Revenue Non-Equivalence in Multidimensional Procurement Auctions under Asymmetry *

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Abstract

Using an example we show that the Revenue Equivalence in the Scoring Auctions, as postulated by Che (1993), no longer holds when the suppliers are asymmetric in their costs of production.

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1 Introduction

Public procurement can be defined as the acquisition of goods and services by the government in order to improve upon the state of the economy. This includes procurement contracts related to transport, health, education, infrastructure, defence equipments etc. The government or government agencies call for tender proposals, rather than simply offering the contracts on first-come-first-serve basis. Once the submission period is over and the proposals are evaluated, the firm quoting the lowest price is declared the winner and offered the contract. The revenue generated in this manner is high compared to any other non-competitive procurement process where no bidding takes place and the good is bought at a fixed or posted price . As Krishna (2010) points out,

> "The process of procurement via competitive bidding is nothing but an auction, except in this case the bidders compete for the right to sell their products or services. Billions of dollars of government purchases are almost exclusively made this way."

It is estimated that public procurement approximately constitutes about 15-20% of GDP in developed and developing jurisdictions.[1] In India, public procurement has been estimated to constitute about 30% of the GDP. Therefore, procurement processes are considered to be of utmost economic importance and are a matter of interest to many researchers across the world.

Quite often, public procurement auctions award the contract based on quality characteristics along with the price. The firms are required to submit two proposals in the form of their bids, a financial and a technical one. The *financial proposal* specifies the price while the *technical proposal* specifies the product in terms of "the cost of operating, maintaining and repairing goods or works, the time for delivery of goods, completion of works or provision of services, the characteristics of the subject matter of the procurement, such as the functional characteristics of goods or works and the environmental characteristics of the subject matter, the terms of payment and of guarantees in respect of the subject matter of the procurement."[2] The technical proposals are then reviewed by an evaluation committee and are assigned quality scores (Lengwiler and Wolfstetter, 2006).

Che (1993) describe a model where both price and the quality score are aggregated by the means of a Scoring Rule designed by the buyer. This kind of a procurement process that is used to buy a differentiated product is often termed as a "Scoring Auction" in the Auction Literature. In his model, the sellers are assumed to be symmetric and the buyer does not delegate the task of procuring the desired good to an agent. Further, Che goes on to define three two-dimensional auctions that are called 'first-score', 'second-score' and 'second-preferred-offer' auctions. In a *first-score auction*, "each firm submits a sealed bid and, upon winning, produces the offered quality at the offered price." In a *second-score auction*, the

[1] The Size of Government Procurement Markets, OECD, 2001.

[2] Stated under the guidelines of Public Procurement Bill 2012,India.

winner is asked to match the second highest score in the auction while in a *second-preferred-offer auction*, the winner is asked to match the exact price-quality combination of the bidder who scored the second lowest. One of the crucial findings of his paper is the two-dimensional Revenue Equivalence Theorem. This theorem states that under a *Naive* Scoring Rule which truly reflects seller's preferences, all the three scoring auctions yield the same expected utility to the buyer.

The main contribution of our paper is to extend the model used by Che (1993), by considering first-score and second-score auctions with asymmetric sellers and to show, using an example that the Revenue Equivalence result no longer holds under a Naive Scoring Rule. According to Maskin and Riley (2000) "...Asymmetries are often important in contract bidding. Each potential contractor has essentially the same information about the nature of the project but a different opportunity cost of completing it. Whenever some aspect of these differences is common knowledge, beliefs are asymmetric...". Asymmetry amongst sellers is quite common in procurement processes in India and can be observed due to differences in location of the firm (local or distant), processing capacity, technology etc. as emphasized by Ramaswami et. al. (2009) and Banker and Mitra (2007).

Scoring auctions are found to dominate many other procurement processes such as menu auctions, beauty contests, and price-only auctions in terms of expected utility to the buyer even when a supplier's private information about his costs is multidimensional (Asker and Cantillon, 2008). The multidimensional and privately known costs can be represented by their 'pseudo-types' and a correspondence can be established between the set of scoring auctions and the set of standard single object one dimensional Independent Private Value (IPV) auctions. In that case, equilibrium in the scoring auction acquires all the properties of the corresponding IPV auction such as existence, uniqueness, efficiency etc. Also, the classic revenue equivalence theorem can be generalized in multidimensional private information setting where the first-score, second-score, ascending score and descending score yield same expected utility to the buyer, provided the scoring rule used is quasi-linear (Asker and Cantillon, 2004).

To implement a scoring rule with an optimal distortion from the buyer's preferences and make an optimal mechanism feasible, a certain degree of commitment is essential which is not an issue in a public procurement process as the buyer has to abide by strict rules and guidelines of the procedure (Asker and Cantillon, 2008). Even so, sometimes these optimal mechanisms are too complicated to be practical. When the supplier's cost including a fixed cost and a marginal cost of producing quality is private information, a quasilinear scoring function can no longer implement the optimal quality scheme. As the optimal mechanism becomes too complex, simpler schemes such as buyer-optimal efficient auctions, negotiation and quasilinear score auction are expected to perform better. However, all the above three procedures under-perform in comparison to the optimal mechanism, negotiation being the least recommended (Asker and Cantillon, 2005). Branco (1997) design multidimensional auctions using correlated costs in the setting similar to Che (1993). He recommends a two-stage optimal procedure where a firm is first selected and later is bargained with to readjust its quality bid.

The rest of the paper is divided into the following sections. Section 2 deals with the first-score procurement auction with asymmetric bidders and section 3 consists of the workings of the second-score procurement auction under Asymmetry. Section 4 discusses the comparison between the two auction formats with respect to the expected utility generated for the buyer, while section 5 concludes.

2 Asymmetric First-Score Procurement Auction

Consider a buyer who wants to procure an indivisible good from two potential suppliers. The suppliers are supposed to submit a bid specifying the 'promised quality' q and the price p of that good. The buyer's utility from the contract $(q, p) \in \mathbb{R}^2_+$ is given by

$$U(q, p) = V(q) - p,$$

where we assume that $V'(q) > 0$, $V''(q) < 0, \lim_{q\to 0} V'(q) = \infty, lim_{q\to\infty} V'(q) = 0)$ in order to ensure an interior solution.

