 14, 16 and 18, we get following 4 constraints respectively.

$$\begin{aligned}
K &\leq \tfrac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta+\delta^7-\delta^2-\delta^3-\delta^4} \\
K &\leq \tfrac{\delta}{1+\delta} \\
K &\leq \tfrac{\delta-\delta^4-\delta^5-\delta^6}{\delta^3} \\
\delta &\geq 0
\end{aligned}$$

It can be checked that the first constraint makes the remaining constraint redundant.

Hence these strategies, by construction, confirm to *one step deviation rule* under the first constraint. This is the equilibrium stated in proposition 1.

We try to construct equilibrium in remaining 15 cases in similar fashion. Only 6 out of 15 cases yield equilibrium as defined in proposition 2 to proposition 7. Hence, proposition 1 to proposition 7 completely characterizes the set of equilibria.

Appendix B

Proof of Corollary 2
Let B_1^1 represents buyer's payoff in E1 with bargaining order $\{1,2\}$. Here, subscript denotes the equilibrium and superscript denotes the bargaining order. Similarly, B_1^2 represents buyer's payoff in E1 with bargaining order $\{2,1\}$. Also, B_2^1 , B_3^1 , B_4^1 , B_5^1 , B_6^1 and B_7^1 denote buyer's payoff in equilibium E2, E3, E4, E5, E6 and E7 respectively, each with bargaining order $\{1,2\}$. B_2^2,B_3^2,B_4^2,B_5^2,B_6^2 and B_7^2 denote buyer's payoff in equilibium E2, E3, E4, E5, E6 and E7 respectively, each with bargaining order $\{2,1\}$.

Claim: $B_1^1 \geq B_1^2$
$\iff \frac{1}{1+\delta}(M-V_2) - \frac{\delta(1-\delta)}{(1+\delta)(1-\delta^4)}[(1+\delta^2)M - \delta^2 V_2] \geq \frac{1}{1+\delta}M - \frac{\delta}{(1+\delta)^2}M - \frac{1+\delta+\delta^3}{\delta(1+\delta)^2(1+\delta^2)}V_2$

$\iff -\frac{V_2}{1+\delta} - \frac{\delta(1-\delta)(1+\delta^2)}{(1+\delta)(1-\delta^4)}M + \frac{\delta^3(1-\delta)}{(1+\delta)(1-\delta^4)}V_2 \geq -\frac{\delta}{(1+\delta)^2}M - \frac{1+\delta+\delta^3}{\delta(1+\delta)^2(1+\delta^2)}V_2$

$\iff -V_2 - \frac{\delta(1-\delta)(1+\delta^2)}{(1-\delta^4)}M + \frac{\delta^3(1-\delta)}{(1-\delta^4)}V_2 \geq -\frac{\delta}{1+\delta}M - \frac{1+\delta+\delta^3}{\delta(1+\delta)(1+\delta^2)}V_2$

$\iff -V_2 - \frac{\delta}{1+\delta}M + \frac{\delta^3}{(1+\delta^2)(1+\delta)}V_2 \geq -\frac{\delta}{1+\delta}M - \frac{1+\delta+\delta^3}{\delta(1+\delta)(1+\delta^2)}V_2$

$\iff \frac{\delta^3}{(1+\delta^2)(1+\delta)}V_2 + \frac{1+\delta+\delta^3}{\delta(1+\delta)(1+\delta^2)}V_2 \geq V_2$

$\iff \frac{1+\delta+\delta^3+\delta^4}{\delta(1+\delta)(1+\delta^2)}V_2 \geq V_2$

$\iff \frac{(1+\delta)(1+\delta^3)}{\delta(1+\delta)(1+\delta^2)} \geq 1$

$\iff 1 \geq \delta$
Which is true, hence the claim is true.

