SOCIAL SECURITY REFORM AND WELFARE IN A TWO SECTOR MODEL

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Abstract

A reform of a pay-as-you-go social security in a one sector model makes the pensioners worse off and the working generations better off in the period of the reform (in a dynamically efficient economy without altruism). The observed reluctance across all age groups to support such reforms is usually explained by the insurance properties of these schemes. I propose an alternative in a two sector setting. Since the old consume labor-intensive goods like health-care etc., the reform causes labor demand to fall and reduces wages. This effect could dominate the lower social security payments for the young. Thus both the young and old oppose the reform (that makes the unborn generations better off--the new steady welfare, with a higher capital stock, is higher).

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1. INTRODUCTION

Most advanced capitalist countries have problems with managing their social security systems. The social security system comprises of many schemes e.g. in the US it pays old-age pension, benefits to the disabled, and also survivor benefits to widows and children of deceased workers. The part of a social security system in need of urgent attention is the old-age pension scheme. The looming crisis is due to the longer lives of the retired, and a declining birth rate manifesting itself in the shrinking size of the labor force. Between 1970 and 2020 in the OECD countries the average life expectancy at 65, increased by six years, and retirement was brought forward by three years. (OECD (2019)). Aaron (2011) sums up the bleak picture thus: "The Social Security Trust Funds face a projected long-term funding gap… (Then) financial rules would require benefit cuts…. If nothing were done before then… it would be necessary in 2037 either to cut benefits by approximately 24 percent or, to sustain benefits, raise earmarked revenues 32 percent" (Aaron (2011) p.385).

An unfunded public pension system or pay-as-you-go (PAYG) scheme provides insurance that private markets are unwilling to offer due to moral hazard and other reasons (Diamond (1977), (2004), Aaron (2011); Feldstein and Liebman (2002) provide a survey). Since the PAYG scheme is funded by a payroll tax, it has implications for labor supply. Moreover, since the working age population saves and invests, capital accumulation is also affected.

Economists, and other policy-makers, have looked at the "reform" of such system. A reform consists of a reduction of transfers from the working population (the "young") to the retired (the "old") in that period--indeed, there is talk of privatizing social security i.e. moving to a fully-funded system from the currently (predominantly) PAYG system.

A large literature exists that looks at the consequences of reforming a PAYG system. The three major themes highlighted above--viz. the crowding out of capital, risk-sharing across generations, and the distortion introduced in labor supply by using payroll taxes to finance the social security scheme—have been discussed at length[1] (see e.g. Krueger and Kubler (2006), Fuster, Imrohoroglu and Imrohoroglu (2007), Nishiyama and Smetters (2007)), Conesa and Garriga (2008), Catalan and Magud (2017)). A reform would ameliorate some of these while exacerbating others. A fair summary of this large literature would be to say that in a dynamically efficient economy, the absence of insurance markets is the main hurdle for the reform of such a system.[2]

The social security reform is a reneging on an implicit promise made by society to the currently old. This is presumed to spur capital accumulation, [3] but the old in the period of reform are paying the price for it.

In the United States, the social pension scheme continues to be the most popular government program.[4]This popularity of the system among the working population requires explanation. We ask in this paper whether at any given date is it true, that in the absence of uncertainty, the interests of the young and the old are always implacably opposed to one another? And will a reform, while hurting the old, benefit the young?

A strand of the literature seeks to explain the existence (and its continuation) of a social security system by appealing to worker's heterogeneous productivity. Low

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1 A sample quotation on this: "(When) wages are not insurable privatization *reduces* efficiency by about $2,400 per future household despite improving labor supply incentives. This loss occurs even though privatization substantially increases the welfare of those born in the *long run* by increasing the capital stock…" (Nishiyama and Smetters (2007) p.1677).

2 See Sinn (2000) for a good analysis of the proposed reform.

3 Empirical evidence does not support the presumption that a move towards a (mandated by law) funded system spurs economic activity, see Altiparmakov and Nedeljkovic (2018). They look at a sample of 36 countries, over the period 1990 to 2013, in Latin America, Eastern Europe and Central Asia. See Bovenberg (2003) for a discussion on Europe.

4 A survey reported that in 2010: "On the 75th anniversary of Social Security, public support for the program remains exceedingly high." And: "Although they are far from claiming Social Security retirement benefits, younger Americans are very supportive of the program. Nine in ten adults under age 30 believe Social Security is an important government program." AARP (2010) p. 1.

productivity workers, in anticipation of a social security receipt that is not (or at most imperfectly) correlated with their contribution to the PAYG system, vote with the retired (see Tabellini (2000), Casamatta, Cremer and Pestieau (2002)).