There are two risk-neutral, (expected) profit maximizing suppliers, labelled strong (s) and weak (w). The cost incurred by a firm in producing the good of the promised quality level q is $c(q, \theta_i)$ where θ_i is the cost or efficiency parameter of the firm and is privately known. Also, let us assume that $c_q > 0, c_\theta < 0, c_{qq} \geq 0, c_{\theta\theta} > 0$, $c_{q\theta} < 0$ and $c_{qq\theta} < 0$. Further, let it be common knowledge that the cost parameters of the two bidders θ_s, θ_w are drawn independently from the uniform distributions on $[\underline{\eta}, \eta_s]$ and $[\underline{\eta}, \eta_w]$ respectively where $\eta_w < \eta_s$ i.e.

$$F_s(\theta) = \frac{\theta - \underline{\eta}}{\eta_s - \underline{\eta}} \;\forall\; \theta \in [\underline{\eta}, \eta_s] \text{ and } F_w(\theta) = \frac{\theta - \underline{\eta}}{\eta_w - \underline{\eta}} \;\forall\; \theta \in [\underline{\eta}, \eta_w].$$

In other words, support for the weak bidder is a subset of the support for the strong bidder which implies that F_w (first-order) stochastically dominates F_s over $[\underline{\eta}, \eta_w]$. Ex-ante profit for the firm with type θ after winning the contract is

$$\pi(q, p|\theta) = \{p - c(q, \theta)\}.Prob[win|S(q, p)].$$

Let a scoring rule be a function $S : \mathbb{R}^2 \to \mathbb{R}$ that associates a score $S(q, p)$ to any potential contract (q, p) between the buyer and a supplier. The Scoring Rule used by the buyer to evaluate the two-dimensional bids submitted in the auction is of the quasi-linear form i.e. $S(q, p) = s(q) - p$ where $s(.)$ is increasing at least for $q \leq argmax_q\ s(q) - c(q, \theta)$ (Che 1993). A *Naive* Scoring Rule is the one that reveals the buyer's true preferences i.e. $S(q, p) = U(q, p)$ or some monotone transformation of U.

2.1 Bidding Strategies

The objective function of the firm is given by

$$Max_{q,p}\ \pi(q,p|\theta) = \{p - c(q,\theta)\}.Prob[win|S(q,p)]. \tag{1}$$

For the firms to make non-negative profits, it should be such that $p \geq c(q,\theta)$. In other words, $S(q,p) = s(q) - p \leq s(q) - c(q,\theta) \leq \max_q s(q) - c(q,\theta)$. We define $S_o(\theta) = \max_q s(q) - c(q,\theta)$ and $q_o(\theta) = argmax_q\ s(q) - c(q,\theta)$ for all future purposes.

Lemma 1 **:** The promised quality level chosen by a firm of type θ_i is

$$q_o(\theta_i) = argmax_q\ s(q) - c(q,\theta_i)$$

for all $\theta_i \in (\underline{\eta}, \eta_i]$; $i = s, w$.

Proof **:** This is immediate from Lemma 1 (Che, 1993). The perceived asymmetry amongst the bidders has no effect on the firms' quality bids. A formal proof is given in the Appendix.

The maximum score that any firm with type θ can offer, $S_o(\theta)$, is termed as the *productive potential* of that firm by Che (1993). Applying Envelope Theorem, $S_o'(\theta) = -c_\theta(q,\theta) > 0$ since the cost of producing a certain level of quality decreases in the efficiency parameter θ of that firm. Thus, $S_o(.)$ is strictly increasing and therefore its inverse exists. Therefore, this $S_o(\theta)$ can be treated as the **pseudo-valuation** of the contract by the firm with type θ. This enables us to transform the two-dimensional procurement auction problem into a unidimensional one.

Let $v = S_o(\theta)$ and follows a cumulative distribution function say, $\mathcal{H}(.)$ with density function $h(.)$, where

$$\mathcal{H}(S) = Prob[S_o(\theta) \leq S] = Prob[\theta \leq S_o^{-1}(S)] = F(S_o^{-1}(S)).$$

Let $b = S(q_o(\theta), p(\theta))$. Then $v - b = \{s(q_o(\theta)) - c(q_o(\theta),\theta)\} - \{s(q_o(\theta)) - p(\theta)\} = p(\theta) - c(q_o(\theta),\theta)$.

The firm's problem can then be written as

$$Max_b\ \pi(b,v) = (v-b)Prob[win|b]. \tag{2}$$

Suppose that in equilibrium, the strong and the weak firm follow strategies $\beta_s(S_o(\theta_s))$ and $\beta_w(S_o(\theta_w))$ respectively. Further, let us assume that these strategy functions are increasing and differentiable. Let their inverses functions be $\phi_s \equiv \beta_s^{-1}$ and $\phi_w \equiv \beta_w^{-1}$.

***Claim 1*:** $\beta_i(S_o(\underline{\eta})) = S_o(\underline{\eta})\ \forall\, i = s, w$ and $\beta_s(S_o(\eta_s)) = \beta_w(S_o(\eta_w))$.

***Proof* :** Refer to the Appendix.

Let $\bar{b} \equiv \beta_s(S_o(\eta_s)) \equiv \beta_w(S_o(\eta_w))$. The expected profit of the firm i when his pseudo-valuation is $v_i \equiv S_o(\theta_i)$ and the bid is $b < \bar{b}$ given that firm j bids using $\beta_j(.)$ is

$$\begin{aligned} \pi(b, v_i) &= (v_i - b)Prob[b > \beta_j(S_o(\theta_j))] \\ &= (v_i - b)Prob[\phi_j(b) > S_o(\theta_j)] \\ \therefore \pi(b, v_i) &= (v_i - b)\mathcal{H}_j(\phi_j(b)). \end{aligned} \tag{3}$$

The first-order condition for firm i, in order to maximize its expected profit is

$$(v_i - b)h_j(\phi_j(b))\phi_j'(b) - \mathcal{H}_j(\phi_j(b)) = 0.$$

At the equilibrium, $b = \beta_i(v_i)$, i.e ,$\beta_i^{-1}(b) = v_i$, or in other words, $\phi_i(b) = v_i$. Therefore, the first-order conditions when both the firms maximize their expected profits simultaneously are,

$$(\phi_i(b) - b)h_j(\phi_j(b))\phi_j'(b) - \mathcal{H}_j(\phi_j(b)) = 0 \qquad \forall\, i, j = s, w. \tag{4}$$

Substituting for $\mathcal{H}(.)$ and $h(.)$ in equation (3), we get

$$(\phi_i(b) - b)f_j(S_o^{-1}(\phi_j(b)))(S_o^{-1}(\phi_j(b)))' = F_j(S_o^{-1}(\phi_j(b))) \qquad \forall\, i, j = s, w. \tag{5}$$

A solution to this system of differential equations, with relevant boundary conditions (Claim 1) constitutes an equilibrium of the first-score auction. Since it is difficult to obtain a general solution without a specific functional form for $S_o(\theta)$, we consider the following example.