Claim: $B_2^1 \geq B_2^2$
$\iff \frac{1}{1+\delta}(M-V_2) - \frac{\delta}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \geq \frac{1}{1+\delta}M - \frac{\delta^3}{1+\delta}(M - V_2) - \frac{V_2}{\delta}$

$\iff -\frac{V_2}{1+\delta} - \frac{\delta}{(1+\delta)}[(1-\delta+\delta^4)M + (\delta-\delta^4)V_2] \geq -\frac{\delta^3}{1+\delta}(M-V_2) - \frac{V_2}{\delta}$

$$\iff -\frac{V_2}{1+\delta} - \frac{\delta(1-\delta+\delta^4)}{1+\delta}M - \frac{\delta^2}{1+\delta}(1-\delta^3)V_2 \geq -\frac{\delta^3}{1+\delta}M + \frac{\delta^3}{1+\delta}V_2 - \frac{V_2}{\delta}$$

$$\iff \frac{\delta^2+\delta^3-\delta-\delta^5}{1+\delta}M \geq \frac{\delta^2}{1+\delta}(1-\delta^3)V_2 + \frac{V_2}{1+\delta} + \frac{\delta^3}{1+\delta}V_2 - \frac{V_2}{\delta}$$

$$\iff \frac{\delta^2+\delta^3-\delta-\delta^5}{1+\delta}M \geq \frac{\delta^3+\delta^4-\delta^6-1}{\delta(1+\delta)}V_2$$

$$\iff \delta^3+\delta^4-\delta^6-\delta^2 \geq (\delta^3+\delta^4-\delta^6-1)K$$

It can be checked that $\delta^3+\delta^4-\delta^6-1 \leq 0 \; \forall \delta$.
Therefore, above expression becomes,

$$\iff K \geq \frac{\delta^3+\delta^4-\delta^6-\delta^2}{\delta^3+\delta^4-\delta^6-1} = H, say. \tag{20}$$

For $\delta^2(1+\delta) \geq 1$, $\delta^3+\delta^4-\delta^6-\delta^2 \geq 0$
Which implies, $H \leq 0$. Hence, inequality 19 holds.

For $\delta^2(1+\delta) < 1$,
Recall following constraint applicable to equilibrium 2 as stated in proposition 2.

$$K \geq \frac{\delta+\delta^7-\delta^2-\delta^4}{1+\delta-\delta^2-\delta^3-\delta^4+\delta^7} \tag{21}$$

or, which can be rewritten as $K \geq \frac{h}{h+1-\delta^3}$
where, $h = \delta+\delta^7-\delta^2-\delta^4$

Claim: $\frac{h}{h+1-\delta^3} > H$

$\iff \frac{h}{h+1-\delta^3} > \frac{m-\delta^2}{m-1}$, where $m = \delta^3+\delta^4-\delta^6$

$\iff h(m-1) < (h+1-\delta^3)(m-\delta^2)$ since, $m-1 \leq 0$ and $h+1-\delta^3 > 0$

$\iff \delta^3 m + \delta^2 h + \delta^2 < h + \delta^5 + m$
This gives,
$\iff \delta^2(1+\delta) < 1$, which indeed is the case.

Hence, $\frac{h}{h+1-\delta^3} > H$.
Therefore we get $K > H$. This proves the initial claim.

Claim: $B_3^1 \geq B_3^2$
Since both offer and counter-offer during the negotiation between the buyer and seller 2 gets rejected. Bargaining order $\{1, 2\}$ is optimal for the buyer (and both sellers).

Claim: $B_4^1 \geq B_4^2$

$\iff \frac{1}{1+\delta}(M - V_2) - \frac{\delta^3}{1+\delta}M \geq \frac{1}{1+\delta}M - \frac{\delta^3}{1+\delta}(M - V_2) - \frac{V_2}{\delta}$

$\iff -\frac{V_2}{1+\delta} \geq \frac{\delta^3}{1+\delta}V_2 - \frac{V_2}{\delta}$

$\iff \frac{1}{\delta} \geq \frac{1+\delta^3}{1+\delta}$

$\iff 1 \geq \delta^4$

Which is true, hence the claim is true.

Claim: $B_5^1 \geq B_5^2$, $B_6^1 \geq B_6^2$ and $B_7^1 \geq B_7^2$
Since both offer and counter-offer during the negotiation between the buyer and seller 1 gets rejected. Bargaining order $\{2, 1\}$ is optimal for the buyer (and both sellers).