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“We haven’t got but one more day”

The Cuban missile crisis as a dynamic chicken game

By

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The Cuban missile crisis of October 1962, when the world came close to nuclear war, has become the classic example of “brinkmanship.” This term is often used in public forums without precise understanding of its full meaning, or of the dynamics and risks of the strategy it involves. And while most game theorists have an intuitive understanding of the theory underlying it, they rarely carry it to the point of writing down a model that captures the specifics of the crisis, let alone putting any numbers into it that will give us a rough idea of how big that risk of nuclear war was. In the belief that theory should constantly make a better contact with reality, and should give us not just qualitative but also quantitative understanding, we offer here an analytic narrative of the crisis, leading to a model where we can get some numbers out. To quote the physicist Hans Bethe, “numbers prove theories.”¹

Here is a quick reminder of the idea of brinkmanship as explained by Schelling (1960, Chs. 7, 8; 1965, Ch. 3). A threat is a response rule, and the threatened action inflicts a cost on both the player making the threat and the player whose action the threat is intended to influence. However, if the threat succeeds in its purpose, this action is not actually carried out. Therefore, there is no apparent upper limit to the cost of the threatened action. But the risk of errors—that is, the risk that the threat may fail to achieve its purpose or that the threatened action may occur by accident—forces the strategist to use the minimal threat that achieves its purpose. If a smaller threat is not naturally available, a large threat can be scaled down by

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¹ <https://www.youtube.com/watch?v=GRuE5vb4Yio>

making its fulfillment probabilistic. You create in advance a probability, but not certainty, that the mutually harmful outcome will happen if the opponent defies you. If you had ex post freedom to act, you would not take that bad action. Therefore you must arrange in advance to lose control over the realization, while controlling the probability. Brinkmanship is the creation and deployment of such a probabilistic threat; it consists of a deliberate and controlled loss of control. That is tricky and risky to achieve successfully.

We begin with a brief narrative of the events. Then we argue why a simple explanation – a threat from the United States that compelled the Soviet Union to withdraw the missiles – is inadequate. Then we build a model of brinkmanship as a dynamic or “real-time” version of chicken, or a war of attrition, and solve it numerically to get an idea of the rough magnitudes involved. We conclude with some general reflections about brinkmanship.

1. Brief Narrative of Events

Our account draws on several books, including some that were written with the benefit of documents and statements released since the collapse of the Soviet Union.² We cannot hope to do justice to the detail, let alone the drama, of the events. We urge you to read the books that tell the story in vivid detail, and watch movies like *Thirteen Days* for the drama.

In late summer and early fall of 1962, the Soviet Union (USSR) started to place medium- and intermediate-range ballistic missiles (MRBMs and IRBMs) in Cuba. The MRBMs had a range of 1,100 miles and could hit Washington, D.C.; the IRBMs, with a range of 2,200 miles, could hit most of the major U.S. cities and military installations.³ The missile sites were guarded by the latest Soviet SA-2–type surface-to-air missiles (SAMs), which could shoot down U.S. high-altitude U-2 reconnaissance planes. There were also IL-28 bombers and tactical nuclear weapons called Luna by the Soviets and FROG (free rocket over ground) by the United States, which could be used against invading troops.

² Our sources (which we list in a separate section of historical references at the end) include Blight and Welch (1989), Thompson (1992), Reeves (1993), Kagan (1995), Fursenko and Naftali (1997), May and Zelikow (eds) (1997), Dobbs (2008), and Ellsberg (2017). All provided us general background information; some are cited for specific details in the text. Allison (1971) remains important not only for its narrative, but also for its analysis and interpretation. Our view differs from his in some important respects, but we remain in debt to his insights.

³ A theme park in Havana displays samples of these missiles (unarmed, we hope!). See Figure 1.

**** FIGURE 1 ABOUT HERE ****

This was the first time that the Soviets had ever attempted to place their missiles and nuclear weapons outside Soviet territory. Had they been successful, it would have increased their offensive capability against the United States manyfold. It is now believed that the Soviets had very few (US aerial reconnaissance showed only 4), operational intercontinental ballistic missiles (ICBMs) in their own country capable of reaching the United States (Kagan 1995, pp. 464, 509–510, Ellsberg 2017, p. 158). Their initial placement in Cuba had about 40 MRBMs and IRBMs, which was a substantial increase. But the United States would still have retained vast superiority over the Soviet Union in the number of nuclear missiles capable of striking the other side. Also, as the Soviets built up their submarine fleet, the relative importance of land-based missiles near the United States would have decreased. But the missiles had more than mere direct military value to the Soviets. Successful placement of missiles so close to the United States would have been an immense boost to Soviet prestige throughout the world, especially in Asia and Africa, where the superpowers were competing for political and military influence. Finally, the Soviets had come to think of Cuba as a “poster child” for socialism. The opportunity to deter a feared U.S. invasion of Cuba and to counter Chinese influence in Cuba weighed importantly in the calculations of the Soviet leader and Premier, Nikita Khrushchev. (See Fursenko and Naftali 1997, pp. 182–183, for an analysis of Soviet motives.)