2.2 An example

Let $V(q) = 2\sqrt{q}$ and $c(q, \theta) = \dfrac{q}{\theta}$. Note that $V(q)$ is increasing and concave in q and $c(q, \theta)$ is decreasing and convex in the efficiency parameter θ. Under *Naive* scoring rule, $s(q) = V(q)$ therefore in this example, $S_o(\theta) = \max_q 2\sqrt{q} - \dfrac{q}{\theta}$ while the first-order condition is

$$\begin{aligned} 2(\frac{1}{2\sqrt{q}}) - \frac{1}{\theta} &= 0 \\ \implies q_o(\theta) &= \theta^2. \end{aligned}$$

Therefore, $S_o(\theta) = 2\theta - \theta = \theta$ and $\mathcal{H}(.) = F(.)$.

***Proposition 1* :**

(a) The bidding strategies of the firms are

$$\beta_j^{FS}(\theta) = \underline{\eta} + \frac{1}{k_j(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_j(\theta - \underline{\eta})^2}\,\right) \qquad \forall j = s, w. \tag{6}$$

(b) In this first-score auction, each firm in equilibrium offers

$$q_s^{FS}(\theta) = q_w^{FS}(\theta) = q_o(\theta) = \theta^2 \tag{7}$$

$$p_i^{FS}(\theta) = 2\theta - \underline{\eta} - \frac{1}{k_i(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_i(\theta - \underline{\eta})^2}\,\right) \tag{8}$$

where $k_j = \dfrac{1}{(\eta_i - \underline{\eta})^2} - \dfrac{1}{(\eta_j - \underline{\eta})^2} \qquad \forall i, j = s, w.$

***Proof*:** The above is derived using the techniques shown in Maskin and Riley (2000). See the Appendix.

It can be verified that $\beta_i(.)$ is increasing in θ for both $i = s, w$ and while β_s is concave, β_w is a convex function in θ.

***Proposition 2*:** $\beta_s^{FS}(\theta) < \beta_w^{FS}(\theta) \quad \forall \theta \in [\underline{\eta}, \eta_w]$

***Proof*:** To prove this, it suffices to show that $\phi_s^{FS}(b) > \phi_w^{FS}(b)$. Since, $k_s = -k_w$,

$$\begin{aligned}
& 1 - k_s(b-\eta)^2 < 1 + k_s(b-\eta)^2 = 1 - k_w(b-\eta)^2 \\
\text{or} \quad & (1 - k_s(b-\eta)^2)^{-1} > (1 - k_w(b-\eta)^2)^{-1} \\
\implies \quad & \underline{\eta} + \frac{2(b - \underline{\eta})}{1 - k_s(b - \underline{\eta})^2} > \underline{\eta} + \frac{2(b - \underline{\eta})}{1 - k_w(b - \underline{\eta})^2} \\
\text{or} \quad & \phi_s^{FS}(b) > \phi_w^{FS}(b) \\
\implies \quad & \beta_s^{FS}(\theta) < \beta_w^{FS}(\theta) \quad \forall \theta \in [\underline{\eta}, \eta_w].
\end{aligned}$$

Hence, *in this first-score auction of with two asymmetric bidders, the weaker firm bids more aggressively than the stronger firm.* Figure 1 depicts the equilibrium bidding strategies when $\underline{\eta} = 0.1$, $\eta_s = 0.9$ and $\eta_w = 0.5$.

***Corollary 1*:**

(a) The expected winning offer in this first-score auction is

$$E(q)^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} q_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta \tag{9}$$

$$E(p)^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} p_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta. \tag{10}$$

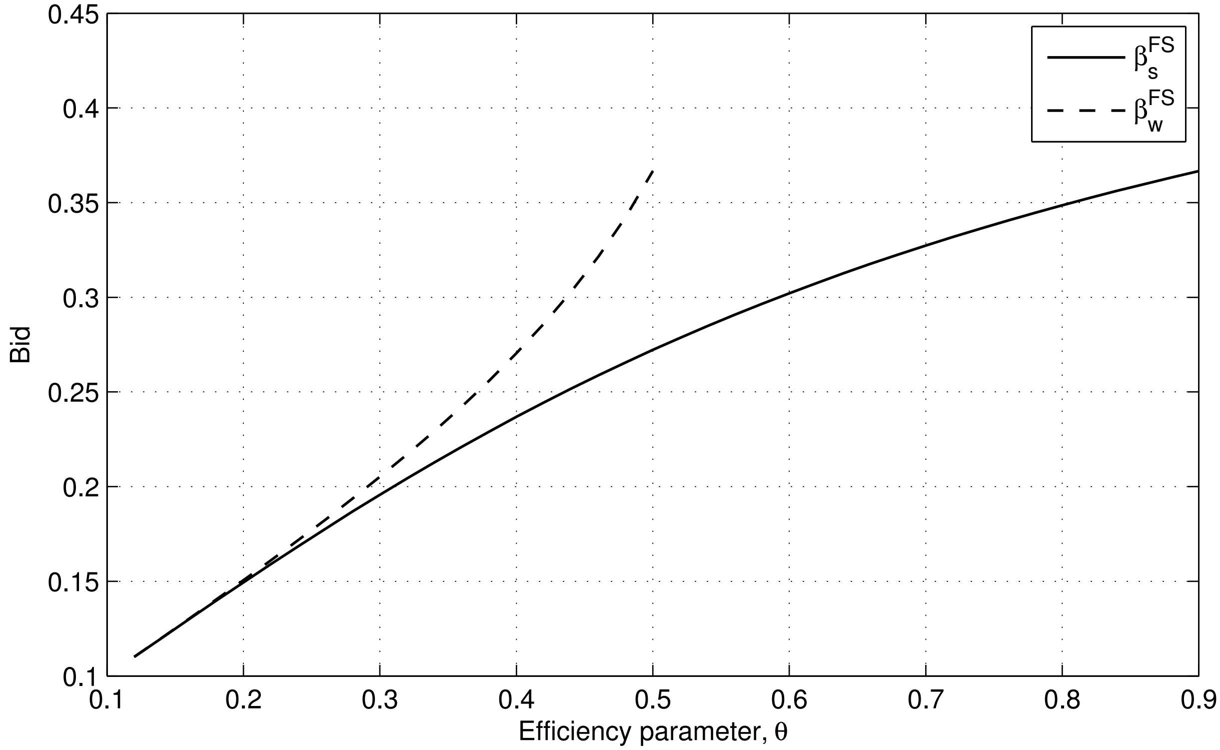

Figure 1: Equilibrium in Asymmetric First-Score Auction ($\underline{\eta} = 0.1$, $\eta_w = 0.5$, $\eta_s = 0.9$)

