A Compromise approach to the measurement of pro-Poorness[1]

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Abstract

A growth vector is often referred to as pro-poor if it generates a reduction in poverty. A second approach to pro-poorness says that growth should be higher for the poor than for the non-poor. In this article we develop an analytical approach by considering a compromise between these two notions of pro-poorness.

JEL classification: D31, I31, P36

Keywords: Pro-poorness, inequality, poverty

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[1] This article has been prepared as a tribute to Professor N.C. Kakwani, who has contributed significantly to the literature on pro-poorness.

1. Introduction

An economic policy may be regarded as pro-poor if it is concerned with the improvement of the well-being of the poor. This can be ensured if the underlying economic growth benefits the poor section of a society. Loosely speaking, this means that economic growth should lead to reduction of poverty in an unambiguous way.

The different approaches to the measurement of pro-poor growth can be broadly categorized into two groups. The contributions in the first group concentrate more on the term ‘growth’, assessing the gains or losses accruing to the poor and to the non-poor. For the contributions in the second group, evaluation of pro-poorness is judged in terms of growth’s potentiality for reduction of poverty with respect to some poverty indices, without considering the changes in the incomes of the non-poor.

Kakwani and Pernia (2000) and Pernia (2003) addressed the important matter of checking how a distribution of growth accomplished by a society at some specific time point has favoured the poor more than the non-poor, that is, the growth benefit has been more than proportionate to the poor than to the non-poor. One way of doing this is to verify whether there has been reduction in relative inequality or absolute inequality in the post growth distribution of income. This in turn brings about the notions of relative and absolute pro-poor growths (Kakwani and Pernia, 2000, Kakwani and Son, 2008). According to Grosse et al. (2008) and Zheng (2011) absolute growth is pro-poor if growth’s support to the poor is higher than that to the non-poor. Ravallion and Chen’s (2003) interpretation of pro-poor growth relies on reduction of poverty with respect to some poverty index, without affecting the incomes of the rest of the population.

Ravallion and Chen (2003) considered a growth incidence curve which shows rates of growths of different quantiles of the distribution of income between two time points. They demonstrated that the area under this up to the headcount index equals the change in the Watts (1968) poverty index times minus one. This method is the same as requiring first order stochastic dominance[2] in terms of pre and post growth income distributions. Kraay (2006) considered a generalization of this using other poverty indices. Dhongde and Silber (2016) proposed a unified approach for the measurement of distributional change using a variant of the relative concentration curve. They employed a Gini-related index with weights based on individual income shares in the growth incidence curve framework to propose indicators of pro-poor growth.

Duclos (2009) and Araar et al. (2009) pointed out that the main features of pro-poor growth are represented by (i) relative or absolute reduction of inequality/poverty, (ii) separation of the set of the poor from set of non-poor, using a line of demarcation, the poverty line and (iii) evaluation of the benefit. A unifying normative framework using stochastic dominance was provided by these authors.

Essama-Nssah (2005) and Essama-Nssah and Lambert (2009) employed the elasticity function of individual income with respect to total income and identified its importance for the measurement of pro-poorness. If this elasticity function takes on the value 1, then the underlying growth pattern is referred to as distributionally neutral. This elasticity function, when multiplied by the growth rate of mean income, defined as the change of mean income as a proportion of the mean income, turns out to be the growth-incidence function defined by Ravallion and Chen (2003). In contrast, the growth rate equivalent to the Kakwani-Son (2008) pro-poorness measure is given by the product of the growth rate of mean income and the ratio between the elasticity of a poverty index with respect to mean income for any growth pattern and the corresponding elasticity for a distributionally neutral growth pattern. A new class of pro-poorness metrics comes out from the general structure. These measures satisfy a factor decomposability postulate; they are decomposable across different sources of income. This postulate enables us to calculate the percentage contributions of different income sources to overall pro-poorness and hence to identify the major income sources that need attention from pro-poorness perspective.