In the Diamond overlapping generations model (Diamond (1965)), following a reform, the disposable income of the young in that period increases (although they, in turn, would see reduced pension payment to them when they are old).[5] Capital accumulation increases. In a one-sector setting, the old in the period when the reform is introduced, lose by the full amount of the reduced transfer since the interest rate in that period is unchanged. And because they consume all their income, their consumption goes down by the full amount of the lowered social security benefits. The young gain (in a dynamically efficient setting) because their lifetime income goes up.[6]

Solow (2005) had lamented on the dearth of two-sector (or multi-sector) models in dynamic macroeconomic settings. That the issue of social security needs this should be self-evident—it is indeed surprising that modelling of social security is confined to a straitjacket of a one sector framework.[7] The consumption basket of the old is different from those of the young. Indeed, the "structural transformation" of an industrialized economy towards a services-based one, happens precisely because the old, whose proportion in the population has grown, consume (certain) services (e.g. health-care) in a higher proportion.

In a one-good model, the capital stock is the only state variable. Given the capital-labor ratio, the factor prices are determined uniquely; these then determine saving and consumption. In a two (or more) sector model(s), the factor prices and the demand for the

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5 "…if the social security system is of the PAYG type, and the rate of interest is higher than the rate of population growth, private saving is more attractive, at least for a young worker." (Casamatta et al (2002) p. 504).

6 If this is not swamped by a fall in the interest rate.

7 Fedotenkov et al. (2019) is an exception. They look at social security in a two-country two sector two good model.

two goods are determined jointly, given the capital stock i.e. the capital stock is insufficient by itself to determine factor prices.[8]

Since Uzawa (1964), it is well-understood that the dynamics of a two sector model depends on assumed factor-intensity rankings.[9] His model was that of an infinite-lived household maximizing its utility. An analysis of social security issues, however, requires lives to be finite. For the two-sector overlapping generations model, Galor (1992) showed that if the consumption good was labor-intensive, the dynamics is a saddle-point; if on the other hand the consumption good were capital-intensive, we get indeterminacy, i.e. both roots are stable and there are an infinity of paths that are convergent. Thus both on the grounds that possibly the consumption of the old is labor-intensive (primarily services) and to avoid indeterminacy, we assume that the consumption good is labor-intensive.[10]

In a two (or more) sector model, following the reform, some of the adverse effect of the loss of income (the “transfer”) to the old would be mitigated by fall in the cost of the goods they consume e.g. health-care. Also, the gain of the increased disposable income of the young would be offset by increased price of the goods that they consume. That is, there are "secondary" effects of the transfer that happen in the period of reform, which cannot be captured in a one-sector model. To put it differently, in any period there are intergenerational linkages that work through the market (in addition to a PAYG transfer).

In the international trade literature, the transfer problem figures prominently. In particular, various authors have examined the possibility whether the donor could gain and/or the recipients lose from a transfer. There it was found that in a dynamic setting and/or a multi-agent setting such a “transfer paradox” could indeed arise (see e.g.

8 It determines the "cone of diversification". But many equilibria are possible within this cone. The unique momentary (or short run) equilibrium is chosen by the saddle path, which makes it forward-looking.

9 This class of models had two goods—a pure consumption good and a pure investment good. The two inputs are labor and capital.

10 This is assumed by others also; see van Groezen et al (2005)—in their model the old consume only labor-services. Fedotenkov et al (2019) also assume this.

Cremers and Sen (2009)). It is surprising that in the social security reform literature, this has not been invoked.

I set up a closed economy model with two goods--a pure consumption good and a pure investment good. With the social security reform, there is increased demand for the investment good (as savings rise), accompanied by a fall in the consumption good demand (as income is transferred from the old, with a marginal propensity to consume of unity, to the young who save a part of their additional income). If the consumption good is labor-intensive, then the wage rate falls.

In such a setting it is possible that everybody alive today would become worse off—the old understandably so, but also the young because their wage falls, and this fall could overwhelm the increased take-home income from the lower payroll tax—the gain from the increased take-home income depends on the difference between the interest rate and population growth rate and is of second order. There would be no one alive today to vote for the reform. This is in spite of the fact that increased capital accumulation makes the future generations better off. The unborn may have liked to vote for reform but they do not have a vote (until they enter the labor force).[11]

The plan of the rest of the paper is as follows: the model is set out in section 2, while the effects of a social security reform are analyzed in section 3. Section 4 looks at some numerical values, while section 5 is the conclusion.

2. THE MODEL

[11] In a one sector model, with the capital stock given, the take home wage rises with a fall in the payroll tax, the interest rate in the next period, as a consequence of higher saving today, falls. And thus welfare effects for the young today are ambiguous. This is a consequence of the wage rate not moving at all, given the predetermined capital stock. In a one sector setting the old definitely lose as consequence of the reform. In a two sector model, the welfare of the old is ambiguous. See Boldrin and Rustichini (2000) on a detailed discussion on the political economy of social security in a one sector setting. Also see Hu (2019) for a discussion of welfare in the one-sector set up. I am grateful to an anonymous referee for helpful comments on this point.

