Middlemen in Cooperative Games : The Intermediary Value

Surajit Borkotokey
Department of Mathematics
Dibrugarh University
Assam, India-786004

International Conclave on Foundations of Decision and Game Theory at IGIDR, Mumbai

14 March 2016

Outline

1. Introduction
   - Cooperative Games
   - The notion of the middlemen in Game theory
2. Main results
   - Middlemen in Cooperative games
   - The I-value and its Characterization
   - Comparison with the existing types
   - Examples
   - Alternative Characterizations
3. Conclusion

Cooperative Games

Definition

- There is a set of agents/players $N = \{1, 2, 3, ..., n\}$.
- Each subset (or coalition) S of agents can work together in various ways, leading to various utilities for the agents.
- Cooperative/coalitional game theory studies which outcome will/should materialize.
- Key criteria:
  - Stability: No coalition of agents should want to deviate from the solution and go their own way.
  - Fairness: Agents should be rewarded for what they contribute to the group.

Cooperative game Formally

- Let $N = \{1, ..., n\}$ be a finite set of players.
- $v : 2^N \to \mathbb{R}$, is the characteristic function from the set of all possible coalitions of players that satisfies $v(\emptyset) = 0$.
- A Cooperative game (transferable utility game) is characterized by two main factors:
  - the player set N and
  - the characteristic function $v : 2^N \to \mathbb{R}$.
- Let $\mathscr{G}(N)$ denote the universal game space consisting of all TU Cooperative games.

Solutions:

- The main assumption in cooperative game theory is that the grand coalition $v(N)$ will form.
- Then the challenge is to allocate the payoff among the players N in some fair way (or Stability ensured).
- A solution concept evaluates how much will be paid to a player for participating in a game.
- A solution concept for TU Cooperative game is a function that assigns a set of payoff vectors to each player in a Cooperative game.

Example

The core solution (Gillies, 1952), The Shapley value (Shapley, 1953), Banzhaf value (Banzhaf, 1965), Compromise value (Tijs,1993), Nucleolus (Schmeidler, 1969), Aumann-Shapley value (Aumann, 1995) and many more...

Shapley Value (Shapley, 1953)

The Shapley value is the aggregation of the marginal contributions of a player in each coalition and is given by,

$$\Phi_i^{Sh}(v) = \sum_{S \subseteq N \,:\, i \in S} \frac{(s-1)!\,(n-s)!}{n!}[v(S) - v(S \backslash i)] \qquad (1)$$

Theorem

The Shapley value is the unique value satisfying the following axioms.

- *Efficiency :* $\sum_{i \in N} \Phi(v) = v(N)$
- *Linearity:* $\Phi_i(\alpha u + \beta v) = \alpha \Phi_i(u) + \beta \Phi_i(v)$
- *Null player property:* $\Phi_i(v) = 0$ *for every null player* $i \in N$
- *Anonymity :* $\Phi_i(v) = \Phi_{\pi i}(\pi v)$.

The Nucleolus (Schmeidler, 1969)

- Consider v :
  $\{\mathbf{x} \in \mathbb{R} \mid \sum_{i \in N} x_i = v(N)$ and $x_i \geq v(i)\ \forall i \in N\} \neq \emptyset$.
- Let $\mathbf{x} \in \mathbb{R}^n$ be an imputation.
- The excess of a coalition $S \subseteq N$ at $\mathbf{x}$ is the real number $e(S, \mathbf{x}) = v(S) - \sum_{i \in S} x_i$.
- At any imputation $\mathbf{x}$, let us denote by $\theta(\mathbf{x}) \in \mathbb{R}^n$ the vector of excesses arranged in non-increasing order, i.e., $\theta_l(\mathbf{x}) \geq \theta_{l'}(\mathbf{x})$ whenever $l < l'$.
- The nucleolus of v denoted by $\Phi^{\mathcal{N}}$ is the (unique) imputation that lexicographically minimizes the vector $\theta(\mathbf{x})$.