Zheng (2011) initiated a consistency property which demands that if one growth pattern is treated as more pro-poor than a second growth pattern at a given growth rate, then pro-poorness ordering between the two growth patterns should remain unaltered if the growth rate comes out

[2] See Shaked and Shanthikumar (2007) for definitions and discussions of different stochastic dominance criteria.

to be higher. Zheng (2011) explicitly established that there does not exist any poverty-growth-elasticity metric that satisfies the growth-rate consistency axiom along the two inequality neutral paths[3]. Zheng (2011) also explored the conditions to be laid down on a poverty metric under which it becomes possible to use a poverty-growth elasticity measure in a consistent way. As Zheng (2011) demonstrated, a sufficient condition for relative growth rate consistency is Jenkins and Lambert's (1997) TIP curve dominance. Assuming that the income distribution is non-decreasingly ordered, its TIP curve is deduced by graphing the cumulative relative shortfalls of the incomes corresponding to the censored income distribution, the distribution obtained by censoring all the incomes above the poverty threshold at the threshold point itself, where the relative shortfalls are taken with respect to the poverty threshold, It may be worthwhile to note that the TIP curve, the growth incidence curve, and the poverty growth curve (Son, 2004) are all quite closely linked to the Lorenz or the generalised Lorenz curve.

The role of generalized Lorenz superiority, which is equivalent to second order stochastic dominance, in the context of pro-poorness has been investigated by Duclos (2009) (see also Son (2004)). The generalized Lorenz superiority of one distribution over another is same as the stipulation that the former can be obtained from the latter by a finite sequence of rank preserving income increments and a finite sequence of rank preserving progressive transfers or simply by a finite number of rank preserving income increments (see Foster and Shorrocks, 1988, Lemma 2) and Chakravarty (2009, Theorem 2.1). It follows from a theorem of Atkinson (1987) that this is equivalent to the condition that the former is regarded as less poverty stricken than the latter for a general class of additive poverty measures for all poverty lines. Therefore, intrinsic to the notions of the curves we have considered here is rank preservation.

In a recent contribution, Chakravarty, Chattopadhyay and D'Ambrosio (2019) proposed a general approach for ordering growth profiles in terms of pro-poorness using 'Progressive Sequential Averaging (PSA)' principle . According to the PSA principle, a growth pattern is regarded as pro-poor if the absolute average benefit of growth is higher for the poor than that for the non-poor, given that the poverty threshold is

[3] For a related discussion, see Klasen (2008) and Klasen and Misselhorn (2008)

variable. Evidently, variation of the poverty cut-off point enables one to consider all possible sets of the poor and non-poor[4]. In the PSA approach the rank preservation assumption of post growth incomes of individuals is not maintained. Consequently, the individuals are identified only by their ill-fare ranks, the ranks in the non-decreasingly ordered initial distribution, and re-ranking of incomes in the post growth profile is allowed.

The PSA ordering is implemented by seeking a dominance relation in terms of the PSA curve, a plot of the cumulative proportions of total growth enjoyed by the bottom cumulative population proportions. All the PSA-ordering equivalent conditions are developed independently of the sizes of the poor and non-poor. Consequently, it becomes possible to compare pro-poorness of the same country over time or of two countries with different populations.

In this chapter we develop a compromise approach to pro-poorness that considers both inequality and poverty reduction. A rigorous evaluation of pro-poorness from such a perspective is yet to be addressed in the literature. We define a growth vector to be pro-poor if either (i) there is a reduction of poverty without any increase in overall inequality or (ii) there is a reduction in overall inequality without any increase in poverty. Thus this notion involves non-increasingness of both poverty and inequality. We refer to it his as a compromise approach because it makes an adjustment between the two following notions of pro-poorness: (i) A growth vector is said to be pro-poor if it induces a reduction in poverty. (A concern here is that lowering of poverty may accompany an increase in inequality. This has been addressed by making adjustments in the change in the poverty level for possible change in inequality (Ravallion and Chen, 2003)), (ii) A growth profile is pro-poor if the growth rate is higher for the poor in comparison with that for the non poor. (See Kakwani and Pernia ,2000, and Pernia , 2003).

[4] In their welfare theoretic generalization of the Sen (1976) poverty index, Blackorby and Donaldson (1980) demonstrated the necessity of complete strict recursivity of the welfare function, a property which demands that the welfare of every subgroup of poor must be separable from that of the non-poor. This implicitly assumes variability of the poverty line.

A requirement for the implementation of this approach is that the type of inequality and poverty indices needs to be similar. In particular, the poverty index may be the inequality index of incomes restricted to the poverty line censored distribution and with shortfalls of income from the given poverty line as the argument.

After presenting the preliminaries and axioms in the next section, in the third section we illustrate our compromise approach using the Gini index and the variance as the inequality metrics and their respective poverty counterparts. Finally, the fourth section concludes.