The closed economy consists of overlapping generations of individuals (or households). No individual is altruistically linked to any future generations i.e., there are no bequests or inheritances. Every individual lives for two periods. In the first period of its life (youth) the individual supplies one unit of labor, pays the social security contribution via a payroll tax, and saves for the second period (old age). In old-age, the individual consumes the saving from the first period plus the return on these savings and the receipts from the social security. The social security system is a PAYG one and hence balanced budget. The population is growing at a constant rate. We shall study the properties of the model by log-linearizing it around the initial steady state.

The representative household born in time period t maximizes the following utility function

$$U_t \equiv \mathrm{U}(D_t^1,\ D_{t+1}^2) \qquad t=0,1,2\ldots. \tag{1}$$

where $D_t^1\,(D_{t+1}^2)$ is the consumption when young (old) of a household born in period t.

The utility function U(.) is increasing and strictly concave in its arguments and satisfies the Inada conditions. Both period consumptions are assumed to be normal.

Its lifetime budget constraint is

$$W_t(1-\tau) + \left(\frac{N\tau W_{t+1}}{\chi_{t+1}}\right) = D_t^1 + \left(\frac{1}{\chi_{t+1}}\right) D_{t+1}^2 \tag{2}$$

where W_t is the wage rate in time period t (in terms of the consumption good, which is the numeraire), N is the population growth factor, n is the population growth rate i.e., $N \equiv 1+n > 0$, χ_{t+1} the own interest factor on one period consumption loans between t and t+1, and τ is the payroll tax. We assume that the system is "dynamically efficient" so $\chi_{t+1} \geq N$ (for all t).

In equation (2) we have used the fact that a PAYG scheme pays the proceeds of the payroll tax to the old in that period. Hence the young in period t expect to receive τW_{t+1} per worker in their old age. The labor-force will be N times the current one. This expected future transfer is discounted to date t by using the discount factor χ_{t+1}.

The maximization yields

$$\frac{\partial \mathrm{U}}{\partial D_t^1} = \chi_{t+1} \frac{\partial U}{\partial D_{t+1}^2} \tag{3}$$

Using equations (2) and (3) we derive the demand functions

$$D_t^1 = D^1(W_t(1-\tau) + \left(\frac{N\tau W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4a}$$

And $$D_{t+1}^2 = D^2(W_t(1-\tau) + \left(\frac{N\tau W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4b}$$

The saving function is given by:

$$S_t \equiv W_t(1-\tau) - D^1(W_t(1-\tau) + \left(\frac{N\tau W_{t+1}}{\chi_{t+1}}\right), \chi_{t+1}) \tag{4c}$$

Savings are assumed to be a non-decreasing function of the real rate of interest.[12] An increase in the interest rate works through three channels—(i) the substitution effect causing a postponement of consumption; (ii) an income effect that would increase consumption in both periods; and (iii) by reducing the present value of future social security receipts it would reduce consumption in both periods.

Firms

[12] This is a standard assumption (see e.g. Azariadis (1993)). But sometimes this is not assumed (see e.g. Casamatta et al (2002)).

The two goods—a pure consumption good (C) and a pure investment good (I)--are produced under conditions of constant returns to scale using the two inputs, capital and labor. K^C (K^I) is the capital employed in the consumption goods (investment goods) sector. Similarly, for L^C (L^I) is the labor employed in the consumption goods (investment goods) sector. All inputs are mobile between sectors instantaneously. The production functions are given by

$$C_t = \mathrm{F}(K_t^C, L_t^C) \tag{5a}$$

$$I_t = \mathrm{G}(K_t^I, L_t^I) \tag{5b}$$

The functions F(.) and G(.) have positive but diminishing marginal productivities and are homogeneous of degree one. They are also assumed to satisfy the Inada conditions.

The consumption good is assumed to be labor-intensive at all relative factor prices. Two justifications are given for assuming this: first, Galor (1992) and Azariadis (1993) have shown that in the other case (i.e. when the consumption good is capital-intensive), there is indeterminacy (multiple perfect foresight paths); and, second, in a two-sector model the old spend all their incomes on consumption goods (a large fraction of these are services). These are labor-intensive.

Firms maximize profits with perfect competition in all markets. In equilibrium, the firms set the minimized unit cost equal to the market price of the product (these tools are standard in international theory since Jones (1965); and in public economics see e.g Atkinson and Stiglitz (1980) Chapter 6)):

$$a_{LC}W_t + a_{KC}R_t = 1 \tag{6a}$$

$$a_{LI}W_t + a_{KI}R_t = p_t \tag{6b}$$

where a_{ij} is the requirement of the i^{th} input (i = K, L) in the production of the j^{th} good (j = C, I). Note that the a_{ij}'s are functions of the relative factor-prices.[13] The relative price of the investment good in terms of (the numeraire) good C is given by p and R is the (gross) return on capital. We assume capital depreciates completely in the process of production.[14] We have in equilibrium $\chi_{t+1} = R_{t+1}/p_t$.