The Soviets hoped to keep the installation secret, concealed under palm trees, until the missiles became operational (Fursenko and Naftali 1997, ch.10). But this did not work. U.S. surveillance of Cuba and of shipping lanes during the late summer and early fall of 1962 had indicated some suspicious activity. When questioned about it by U.S. diplomats, the Soviets denied any intentions to place missiles in Cuba. Later, faced with irrefutable evidence, they said that their intention was defensive, to deter the United States from invading Cuba. It is hard to believe this, although we know that an offensive weapon can serve as a defensive deterrent threat.

An American U-2 “spy plane” took photographs over western Cuba on Sunday and Monday, October 14 and 15; they showed unmistakable signs of construction on MRBM launching sites. (Evidence of IRBMs was found later, on October 17.) They were shown to President Kennedy the following day (October 16). He immediately convened an ad hoc group of top-level advisers,

later called the Executive Committee of the National Security Council (ExComm), to discuss the alternatives.⁴ He decided to keep the matter totally secret until he was ready to act, mainly because if the Soviets knew that the Americans knew, they might speed up the installation and deployment of the missiles before the Americans were ready to act, but also because spreading the news without announcing a clear response would create panic in the United States. In the rest of that week (October 16 through 21), ExComm met numerous times. To preserve secrecy, the President continued his normal schedule, including travel to speak for Democratic candidates in the upcoming midterm congressional elections. He kept in constant touch with ExComm. He dodged press questions about Cuba and persuaded one or two trusted media owners or editors to preserve the facade of business as usual.

Different members of ExComm had widely differing assessments of the situation and supported different actions. The military Chiefs of Staff thought that the missile placement changed the balance of military power substantially; Defense Secretary McNamara thought it changed “not at all” but regarded the problem as politically important nonetheless (May and Zelikow (eds) 1997, p. 89). Kennedy pointed out that the first placement, if ignored by the United States, could grow into something much bigger and that the Soviets could use the threat of missiles so close to the United States to try to force the withdrawal of the U.S., British, and French presence in West Berlin. Kennedy was also aware that it was a part of the geopolitical struggle between the United States and the Soviet Union (May and Zelikow (eds) 1997, p. 92).

It now appears that he was very much on the mark in this assessment. The Soviets planned to expand their presence in Cuba into a major military base (May and Zelikow (eds) 1997, p. 677). They expected to complete the missile placement by mid-November. Khrushchev had planned to sign a treaty with Castro in late November, then travel to New York to address the United Nations and issue an ultimatum for a settlement of the Berlin issue (May and Zelikow (eds) 1997, p. 679; Fursenko and Naftali 1997, p. 182), using the missiles in Cuba as a threat for

⁴ Members of ExComm who figured most prominently in the discussions were the Secretary of Defense, Robert McNamara; the National Security Adviser, McGeorge Bundy; the Chairman of the Joint Chiefs of Staff, General Maxwell Taylor; the Secretary of State, Dean Rusk, and Undersecretary George Ball; the Attorney General, Robert Kennedy (who was also the President’s brother); the Secretary of the Treasury, Douglas Dillon (also the only Republican in the Cabinet); and Llewellyn Thompson, who had recently returned from being U.S. Ambassador in Moscow. During the two weeks that followed, they would be joined by or would consult with several others, including the U.S. Ambassador to the United Nations, Adlai Stevenson; the former Secretary of State and a senior statesman of U.S. foreign policy, Dean Acheson; and the Chief of the U.S. Air Force, General Curtis LeMay.

this purpose. Khrushchev thought Kennedy would accept the missile placement as a fait accompli. Khrushchev appears to have made these plans on his own. Some of his top advisers privately thought them too adventurous, but the top governmental decision-making body of the Soviet Union, the Presidium, supported him, although its response was largely a rubber stamp (Fursenko and Naftali 1997, p. 180). Castro was at first reluctant to accept the missiles, fearing that they would trigger a U.S. invasion (May and Zelikow (eds) 1997, pp. 676–678), but in the end he, too, accepted them. The prospect gave him great confidence and lent some swagger to his statements about the United States (Fursenko and Naftali 1997, pp. 186–187, 229–230).