Marginalists vs Bargaining minus Stability

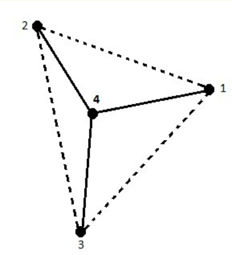
- The Shapley value is based on the well known marginalist principle in economic theory.
- The nucleolus is based on some endogenous bargaining process.
- The nucleolus need not be equal to the Shapley value.
- The Shapley value builds on the principle of fairness.
- The nucleolus does not consider fairness as a reasonable mean.

Remark

- Stability implies that no subset of players has an incentive to break off and work on its own.
- The Shapley value and the nucleolus are not necessarily stable.

An Example

- Network to form among four adjacent countries $1, 2, 3$ and 4.
    - The trans-national gas pipelines through Iran, Afganisthan and China...
    - The 32 nation Great Asian Highway project and road links to Europe.
- The benefits measured in terms of trade.
- The network needs a hub spoke architecture to attain maximum profit.
- If each country trades directly with each other then it is not so profitable.



âĂŃ

The Higway Problem

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$	$\emptyset$
v(S)	1.5	1	1	2	2	2	4	0
S	$\{4\}$	$\{1,4\}$	$\{2,4\}$	$\{3,4\}$	$\{1,2,4\}$	$\{1,3,4\}$	$\{2,3,4\}$	$\{1,2,3,4\}$
v(S)	0	5.5	5	5	5	5	5	6

Table: The Highway Problem

- The Shapley value with $N = \{1,2,3,4\} :: (1.42, 1.17, 1.17, 2.24)$.
- The Shapley value with $N = \{1,2,3\} :: (1.5, 1.25, 1.25)$.
- The Nucleolus with $N = \{1,2,3,4\} :: (1.3, 0.8, 0.8, 3.1)$.
- The Nucleolus with $N = \{1,2,3\} :: (1.6667, 1.16666, 1.16666)$.

Question?

Can this game be seen on a different light?

Players as catalyst!

Situations where players are not individually productive. But crucial in bringing out the synergies among other players.

- Grubhub Food Delivery and Takeout Service in the USA and Europe.
  - A mobile and online food ordering company that connects diners and corporate businesses with thousands of takeout restaurants.
  - It is not productive by itself but creates synergies among the customers.
- Similar examples include Uber Cab Services, Groupons, Ola etc.

Question?

Can we call them “The Middlemen"?

Middlemen: Definitions

Non-cooperative games

- A time saving institution who buys goods from the seller and then sells them to the buyer when direct communication between the buyer and the seller is not possible.
- An intermediary/middleman (or go-between) is a third party that offers intermediation services between two trading parties.
- The intermediary acts as a conduit for goods or services offered by a supplier to a consumer.
- In a larger sense, an intermediary can be a person or organization who or which facilitates a contract between two other parties.
- Middleman is responsible for trade or any other negotiations.

Intermediary Activities

- The notion of a *middleman* under non-cooperative framework is primarily attributed to the seminal work of Rubinstein and Wolinsky, 1987.
- Middlemen trade but don't originally own a good, don't physically alter the good, receive no consumption value from processing the good. (Biglaiser, 1993)
- The *middleman* is introduced as an intermediate node in the network through which all resources pass by (Yavas, 1994).
- Bailey and Bakos (1997) analyzed a number of case studies and identified four roles of electronic intermediaries including **information aggregating, providing trust, facilitating and matching**.
- Competition among intermediary service providers (Cailland and Jullien, 2003).
- Pricing in complex structures of Intermediation (Choi et al., 2015).

Traces of the middlemen in Cooperative Games

Buyer-Seller Model (ROTH 1988)-Middlemen

- Seller reservation price 100. Buyer 1 has reservation price 150 and Buyer 2 has reservation price 175.
- $v(S) =$ Max of the difference of the seller-buyer reservation prices.
- Trade will be in between the Seller and the buyer with maximum $v(S)$.