Market-Clearing

In any period, there are two goods markets and two factor markets. By Walras' Law, if three of these are in equilibrium in any period, then so is the fourth one. We thus have

$$a_{LC}C_t + a_{LI}I_t = 1 \tag{7a}$$

$$a_{KC}C_t + a_{KI}I_t = k_t \tag{7b}$$

$$S_t = p_t.I_t \tag{7c}$$

Equations (7a), (7b) and (7c) are the market-clearing conditions for the labor, capital and investment goods markets respectively. In equations (7c) (and in (8a) and (8b) below) we have incorporated the assumption of one hundred per cent depreciation. The variable C_t is the production per worker of the consumption good, I_t is the output per worker of the investment good and k_t is the capital stock per worker (all in time period t).

Dynamics

The dynamics of the capital stock comes from the fact that the investment good this period is next period's capital stock. Taking into account the growth in the labor force,

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13 We rule out Leontief technologies, where the a_{ij}'s are constant, by assumption (because if the consumption good is labor-intensive, Leontief technologies do not give dynamic stability).

14 This is an innocuous assumption and can be dispensed with easily. It is also made routinely in the literature.

per worker capital stock evolves as (N, as noted above, is the population growth factor), we have:

$$\mathrm{N}k_{t+1} = I_t \tag{8a}$$

Equation (7c) written out in full (using (4c)) gives us another dynamic equation (8b):

$$W(p_t)(1-\tau) - D^1\left(\left(W(p_t)(1-\tau) + \frac{\tau NW(p_{t+1})p_t}{R(p_{t+1})}\right), \frac{R(p_{t+1})}{p_t}\right) = p_t \mathrm{I}(p_t, k_t) \tag{8b}$$

Competitive Equilibrium

Definition: A competitive equilibrium is, given τ and the initial stock of capital $k(0)$, a sequence of prices and capital stocks $(p_t, k_t)_{t=0}^{\infty}$, of wages and the rental rates $(W_t, R_t)_{t=0}^{\infty}$, and the consumption pairs $(D_t^1, D_{t+1}^2)_{t=0}^{\infty}$ such that:

(i) households maximize utility (equations (2) and (3)),

(ii) firms maximize profits (equations (6a) and (6b)),

(iii) markets clear (equations (7a), (7b) and (7c)),

(iv) and the capital stock dynamics is given by (8a).[15]

Existence of equilibrium for the above system is shown in Galor (1992) and Azariadis (1993) (see section 15.5).

The Dynamical System

Equation (8a) and (8b) are a system of two difference equations expressing p_{t+1} and k_{t+1} in terms of p_t and k_t and τ.

15 For the existence of a steady state with a positive capital stock τ cannot be very "large".

Write the log-linearized dynamics compactly as (a ^ over a variable is the percentage deviation from the steady state):

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{p}_{t+1} \end{bmatrix} = A \begin{bmatrix} \hat{k}_t \\ \hat{p}_t \end{bmatrix} + Hd\tau \tag{9}$$

The elements of matrix A and H are given in Appendix 2. Matrix A has, under some reasonable assumptions, two positive roots, lying on either side of unity or equivalently one root of A-I is positive, while the other is negative (see Appendix 2).

We can draw a phase diagram (Figure 1) under the assumption that the short run dynamics is Walrasian—a rise in p_t causes an excess supply of the investment good (again see Appendix 2). It shows that both $\hat{k}_{t+1} - \hat{k}_t = 0$ (the KK curve) and $\hat{p}_{t+1} - \hat{p}_t = 0$ (the IS curve) are downward sloping, with the latter curve being the steeper of the two. The horizontal arrows point away from the KK curve and the vertical arrows point away from the IS curve. The steady state is a saddle point and the stable arm is flatter than the IS line.

Steady State

The steady state of this economy is obtained by setting $k_{t+1} = k_t = k$ and $p_{t+1} = p_t = p$ (a steady state value is denoted without a time subscript) and solving for the other (now time-invariant) variables. From equations (8a) and (8b) we have:

$$Nk = I(p,k) \tag{10a}$$

$$W(p)(1-\tau) - D^1\left(\mathrm{W(p)}(1-\tau) + \left(\frac{N\tau \mathrm{W(p)p}}{R(p)}\right), \frac{R(p)}{p}\right) = pI(p,k) \tag{10b}$$

3. A SOCIAL SECURITY REFORM

Let us analyze a reform defined to be a cut in the payroll tax (dτ<0) implemented on date 1. On this date there are old (generation '0') and the young born in period 1. Given the wage rate and the interest rate, savings by generation 1 onwards rise with the social security reform for two reasons: (1) because individuals receive a higher take-home wage in the first period of their lives; and (2) because in their old age they would receive lower transfers from the next generation.