(b) Under Naive-Scoring rule, the buyer's expected utility under this first-score auction is

$$EU^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} \beta_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta. \tag{11}$$

3 Asymmetric Second-Score Procurement Auction

In a *second-score auction*, the winning firm is asked to match the second highest score in the auction. However, it is not essential to offer the exact price and quality combination that the firm with the second highest score did.

3.1 Bidding Strategies

Again, we define $S_o(\theta) = \max_q s(q) - c(q, \theta)$ and $q_o(\theta) = argmax_q s(q) - c(q, \theta)$ to convert the two-dimensional second-score auction into a unidimensional exercise.

Lemma 2 **:** The promised quality level chosen by a firm of type θ_i in this second-score auction is

$$q_o(\theta_i) = argmax_q s(q) - c(q, \theta_i)$$

for all $\theta_i \in (\underline{\eta}, \eta_i]$; $i = s, w$.

Proof**:** Similar to the proof of Lemma 1.

The expected profit of the firm i in the second-score auction when there are only two bidders with pseudo-valuations is $v_i \equiv S_o(\theta_i)$ $\forall i$ and the bids are β_i, is

$$\pi(\beta_i, v_i) = E\left[(v_i - \beta_j) I_{\beta_i > \beta_j}\right] \qquad \forall\, i, j = s, w. \tag{12}$$

Proposition 3 **:**

(a) The bidding strategies of the firms are

$$\beta_i^{SS}(\theta) = S_o(\theta) \qquad \forall i = s, w. \tag{13}$$

(b) In this second-score auction, each firm in equilibrium offers

$$q_i^{SS}(\theta) = q_o(\theta) \tag{14}$$

$$p_i^{SS}(\theta) = c(q_o(\theta), \theta) \qquad \forall i = s, w. \tag{15}$$

Proof**:** Truthful bidding remains a weakly dominant strategy even when we introduce asymmetry amongst the bidders. See the Appendix.

Let $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$. Then, the Expected Utility under second-score auction is given as

$$EU^{SS} = E\{V q_o(\theta_1) - p(\theta_1, \theta_2)\} \tag{16}$$

where $p(\theta_1, \theta_2)$ is the payment made by buyer and is equal to $s(q_o(\theta_1)) - s(q_o(\theta_2)) + c(q_o(\theta_2), \theta_2)$.

The expected winning offer in the second-score auction is

$$E(q)^{SS} = E\{q_o(\theta_1)\} \tag{17}$$

$$E(p)^{SS} = E\{s(q_o(\theta_1)) - s(q_o(\theta_2)) + c(q_o(\theta_2), \theta_2)\} \tag{18}$$

Under Naive Scoring Rule, the buyer's expected utility under the second-score auction becomes

$$\begin{aligned} EU^{SS} &= E\{s(q_o(\theta_2)) - c(q_o(\theta_2)\theta_2) \\ &= E\{S_o(\theta_2)\}. \end{aligned} \tag{19}$$

3.2 An example

We assume the same functional forms for $V(q)$ and $c(q,\theta)$ as we did in Section 2.2 in order to solve for the equilibrium bidding strategies in the second-score auction i.e. $s(q) = V(q) = 2\sqrt{q}$ and $c(q,\theta) = \dfrac{q}{\theta}$. Using these functional forms, we have $q_o(\theta) = \theta^2$, $S_o(\theta) = \theta$, $\mathcal{H}(.) = F(.)$ and therefore the bidding functions are

$$\beta_i^{SS}(\theta) = \theta \ \forall i = s, w. \tag{20}$$

The expected winning offer is

$$E(q)^{SS} = E\{\theta_1^2\} \tag{21}$$

$$E(p)^{SS} = E\{2\theta_1 - \theta_2\} \tag{22}$$

where $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$.

And the buyer's expected utility in this case is

$$EU^{SS} = E\{\theta_2\}. \tag{23}$$

4 Expected Utility Comparison

We now compare the expected utilities of the buyer under the two auction formats. Under the naive scoring rule, the expected utilities to the buyer in the first-score and the second-score auctions are

$$EU^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} \beta_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta \tag{24}$$

$$EU^{SS} = E\{Min(\theta_s, \theta_w)\} \tag{25}$$

where $\beta_i^{FS}(\theta) = \underline{\eta} + \dfrac{1}{k_i(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_i(\theta - \underline{\eta})^2}\right)$ and $\phi_i^{FS}(\theta) = \underline{\eta} + \dfrac{2(b - \underline{\eta})}{1 - k_j(b - \underline{\eta})^2}$. Figure 2 depicts the equilibrium strategies for both the firms in first-score and second-score auctions for $\underline{\eta} = 0.1$, $\eta_s = 0.9$ and $\eta_w = 0.5$.

