

From Assessment to Placement: Uncertainty and Effort Choice

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The link....

between education and employment is crucial for economic development

Education generates human capital, a necessary input in the development process, by enhancing an individual's embodied skills above their raw labour ability (**Belfield and Levin, 2003, Cohn and Addison, 1998, Goldin and Katz, 1998**).

Education generates valuable job market information about the skill levels of the potential recruits

Higher Education: a three-activity process

Training (effort by teacher and student) → skill formation (unobservable but verifiable)

Assessment (effort by teacher) → certificate of student skill (observable)

Placement after external assessment by recruiters → jobs (observable)

Nature of teacher's efforts involved in teaching and assessment are very different therefore we consider these separately

Examinations may be part of the training process itself although here examinations play the role of only assessment leading to certificates

Student's effort in participating in examination process is not considered:

Assessment process or examination is not modelled separately. Its outcome i.e. student grade depends only on true student ex-post skill (which again is dependent on teacher effort and student effort in learning) and teacher effort in assessment of that skill.

Student effort in exam cannot influence learning / ex-post skill or the grades; (alternatively we could assume student's additional effort cost for participating in exams is zero as this is similar to learning effort)

Various newspapers and industry magazines publish lists of **rankings of business schools in India**. Whilst criteria for each publication differs, business schools included in rankings are those with full-time Master of Business Administration (MBA) or Post-Graduate Diploma in Management (PGDM) programs.

Factors normally taken into account include **placement**, quality of faculty, quality of students, quality of infrastructure, industry brand image and fees.

The Development Question

Higher Education Process: Training, Assessment, Placement

Two observations:

- ✓ **Stress on student placement as an indicator of quality of academic process and teaching quality**
- ✓ **The huge resource expenditure and large variety of measures related to the recruitment process and the declining credibility or importance of certification by institutions of higher education, in the recruitment process.**

The Development Question (contd)

Questions:

- ✓ How to design an incentive scheme or pay structure that will improve
 - 1 - teaching – learning effort and level of **skill formation** &
 - 2 – assessment effort and ensure credible **certification** of student skill level?

- ✓ Does rewarding teaching effort based on student placements help to achieve 1 above?
- ✓ Will rewarding assessment effort based on match between certificate and placement level help to achieve 2 above?

Framework that incorporates observed features of the HE process:

- Joint role played by teachers and students in training and skill formation process
- Assessment as a separate activity performed by teachers in addition to teaching
- Assessment as a more **laborious activity** compared to teaching; **easier to grade extreme quality**
- Placement based on recruiter assessment in addition to institutional assessment (by teachers) who impart the skill
- Teacher pay based on student placement and also on match between institutional and recruiter assessment

Early Literature

Education generates human capital by enhancing an individual's embodied skills above their raw labour ability (**Belfield and Levin, 2003, Cohn and Addison, 1998, Goldin and Katz, 1998**).

Rothschild and White (1995) and **Dolan et. al. (1985)** stress on **the joint role** played by students (willing to invest their time and effort to learn skills) and teachers (who have the skill and are able to impart the training) in the generation of human capital.

University managers influence the **technical efficiency** of the universities by allocating resources within the institution and acting as principals in **structuring incentives** for the agents (**Johnes, 1999**).

Performance related pay (PRP) for teachers: evidence for a positive impact of teacher salary levels on students' outcomes at the school level (**Loeb and Page, 1999, Dewey et al, 2000**).

Teacher pay affects teacher performance by influencing the recruitment and retention of more able teachers (**Jacobson, 1995, Dolton and Van der Klaauw, 1996, 99**)

Induces better performance by continuing teachers (**Hanushek et al, 1999**).

The Model

Game with two players – a teacher and a student.

The players perform two activities resulting in two outputs.

Teaching: Skill formation or quality q (unobservable) which involves effort by both the teacher and the student; t and e

Assessment: Certificate or student grade \hat{q}_T , requires effort by only the teacher, in assessing student quality; a

All efforts are unobservable.