Steady State Effects

Following a social security reform, the steady state effects of the reform (d τ <0) are (from (10a) and (10b)):

$$\hat{k}\,/d\tau = -W\{1 - D_W^1\Phi\}\eta_{Ip}/(\Gamma\Delta) < 0 \tag{11a}$$

$$\hat{p}\,/d\tau = W(\eta_{Ik} - 1)(1 - D_W^1\Phi)/(\Gamma\Delta) > 0 \tag{11b}$$

where

$$\Gamma \equiv \eta_{Rp} D_\chi^1 \chi - D_W^1 (\frac{\tau W}{\chi})\}(\eta_{Rp} - \eta_{Wp}) < 0$$

and $\Delta \equiv \left(1 - \eta_{Ip}\right)\Gamma^{-1}\left\{\left(1 - D_W^1\right)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right) - \Gamma - \mathrm{pI}\right\}$

$$+\Gamma^{-1}\, pI(1 + \eta_{Ip})$$

Δ<0 (this is the product of the two roots of A-I and is negative from saddle-point stability) is the determinant of (A-I) and $\eta_{ij}$is an elasticity of the ith variable with respect to the jth variable (e.g. η_{Wp} is the elasticity of the wage with respect to the price of investment good)—see the Appendix 1 and 2 for details. In equation (11) to save on notation, we have used $\Phi \equiv (\chi - N)/\chi)\}$; dynamically efficiency implies $\chi \geq N$ or $\Phi \geq 0$.

Thus, across steady states, the social security reform (d τ <0) crowds "in" capital and raises the wage rate—exactly what the proponents of reform say it would do.

Dynamics

In figure 1, a fall in τ moves the IS curve out and the new steady state is at ε_1 (the initial steady state was at ε_0). The system jumps up to ε_{01} (with k_0 predetermined) to the new stable arm, and then adjusts (as capital is accumulated) monotonically along the stable arm to ε_1.

The dynamic behavior of p_t, following a cut in τ, shows that first p_t increases, then it falls along the stable arm. Thus initially the wage rate falls. The take home wage (i.e. net of the payroll tax) rises (loosely, because capital is accumulated).

Welfare

What is the effect of this on the welfare of a representative member of any generation t (t=0, 1, 2….)? We look at the changes in (i) steady state welfare, (ii) the welfare of the old when the policy is introduced (i.e., the generation born on date 0 which receives a smaller social security check); and (iii) the generation born in period 1, who inherit a capital stock from a high τ regime, but now pay a smaller social security contribution and will receive smaller old-age pensions.

The indirect utility function (from equations (1) and (2)) is given by:

$$V_t(\equiv \operatorname{argmax}_{S_t} U\big((W_t(1-\tau)-S_t),(\,\chi_{t+1}S_t+N\tau W_{t+1})\big))$$
$$= \mathrm{V}\left(W_t(1-\tau)+\left(\frac{\tau N W_{t+1}}{\chi_{t+1}}\right),\chi_{t+1}\right)$$

Or[16]

16 We have used the properties of the indirect utility function: $\partial V_t/\partial W_t = \frac{\partial U_t}{\partial c_t^1}$, and $\partial V_t/\partial \chi_{t+1} = \mathrm{S}\frac{\partial U_t}{\partial c_{t+1}^2}$.

$$\frac{dV_t}{V_W} = \left[\left((1-\tau)\mathrm{W}\eta_{Wp}\hat{p}_t + \left(\frac{\tau NW}{\chi}\right)\{(\eta_{Wp} - \eta_{Rp})\hat{p}_{t+1} + \hat{p}_t\} - \Phi W d\tau\right) + S(\eta_{Rp}\hat{p}_{t+1} - \hat{p}_t)\right] \quad (12)$$

where V_W is the marginal utility of income i.e. $\frac{\partial V}{\partial W}$.

In the steady state the change in utility is:

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$$\frac{\mathrm{dV}}{V_W} = \left\{W\eta_{Wp}(1-\tau)\left(1 + 2\left(\frac{\tau N}{\chi(1-\tau)}\right) - \left(\frac{S}{W(1-\tau)}\right)\right)(\hat{p}/d\tau) - \Phi\mathrm{W}\right\}d\tau \quad (13)$$

This is unambiguously positive for d τ <0.[17]

Period 1

So in the new steady state, with a higher capital stock, welfare rises. But what happens in the earlier periods when the contribution of a higher capital stock is yet to kick in? We now turn to the period when the reform is implemented.

In the period the reform is introduced (period 1), there are the old (generation 0) who receive a smaller old age pension, and the young (generation 1) who pay less to the old but in turn will receive less when they, in turn, are old in period 2.

The jump in p_1 is given in figure 1 by the vertical difference between ε_{01} and ε_0

$$\frac{d\hat{p}_1}{d\tau} = \frac{W\left(1 - D_W^1\Phi\right)}{(\xi_U - 1)\Gamma} \quad (14)$$

where $\xi_U > 1$ is the unstable root of matrix A in equation (9).

17 This is because $\eta_{Wp}W(1-\tau)[1 - \left(\frac{S}{(1-\tau)W}\right) + 2\left(\frac{\tau N}{\chi(1-\tau)}\right) < 0$, in addition to $-\Phi\mathrm{W} < 0$.