***Proposition 4*:** $\beta_i^{FS}(\theta) < \beta_i^{SS}(\theta) \quad \forall i = s, w.$

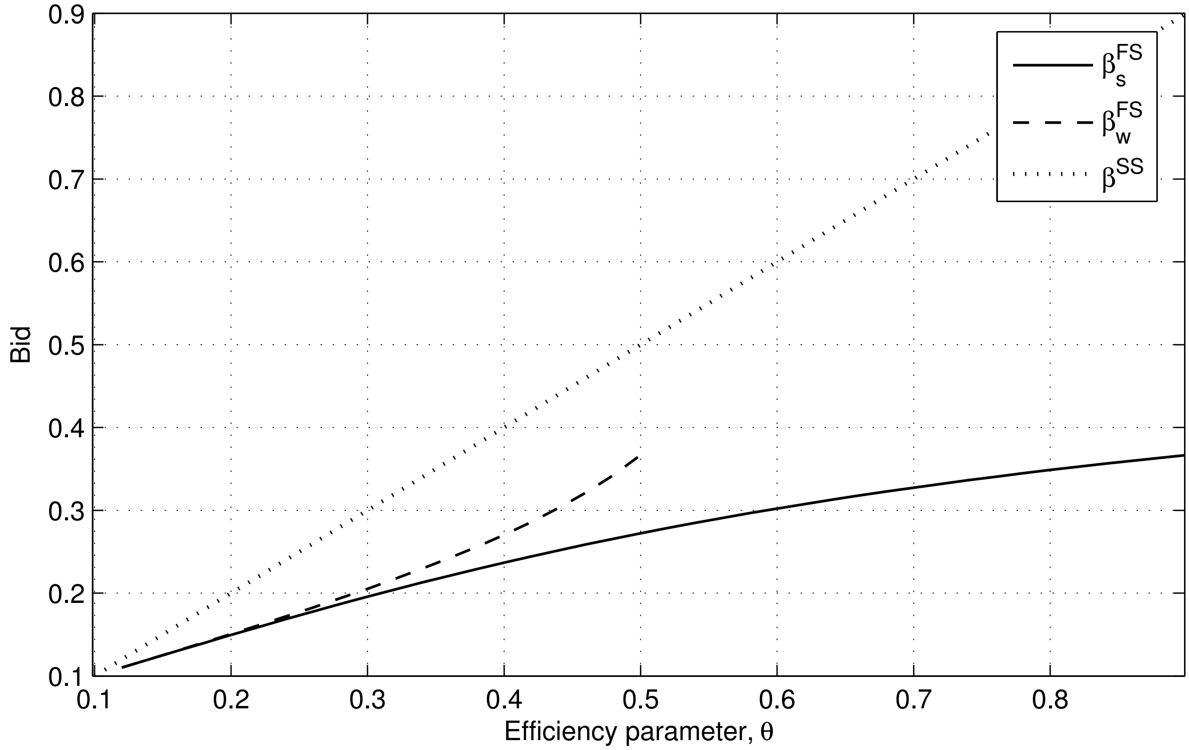

Figure 2: Equilibrium in Asymmetric First-Score and Second-Score Auction

***Proof*:** We know that $\beta_i^{SS}(\theta) = \theta \quad \forall i$. Now, $\beta_w^{FS}(\theta)$ cannot lie strictly above the $b = \theta$ axis since it intersects at a point say, $t_w \in (\underline{\eta}, \eta_w]$, as calculated below.

$$\underline{\eta} + \frac{1}{k_w(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_w(\theta - \underline{\eta})^2}\,\right) = \theta$$
$$-1 + \sqrt{1 + k_w(\theta - \underline{\eta})^2} = k_w(\theta - \underline{\eta})^2$$
$$(\sqrt{1 + k_w(\theta - \underline{\eta})^2} - 1)\sqrt{1 + k_w(\theta - \underline{\eta})^2} = 0.$$

$\therefore$ Either $\theta = \underline{\eta}$ or $\theta = \underline{\eta} \pm \dfrac{1}{\sqrt{-k_w}}$. Since, $k_w < 0$, these expressions give us one and only one real point of intersection $t_w = \underline{\eta} + \dfrac{1}{\sqrt{-k_w}}$ within the interval $(\underline{\eta}, \eta_s]$.

Therefore ,the curve $\beta_w^{FS}(\theta)$ lies strictly below the $b = \theta$ axis, when θ takes values form the interval $[\underline{\eta}, t_w]$. Now the bid corresponding to t_w where the curves $\beta_w^{SS}(\theta) = \theta$ and $\beta_w^{FS}(\theta)$ intersect is greater than the upper bound of the bids, $\bar{b}$. Thus, the two curves will never intersect within the interval $[\underline{\eta}, \eta_w]$ and $\beta_w^{FS}(\theta)$ would strictly lie below $\beta_w^{SS}(\theta) \quad \forall\, \theta \in (\underline{\eta}, \eta_w]$.

Similarly we will have the above expressions for the strong firm. However, the only valid point of intersection is $\theta = \underline{\eta}$ which lies outside the interval $(\underline{\eta}, \eta_s]$. The other two expressions do not return real values since $k_s > 0$. Therefore, the curve $\beta_s^{FS}(.)$ and $\beta_s^{SS}(.)$ do not intersect other than their common point of origin. Since, $\beta_s^{FS}(.)$ is increasing and concave in θ, this possible only when $\beta_s^{FS}(.)$ lies strictly below $\beta_s^{SS}(.)$.

Since, $\beta_i^{FS}(\theta) < \theta$, $\phi_j^{FS}(\beta_i^{FS}(\theta)) \neq \theta$. In fact, $\phi_w^{FS}(\beta_s^{FS}(\theta)) < \theta$ and $\phi_s^{FS}(\beta_w^{FS}(\theta)) > \theta$. Note that

$$\begin{aligned} \phi_s^{FS}(b) &> \phi_w^{FS}(b) \\ \phi_s^{FS}(\beta_s^{FS}(\theta)) &> \phi_w^{FS}(\beta_s^{FS}(\theta)) \\ \implies \theta &> \phi_w^{FS}(\beta_s^{FS}(\theta)) \qquad \forall \theta \in [\underline{\eta}, \eta_s]. \end{aligned}$$

Similarly, for all θ in $[\underline{\eta}, \eta_w]$, $\phi_s^{FS}(\beta_w^{FS}(\theta)) > \theta$.