All efforts are costly to the agents; Teacher prefers teaching to assessment

Framework: multi-tasking with multiple agents.

Teacher chooses t and a simultaneously with student choosing e .

The Model (contd.)

Placements: happen based on teacher's certificates of student skill / student grades and external assessment by recruiting firm

Firm's effort in assessment, τ is given exogenously; firms play a passive role

Educational Production Function:

$$q = Q(t, e) = te, \quad t, e \in [0, 1] \dots\dots\dots(1)$$

***t* and *e*: effort levels chosen by the teacher and the student in teaching and learning respectively, which are unobservable; *q* refers to the skill or “quality” generated by the training process.**

Quality Assessment:

Evaluation of student quality happens twice – internal assessment by teacher and external by recruiting firm when student enters job market

Evaluation procedure is not perfect and grades assigned are noisy estimates

Internal Assessment / Grading by Teacher:

\hat{q}_T , the teacher's estimate of student quality or the grade assigned to the student by the teacher,

$\hat{q}_T \in (0, 1): E(\hat{q}_T) = q \text{ and } V(\hat{q}_T) = \sigma(1 - a)q(1 - q) \quad \sigma \in (0, 1), \quad a \in [0, 1] \dots\dots\dots(2)$

Expected value of grade correctly measures true quality.

Accuracy of grade increases (variance decreases) with assessment effort and the extremity of the true quality; and with improvement in state of evaluation technology (exogenously given imprecision parameter σ has a smaller value)

External Assessment / Grading by Recruiting Firm:

\hat{q}_F has the following properties:

$E(\hat{q}_F) = q \text{ and } V(\hat{q}_F) = \sigma(1 - \tau)q(1 - q) \quad \tau \in (0, 1) \dots\dots\dots(3) \quad (\tau \text{ is exogenous})$

Student's Payoff:

The pay packet offered by the recruiter, $\pi \in (0, 1)$, is given by

$$\pi = \varphi \hat{q}_F + (1 - \varphi) \hat{q}_T \quad \varphi \in (0, 1) \dots \dots \dots (4)$$

Student payoff is the salary the student receives from the firm less the effort cost in

learning: $c(e) = \frac{1}{2} e^2$

$$S = \pi - c(e) = \pi - \frac{1}{2} e^2 \dots \dots \dots (5)$$

Tuition fee is not considered in this model. A lump sum fee at the time of entry would not affect any of the results and would only influence student participation in the education process.

Teacher's Payoff:

The teacher's pay has two components – reward for teaching at the rate λ , based on placement level and reward for assessment based on match between placement and grade (or penalty for wrong assessment γ based on discrepancy) :

$$P = \lambda\pi - \gamma d^2 \quad \lambda, \gamma > 0 \dots\dots\dots(6)$$

where $d = \hat{q}_F - \hat{q}_T \dots\dots\dots(6a)$

The teacher's dis-utilities from teaching and assessment efforts:

$$C(t) = \frac{1}{2}\theta t^2; \quad c(a) = \frac{1}{2}a^2 ; \quad 0 < \theta < 1$$

Here θ is assumed to be less than one ($\theta < 1$), to indicate the teacher's marginal disutility from teaching is less than that for assessment.

Teacher's Payoff:

The teacher's payoff is the teacher's salary less the dis-utilities from teaching and assessment efforts:

$$T = \lambda\pi - \gamma d^2 - \frac{1}{2}\theta t^2 - \frac{1}{2}a^2 \quad \theta \in (0, 1) \dots\dots\dots(7)$$

Substituting for π and d using (4) and (6a) the teacher's payoff may be written as:

$$T = \lambda[\varphi\hat{q}_F + (1 - \varphi)\hat{q}_T] - \gamma(\hat{q}_F - \hat{q}_T)^2 - \frac{1}{2}\theta t^2 - \frac{1}{2}a^2 \dots\dots\dots(8)$$

Working of the Model: Effort Choice

All agents maximize their expected payoffs; solve for Nash equilibria of the game.