The welfare of the old in period 1 (generation ‘0’) is given by their consumption level:

$$V_0 = R_1 k_1 + N\tau W_1 \tag{15}$$

Or,

$$dV_0 = \{(Rk\eta_{Rp} + N\tau W\eta_{Wp})\frac{d\hat{p}_1}{d\tau} + NW\}d\tau \tag{15'}$$

The old receive a double whammy from the social security transfers—a fall in τ and a fall in W_1. The direct effect of a cut in τ is offset up by a rise in interest rate (this rise is accompanied by a fall in the wage rate). A priori, this is ambiguous but for the numerical values in the next section, welfare of the old in period zero falls.[18]

Going back to equation (12), for generation 1

$$\frac{dV_1}{V_W} = \left[\left(\left\{(1-\tau)\mathrm{W}\eta_{Wp} + \left(\frac{\tau NW}{\chi}\right) - S\right\}\hat{p}_1 + \{\left(\frac{\tau NW}{\chi}\right)\left(\eta_{Wp} - \eta_{Rp}\right) + S\eta_{Rp}\}\hat{p}_2 - \Phi W d\tau\right)\right] \tag{16}$$

In equation (16), as p_1 rises, wages fall. A rise in p_1(given p_2) also lowers the (consumption) real rate of interest ($\chi_{t+1} \equiv R_{t+1}/p_t$)—this raises the present value of future social security receipts but reduces the interest income on savings. A rise in p_2 raises the interest on savings but reduces the social security receipts through a fall in future wage rate and its present value by raising R_2. The effect of a cut in τ is ambiguous theoretically.

To get a handle on the possible sign of dV_1/V_W, examine the expression in (16) at the “Golden Rule”. The last term ΦWdτ =0 (because $\Phi = 0$). If we ignore the terms involving τ (τ is “small”), we need $\left\{(1-\tau)\mathrm{W}\eta_{Wp} - S\right\}\hat{p}_1 + S\eta_{Rp}\hat{p}_2 < 0$—that is indeed possible. We next turn to examine the sign of the expression in (16) away from the “Golden Rule” for some reasonable parameter values.

18 For our assumed numerical values, this is negative. I have looked at the possibility of the numerical values that make it positive—something that is impossible in a one-sector model. This is a subject of ongoing research.

4. NUMERICAL ILLUSTRATIONS

I provide four examples with different factor shares and elasticities of substitution in production and consumption. I assume an isoelastic utility function

$$U(D_t^1, D_{t+1}^2) = {D_t^1}^{(1-\sigma)} + \beta {D_{t+1}^2}^{(1-\sigma)}$$

The annual rate of interest for illustration is chosen to be 2% (2.5% gives the similar results qualitatively) and population growth is 1%. For each period of 30 year length this gives $\Phi = \frac{1.02^{30} - 1.01^{30}}{1.02^{30}} = 0.26$ (if we had chosen the annual real interest rate to be 2.5%, $\Phi = 0.36$). I set $\beta\chi = 1$. The value of τ is set equal to 0.1. In Table 1 below, θ_{LC} (θ_{LI}) is the share of wages in sector C (respectively I), λ_{LC} (λ_{KC}) the share of employment of L (respectively K) in sector C, ε_i is the elasticity of substitution between factors in sector I (i=C,I), 1/σ is the intertemporal elasticity of substitution in consumption, and $\xi_S$and, ξ_U are respectively the stable and unstable roots for that row. Finally, $\mathrm{dV}/V_W$is the change in welfare (and is of the same sign as $\mathrm{d}\tau$).

Table 1 Some Numerical Examples

θ_{LC}	θ_{LI}	λ_{LC}	λ_{KC}	$\varepsilon_C = \varepsilon_I$	1/σ	ξ_S	ξ_U	dV/V_W
0.80	0.50	0.75	0.50	1	2	0.54	3.32	$0.12\mathrm{d}\tau$
0.85	0.50	0.80	0.50	0.50	1.4	0.75	3.96	$0.23\mathrm{d}\tau$
0.80	0.47	0.80	0.40	0.70	2	0.53	3.57	$0.16\mathrm{d}\tau$
0.85	0.60	0.80	0.50	0.25	2	0.67	2.66	$0.22\mathrm{d}\tau$

5. CONCLUSIONS

We have shown the possibility that in the period of reform, the younger generation (that pays a lower payroll tax) could end up by having a lower lifetime utility. The older generation in that period also is likely to lose. This happens, even though when the reform kicks in, capital accumulation and increases wages in the future.

How is it possible that both agents may lose in the initial period of the reform? The fact that the old lose is unsurprising—under our assumed numerical values, the interest rate rises but is insufficient to compensate for the loss from the transfer.[19] What is surprising is that the young also lose. The intuition is that the direct gain from the transfer is of a second-order (depends on the difference between the interest rate and population growth rate), whereas the fall in wages is of first order (notwithstanding the subsequent rise in the interest rate). From the discussion in section 3, generation 1 members will save more, because of lower social security payments in period 1 and also lower social security receipts in period 2. If they save "a lot", then p_1 will move "a lot". As a consequence, W_1 falls a lot. And this fall in the wage rate cannot be compensated for by the rise in the real rate of interest, and the present value of the reduced social security contributions.