2. **Preliminaries and axioms**

2.1 Preliminaries

Let $x_i > 0$ stand for the income of person i in an $m-$person society. We assume, without loss of generality, that the incomes are non-decreasingly ordered, that is, $x_1 \leq x_2 \ldots \leq x_m$. We write D^m for the set of income distributions in this society. A typical element of D^m is denoted by $x = (x_1, x_2, \ldots, x_m)$. The set of all possible income distributions is given by $D = \bigcup_{m \in N} D^m$, where N is the set of positive integers.

Let $z > 0$ be the arbitrarily given poverty cut-off limit. The cut-off point z is assumed to take values in the interval $[z_-, z_+]$. A person is defined as poor if his income falls below the poverty threshold z , otherwise he is called non-poor. For any $x \in D^m$, the number of poor persons is denoted by $q(x)$ (or, simply by q). Since x is non-decreasingly ordered, it follows that $x_i < z$ for $i = 1,2,\ldots\ldots,q$ and $x_i \geq z$ for $i = q+1, q+2,\ldots\ldots,m$. Thus, the set of poor persons corresponding to x is given by $\{1,2,\ldots\ldots,q\}$. For any $x \in D^m$, the associated income distribution of the poor is denoted by $x^p = (x_1, x_2, \ldots, x_q)$. For any $x \in D^m$ and $x^p \in D^q$, we denote their mean incomes respectively by $\bar{x}$ and $\bar{x}^p$.

Assume that the economy has undergone some income growth and the pre-growth income distribution $x \in D^m$ gets transformed into $y \in D^m$. We can define the underlying individual absolute growths using the growth functions $g_i : D^1 \times D^1 \to \Re^1$, where $i = 1,2,......,m$. By assuming that the form of $g_i = (y_i - x_i)$ may vary across persons, we implicitly assume that the growth patterns need not be identical. Let $b_i = g_i(x_i) = (y_i - x_i)$. By assuming that $b_i \in \Re^1$, we allow the possibility that growths may be negative. For the pre-growth income distribution $x \in D^m$, the growth levels are denoted by $b = (b_1, b_2, \ldots, b_m) \in \Re^m$. Thus, the notions of poor and the growth profile are defined with respect to the pre-growth income distribution x. Evidently, the growth vector $b \in \Re^m$ need not be ordered and hence a poor person may experience a higher growth than a rich person. We write b^p for the growth vector of the poor. The growths are incentive preserving; that is, if $x_i \le x_j$, then $y_i \le y_j$. In words, the pre-growth ranks of the individuals are maintained in the post-growth situation. Although a person may experience a negative growth, we assume his status in the pre-growth distribution is maintained in the post-growth profile. That is, if a person is poor (rich) in the pre-growth distribution then he is poor (rich) as well in the post-growth distribution. The rank preservation assumption of growth is made in order to have wider choices of indices of poverty and inequality. If the indices satisfy anonymity this assumption is indirectly incorporated.(See Subsection 2b.)

A pro-poorness index Π is a real valued function of incomes, poverty threshold and growths. Formally, $\Pi : D^m \times [z_-, z_+] \times \Re^m \to \Re^1$. That is, for any $m \in N$, $(x, z, b) \in D^m \times [z_-, z_+] \times \Re^m$, the real number $\Pi(x, z, b)$ indicates extent of pro-poorness underlying the growth vector $b \in \Re^m$, given the pre-growth income distribution $x \in D^m$ and the poverty cut-off limit $z \in [z_-, z_+]$.

2b.Axioms

Since our compromise approach incorporates both inequality and poverty, the pro-poorness index should satisfy several axioms that are consistent with inequality and poverty.

The first axiom is anonymity which requires the pro-poorness metric to remain invariant under any reordering of incomes. This postulate enables us to define the pro-poorness evaluator directly on ordered incomes. In view of this axiom, we can say that all attributes other than income are inappropriate to the assessment of pro-poorness. The second axiom we incorporate is the well-known focus axiom, a stipulation which claims that poverty should remain unchanged under a reduction in the income of a rich as long as the reduction does not make the person poor. We also stipulate that inequality should decrease under a rank preserving progressive transfer that does not change the number of poor, while the corresponding poverty should either remain unchanged or decrease depending upon the location of the transfer.