In all ExComm meetings up to and including the one on the morning of Thursday, October 18, everyone appears to have assumed that the U.S. response would be purely military. The only options that they discussed seriously during this time were (1) an air strike directed exclusively at the missile sites and (probably) the SAM sites nearby, (2) a wider air strike including Soviet and Cuban aircraft parked at airfields, and (3) a full-scale invasion of Cuba. If anything, attitudes hardened when the evidence of the presence of the longer-range IRBMs arrived. In fact, at the Thursday meeting, Kennedy discussed a timetable for air strikes to commence that weekend (May and Zelikow (eds) 1997, p. 148).

McNamara had first mentioned a blockade toward the end of the meeting on Tuesday, October 16, and developed the idea (in a form uncannily close to the course of action actually taken) in a small group after the formal meeting had ended (May and Zelikow (eds) 1997, pp. 86, 113). Ball argued that an air strike without warning would be a “Pearl Harbor” and that the United States should not do it (May and Zelikow (eds) 1997, p. 115); he got important support from Robert Kennedy (May and Zelikow (eds) 1997, p. 149). The civilian members of ExComm further shifted toward the blockade option when they found that what the military Joint Chiefs of Staff wanted was a massive air strike; the military regarded a limited strike aimed at only the missile sites so dangerous and ineffective that “they would prefer taking no military action than to take that limited strike” (May and Zelikow (eds) 1997, p. 97).

Between October 18 and Saturday, October 20, the majority opinion within ExComm gradually coalesced around the idea of starting with a blockade, simultaneously issuing an ultimatum with a short deadline (from 48 to 72 hours was mentioned), and proceeding to military action if necessary after this deadline expired. International law required a declaration of war to

set up a blockade, but this problem was ingeniously resolved by calling it a “naval quarantine” of Cuba (May and Zelikow (eds) 1997, p. 190–196).

Some people held the same positions throughout these discussions (from October 16 through 21)—for example, the military Chiefs of Staff constantly favored a major air strike—but others shifted their views, at times dramatically. Bundy initially favored doing nothing (May and Zelikow (eds) 1997, p. 172) and then switched toward a preemptive surprise air attack (May and Zelikow (eds) 1997, p. 189). President Kennedy’s own positions also shifted away from an air strike toward a blockade. He wanted the U.S. response to be firm. Although his reasons undoubtedly were mainly military and geopolitical, as a good politician he was also fully aware that a weak response would hurt the Democratic party in the imminent congressional elections. In contrast, the responsibility of starting an action that might lead to nuclear war weighed very heavily on him. He was impressed by the CIA’s assessment that some of the missiles were already operational, which increased the risk that any air strike or invasion could lead to the Soviets’ firing these missiles and to large U.S. civilian casualties (Fursenko and Naftali 1997, p. 235). In the second week of the crisis (October 22 through 28), his decisions seemed constantly to favor the lowest-key options discussed by ExComm.

By the end of the first week’s discussions, the choice lay between a blockade and an air strike. In a straw vote on October 20 the blockade won 11 to 6 (Kagan 1995, p. 516). Kennedy made the decision to start by imposing a blockade and announced it in a television address to the nation on Monday, October 22. He demanded a halt to the shipment of Soviet missiles to Cuba and a prompt withdrawal of those already there.

Kennedy’s speech brought the whole drama and tension into the public arena. The United Nations held several dramatic but unproductive debates. Other world leaders and the usual busybodies of international affairs offered advice and mediation.

Between October 23 and October 25, the Soviets at first tried bluster and denial; Khrushchev called the blockade “banditry, a folly of international imperialism” and said that his ships would ignore it. The Soviets, in the United Nations and elsewhere, claimed that their intentions were purely defensive and issued statements of defiance. In secret, they explored ways to end the crisis. This exploration included some direct messages from Khrushchev to Kennedy. It also included some very indirect and lower-level approaches by the Soviets. In fact, as early as Monday, October 22—before Kennedy’s TV address—the Soviet Presidium had decided not to

let this crisis lead to war. By Thursday, October 25, they had decided that they were willing to withdraw from Cuba in exchange for a promise by the United States not to invade Cuba, but they had also agreed to “look around” for better deals (Fursenko and Naftali 1997, pp. 241, 259). The United States did not know any of the Soviet thinking about this.