S	$\{S\}$	$\{B1\}$	$\{B2\}$	$\{S, B1\}$	$\{S, B2\}$	$\{B1, B2\}$	$\{S, B1, B2\}$
v(S)	0	0	0	50	75	0	75

Table: The Buyer-Seller Model : The Shapley value is $(45.83, 8.3, 20.3)$.

Question?

Why should we pay Buyer 1 when the actual trade took place between the Seller and Buyer 2 only?

Traces of the middlemen in Cooperative Games

Kalai, E., A. Postlewaite and J. Roberts, “Barriers to Trade and Disadvantageous Middlemen: Nonmonotonicity of the Core," Journal of Economic Theory, 19, No. 1, 1979, 200-209.

1. A player will choose whether its better to be middleman.
2. Core solution gives a kind of stability among the players.

Borkotokey, S., R. Kumar, S. Sarangi, A Solution Concept for Network Games : The Role of Multilateral Interactions, European Journal of Operational Research, 243, 2015, 912-920.

1. Players' multilateral interactions in a network studied.
2. Role of a player in various key positions (centrality) in generating worth.

The Highway Problem Revisited

- The Highway through Country 4 gives the maximum profit.
- Alternative passages are insignificant in comparison to the new highway.
- Country 4 facilitates some extra earning to all the countries and can charge some **intermediary fee** by giving her territorial land support.

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$	$\emptyset$
v(S)	1.5	1	1	2	2	2	4	0
S	$\{4\}$	$\{1,4\}$	$\{2,4\}$	$\{3,4\}$	$\{1,2,4\}$	$\{1,3,4\}$	$\{2,3,4\}$	$\{1,2,3,4\}$
v(S)	0	5.5	5	5	5	5	5	6

Table: The Highway Problem

Country 4 acts as the MIDDLEMAN!

The Formal Model : Middleman

Definition

A player $i \in N$ is said to engage in intermediary activities in a game $v \in \mathscr{G}(N)$ if

$$v(S \cup i) > v(S), \ \ \forall\, S \in \mathscr{N}_i \ \text{ and } v(\{i\}) = 0.$$

Definition

Given a $v \in \mathscr{G}(N)$, player $i \in N$ engages in intermediary activities for v if and only if i is endowed with a set $\boldsymbol{\zeta} = \left\{\zeta_S^i \in (0, \infty) | S \in \mathscr{N}_i\right\}$ such that

$$v(S \cup i) = \zeta_S^i + v(S) \ \text{ and } v(\{i\}) = 0.$$

Definition

The set $\boldsymbol{\zeta}$ is called a Scheme of Intermediary Activities (SIA) of player i.

The Middleman

Definition

An SIA, $\boldsymbol{\zeta} = \{\zeta_S^i \in (0,\infty) | S \in \mathscr{N}_i\}$ of player i is called equitable if $\zeta_S^i = \zeta_{S'}^i$ for every $S, S' \subseteq N$ with $s = s'$.

Definition

A player $i \in N$ is called a middleman for a game $v \in \mathscr{G}(N)$ if it leads to intermediary activities in v with a unique SIA.

Let the class of TU games with middlemen be denoted by $\mathscr{GM}$.

Notations

Every member of $\mathscr{GM}$ is represented by the quadruple $(N, \boldsymbol{\zeta}, \xi, v)$

Middlemen in TU Cooperative games

Assumption

- Every TU game with middlemen has exactly one middleman, i.e.,
  1. $\exists i \in N$ such that $\forall S \subset N \setminus i,\ v(S \cup i) > v(S)$ and
  2. $\forall j \neq i, j \in N\ v(\{j\}) \neq 0$ or $\exists\ T \subseteq N \setminus j\ :\ v(T \cup j) \ngtr v(T)$.
- The unique middleman i's equitable SIA is $\{\zeta_S : \forall\ \ S \subseteq N \setminus i\}$.
- Fixed intermediary fee $\xi v(N)$. ξ : Intermediary Factor (IF).
- A middleman is not awarded any non-zero payoff from the game (other than his intermediary fee).
- The remaining of the grand coalition distributed to the other players.