In a two sector model, if a social security scheme is being introduced from scratch (a la Boldrin and Rustichini (2000)), an increase in the payroll tax reduces the take-home pay and lowers the demand for the investment good. This reduces the price of the investment good, contributing to an increased welfare of the young via an increase in the wage rate, while reducing the welfare of the retired via a fall in the interest rate, ceteris paribus. This also leads to an increase in the sectoral capital intensities, since the investment good is capital-intensive (and its demand falls). The Condorcet payroll tax

[19] There are numerical values for which the old would gain, a possibility ruled out in a one-sector model. In this paper we are concerned with the welfare of the young and the underlying mechanism that is peculiar to a two-sector model (see Hu (2019) for an extended discussion).

may be higher for a two-sector case compared to a one sector model where these channels are missing.[20]

The arguments above have been conducted in a balanced budget setting. A social security fund, in reality, is infinitely more complex. The paper, at the very least, flags the need to examine a social security reform in a two (or more) sector setting. The other aspects of modelling such a change that have been deliberately switched off in this paper (e.g. elastic labor supply, many period lives, lack of insurance etc.), should be incorporated in such a (more realistic) model.[21]

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[20] I am grateful to an anonymous referee for suggesting this.

[21] The argument in this paper, however, seems to work against the proposal of postponing retirement as a panacea for the viability of the social security system. This is shown in a paper available from the author on request.

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APPENDIX 1

This is essentially reproduced from Atkinson and Stiglitz (1980) pp. 146-152. Equations (6a) and (6b) yield by logarithmic differentiation

$$\theta_{LC}\widehat{W}_t + \theta_{KC}\hat{R}_t = 0 \tag{A1.1a}$$

$$\theta_{LI}\widehat{W}_t + \theta_{KI}\hat{R}_t = \hat{p}_t \tag{A1.1b}$$

From (A1.1a) and (A1.1b), we can solve for $\hat{W}_t$ and $\hat{R}_t$ in terms of $\hat{p}_t$. We thus have

$$\eta_{Wp} \equiv \hat{W}_t / \hat{p}_t = -\theta_{KC} / \Delta \tag{A1.2a}$$

$$\eta_{Rp} \equiv \hat{R}_t / \hat{p}_t = \theta_{LC} / \Delta \tag{A1.2b}$$

where $\Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC}$ and η_{ij} is the (partial) elasticity of variable i with respect to j. From equations (A1.2a) and (A1.2b) we see that η_{Wp} and η_{Rp} depend on capital intensities. Given our assumption that the consumption good is labor-intensive, $\Delta>0$. And hence by the Stolper-Samuelson Theorem, $\eta_{Wp} < 0, \eta_{Rp} > 1$.

Similarly, by logarithmically differentiating (6a), (6b) and (6c) we have

$$\lambda_{LC} \cdot \hat{C}_t + \lambda_{LI} \cdot \hat{I}_t = [\hat{W}_t - \hat{R}_t][\lambda_{LC} .\theta_{KC} .\varepsilon_C + \lambda_{LI} .\theta_{KI} .\varepsilon_I] \tag{A1.3a}$$

$$\lambda_{KC} \cdot \hat{C}_t + \lambda_{KI} \cdot \hat{I}_t = \hat{k}_t - [\hat{W}_t - \hat{R}_t][\lambda_{KC} .\theta_{LC} .\varepsilon_C + \lambda_{KI} .\theta_{LI} .\varepsilon_I] \tag{A1.3b}$$

where λ_{ij} is the share of sector j in the total employment of input i and ε_j is the elasticity of substitution between inputs in the j^{th} industry.

From equations (A1.3a) and (A1.3b), we have the Rybczinski effects (which depend on assumed capital intensities)

$$\eta_{Ik} \equiv \hat{I}_t / \hat{k}_t = \lambda_{LC} / \Omega > 0 \tag{A1.4a}$$

$$\eta_{Ck} \equiv \hat{C}_t / \hat{k}_t = -\lambda_{LI} / \Omega < 0 \tag{A1.4b}$$

where $\Omega \equiv \lambda_{LC} - \lambda_{KC} > 0$ (by assumption).

