A Negative Result on the Capabilities Approach

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International Conclave on Foundations of Decision and Game Theory
IGIDR, Mumbai
14th March, 2016

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- 2 Core Concepts - Functionings and Capabilities.

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- An example - owning a laptop v/s using a laptop.

Capabilities and Capability Sets

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- It reflects *the opportunities* to choose between different functioning combinations.
- Different functioning bundles available v/s the achieved functioning bundle.
- Achieved functioning reflects how an individual makes use of the opportunities available to him.

Standard of Living

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- Exclusive determination by his/her achieved functioning bundle.

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Standard of Living

Three possible ways to determine the Standard of Living of individuals using the Capabilities Approach:

- Exclusive determination by his/her achieved functioning bundle.
- Exclusive determination by the Capability Set available to him/her.
- Determination of the Standard of Living using both: the Capability Set of the individual and the achieved functioning bundle of the individual from this Capability Set. Such a generalized set-up is also called the Achievement-Opportunity Combinations (AOCs).

In this presentation, we have looked at only the first of the above 3 methods.

Some Basic Notations and Assumptions

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- Each functioning k takes values from an arbitrary set, $F_k \subseteq \mathbb{R}$.

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- Since there are m functioning bundles, we therefore have, $X = F_1 \times F_2 \times ... \times F_m$.
- The functioning bundles are usually denoted as x, y, z etc.
- $\forall x, y \in X$, we denote that functioning bundle x is strictly greater than the functioning bundle y by writing $x > y$ if, $x_k \geq y_k \ \forall \ k \ \in M$ and $x \neq y$.

Some Basic Notations and Assumptions (Contd...)

- the set of all *convex, compact and comprehensive* subsets of X will be used as the class of all possible *capability sets* available to *any* individual $i \in N$.

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- Let this above class of all possible capability sets be denoted by Z.
- One can think of every individual in set N as choosing a functioning bundle x from some capability set A which belongs to Z.

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- $\succeq$ is assumed to be *reflexive* and *transitive*, but not necessarily *complete*.
- The EE is not ranking the social states; rather, she is simply comparing one individual's standard of living with the standard of living of another individual.

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- If for any pair of individuals $i, j \in N$, there exists functionings $x, y \in X$, which satisfy the above property, then we call the pair (x, y), a minimal relativism functioning pair (MRFP).

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- If for any pair of individuals $i, j \in N$, there exists functionings $x, y \in X$, which satisfy the above property, then we call the pair (x, y), a minimal relativism functioning pair (MRFP).
- In the absence of this property, the comparisons of living standards would trickle down to a one player game.

Weak Dominance and Dominance

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- $\succeq$ satisfies **dominance** if and only if, $\forall i, j \in N$ and $\forall x, y \in X$, $x > y$ implies that $(i, x) \succ (j, y)$.
- Dominance is logically a stronger notion. For our purposes, weak dominance itself will suffice.

- The binary relationship $\succeq$ satisfies the continuity property if and only if, $\forall i \in N$ and $\forall x, y, z \in X$, if $(i, x) \succ (i, y)$, then there exists $\epsilon_1, \epsilon_2 > 0$, such that $||x - z|| < \epsilon_1$ implies that $(i, z) \succ (i, y)$ and $||y - z|| < \epsilon_2$ implies that $(i, x) \succ (i, z)$.

Our Assumption

- The binary relationship $\succeq$ satisfies our assumption if and only if, $\forall i \in N$ and for the functioning bundles $x, y \in X$, if $(i, x) \succ (i, y)$, then there exists a functioning bundle y', such that either $x > y' > y$ with $(i, x) \succ (i, y') \succeq (i, y)$ or $y' = y$ with $(i, x) \succ (i, y)$.

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- In words, our assumption states that, if for an individual i, $(i, x) \succ (i, y)$, then two mutually exclusive situations can occur. Either there exists a functioning bundle y' in between x and y, such that i's standard of living when she has bundle x is strictly greater than when she has bundle y'; and her standard of living when she has bundle y' is at least as good as her living standard when she has the bundle y. Or the functioning bundle y' is the same as the functioning bundle y, in which case we already knew that $(i, x) \succ (i, y)$.

Our Assumption

- We note that the case of $y' = y$, two plausible cases arise: either the functioning bundles x and y are comparable, i.e., $x > y$ or $y > x$ or $x = y$; or, x and y are not comparable.

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- We note that the case of $y' = y$, two plausible cases arise: either the functioning bundles x and y are comparable, i.e., $x > y$ or $y > x$ or $x = y$; or, x and y are not comparable.
- We also note that our assumption is defined in a manner such that, whatever be the structure of $X = F_1 \times F_2 \times ... \times F_m$, the assumption would always hold.

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- Looking at the definition of mild continuity, it can be seen that the definition is *for all* functioning bundles in the ϵ neighbourhood of the functioning bundle x. Our assumption on the other hand asks for just *one* functioning bundle, which is also guaranteed to exist.