They are intuitive...

How to distribute the Remaining?

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{4\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$
v(S)	1.5	1	1	0	2	2	2
S	$\{1,2,3\}$	$\{1,4\}$	$\{2,4\}$	$\{3,4\}$	$\{1,2,4\}$	$\{2,3,4\}$	$\{1,3,4\}$
v(S)	4	5.5	5	5	5	5	5

Table: The Highway Game : $(N, \zeta, \xi, v) \in \mathscr{GM}$: $v\{1,2,3,4\} = 6.$

- The SIA of 4 is $\zeta_{\{i\}} = 4$, $\zeta_{\{i,j\}} = 3$, $\zeta_{\{i,j,k\}} = 2$.
- $\Phi^S h(v) = (1.416, 1.167, 1.167, 2.25)$, $\Phi^N(v) = (1.3, 0.8, 0.8, 3.1)$.
- $\Phi^S h(v) = (1.5, 1.25, 1.25)$, $\Phi^N(v) = (1.68, 1.16, 1.16)$.
- Thus players will prefer to play without a middleman. Scope for negotiation...

The Bottomline is-

We need an alternative Allocation Scheme...

The I-value for the class $\mathscr{G}\mathscr{M}$: Linearity

Lemma

Let Φ be a value for $\mathscr{G}(N)$ that satisfies Lin. Then there exist real constants α_S^j for all $j \in N$ and $S \subseteq N$ such that for every $v \in \mathscr{G}(N)$,

$$\Phi_j(v) = \sum_{\emptyset \neq S \subseteq N} \alpha_S^j \, v(S) \tag{2}$$

The I-value for the class $\mathscr{GM}$: Middleman Axiom

The axiom of *middleman* (*MA*): A value $\Phi : \mathscr{GM}^A \to \mathbb{R}^n$ satisfies *MA* if $\Phi_i(v) = 0$ whenever $i \in N$ is a *middleman* for v.

Lemma

Let the value Φ satisfy Lin and MA. Then for each $i \in N$ there exist real constants δ_S^i for all $S \subseteq N \setminus i$ such that for every $(N, \zeta, \xi, v) \in \mathscr{GM}$,

$$\Phi_i(N, \zeta, \xi, v) = \sum_{S \subseteq N \setminus i} \delta_S^i \, \{v(S \cup i) - v(S)\}\,. \tag{3}$$

Moreover $\delta_S^i = 0$ for each $S \subseteq N \setminus i$, if i is a middleman.

The I-value for the class $\mathscr{GM}$: Monotonicity

Axiom of Monotonicity(*M*) : A value $\Phi : \mathscr{GM} \to \mathbb{R}^n$ satisfies monotonicity if $\Phi_i(N, \boldsymbol{\zeta}, \xi, v) \geq 0$ for every monotonic game $(N, \boldsymbol{\zeta}, \xi, v) \in \mathscr{GM}$.

Lemma

Let Φ be a value for $\mathscr{GM}$ and assume that Φ satisfy Lin, MA and M. Then for all $i \in N$ and $S \subseteq N \setminus i$ there exist real constants δ_S^i such that for every $v \in \mathscr{GM}$,

$$\Phi_i(N, \boldsymbol{\zeta}, \xi, v) = \sum_{S \subseteq N \setminus i} \delta_S^i \ \{v(S \cup i) - v(S)\}. \ \textit{with } \delta_S^i \geq 0. \tag{4}$$

The I-value for the class $\mathscr{G}\mathscr{M}$: Anonymity

Lemma

Under Lin, MA, M and AN for all $j \in N \setminus i$, there exist real constants δ_s for all $S \subseteq N \setminus j$ such that for every $(N, \boldsymbol{\zeta}, \xi, v) \in \mathscr{G}\mathscr{M}$ we have,

$$\Phi_j(N, \boldsymbol{\zeta}, \xi, v) = \sum_{S \subseteq N \setminus j} \delta_s \, \{v(S \cup j) - v(S)\} \, . \tag{5}$$

The I-value for the class $\mathscr{GM}$: I-Efficiency

- The *intermediary fee* of $\xi_i v(N)$ for the *middleman* i is paid from $v(N)$.
- There remains only $v(N) - \xi_i v(N)$ to allocate under Φ.