Now,

$$EU^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} \beta_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta \tag{26}$$

$$= \int_{\underline{\eta}}^{\eta_s} \beta_s^{FS}(\theta) \frac{\phi_w^{FS}(\beta_s^{FS}(\theta)) - \underline{\eta}}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} d\theta + \int_{\underline{\eta}}^{\eta_w} \beta_w^{FS}(\theta) \frac{\phi_s^{FS}(\beta_w^{FS}(\theta)) - \underline{\eta}}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} d\theta \tag{27}$$

We perform the change of variables, $\phi_s^{FS}(b) = \theta$ and $\phi_w^{FS}(b)$ in the two integrals respectively.

$$EU^{FS} = \int_{\underline{\eta}}^{\bar{b}} b \frac{\phi_w^{FS}(b) - \underline{\eta}}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \frac{d\phi_s^{FS}(b)}{db} db + \int_{\underline{\eta}}^{\bar{b}} b \frac{\phi_s^{FS}(b) - \underline{\eta}}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \frac{d\phi_w^{FS}(b)}{db} db \tag{28}$$

$$= \frac{1}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \int_{\underline{\eta}}^{\bar{b}} b \left[(\phi_w^{FS}(b) - \underline{\eta}) \frac{d\phi_s^{FS}(b)}{db} + (\phi_s^{FS}(b) - \underline{\eta}) \frac{d\phi_w^{FS}(b)}{db} \right] db \tag{29}$$

$$= \frac{1}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \int_{\underline{\eta}}^{\bar{b}} b \left[\frac{d}{db} (\phi_w^{FS}(b) - \underline{\eta})(\phi_s^{FS}(b) - \underline{\eta}) \right] db \tag{30}$$

$$= \bar{b} - \frac{1}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \int_{\underline{\eta}}^{\bar{b}} (\phi_w^{FS}(b) - \underline{\eta})(\phi_s^{FS}(b) - \underline{\eta}) db \tag{31}$$

Since the term within the integral is always positive for all $b \in [\underline{\eta}, \bar{b}]$, $EU^{FS} \leq \bar{b}$.

In fact, the expected utility in the first-score auction turns out to be

$$EU^{FS} = \bar{b} - \frac{(\eta_s - \underline{\eta})^2(\eta_w - \underline{\eta})^2}{(\eta_s - \eta_w)^{3/2}(\eta_w + \eta_s - 2\underline{\eta})^{3/2}} \left[\log\left(\frac{\sqrt{\eta_w + \eta_s - 2\underline{\eta}} + \sqrt{\eta_s - \eta_w}}{\sqrt{\eta_w + \eta_s - 2\underline{\eta}} - \sqrt{\eta_s - \eta_w}} \right) - 2 \arctan \sqrt{\frac{\eta_s - \eta_w}{\eta_w + \eta_s - 2\underline{\eta}}} \right] \quad (32)$$

Similarly, we can calculate the expected utility to the buyer in the second-score auction using the expression in equation (25).

$$EU^{SS} = \frac{1}{6(\eta_s - \underline{\eta})} \left[(3\eta_s - \eta_w)(\eta_w + \underline{\eta}) - 4\underline{\eta}^2 \right] \quad (33)$$

Since it is difficult to compare EU^{FS} and EU^{SS} for arbitrary values of $\underline{\eta}, \eta_s$ and η_w we assume that $\eta_w = \underline{\eta} + \dfrac{1}{1+\alpha}$ and $\eta_s = \underline{\eta} + \dfrac{1}{1-\alpha}$ where $0 < \alpha < 1$ denotes the level of asymmetry between the two firms. For $\alpha = 0$, the firms are symmetric, i.e. $\eta_s = \eta_w = \eta$ (say), and the expected utility to the buyer under both the scoring auctions is same.

$$EU^{sym} = \frac{\eta + 2\underline{\eta}}{3} \quad (34)$$

The expected utility under second score auction when $\alpha > 0$ is

$$EU^{SS} = \frac{1 + 2\alpha + 3\underline{\eta}(1 - \alpha^2)}{3(1 + \alpha)^2}$$

which can be easily seen to be decreasing in α. It is not straightforward to check whether the expected utility under the first-score auction is increasing or decreasing in α. We consider the distribution of the winning bid, say $M(.)$ where

$$\begin{aligned} M(b) &= Prob[Max\{\beta_s(\theta_s), \beta_w(\theta_w)\} \leq b] \qquad (\because S_o(\theta) = \theta) \\ &= Prob[\theta_s \leq \phi_s(b)]Prob[\theta_w \leq \phi_w(b)] \\ &= F_s(\phi_s(b))F_w(\phi_w(b)) \\ &= (1 - \alpha^2)(\phi_s(b) - \underline{\eta})(\phi_w(b) - \underline{\eta}) \end{aligned}$$

For the above values of η_s and η_w, $k_s = -k_w = 4\alpha$ and $\bar{b} = \underline{\eta} + 1/2$. Therefore, the distribution of the equilibrium bid is

$$M(b) = \frac{4(1 - \alpha^2)(b - \underline{\eta})^2}{1 - 16\alpha^2(b - \underline{\eta})^4}$$

$\underline{\eta}$	η_w	η_s	EU^{FS}	EU^{SS}
.1	.5	.9	.28	.27
1	2	3	1.46	1.42
1	2	5	1.59	1.46
1	3	5	3.15	1.83
1	4	5	7.03	2.12
1.3	1.6	7.8	1.55	1.45
10	47	59	24.13	23.84

Table 1: Expected Utilities in the First Score and Second-Score Auction

which is decreasing in α.

Now, the expected utility under first-score auction is nothing but the expected value of the equilibrium bid i.e.

$$\begin{aligned} EU^{FS} &= \int_{\underline{\eta}}^{\underline{\eta}+1/2} b \; dM(b) \\ &= \underline{\eta} + 1/2 - \int_{\underline{\eta}}^{\underline{\eta}+1/2} dM(b) \qquad (\because \text{Integrating by parts}) \end{aligned}$$

Therefore the expected utility under first-score auction is increasing in α since distribution function $M(.)$ is decreasing in α. Now for $\alpha = 0$, the firms are symmetric and the expected utilities under both the auction are same. However, an increase in α would lead to a decrease in EU^{SS} and an increase in EU^{FS}. Thus we find that $EU^{FS} > EU^{SS}$ for all values of α i.e. the expected utility generated from the second-score auction is lower than that from the first-score auction which is congruous with the result shown in Maskin and Riley (2000) who show that if the strong bidder's type distribution is a "shifted" or a "stretched" version of that of the weak bidder, the expected revenue from the first price auction dominates that from the second.

We also summarize some numerical examples in Table 1 for generic values of $\underline{\eta}, \eta_w$ and η_s which show that the expected utilities generated from both the first-score and the second-score auctions are not equal to one another.

