Bi-cooperative Network games: A Link-based Allocation Rule

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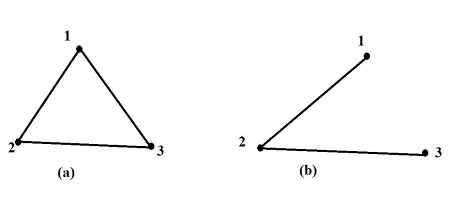
Outline

1. Introduction
   - Network games
   - Bi-cooperative games
2. Main results
   - The notion of Bi-cooperative network games
   - Solution concept: Link-based allocation rule
   - Characterization
3. Concluding remarks

Network game

- A player connects with another player if the benefits are available to her via her own links.
- Myerson (1977) introduced a game and called as communication situation.
- In this game, the players are restricted to cooperate only through their existing links.
- Jackson and Wolinsky (1996) extend communication situation.
- The Network game is an extension of communication structure.
- In this game, the value generated depends directly on the network structure.



Is the value necessarily same?

Bi-cooperative game

Introduction and motivation of Bi-cooperative games

- Do the players in $N \setminus S$ have no influence on the worth of S if the coalition S is formed?
- Bilbao introduced the Bi-cooperative games as a generalization of cooperative games.
- In this game, each player has three options:
  1. she can participate the game positively (defender),
  2. she can participate the game negatively (defeater),
  3. she does not participate the game (abtentionist).

Solution Concepts

Solution concept of Network games

- The Myerson value, a player-based allocation rule is a extension of the Shapley value from cooperative games to communication situation.
- The Position value, based on the Shapley value is a link-based allocation rule for Network games.

Solution concept of Bi-cooperative games

- Different solution concepts for bi-cooperative games have so far been found in the literature.
- Among them the ones proposed by Bilbao et al. (2000) and Labreuche and Grabisch (2008) are of great importance.
- However these two solution concepts differ due to the relations among bi-coalitions that impose two different lattice structures on them.
- The order relation taken by Bilbao et al.
- Labreuche and Grabisch proposed an alternative order among the bi-coalitions : the product order and obtained a Shapley like value for the corresponding class of bi-cooperative games, which we call here the LG value in short.

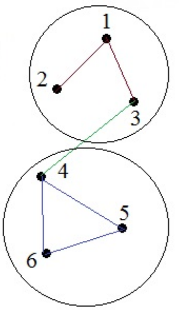
Bi-cooperative Network game

Motivations of Bi-cooperative network games

- Like Cooperative games, we extend the network games by combining a bi-cooperative structure in it.
- Do the network $g^N \setminus g$ have no influence on the value accrued by the network g?

Bi-cooperative Network game

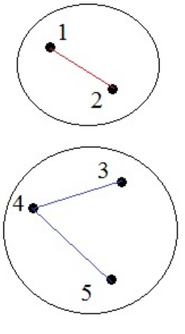
- A geo-political situation where some countries with different motives over an issue agree to negotiate, see for example (Park 2000, International trade agreements between countries of asymmetric size), where the size of countries differ.
- A treaty proposed by some members may not be suitable for the rest and they ask for amendments.
- Such amendments will be influenced by the diplomatic capabilities of each country.
- It is natural to assume that each nation is connected to each other through mutual trade relations.
- Evidently they form two connected sub-networks of opposite motives within the same trade network.



Network g_1

Bi-cooperative Network game

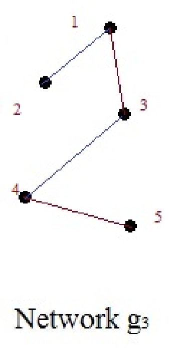
- A second situation is considered when players of opposite polarity form distinct components in the network.
- This may be the case when players of each polarity are devoid of any information from their opponent.
- See (Manea 2011, Bargaining in Stationary Networks) for an interesting study of bargaining between randomly matched players when agreements lead to link formation.



Network g_2

Bi-cooperative Network game

- The **double agents** working for various spying agencies, see
  - Isby, D. C (2004), War II: Double Agent's D-Day Victory, World War II", June 2004; accessed at http://www.historynet.com/world-war-ii-double-agents-d-day-victory.htm.
  - http://www.history.com/news/history-lists/6-daring-double-agents
- The duality of involvement of a player in different networks according to her affinity to different groups of people.
- Real life examples of such interactive situations are found in social networks like Twitter and facebook.



Bi-cooperative Network game

Motivations of Bi-cooperative network games

Fig 1 : Three Types of Bi-cooperative Network Games*

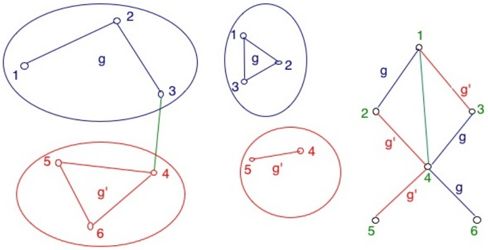


Fig 1(a) Fig 1(b) Fig 1(c)

*Here the blue links are positive, red are negative and the green links are indifferent.