I-Efficiency Axiom (I-Eff):
A value Φ on $\mathscr{GM}$ satisfy I-efficiency (*I-Eff*) i.e.,

$$\sum_{j \in N \setminus i} \Phi_i(N, \zeta, \xi, v) = v(N) - \xi_i v(N).$$

The I-value for the class $\mathscr{GM}$

Lemma

Let Φ be a value for $\mathscr{GM}$ and assume that Φ satisfy Lin, MA, M, AN and Eff. Then for every $v \in \mathscr{GM}$ there exist real constants γ'_s for all $S \subseteq N$ given by

$$\Phi_i(N, \zeta, \xi, v) = \begin{cases} \sum_{\substack{S \subseteq N \setminus j \\ i \in S}} \gamma'_s \, \{v(S \cup j) - v(S)\}, & \text{if } j \neq i \\ 0, & \text{if } j = i \end{cases}$$

where,

$$\gamma'_s = \frac{(s-1)!(n-s-1)!}{(n-1)!}(1-\xi)$$

The I-value for the class $\mathscr{G}\mathscr{M}$

Combining Lemma (1)-(5), we have the following important theorem.

Theorem

A value Φ defined over $\mathscr{G}\mathscr{M}$ satisfies the axioms Lin, MA, M, AN and Eff if and only if it is given by (6).

- Theorem provides separate characterizations for $\Phi(N, \zeta, \xi, v)$ specific to the quantities ζ and ξ.
- Opposed to the Shapley value or the nucleolus which are defined on the entire class $\mathscr{G}(N)$ of TU games.
- Denote the value by $\Phi^I(N, \zeta, \xi, v)$.
- We call the Intermediary value or the I-value in short.

Independence of the Axioms

Example

Let $\Phi^1(N, \zeta, \xi, v) = (0, 0, ..., 0)$. Then Φ^1 satisfies all axioms except *I-Eff*.

Example

The function $\Phi^2(N, \zeta, \xi, v) = (1 - \xi)\Phi^{Sh}(N, v)$ satisfies all the axioms other than *MA*.

Example

- Let $\bar{n}(N)$ be the lowest labelled player such that $\bar{n}(N) \neq i$ and for each $j \neq i$ in N, $j > \bar{n}(N)$, see van den Brink et al. (2013).
- Let $\Phi^3_{\bar{n}(N)}(N, \zeta, \xi, v) = (1 - \xi)v(N)$, and $\Phi_j(N, \zeta, \xi, v) = 0$ for each $j \neq \bar{n}(N)$. Then Φ^3 satisfies all the axioms except *AN*.

Independence Contd.

Example

$$\Phi_j^4(N, \zeta, \xi, v) = \begin{cases} (1-\xi)\left\{v(\{j\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{n-1}\right\} & \text{if } j \neq i \\ 0 \ \text{ if } j = i \end{cases}$$

Then Φ_j^4 satisfies all the axioms except *M*.

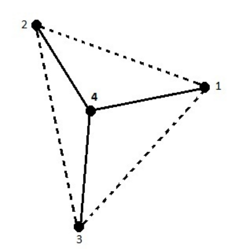
Example

Fix an $\alpha > 0$.

$$\Phi_j^5(N, \zeta, \xi, v) = \begin{cases} \frac{v(N)(1-\xi)}{n-1} \text{ when } j \neq i \text{ and } v(N) > \alpha \\ \Phi_j^I(N, \zeta, \xi, v) \ \text{ when } j \neq i \text{ and } v(N) \leq \alpha \\ 0 \ \text{ when } j = i \end{cases}$$

Thus Φ^5 satisfies all the properties except *Lin*.