5 Conclusion

In this paper, we compute the equilibrium strategies of the firms in two procurement mechanisms, the first-score auction and the second-score auction, where the suppliers are asymmetric in their costs of productions. In our constructed example, while the types of both the strong and weak sellers are drawn from a uniform distribution, the strong seller's type distribution is a "stretched" version of that of the weak seller. The bidders are required to submit a two-dimensional bid quoting the level of quality promised to be delivered and the payment in return. These bids are then evaluated using a scoring rule and the firm with the highest score is declared the winner. We find that under these two auction formats, the expected utility to the buyer no longer remains the same when the bidding firms are asymmetric in nature. Thus the Revenue Equivalence of the Scoring Auctions as postulated by Che (1993) breaks down in the presence of asymmetry.

Maskin and Riley (2000) proved that with asymmetric uniformly distributed values, the first-price auction is revenue superior than the open or Vickrey auction. We find a similar result in our setup, as bidders in our model bid in an identical environment as they map their pseudo-valuations into scores. Since the probability that the weaker seller wins the procurement auction is positive in the first-score auction, it is deemed to be inefficient; this is in contrast to the second-score auction, where it is a weakly dominant strategy for both the firms to bid their pseudo-valuation.

Appendix

Proof of Lemma 1 :
Suppose for firm j with $\theta_j < \eta_s$, the equilibrium bid is (q_j, p_j) where $q_j \neq q_o(\theta_j)$. Then we show that this bid-tuple (q_j, p_j) is strictly dominated by another tuple (Q_j, P_j) where $Q_j = q_o(\theta_j)$ and $P_j = p_j + s(q_o(\theta_j)) - s(q_j)$. Note that $S(q_j, p_j) = S(Q_j, P_j)$ as $S(Q_j, P_j) = s(Q_j) - P_j = s(q_o(\theta_j)) - [p_j + s(q_o(\theta_j)) - s(q_j)] = s(q_j) - p_j = S(q_j, p_j)$

To show the strict dominance, we must show that $\pi(Q_j, P_j|\theta_j) > \pi(q_j, p_j|\theta_j)$.
Now,

$$\begin{aligned}\pi(Q_j, P_j|\theta_j) &= \{P_j - c(Q_j, \theta_j)\}.Prob\,[\,win\,|S(Q_j, P_j)] \\ &= \{p_j + s(q_o(\theta_j)) - s(q_j) - c(q_o(\theta_j), \theta_j)\}.Prob\,[\,win\,|S(q_j, p_j)] \\ &= \{p_j - c(q_j, \theta_j) + (s(q_o(\theta_j)) - c(q_o(\theta_j), \theta_j)) - (s(q_j) - c(q_j, \theta_j))\}.Prob[win|S(q_j, p_j)]\end{aligned}$$

Since $q_o(\theta_j) = argmax\, s(q_j) - c(q_j, \theta_j)$ and $q_j \neq q_o(\theta_j)$, $\quad s(q_o(\theta_j)) - c(q_o(\theta_j), \theta_j) > s(q_j) - c(q_j, \theta_j)$.

$\therefore$ $\pi(Q_j, P_j|\theta_j) > \{p_j - c(q_j, \theta_j)\}.Prob\,[\,win\,|S(q_j, p_j)]$, which is nothing but equal to the $\pi(q_j, p_j|\theta_j)$,provided $Prob\,[\,win\,|S(q_j, p_j)] > 0$.

Claim: $Prob\,[\,win\,|S(q_j, p_j)] > 0$

To prove this claim, define $\underline{S} = inf\{S|Prob\,[\,win\,|S(q_j, p_j)] > 0\}$. Then $S_o(.)$ is a increasing function in θ_j. Also, $\underline{S} \leq S_o(\underline{\eta})$ for the trade to always take place. We shall prove the claim by contradiction. Suppose $\exists$ a $\theta_m > \underline{\eta}$ such that $Prob\,[\,win\,|S(q_m, p_m)] = 0$. Then the chosen score $S_m = S(q_m, p_m)$ must be such that $S_m \leq \underline{S}$. However, since $\underline{S} \leq S_o(\underline{\eta}) \leq S_o(\theta_m)$, firm with cost type θ_m can bid a score $S'_m \in (\underline{S}, S_o(\theta_m))$ which allows positive profits for that firm, thereby contradicting the optimality of the score choice S_m. Hence the claim.

So, with the proof of this claim, we have shown that $\pi(Q_j, P_j|\theta_j) > \pi(q_j, p_j|\theta_j)$ which contradicts the fact the bid-tuple (q_j, p_j) is the optimal choice for the firm j.

Proof of Claim 1 :
We prove the above claim by contradiction. Suppose $\beta_i(S_o(\underline{\eta})) \neq S_o(\underline{\eta})\ \forall\ i = s, w$. Then $\beta_i(S_o(\underline{\eta}))$ cannot be greater than $S_o(\underline{\eta})$, for the firm then will incur a loss if it wins the auction. Also, it is not a dominant strategy for the firm of the lowest type $\underline{\eta}$ to bid less than $S_o(\underline{\eta})$ since the probability of winning the auction reduces with the bid moving further away from the valuation. To elaborate further, suppose both the realized types are $\underline{\eta}$ and while $\beta_i(S_o(\underline{\eta})) < S_o(\underline{\eta})$, let us suppose $\beta_j(S_o(\underline{\eta})) = S_o(\underline{\eta}) \quad \forall i \neq j$. So, firm i will lose the auction even when it could have won, by bidding $\beta_i(S_o(\underline{\eta})) = S_o(\underline{\eta})$.
Moreover, if $\beta_s(S_o(\eta_s)) > \beta_w(S_o(\eta_w))$, then the strong bidder of type η_s would win with probability 1. However, it can increase its payoff by bidding slightly less than $\beta_s(S_o(\eta_s))$ and likewise, will get maximum benefit by bidding equal to $\beta_w(S_o(\eta_w))$.