Student

Optimal choice of effort in learning is obtained as solution to the following optimization problem:

$$\text{Max}_e E(S) = q - \frac{1}{2}e^2 \quad \dots\dots\dots(9)$$

The first order condition for the above maximization problem yields,

$$\begin{aligned} \frac{\partial E(S)}{\partial e} &= t - e = 0 \\ \Rightarrow e &= t \equiv e^* \quad \dots\dots\dots(10) \end{aligned}$$

Working of the Model: Effort Choice

Teacher

Optimal choice of effort in teaching and assessment is obtained as solution to the following optimization problem:

$$\text{Max}_{t,a} E(T) \quad \dots\dots\dots(11)$$

$$E(T) = \lambda q - \gamma \sigma q(1 - q)[(1 - a) + (1 - \tau)] - \frac{1}{2} \theta t^2 - \frac{1}{2} a^2 \quad \dots\dots\dots(12)$$

The first order conditions:

$$\frac{\partial E(T)}{\partial a} = 0 \Rightarrow a = \gamma \sigma q(1 - q) \equiv a^* \quad \dots\dots\dots(13)$$

$$\frac{\partial E(T)}{\partial t} = 0 \Rightarrow \lambda e - \gamma \sigma(2 - \tau - a)(e - 2te^2) - \theta t = 0 \quad \dots\dots\dots(14)$$

Teacher's expected payoff:

$$E(T) = \lambda q - \gamma[V(\hat{q}_F) + V(\hat{q}_T)] - \frac{1}{2}\theta t^2 - \frac{1}{2}a^2$$

$$\Rightarrow E(T) = \lambda q - \gamma\sigma q(1 - q)[(1 - \alpha) + (1 - \tau)] - \frac{1}{2}\theta t^2 - \frac{1}{2}a^2$$

Since $E(\hat{q}_F) = E(\hat{q}_T) = q$ and

$$E[(\hat{q}_F - \hat{q}_T)^2] = E[(\hat{q}_F)^2] - [E(\hat{q}_F)]^2 + E[(\hat{q}_T)^2] - [E(\hat{q}_T)]^2 - 2E(\hat{q}_F)E(\hat{q}_T) + 2q^2 = V(\hat{q}_F) + V(\hat{q}_T)$$

Substituting e^* and a^* for e and a in (14) yields the following polynomial in t :

$$t(\lambda - \theta) - \gamma\sigma t(1 - 2t^2)[(2 - \tau) - \gamma\sigma t^2(1 - t^2)] = 0$$

This may be expressed as product of t and a third degree polynomial in $t^2 (= q)$, W :

$$t \cdot W = 0$$

$$W = (A - mB) + m(m + 2B)q - 3m^2q^2 + 2m^2q^3$$

$$\text{where } A = \lambda - \theta; m = \gamma\sigma; B = (2 - \tau)$$

So either $t = 0$ or t is obtained as solution to the equation $W = 0$; this will have one or three positive solutions and hence one or three real positive solutions in t