In public as well as in private communications, the USSR suggested a swap: withdrawal of U.S. missiles from Turkey and of Soviet ones from Cuba. This possibility had already been discussed by ExComm. The missiles in Turkey were obsolete; the United States wanted to remove them anyway and replace them with a Polaris submarine stationed in the Mediterranean Sea. But it was thought that the Turks would regard the presence of U.S. missiles as a matter of prestige and so it might be difficult to persuade them to accept the change. (The Turks might also correctly regard missiles, fixed on Turkish soil, as a firmer signal of the U.S. commitment to Turkey’s defense than an offshore submarine, which could move away on short notice; see May and Zelikow (eds) 1997, p. 568.)

The blockade went into effect on Wednesday, October 24. Despite their public bluster, the Soviets were cautious in testing it. Apparently, they were surprised that the United States had discovered the missiles in Cuba before the whole installation program was completed; Soviet personnel in Cuba had observed the U-2 overflights but had not reported them to Moscow (May and Zelikow (eds) 1997, p. 681). The Soviet Presidium ordered the ships carrying the most sensitive materials (actually the IRBM missiles) to stop or turn around. But it also ordered General Issa Pliyev, the commander of the Soviet troops in Cuba, to get his troops combat-ready and to use all means except nuclear weapons to meet any attack (May and Zelikow (eds) 1997, p. 682). In fact, the Presidium twice prepared (then canceled without sending) orders authorizing him to use tactical nuclear weapons in the event of a U.S. invasion (Fursenko and Naftali 1997, pp. 242–243, 272, 276). The U.S. side saw only that several Soviet ships (which were actually carrying oil and other nonmilitary cargo) continued to sail toward the blockade zone. The U.S. Navy showed some moderation in its enforcement of the blockade. A tanker was allowed to pass without being boarded; the tramp steamer *Marucla* carrying industrial cargo was boarded but allowed to proceed after only a cursory inspection. But tension was mounting, and neither side’s actions were as cautious as the top-level politicians on both sides would have liked.

On the morning of Friday, October 26, Khrushchev sent Kennedy a conciliatory private letter offering to withdraw the missiles in exchange for a U.S. promise not to invade Cuba. But later

that day he toughened his stance. It seems that he was emboldened by two items of evidence. First, the U.S. Navy was not being excessively aggressive in enforcing the blockade. It had let through some obviously civilian freighters; they boarded only one ship, the *Marucla*, and let it pass after a cursory inspection. Second, some dovish statements had appeared in U.S. newspapers. Most notable among them was an article by the influential and well-connected syndicated columnist Walter Lippman, who suggested the swap whereby the United States would withdraw its missiles in Turkey in exchange for the USSR's withdrawing its missiles in Cuba (Fursenko and Naftali 1997, p. 275). Khrushchev sent another letter to Kennedy on Saturday, October 26, offering this swap, and this time he made the letter public. The new letter was presumably a part of the Presidium's strategy of "looking around" for the best deal. Members of ExComm concluded that the first letter was Khrushchev's own thoughts but that the second was written under pressure from hard-liners in the Presidium—or was even evidence that Khrushchev was no longer in control (May and Zelikow (eds) 1997, pp. 498, 512–513). In fact, both of Khrushchev's letters were discussed and approved by the Presidium (Fursenko and Naftali 1997, pp. 263, 275).

ExComm continued to meet, and opinions within it hardened. There was growing feeling that the blockade by itself would not work. Kennedy's television speech had imposed no firm deadline; in the absence of a deadline a compelling threat is vulnerable to the opponent's procrastination, or "salami tactics" (Schelling 1965, pp. 66-69) Kennedy had seen this quite clearly, and as early as Monday, October 22 he commented, "I don't think we're gonna be better off if they're just sitting there" (May and Zelikow (eds) 1997, p. 216). But a hard, short deadline was presumably thought to be too rigid. By Thursday, others in ExComm were realizing the problem; for example, Bundy said, "A plateau here is the most dangerous thing" (May and Zelikow (eds) 1997, p. 423). The hardening of the Soviet position, as shown by the public "Saturday letter" that followed the conciliatory private "Friday letter," was another concern. More ominously, that Friday, U.S. surveillance had discovered that there were tactical nuclear weapons (FROGs) in Cuba (May and Zelikow (eds) 1997, p. 475). This discovery showed the Soviet presence there to be vastly greater than thought before, but it also made invasion more dangerous to U.S. troops. Also on Saturday, a U.S. U-2 plane was shot down over Cuba. In addition, Cuban anti-aircraft defenses fired at low-level U.S. reconnaissance planes. The grim

mood in ExComm throughout that Saturday was well encapsulated by Dillon: “We haven’t got but one more day” (May and Zelikow (eds) 1997, p. 534).