Fig 2

Bi-cooperative Network game

Definition

Player: Let $N = \{1, 2, ..., n\}$ be a fixed player set who are connected in some network relationships.

Definition

Network: Let g^N be the set of all subsets of size 2.
Let $G = \{g | g \subseteq g^N\} =$ the set of all possible networks on N.
Let $g_1, g_2 \in G$ such that $g_1 \cap g_2 = \emptyset$. Then the pair (g_1, g_2) is called a bi-network. Let $\mathcal{Q}(\mathcal{G}) = \{(g_1, g_2) | g_1 \cap g_2 = \emptyset, g_1, g_2 \in G\}$ be the set of all bi-networks.

Definition

Value function: A value function is a function $b : \mathcal{Q}(G) \to \mathbb{R}$, with $b(\emptyset, \emptyset) = 0$.
The set of all possible value functions is denoted by $\mathcal{B}$.

Bi-cooperative Network game

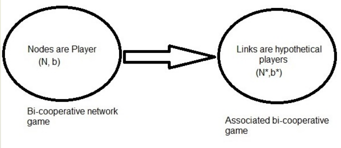
Definition

Bi-cooperative network game: A Bi-cooperative network game is a pair (N, b), of a set of players and a value function.
The set of all bi-cooperative network games is denoted by $\mathcal{BG}$

Bi-cooperative Network game

Definition: Associated Link games

- The associated link game is defined by considering the set of links as a player set.
- Given $(g_1, g_2) \in \mathcal{Q}(G)$, denote by $[g_1]$ the set of all hypothetical players representing the links in g_1.
- Let N^* be the set of all such hypothetical players.
- Given a Bi-cooperative network game $(N, b) \in \mathcal{BG}$, define the associated Bi-cooperative game (N^*, b^*) of (N, b) as follows.
- Given $(S, T) \in \mathcal{Q}(N^*)$, there exist $(g_1, g_2) \in \mathcal{Q}(G)$ such that $S = [g_1]$ and $T = [g_2]$. Define $b^* : \mathcal{Q}(N^*) \to \mathbb{R}$ by $b^*(S, T) = b(g_1, g_2)$.
- Given $(g_1, g_2) \in \mathcal{Q}(G)$, set $\mathcal{Q}([g_1], [g_2]) = \{(S, T) \mid S \subseteq [g_1], T \subseteq [g_2] : S \cap T = \emptyset\}$.
- It follows that for $(S, T) \in \mathcal{Q}([g_1], [g_2])$, there is a $(g_1', g_2') \in \mathcal{Q}(G)$ with $g_1' \subseteq g_1$ and $g_2' \subseteq g_2$ such that $S = [g_1'], T = [g_2']$.
- The class of the associated Bi-cooperative games is denoted by $\mathcal{BG}^*$.



Bi-cooperative Network game

Definition: Link-based allocation rule for Bicooperative network games

Given $(N, b) \in \mathcal{BG}$, an allocation rule $Y =: \mathcal{Q}(G) \times \mathcal{BG} \to \mathbb{R}^n$ with respect to $(g_1, g_2) \in \mathcal{Q}(G)$ is a link based allocation rule for the class $\mathcal{BG}$ of Bi-cooperative Network games if there is a consistent value $\Psi : \mathcal{BG}^* \to (\mathbb{R}^{\frac{n(n-1)}{2}})^{\mathcal{Q}(N^*)}$ such that,

$$\sum_{l \in (g_1+g_2)} \Psi_l(N^*, b^*)([g_1], [g_2]) = b^*([g_1], [g_2])$$

and $$Y_i((g_1, g_2), b) = \sum_{l \in L_i(g_1, g_2)} \frac{\Psi_l(N^*, b^*)([g_1], [g_2])}{2}$$

Bi-cooperative Network game

Definition: The Position value for Bicooperative network games

The Position value Y^{BNPV} is a link based allocation rule.

$$(Y^{BNPV})_i((g_1, g_2), b) = \sum_{\substack{l \in L_i(g_1, g_2) \\ S=[g_1], T=[g_2]}} \frac{1}{2} \Phi_l^{LG}(N^*, b^*)(S, T)$$

Characterization of the Position value

Axiom BN1

Efficiency: If $(N, b) \in \mathcal{BG}$ and $(g_1, g_2) \in \mathcal{Q}(G^N)$, it holds

$$\sum_{i \in N(g_1, g_2)} Y_i((g_1, g_2), b) = b(g_1, g_2).$$

Lemma

The Position value satisfies efficiency.