Lemma 1: The function W is strictly monotonically increasing in q

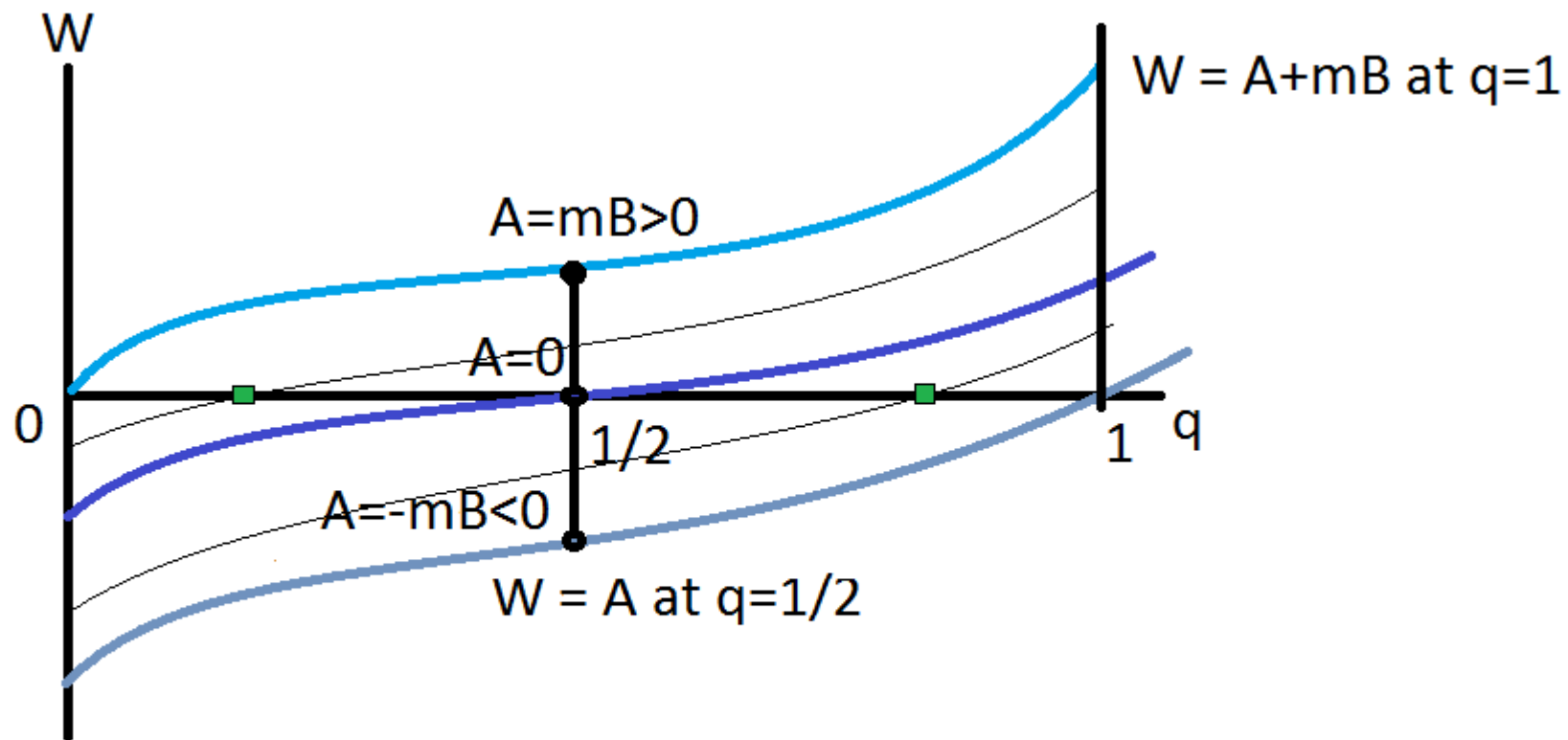
Lemma 2: (a) There exists equilibrium t , iff

$$\theta - \sigma(2 - \tau)\gamma \leq \lambda \leq \theta + \sigma(2 - \tau)\gamma$$

(b) Only one real solution will exist

Proposition: $q^* \geq \theta$ or $q^* < \frac{1}{2}\theta$ according as $\lambda \leq \theta$ or $\lambda > \theta$