Often we may need to compare the pro-poorness of the same society over time or across societies. This necessitates that the pro-poorness evaluation should be population replication invariant; given the poverty threshold, an income-by-income replication of the population should keep pro-poorness level unchanged.

3. Illustrations

In order to illustrate the compromise approach, we consider two examples. The first example takes the variance as the inequality metric. For any income distribution $x \in D^m$, its variance $V(x)$ is defined as $V(x) = \frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2$. As per the first characteristic of the compromise approach, we need satisfaction of the inequality

$$\frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2 \geq \frac{1}{m}\sum_{i=1}^{m}\left(x_i + b_i - \bar{x} - \bar{b}\right)^2 , \quad (1)$$

which implies that

$$\frac{1}{m}\sum_{i=1}^{m}\left(b_i-\bar{b}\right)^2+\frac{2}{m}\sum_{i=1}^{m}\left(x_i-\bar{x}\right)\left(b_i-\bar{b}\right)\leq 0\ . \tag{2}$$

This gives

$$\frac{\frac{1}{m}\sum_{i=1}^{m}\left(x_i-\bar{x}\right)\left(b_i-\bar{b}\right)}{\frac{1}{m}\sum_{i=1}^{m}\left(x_i-\bar{x}\right)^2}\leq-\frac{1}{2}\frac{\frac{1}{m}\sum_{i=1}^{m}\left(b_i-\bar{b}\right)^2}{\frac{1}{m}\sum_{i=1}^{m}\left(x_i-\bar{x}\right)^2}\ . \tag{3}$$

The left hand side of the above inequality is simply the regression coefficient of b on x. A negatively sloped regression line indicates that the benefits are inversely related to the incomes on an average.

The second component of our approach requires simultaneous poverty reduction using an analogous poverty index. Consequently, the poverty function we consider here is the average of the squared deviations of incomes of the poor from the poverty line, that is, $\frac{1}{q}\sum_{i=1}^{q}\left(z-x_i\right)^2$.

We, therefore, need fulfilment of the following inequality

$$\frac{1}{q}\sum_{i=1}^{q}\left(z-x_i\right)^2\geq\frac{1}{q}\sum_{i=1}^{q}\left(z-x_i-b_i\right)^2. \tag{4}$$

This inequality implies that

$\frac{1}{q}\sum_{i=1}^{q}b_i^{\,2}+\frac{2}{q}\sum_{i=1}^{q}\left(z-x_i\right)b_i\leq 0$, from which it follows that

$$\frac{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)(b_i-\bar{b}_p)}{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)^2} \le \frac{\frac{1}{q}\sum_{i=1}^{m}(b_i-\bar{b}_p)^2}{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)^2}\left[-\frac{1}{2}+\bar{b}^p\left(z-\bar{x}^p-\frac{\bar{b}^p}{2}\right)\right].$$

We rewrite the above inequality as

$$\frac{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)(b_i-\bar{b}_p)}{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)^2} \le \frac{-\frac{1}{2q}\sum_{i=1}^{m}(b_i-\bar{b}_p)^2+\left[\bar{b}^p\left(z-\bar{x}^p-\frac{\bar{b}^p}{2}\right)\right]}{\frac{1}{q}\sum_{i=1}^{m}(x_i-\bar{x}_p)^2}. \tag{5}$$

Inequality (5) requires that the regression coefficient of b on x ,where the data points are for the poor only, to be more negative than the overall coefficient, since the additional term is likely to be negative.

We are now in a position to combine the above results to arrive at an overall metric of pro-poorness in the following manner. We can rewrite the inequality reducing pro-poorness condition (3) a

$$\frac{\frac{1}{m}\sum_{i=1}^{m}(x_i-\bar{x})(b_i-\bar{b})}{\sqrt{\frac{1}{m}\sum_{i=1}^{m}(x_i-\bar{x})^2\frac{1}{m}\sum_{i=1}^{m}(b_i-\bar{b})^2}} \le -\frac{1}{2}\frac{\sqrt{\frac{1}{m}\sum_{i=1}^{m}(b_i-\bar{b})^2}}{\sqrt{\frac{1}{m}\sum_{i=1}^{m}(x_i-\bar{x})^2}}. \tag{6}$$

From (6) it follows that $2\rho(x,b)+\frac{\sigma_b}{\sigma_x}$ can be an indicator of pro-poorness addressing inequality non-increasingness, where $\rho(x,b)$ is the correlation coefficient between the benefits and the incomes, and σ_b and σ_x are respectively the standard deviations of the benefits and the incomes. For pro-poorness we require this measure to be negative.