Pattanaik and Xu (2007)

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- For $X = [0, b(1)] \times \cdots \times [0, b(m)]$, when $\succeq$ is made to satisfy the mild continuity assumption, the plausible properties of minimal relativism and weak dominance are at odds with each other.
- In their proof of the proposition, the transitivity of $\succeq$ fails to hold.

An Important Remark

- If every MRFP (x, y) is such that x and y are incomparable, it makes no sense to talk about minimal relativism and weak dominance holding simultaneously for $\succeq$.

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- If every MRFP (x, y) is such that x and y are incomparable, it makes no sense to talk about minimal relativism and weak dominance holding simultaneously for $\succeq$.
- The above is easy to see. Weak dominance is the only property for which interpersonal comparisons can be made. For weak dominance to hold, we must have two comparable functioning bundles. If that is never the case for any minimal relativism functional pair, we can never compare minimal relativism and weak dominance simultaneously.

Our Proposition

- For any arbitrary set X, $\succeq$ cannot **simultaneously** satisfy the desirable properties of minimal relativism and dominance when they are made to satisfy our assumption, whenever such a comparison makes sense.

Proof of the Proposition (By Contradiction)

- Assume that $\succeq$ is reflexive, transitive, satisfies minimal relativism, weak dominance and our assumption.

$\succeq$ satisfies minimal relativism and hence we know that there exist individuals $i, j \in N$ and $x, y \in X$, such that,

$$(i, x) \succ (i, y) and (j, y) \succ (j, x) \tag{1}$$

Furthermore, (x, y) is an MRFP.

Proof of the Proposition (By Contradiction)

- Assume that $\succeq$ is reflexive, transitive, satisfies minimal relativism, weak dominance and our assumption.
- First, consider X to be a dense subset of $\mathbb{R}^k$.

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$$(i, x) \succ (i, y) and (j, y) \succ (j, x) \tag{1}$$

Furthermore, (x, y) is an MRFP.

Proof of the Proposition (By Contradiction)

Let us now consider individual i. Now, since each component of any functioning bundle belongs to F_k, which is dense in $\mathbb{R}$, $\forall k = 1, 2, ..., m$, using Equation (1) and applying our assumption, we know that,

$$\exists y' \in X, \text{such that}$$

$$x > y' > y \implies (i, x) \succ (i, y') \succeq (i, y) \tag{2}$$

Now, using the assumption that $\succeq$ satisfies weak dominance, we have,

$$\textit{for any } y' > y, (i, y') \succeq (j, y) \tag{3}$$

Combining Equations (2) and (3), we have, for individuals $i, j \in N$ satisfying minimal relativism, there exists a functioning bundle $y < y' < x$, such that,

$$(i, x) \succ (i, y') \succeq (j, y) \tag{4}$$

Proof of the Proposition (By Contradiction)

Now, let us consider individual j, for whom we know from Equation (1) that $(j,y) \succ (j,x)$. So, working along the same line of argument as done for individual i, we have,

$$(j,y) \succ (j,x) \implies \exists x' \in X, \text{such that}$$

$$y > x' > x \implies (j,y) \succ (j,x') \succeq (j,x) \tag{5}$$

Once again using Definition (3) and making use of the fact that we have assumed $\succeq$ satisfies weak dominance, we have,

$$for\ any\ x < x', (j,x') \succeq (i,x) \tag{6}$$

Combining Equations (5) and (6), therefore, we have, for individuals $j, i \in N$ satisfying minimal relativism, there exists a functioning bundle $x < x' < y$, such that,

$$(j,y) \succ (j,x') \succeq (i,x) \tag{7}$$

Proof of the Proposition (By Contradiction)

Combining equations (4) and (7), we therefore have the following

$$(i, x) \succ (i, y\prime) \succeq (j, y) \succ (j, x\prime) \succeq (i, x) \qquad (8)$$

Therefore, as seen from Equation (8), one can see that it constitutes a contradiction on the reflexivity property of the binary relationship $\succeq$.

Proof of the Proposition (By Contradiction)

Now, suppose that X is not dense in $\mathbb{R}^k$. Then for individuals i and j, either there exists a y' and x' respectively, such that $(i, x) \succ (i, y\prime) \succeq (i, y)$ and $(j, y) \succ (j, x') \succeq (j, x)$ and exactly similar arguments follow as before. Or, for the MRFP with x and y comparable, with $x = y$, $(i, x) \succ (i, y)$ is not true; or with $x > y$, using weak dominance, $(j, x) \succeq (j, y)$, which is at odds with $(j, y) \succ (j, x)$, as coming from Equation (1).
Hence, for every possible case, the initial assumption of $\succeq$ satisfying minimal relativism and dominance simultaneously cannot be true. Thus, by contradiction, the proposition is proved.

Conclusion and the Way Ahead

- It was then shown that while considering weak dominance and the property of minimal relativism, even though no continuity properties were required to be fulfilled and only that weaker assumption's fulfilment was considered, there is still a tension between the two. This is unfortunate because both the properties are highly attractive.

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- The generalized setting of AOCs can also be proved!
- Most importantly, the proposition holds true for any arbitrary X and not for only the specific X considered in Pattanaik and Xu(2007).