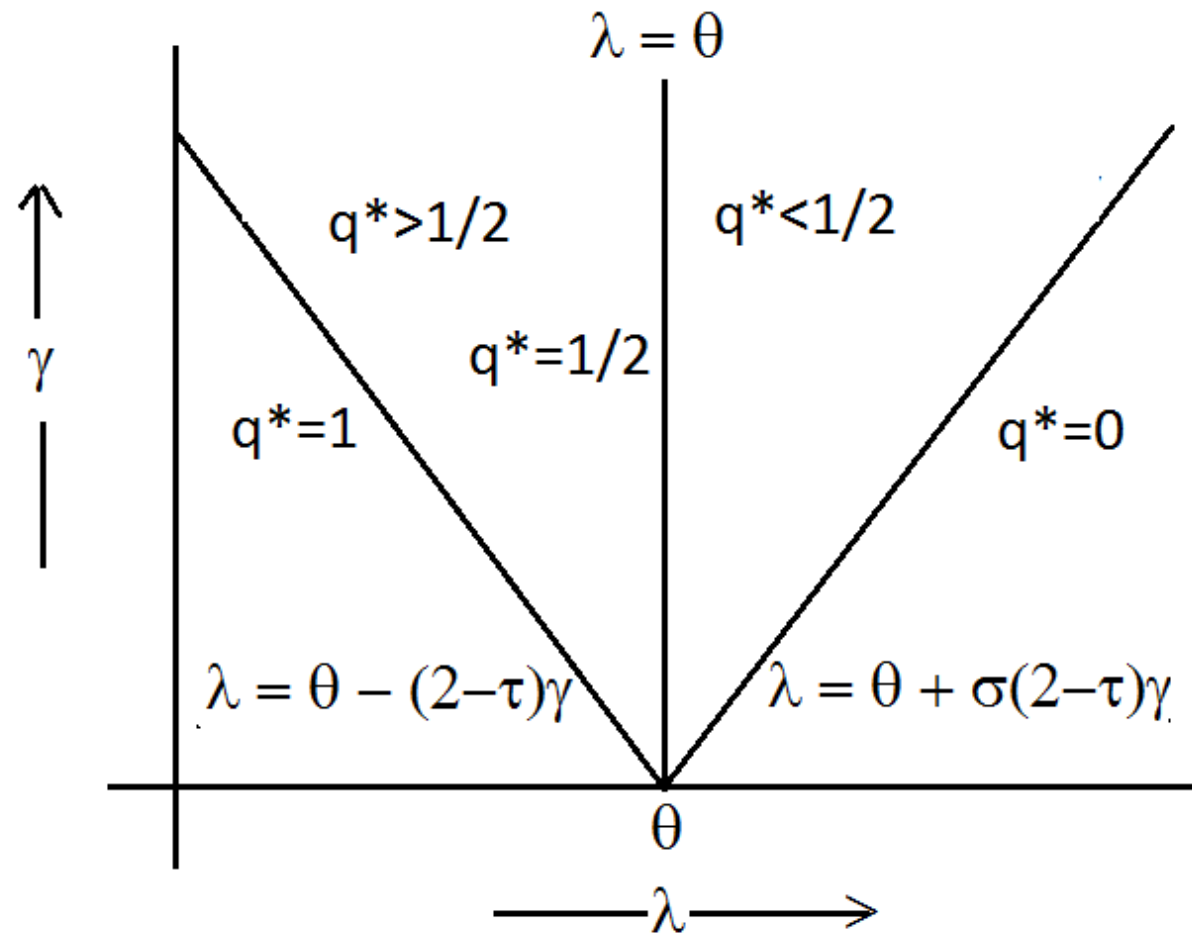
Solution Characterisation:



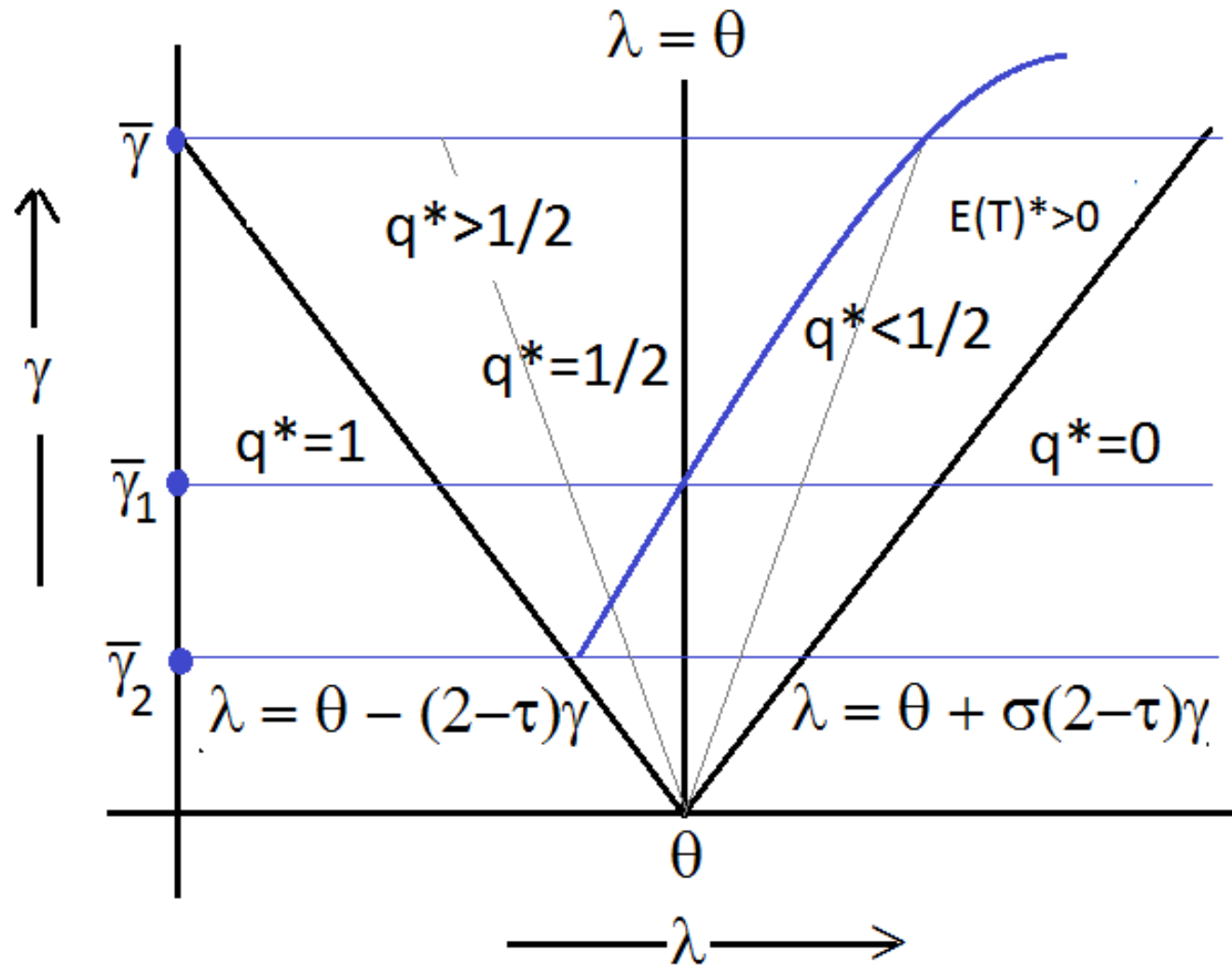
$$W = A - mB \text{ at } q=0$$

$$-mB \leq A \leq mB \Rightarrow \theta - \sigma(2 - \tau)\gamma \leq \lambda \leq \theta + \sigma(2 - \tau)\gamma$$

Solution Characterisation:



Solution Characterisation: Participation Constraint



Student chooses same effort as teacher

Assessment effort will be higher if:

Technology is imprecise

Penalty rate is higher

Carrot and stick policy:

Too much stress on placements as an indicator of teaching effort may be counterproductive i.e. rewarding teaching effort based on student placements will result in poor quality or skill level

Penalty for wrong assessment may induce teachers to teach well and ensure high level of quality or skill formation (as students will match teacher effort) and improve probability of correct assessment.

The Development Question

Higher Education Process: Training, Assessment, Placement

What we know and observe:

Activities involved

Effort

Disutility from effort

Skill formation

Certification

Performance measurement, its indicators and pay structure

What we suggest:

Alternative pay structure

What we may infer:

Effect on quality of education process – skill level and credibility of certification

Whether PRP for teachers will work or not?