Likewise, the poverty reduction condition (5) for pro-poorness can be rewritten as

$$\rho\left(x^p, b^p\right) \leq -\frac{1}{2}\frac{\sigma_b^p}{\sigma_x^p} + \frac{\bar{b}^p\left(z - \bar{x}^p - \frac{\bar{b}^p}{2}\right)}{\sigma_b^p \sigma_x^p} \tag{7}$$

giving a measure $2\rho\left(x^p, b^p\right) + \frac{\sigma_b^p}{\sigma_x^p} - \frac{2\bar{b}^p\left(z - \bar{x}^p - \frac{\bar{b}^p}{2}\right)}{\sigma_x^p \sigma_b^p}$, where $\rho\left(x^p, b^p\right)$ is the correlation coefficient between the incomes of the poor and their benefits; and σ_x^p and σ_b^p are respectively the standard deviations of the incomes of the poor and their benefits. As in the case of inequality reduction, we require this metric to be negative.

When both indicators are negative we have unambiguous pro-poorness. Further, if we wish to have a single compromise measure, we can take a convex combination of the two measures. Formally, the formula for the compromise measure is given b

$$\Pi_V^\theta(x, z, b) = \theta\left(2\rho(x, b) + \frac{\sigma_b}{\sigma_x}\right) + (1 - \theta)\left[2\rho\left(x^p, b^p\right) + \frac{\sigma_b^p}{\sigma_x^p} - \frac{2\bar{b}^p\left(z - \bar{x}^p - \frac{\bar{b}^p}{2}\right)}{\sigma_x^p \sigma_b^p}\right]. \tag{8}$$

where $0 \leq \theta \leq 1$ is a parameter. The subscript V in $\Pi_V^\theta(x, z, b)$ indicates that the underlying inequality metric is the variance. As the value of the parameter θ decreases over the closed interval $[0,1]$, more weight is assigned to poverty reduction than to inequality reduction. Therefore, θ may be interpreted as a policy parameter.

Remark 1: We now look at the implication of the assumption that the sizes of the poor and the non-poor remain the same in the pre and post growth situations. While for the inequality condition we do not require this, for poverty reduction this is indeed a requirement. To illustrate, consider the average quadratic shortfall poverty index. Since the income distribution is illfare ranked, we have

$$(z-x_q)^2 \leq (z-x_{q-1})^2 \leq ....\leq (z-x_1)^2,$$

from which it follows that

$$\frac{1}{q}\sum_{i=1}^{q}(z-x_i)^2 \leq \frac{1}{q-1}\sum_{i=1}^{q-1}(z-x_i)^2.$$

Thus if the headcount drops with the person nearest to the poverty line becoming non-poor, with other individuals remaining in their positions, poverty increases. This situation can be averted if we assume that the headcount remains the same. We maintain this assumption throughout the chapter.

The variance is an absolute measure of inequality; it remains invariant under equal absolute changes in incomes. Likewise, its poverty sister chosen above is also an absolute metric; it stands unaltered under equal absolute changes in incomes and the poverty limit itself. These contrast with relative notions of inequality and poverty, where relativity of inequality demands inequality invariance with respect to equi-proportional variations in incomes and relative poverty requires poverty to remain unchanged for equi-proportional variations in incomes and the threshold limit as well.

Our next illustration is based on the Gini index I_G, the most frequently used relative inequality standard. It is known that the Gini index can be written in terms of the covariance of incomes and their proportional ranks (Yitzhaki, 1998). We use this formulation of the Gini for easy exposition. Now, $I_G(x) = \frac{2}{\bar{x}}Cov(x, F(x))$, where $x \in D^m$ and F is the cumulative distribution function of x. Given that incomes are illfare ranked, in the current context $F(x)$ stands for the vector $\left(\frac{1}{m}\frac{2}{m},.....,\frac{m-1}{m},\frac{m}{m}\right)$. In view of our assumption, $F(x)=F(x+b)$, where $F(x+b)$ is the cumulative distribution function of $(x+b)$.

$$I_G(x+b)=\frac{2}{\bar{x}+\bar{b}}Cov(x+b,F(x+b))$$
$$=\frac{2}{\bar{x}+\bar{b}}Cov(x+b,F(x))$$
$$=\frac{2}{\bar{x}+\bar{b}}[Cov(x,F(x))+Cov(b,F(x))]. \qquad (9)$$