Characterization of the Position value

Definition

A player $i \in N$ is called a null player for $b \in \mathcal{BG}$ if for every $l \in L_i(g_1 + g_2)$ and every $(g_1', g_2') \in \mathcal{Q}(G^N)$ such that $l \notin g_1' + g_2'$,

$$b(g_1' + l, g_2') = b(g_1', g_2') = b(g_1', g_2' + l).$$

Axiom BN2

Null player property: If i is a null player for $b \in \mathcal{BG}(N)$, then

$$Y_i((g_1, g_2), b) = 0.$$

Lemma

The Position value satisfies null player property.

Characterization of the Position value

Axiom BN3

Intra-Network Symmetry: If $b \in \mathcal{BG}$, $(g_1, g_2) \in \mathcal{Q}(G)$ and σ be a permutation on N such that $\sigma(g_1) = g_1$ and $\sigma(g_2) = g_2$, for $i \in N$, then it holds that for all $i \in N$

$$Y_{\sigma(i)}((g_1, g_2), b \circ \sigma^{-1}) = Y_i((g_1, g_2), b).$$

Lemma

The Position value satisfies Intra-Network Symmetry.

Characterization of the Position value

Axiom BN4

Inter-Network Symmetry: Let $i \in N(g_1)$ and $j \in N(g_2)$ and $b_i, b_j \in \mathcal{BG}$ such that for all $(g_1', g_2') \in \mathcal{Q}(G)$ and $l_i \in L_i(g_1), l_j \in L_j(g_2)$ such that $l_i \notin g_1', l_j \notin g_2'$,

$$b_i(g_1' + l_i, g_2') - b_i(g_1', g_2') = b_j(g_1', g_2') - b_j(g_1', g_2' + l_j)$$
$$b_i(g_1' + l_i, g_2' + l_j) - b_i(g_1', g_2' + l_j) = b_j(g_1' + l_i, g_2') - b_j(g_1' + l_i, g_2' + l_j),$$

then we must have, $Y_i((g_1, g_2), b_i) = -Y_j((g_1, g_2), b_j)$.

Lemma

The Position value satisfies Inter-Network Symmetry.

Characterization of the Position value

Axiom BN5

Linearity: For all $\alpha, \beta \in \mathbb{R}$ and $b_1, b_2 \in \mathcal{BG}$ and $(g_1, g_2) \in \mathcal{Q}(G)$,

$$Y((g_1, g_2), \alpha b_1 + \beta b_2) = \alpha Y((g_1, g_2), b_1) + \beta Y((g_1, g_2), b_2).$$

Lemma

The Position value is linear.

Characterization of the Position value

Axiom BN6

Monotonicity: For $(g_1, g_2) \in \mathcal{Q}(G)$, $i \in N(g_1 + g_2)$ and b_1, b_2 be two Bi-cooperative network games such that,

$$\begin{aligned} b_1(g_1', g_2') &= b_2(g_1', g_2') \\ b_1(g_1' + l, g_2') &\geq b_2(g_1' + l, g_2') \\ b_1(g_1', g_2' + l) &\geq b_2(g_1', g_2' + l) \end{aligned}$$

for all $l \in L_i(g_1 + g_2)$ and $(g_1', g_2') \in \mathcal{Q}(G^N)$ such that $l \notin g_1' + g_2'$, then

$$Y((g_1, g_2), b_1) \geq Y((g_1, g_2), b_2).$$

Lemma

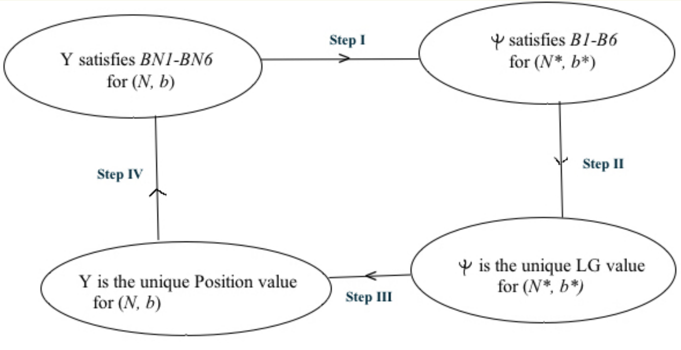
The Position value is monotone.

Characterization of the Position value

Theorem

The Position value is the only one link-based allocation rule which satisfies efficiency, null player property, linearity, monotonicity, intra-network symmetry and inter-network symmetry.

Outline of the proof



Conclusions

Conclusion

- *We introduce the notion of Bi-cooperative network games.*
- *We extend the link-based allocation rule, the Position value for Bi-cooperative network games.*
- *We characterize this allocation rule using the axioms efficiency, linearity, monotonicity, intra-network symmetry, inter-network symmetry, null player property.*

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