The Highway Game Revisited



- $\Phi^{Sh}(v) = (1.416, 1.167, 1.167, 2.25)$.
- Without a middleman, $\Phi^{Sh}(v) = (1.5, 1.25, 1.25)$.
- $\Phi^{N}(v) = (1.3, 0.8, 0.8, 3.1)$.
- Without a middleman, $\Phi^{N}(v) = (1.68, 1.16, 1.16)$
- Thus if the players and the middleman agree to an intermediary fee of $\frac{v(N)}{n} = 1.5$,

$$\Phi^{I}(N, \zeta, \xi, v) = (1.625, 1.4375, 1.4375).$$

- If the players and the middleman agree to an intermediary fee of 1.3,

$$\Phi^{I}(N, \zeta, \xi, v) = (1.7, 1.5, 1.5).$$

Comparison with Other types of Players

- δ-reducing player (Calvo and Gutiérrez-López, 2016) : $v(S \cup i) = \delta v(S)$ for all $S \subseteq N \setminus i$.
- The δ-reducing player does exactly the opposite of what we have assumed in our model.
- $\boldsymbol{\xi}$-player (Casajus and Huettner, 2014) in $v \in \mathcal{G}(N)$ if $v(i) = 0$ and $v(S \cup i) - v(S) = \xi_s \frac{v(S)}{s}$ for all $S \subseteq N \setminus i$, $S \neq \emptyset$.
- The $\boldsymbol{\xi}$-player increases or decreases the worth of a coalition $S \subseteq N \setminus i$, ξ_s times her per capita worth.
- Player $i \in N$ is a proportional player in $v \in \mathcal{G}(N)$, if $v(i) = 0$ and $\frac{v(S \cup i)}{s+1} = \frac{v(S)}{s}$ for all $S \subseteq N \setminus i$.
- Player $i \in N$ is a quasi proportional player in $v \in \mathcal{G}(N)$, if $v(i) = 0$ and $\frac{v(S \cup i)}{s+2} = \frac{v(S)}{s+1}$ for all $S \subseteq N \setminus i$.
- Observe that the $\boldsymbol{\xi}$-player is a proportional player for $\xi_s = 1$ and a quasi proportional player for $\xi_s = \frac{s}{s+1}$ for all $s \in N \setminus i$.

Voting Game

Observation

- $v \in V \cap \mathscr{GM}$ if and only if $\zeta_S = 1$ for every $S \subseteq N \setminus i$.
- Any coalition $S \subseteq N \setminus i$ is a loosing coalition
- $S \cup i$ is always winning.

Remark

- If i is a middleman then he is also a critical player.
- A middleman cannot be a veto player.
- v does not have a null player as for each $j \in N \setminus i$, we have $v(i, j) = 1 \neq 0 = v(j)$.
- The only minimal winning coalitions in (N, v) are the pairs $\{i, j\}$ where i is a middleman and j is any other player.

The Potential to characterize the I-value

Definition

Given a function $P : \mathscr{G}\mathscr{M} \to \mathbb{R}$ associated with a real number $P(N, \boldsymbol{\zeta}, \xi, v)$ to each $(N, \boldsymbol{\zeta}, \xi, v) \in \mathscr{G}\mathscr{M}$ with middleman $i \in N$, the *intermediary-gradient* of a player $j \in N$ is defined as

$$D_j P(N, \boldsymbol{\zeta}, \xi, v) := \begin{cases} P(N, \boldsymbol{\zeta}, \xi, v) - P(N \setminus j, \boldsymbol{\zeta}, \xi, v) & \text{if } j \neq i. \\ 0 \;\; \text{if } j = i \end{cases} \tag{6}$$

Definition

Let $i \in N$ be a middleman in $(N, \boldsymbol{\zeta}, \xi, v) \in \mathscr{G}\mathscr{M}$. A function $P : \mathscr{G}\mathscr{M} \to \mathbb{R}$ starting from $P(\{i, j\}, \boldsymbol{\zeta}, \xi, v) = (1 - \xi)v(\{i, j\})$ for all $j \in N \setminus i$ is called an intermediary-potential if

$$\sum_{j \in N \setminus i} D_j P(N, \boldsymbol{\zeta}, \xi, v) = (1 - \xi)v(N). \tag{7}$$

The Potential

Theorem

There exists a unique intermediary-potential P for every $(N, \zeta, \xi, v) \in \mathcal{GM}$ with middleman $i \in N$, the payoffs $(D_j P(N, \zeta, \xi, v))_{j \in N \setminus i}$ coincide with the I-value of the game and the intermediary-potential of (N, ζ, ξ, v) is uniquely determined by (7).