Hence inequality reduction requires

$I_G(x)-I_G(x+b)=Cov(x,F(x))\left[\frac{2}{\bar{x}}-\frac{2}{\bar{x}+\bar{b}}\right]-\frac{2}{\bar{x}+\bar{b}}Cov(b,F(x))\geq 0$, which on further simplification gives $Cov(b,F(x))\leq\frac{\bar{b}}{\bar{x}}Cov(x,F(x))$. This inequality is equivalent to

$\rho(b,F(x))\leq\frac{\bar{b}\,\sigma_x}{\bar{x}\sigma_b}\rho(x,F(x))=\frac{CV(x)}{CV(b)}\rho(x,F(x))$, where CV denotes the coefficient of variation. Consequently, a pro-poorness metric based on non-increasingness of inequality, as measured by the Gini index, can be

$$CV(b)\rho(b,F(x))-CV(x)\rho(x,F(x)). \qquad (10)$$

A negative value of this index will represent pro-poorness.

For poverty reduction, we can compute the analogous Gini with income shortfalls from the poverty thresholds as the argument and restricting the set to be population of the poor. However, since the covariance is not affected by a change of origin, the Gini computed from the income shortfalls of the poor will be the just a scalar multiple of the Gini computed from the incomes with the mean subtracted from z. The required condition, therefore, comes to be $Cov\left(b^p, F\left(x^p\right)\right) \le \frac{\bar{b}}{z - \bar{x}^p} Cov\left(x^p, F\left(x^p\right)\right)$. Thus, a measure of pro-poorness based on poverty non-increasingness will be

$$CV\left(b^p\right)\rho\left(b^p, F\left(x^p\right)\right) - CV\left(x^p\right)\rho\left(x^p, F\left(x^p\right)\right). \tag{11}$$

A negative value of this measure will indicate pro-poorness.

A convex combination $\Pi_G^\theta(x,z,b)$ of the two indicators proposed in (10) and (11) can be taken as a Gini-based single compromise standard of pro-poorness. Formally,

$$\Pi_G^\theta(x,z,b) = \theta\left[CV(b)\rho(b,F(x)) - CV(x)\rho(x,F(x))\right] + (1-\theta)\left[CV\left(b^p\right)\rho\left(b^p,F\left(x^p\right)\right) - CV\left(x^p\right)\rho\left(x^p,F\left(x^p\right)\right)\right], \tag{12}$$

where, as in (8), $0 \le \theta \le 1$ is a policy parameter.

Remark 2: The poverty index used in (11) was suggested by Thon (1979). This index is a violator of several important poverty axioms, including population replication invariance. (See Zheng, 1997, for a detailed discussion.)

4. Conclusions

The two compromise indicators of pro-poorness we have analyzed in this chapter are based on the variance and the Gini index, and their corresponding poverty counterparts. It will be worthwhile to develop a unified approach to the measurement of pro-poorness along this line. We leave this as a future research program.

We conclude the chapter by exploring the possibility of employing the sociological phenomenon of relative deprivation for evaluation of pro-poorness. (See Runciman, 1966, for a discussion on relative deprivation from different perspectives.) Given the illfare ranked income distribution $x \in D^m$, person i has a feeling of deprivation about incomes higher x_i . A simple metric of person $i's$ deprivation is $d_i = \sum_{j=i+1}^{n}(x_j - x_i)$. In words, person $i's$ deprivation about higher incomes is the sum of shortfalls of his incomes from all higher incomes[5]. The post-growth deprivation of person i is given by $\bar{d}_i = \sum_{j=i+1}^{n}(x_j + b_j - x_i - b_i)$. By our assumption, $(x_j + b_j - x_i - b_i) \geq 0$ for all $j \in \{i+1.i+2,......,m\}$. Consequently, for deprivation of person i to decrease under income growth, the following inequality is required to be fulfilled

$$(d_i - \bar{d}_i) \geq 0 . \tag{13}$$

The above inequality can be simplified as

$$b_i \geq \frac{1}{n-i}\sum_{j=i+1}^{n} b_j . \tag{14}$$

The simultaneous satisfaction of the condition in (14) for all individuals imply that the average benefit of every subgroup of poor cumulated from bottom is not less than the average benefit of the corresponding complementary class of (rich) persons. This is the PSA condition of pro-poorness introduced by Chakravarty , Chattopadhyay and D'Ambrosio(2019). This provides an interpretation of the PSA condition from a relative deprivation perspective. Since the notion of deprivation has not been used previously in the context of pro-poorness, it would be worthwhile to investigate its implications in greater details in the pro-poorness set up.