Proof of Proposition 1 :
Substituting $S_o(\theta) = \theta$ and $\mathcal{H}(.) = F(.)$ in equation (5), we get

$$\begin{aligned}
(\phi_i(b) - b) f_j(\phi_j(b))\phi'_j(b) &= F_j(\phi_j(b)) \qquad \forall\, i, j = s, w \\
\implies (\phi_i(b) - b)\frac{1}{\eta_j - \underline{\eta}}\phi'_j(b) &= \frac{\phi_j(b) - \underline{\eta}}{\eta_j - \underline{\eta}} \\
\implies (\phi_i(b) - b)\phi'_j(b) &= \phi_j(b) - \underline{\eta} \qquad \text{(A-1)}
\end{aligned}$$

which is equivalent to

$$(\phi_i(b) - b)(\phi'_j(b) - 1) = \phi_j(b) - \underline{\eta} - \phi_i(b) + b$$

Adding the two equations for $i, j = s, w$, we get

$$\frac{d}{db}(\phi_s(b) - b)(\phi_w(b) - b) = 2b - 2\underline{\eta} \qquad \text{(A-2)}$$

Integrating this, we obtain

$$(\phi_s(b) - b)(\phi_w(b) - b) = b^2 - 2\underline{\eta}b + K \qquad \text{(A-3)}$$

where K is the constant of integration. Substituting $\phi_s(\underline{\eta}) = \phi_w(\underline{\eta}) = \underline{\eta}$, we get

$0 = \underline{\eta}^2 - 2\underline{\eta}^2 + K$ which implies that $K = \underline{\eta}^2$.

Therefore the above equation becomes,

$$(\phi_s(b) - b)(\phi_w(b) - b) = (b - \underline{\eta})^2 \tag{A-4}$$

Using Claim 1, we can calculate $\bar{b}$ by substituting $\phi_i(\bar{b}) = \eta_i \qquad \forall i = s, w$.

$$(\eta_s - \bar{b})(\eta_w - \bar{b}) = (\bar{b} - \underline{\eta})^2$$

which implies that,

$$\bar{b} = \frac{\eta_s \eta_w - \underline{\eta}^2}{\eta_s + \eta_w - 2\underline{\eta}} \tag{A-5}$$

Now, we can rewrite equation (A-1) as,

$$\phi_j'(b) = \frac{(\phi_j(b) - \underline{\eta})(\phi_j(b) - b)}{(b - \underline{\eta})^2} \tag{A-6}$$

We apply a change of variables by defining $(\phi_j(b) - \underline{\eta}) = \Phi_j(b)$ and $(b - \underline{\eta}) = B$. Therefore, the above equation reduces to,

$$\Phi_j'(B) = \frac{\Phi_j(B)(\Phi_j(B) - B)}{B^2} \tag{A-7}$$

Let $\Phi_j(B) - B = B\Gamma_j(B)$. Then

$$\Phi_j'(B) - 1 = \Gamma_j(B) + B\Gamma_j'(B)$$

Using this, equation (A-7) becomes,

$$\Gamma_j(B) + B\Gamma_j'(B) + 1 = \Gamma_j(B)(\Gamma_j(B) + 1) \tag{A-8}$$

or

$$\frac{\Gamma_j'(B)}{(\Gamma_j^2(B) - 1)} = \frac{1}{B}$$

Using integration by partial fractions, we obtain,

$$\Gamma_j(B) = \frac{1 + k_j B^2}{1 - k_j B^2} \tag{A-9}$$

where k_j is a constant of integration $\forall j = s, w$

Reverting back to the original variables,

$$\begin{aligned} \frac{\Phi_j(B)}{B} - 1 &= \frac{1 + k_j B^2}{1 - k_j B^2} \\ \text{or} \qquad \Phi_j(B) &= \frac{2B}{1 - k_j B^2} \end{aligned} \tag{A-10}$$

$\therefore \qquad \forall j = s, w,$

$$\phi_j^{FS}(b) = \underline{\eta} + \frac{2(b - \underline{\eta})}{1 - k_j (b - \underline{\eta})^2} \tag{A-11}$$

Since $\phi_j(\bar{b}) = \eta_j$, where $\bar{b}$ is defined in (A-5), we obtain the constants of integration as

$$k_j = \frac{1}{(\eta_i - \underline{\eta})^2} - \frac{1}{(\eta_j - \underline{\eta})^2} \qquad \forall j = s, w \tag{A-12}$$

The bidding strategies, obtained by inverting (A-11) are,

$$\beta_j^{FS}(\theta) = \underline{\eta} + \frac{1}{k_j(\theta - \underline{\eta})} \left(-1 + \sqrt{1 + k_j(\theta - \underline{\eta})^2} \right) \qquad \forall j = s, w \tag{A-13}$$

Proof of Proposition 3:
To prove that, let $\beta_j = v_j$ be the equilibrium strategy for the firm j. What is the optimal response for firm i ?

Equation , then, can be rewritten as

$$\pi(\beta_i, v_i) = E\left[(v_i - \tilde{v_j}) I_{\beta_i > \tilde{v_j}}\right] \qquad \forall\, i, j = s, w \tag{A-14}$$

where $\tilde{v_j}$ is the observable v_j, a random variable ,since the actual pseudo-valuation of the firm i is unknown. Therefore,

$$\begin{aligned} \pi(\beta_i, v_i) &= \int_{S_o(\underline{\eta})}^{\beta_i} (v_i - x)\, h_j(x) dx \\ &= \int_{S_o(\underline{\eta})}^{\beta_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx \end{aligned} \tag{A-15}$$

Firm i's problem is choose β_i so as to maximize equation (A-15). We show that β_i is neither greater than nor lesser than v_i.

Let $\boldsymbol{\beta_i < v_i}$
If β_i is increased to v_i, the change in the integral in equation (A-15) is

$$\Delta\pi = \int_{\beta_i}^{v_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx$$

Since, $f_j(.)$ and $S_o^{-1}(.)$ are both increasing and $\beta_i < x < v_i$, we see that $\Delta\pi > 0$. Therefore, β_i , cannot be less than v_i.

Let $\boldsymbol{\beta_i > v_i}$
The difference in profit in that case is

$$\Delta\pi = \int_{v_i}^{\beta_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx$$

which is negative for $v_i < x < \beta_i$. Firm i would deviate from its previous strategy $\beta_i > v_i$ to $\beta_i = v_i$.

Hence, the expected profit maximizing bids are

$$\beta_i^{SS} = v_i \qquad \forall i = s, w \tag{A-16}$$

This implies that,

$$\begin{aligned} s(q_o(\theta)) - p(\theta) = S_o(\theta) = s(q_o(\theta)) - c(q_o(\theta), \theta) \\ \therefore p(\theta) = c(q_o(\theta), \theta) \end{aligned}$$

Therefore, in the second-score auction, each firm in equilibrium offers

$$\begin{aligned} &q_i^{SS}(\theta) = q_o(\theta) \\ &p_i^{SS}(\theta) = c(q_o(\theta), \theta) \qquad \forall i = s, w \end{aligned} \tag{A-17}$$

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