Definition

Let Φ be a solution of $(N, \zeta, \xi, v) \in \mathcal{GM}$ with middleman $i \in N$. Given $T \subseteq N \ : \ i \in T$, define the reduced game (T, v_T^{Φ}) as follows.

$$v_T^{\Phi}(S) = \begin{cases} v(S \cup T^c) - \frac{1}{(1-\xi)} \sum_{j \in T^c} \Phi_j(S \cup T^c, \zeta, \xi, v), & \text{if } i \in S \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where $T^c = N \setminus T$.

Consistency

Definition

A solution function Φ is consistent if for any $(N, \zeta, \xi, v) \in \mathscr{G}\mathscr{M}$ with middleman i and $T \subseteq N$ such that $i \in T$,

$$\Phi_j(T, \zeta, \xi, v_T^{\Phi}) = \Phi_j(N, \zeta, \xi, v), \ \forall j \in T \setminus i \tag{9}$$

Theorem

The I-value is consistent.

Standard for Three Person Game

Definition

A solution function Φ is standard for three-person games $(\{i,j,k\},v)$ with middleman i if

$$\Phi_j(\{i,j,k\},\zeta,\xi,v) = (1-\xi)\left[v(\{i,j\}) + \frac{1}{2}(v(\{i,j,k\}) - v(\{i,j\}) - v(\{i,k\}))\right] \;\forall j \neq k.$$

Lemma

A solution Φ is I-efficient if it satisfies consistency and standard for three-person games with a middleman.

Characterization : Main Theorem 2

Theorem

let Φ be a solution function. Then Φ is the I-value if and only if it is consistent and standard for three-person games with a middleman.

Independence of the Axioms

- $\Phi^1 = (0, 0, 0..., 0)$ satisfies consistency but is not standard for the three person game with a middleman.
- Define,

$$\Phi'(N, \zeta, \xi, v) = \begin{cases} \Phi^I(N, \zeta, \xi, v) \text{ when } n = 3 \\ \Phi(N, \zeta, \xi, v) \text{ when } n > 3 \end{cases}$$

Then Φ' satisfies consistency but not standard for three person games with middleman.

Conclusion

- We introduce the notion of *middlemen* in TU cooperative games.
- We propose a new value for TU cooperative games with *middlemen*.
- The axioms which are used to characterize this value are linearity, anonymity, efficiency and a new axiom : the axiom of middlemen.
- Future of Middlemen? The multiplicative model.

References I

- Aumann, R. J. and Hart, S. (eds.), 2002, Handbook of game theory, with economic applications, vol 3. North- Holland, Amsterdam.
- Banzhaf, J.F.I., 1965, Weighted voting does not work: A mathematical analysis, Rutgers Law Review, 19, 317-343.
- Branzei, R., Dimitrov, D. and Tijs, S., Models in Cooperative Game Theory: Crisp, Fuzzy and Multichoice Games, Springer, Berlin Heidelberg, 2008.
- Bailey, J. P., and Bakos, J. Y., 1997, An Exploratory Study of the Emerging Role of Electronic Intermediariations and Policy, International Journal of Electronic Commerce 1(3), 7-20.
- Chun, Y., 1991, On the symmetric and weighted Shapley values, Int. J. Game Theory, 20, 183-190.
- Derks, J., Haller, H. and Peters, H., 2000, The selectope for cooperative games, Int. J. Game Theory, 29, 23-38.

Thank You...Your Comments