From (A1.3a) and (A1.3b), we have the supply elasticities (which are independent of capital intensities)

$$\eta_{Ip} \equiv \hat{I}_t / \hat{p}_t = \{\lambda_{LC}\lambda_{KC}\varepsilon_I + (\lambda_{LC}\lambda_{KI}\theta_{LI} + \lambda_{KC}\lambda_{LI}\theta_{KI})\varepsilon_C\}/(\Delta\Omega) \geq 0 \tag{A1.4c}$$

APPENDIX 2

Matrix A and the vector H in equation (9) are given by

:

$$A \equiv \begin{bmatrix} \eta_{Ik} & \eta_{Ip} \\ -pI\eta_{Ik}/\Gamma & [(1-D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1(\frac{\tau NW}{\chi})\} - pI(1+\eta_{Ip})]/\Gamma \end{bmatrix},$$

$$\mathrm{H} \equiv \begin{bmatrix} 0 \\ -W(1 - D_W^1\Phi)/\Gamma \end{bmatrix}$$

$$\Gamma \equiv \eta_{Rp}\{D_\chi^1\chi - D_W^1(\frac{\tau NW}{\chi})\} + D_W^1(\frac{\tau NW}{\chi})\eta_{Wp} < 0 \qquad \text{(A2.1)}$$

where $D_j^1 \equiv \partial D_t^1 / \partial j_t$ is the derivative with respect to variable j (j=W,χ).

All the elements of matrix A are positive. The two roots are

ξ_S and ξ_U.

$$Tr\ A = \xi_S + \xi_U = a_{11} + a_{22} > 0$$

$$DetA = \xi_S\xi_U = \eta_{Ik}[(1 - D_W^1)(1-\tau)W\eta_{Wp} + \{D_\chi^1\chi - D_W^1(\frac{\tau NW}{\chi})\} - pI]/\Gamma > 0$$

Now

$$(TrA)^2 - 4DetA > 0$$

(Proof: All the elements of matrix A are positive. Hence

$(a_{11} + a_{22})^2 - 4\{a_{11}a_{22} - a_{12}a_{21}\} = (a_{11} - a_{22})^2 + 4(a_{12}a_{21}) > 0\ \blacksquare$)

So the (two positive) roots are real.

The requirement

$$\Delta \equiv 1 - TrA + DetA < 0$$

(this is the condition that two the roots of the matrix A-I are of opposite signs; or the roots of A lie on the opposite side of unity)

In our model, $\Delta = \left(1 - \eta_{Ip}\right)\Gamma^{-1}\left\{(1 - D_W^1)(1-\tau)W\eta_{Wp} + D_\chi^1\chi - D_W^1\left(\frac{\tau W}{\chi}\right) - \Gamma - \mathrm{pI}\right\}$

$+\Gamma^{-1}\, pI(1 + \eta_{Ip})$.

A “high enough” value of η_{Ip} is a sufficient condition to deliver this (requiring “high” elasticities of substitution in production). Cobb-Douglas technologies in the two sectors will satisfy this condition, but Leontief in both sectors will not.

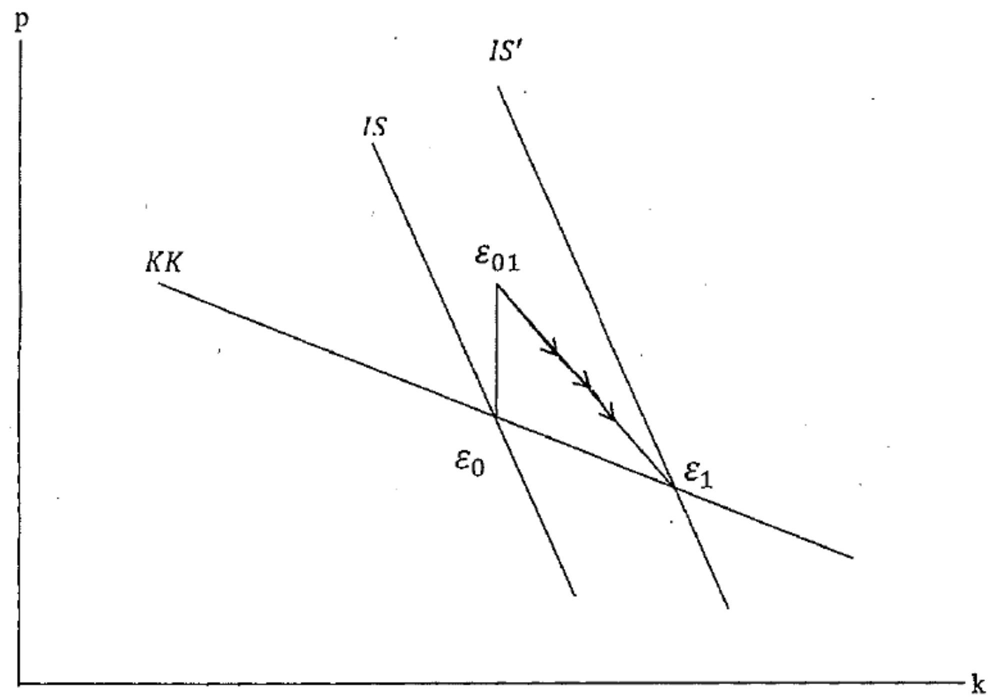

Figure 1 The Dynamic Effects of a Cut in Payroll Tax