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[5] Alternatively, one may also consider using the proportion of the population richer than a person as an indicator of his deprivation.

Acknowledgements

For helpful comments and suggestions, we thank an anonymous reviewer. We also thank Jacques Silber who went through two earlier drafts of the paper and offered several suggestions.

References

Araar, A., Duclos, J.-Y. , Audet, M. & Makdissi, P. (2009). Testing for pro-poorness of growth, with an application to Mexico. *Review of Income and Wealth*, 55, 853- 881.

Atkinson, A.B. (1987). On the measurement of poverty. *Econometrica*, 55, 749-764.

Blackorby, C. & Donaldson, D. (1980). Ethical indices for the measurement of poverty. *Econometrica*, 48, 1053-61.

Chakravarty, S.R. (2009). *Inequality, polarization and poverty: Advances in distributional analysis.* New York: Springer.

Chakravarty, S.R., Chattopadhyay , N. & D'Ambrosio,C.(2019). Pro-poorness orderings. *Review of Income and Wealth*, 65, 785- 803.

Dhongde, S. & Silber, J. (2016). On Distributional change, pro-poor growth and convergence. *Journal of Economic Inequality*, 14, 249-267.

Duclos, J.-Y. (2009). What is pro-poor? *Social Choice and Welfare*, 32, 37-58.

Essama-Nssah, B. (2005). A unified framework for pro-poor growth analysis. *Economic Letters*, 89, 216-221.

Essama-Nssah, B & Lambert, P.J. (2009). Measuring pro-poorness: A unifying approach with new results. *Review of Income and Wealth*, 55, 752-778.

Foster, J. & Shorrocks, A.F. (1988). Poverty orderings and welfare dominance. *Social Choice and Welfare*, 5,

Grosse, M., Harttgen, K. & Klasen, S. (2008). Measuring pro-poor growth in non-income dimensions. *World Development*, 36, 1021-1047.

Jenkins, S. & Lambert, P.J. (1997). Three 'I's poverty curves, with an application of UK poverty trends. *Oxford Economic Papers*, 49, 317-327.

Kakwani, N & Pernia, E.M. (2000). What is pro-poor growth? *Asian Development Review*, 18, 1-16.

Kakwani, N & Son, H.H. (2008). Poverty equivalent growth rate. *Review of Income and Wealth*, 54, 643-655.

Klasen, S. (2008). Economic growth and poverty reduction: measurement issues using income and non-income indicators. *World Development,* 36, 420-445.

Klasen, S. & Misselhorn, M. (2008). Determinants of the growth semi-elasticity of poverty reduction. *EUDN/WP* 2008-11.

Kraay, A. (2006). When is growth pro-poor? Evidence from a panel of countries. *Journal of Development Economics*, 80, 198-227.

Pernia, E.M. (2003). Pro-poor growth what is it and how is it important? Manila: *Asian Development Bank.*

http://www.adb.org/Documents/EDRC/Policy%5FBriefs/PB017.

Ravallion, M & Chen, S. (2003). Measuring pro-poor growth, *Economic Letters*, 78, 93-99.

Runciman, W.G.(1966). *Relative deprivation and social justice*. Routledge, London.

Sen, A. K.(1976).Poverty: An ordinal approach to measurement, *Econometrica,* 44, 219–31.

Shaked, M & Shanthikumar, G. (2007). *Stochastic orders*. Berlin:*Springer.*

Son, H.H (2004). A note on pro-poor growth. *Economic Letters,* 82, 307-314.

Watts, H.W. (1968). An economic definition of poverty. In: Moynihan, D.P. (Ed) *On understanding poverty: Perspectives from the social*

\ *sciences.* New York: Basic Books, 316-329.

Yitzhaki, S.(1998). More than a dozen alternative ways of spelling the Gini. *Research on Economic Inequalit* ,8, 13-20.

Zheng, B. (1997). Aggregate poverty measures. *Journal of Economic Surveys*, 11, 123-162.

Zheng, B. (2011). Consistent comparison of pro-poor growth. *Social Choice and Welfare*, 37, 61-79.