On Saturday, plans leading to escalation were being put in place. An air strike was planned for the following Monday, or Tuesday at the latest, and Air Force reserves were called up (May and Zelikow (eds) 1997, pp. 612–613). Invasion was seen as the inevitable culmination of events (May and Zelikow (eds) 1997, pp. 537–538). A tough private letter to Khrushchev from President Kennedy was drafted and was handed over by Robert Kennedy to the Soviet Ambassador in Washington, Anatoly Dobrynin. In it, Kennedy made the following offer: (1) The Soviet Union withdraws its missiles and IL-28 bombers from Cuba with adequate verification (and ships no new ones). (2) The United States promises not to invade Cuba. (3) The U.S. missiles in Turkey will be removed after a few months, but this offer is void if the Soviets mention it in public or link it to the Cuban deal. An answer was required within 12 to 24 hours; otherwise “there would be drastic consequences” (May and Zelikow (eds) 1997, pp. 605–607).

On the morning of Sunday, October 28, just as prayers and sermons for peace were being offered in many churches in the United States, Soviet radio broadcast the text of a letter that Khrushchev was sending to Kennedy, in which he announced that construction of the missile sites was being halted immediately and that the missiles already installed would be dismantled and shipped back to the Soviet Union. Kennedy immediately sent a reply welcoming this decision, which was broadcast to Moscow by the Voice of America radio. It now appears that Khrushchev’s decision to back down was made before he received Kennedy’s letter through Dobrynin but that the letter only reinforced it (May and Zelikow (eds) 1997, p. 689).

That did not quite end the crisis. The U.S. Joint Chiefs of Staff remained skeptical of the Soviets and wanted to go ahead with their air strike (May and Zelikow (eds) 1997, p. 635). In fact, the construction activity at the Cuban missile sites continued for a few days. Verification by the United Nations proved problematic. The Soviets tried to make the Turkey part of the deal semipublic. They also tried to keep the IL-28 bombers in Cuba out of the withdrawal. Not until November 20 was the deal finally clinched and the withdrawal begun (May and Zelikow (eds) 1997, pp. 663–665; Fursenko and Naftali, pp. 298–310).

2. Inadequacy of Simple Game-Theoretic Explanation

At first sight, the game theory of the crisis looks very simple. The United States wanted the Soviet Union to withdraw its missiles from Cuba; thus the U.S. objective was to achieve compellence (Schelling, 1960 pp.195-199, 1965 pp. 69-91). For this purpose, the United States deployed a threat: Soviet failure to remove the missiles would lead to a nuclear war.⁵ This was sufficiently frightening to Khrushchev that he complied. The prospect of nuclear annihilation was equally frightening to Kennedy, but that is in the nature of a threat. All that is needed is that the threat be sufficiently costly to Khrushchev to induce him to act in accordance with Kennedy's wishes; then Kennedy doesn't have to carry out the bad action anyway.

But a moment's further thought shows this interpretation to be unsatisfactory. One might start by asking why the Soviets would deploy the missiles in Cuba at all, when they could look ahead to this unfolding of the subsequent game in which they would come out the losers. They must have thought it possible that the United States would accept the missiles as a *fait accompli*, just as they themselves had accepted US missiles in Turkey. This belief may have been bolstered by their perception that President Kennedy had appeared weak, both during his meeting with Khrushchev in Vienna and during the Bay of Pigs episode in 1961. And the members of ExComm were never sure about how the Soviets would respond to various US actions. Thus we have to allow asymmetric information about the other side's payoffs into a game-theoretic analysis.

But even more important, several facts about the situation and several events in the course of its unfolding do not fit into the picture of a simple threat. As we pointed out before, the idea that a threat has only a lower limit on its size—namely, that it be large enough to frighten the opponent—is correct only if the threatener can be absolutely sure that everything will go as planned. But almost all games have some element of uncertainty. You cannot know your opponent's value system for sure, and you cannot be completely sure that the players' intended actions will be accurately implemented. Therefore, a threat carries a twofold risk. Your opponent may defy it, requiring you to carry out the costly threatened action; or your opponent may

⁵ We do not mean that the US would have immediately launched a nuclear attack on the Soviet Union, but that a US air strike on the missiles or an invasion of Cuba to remove the missiles would have started a sequence of events – a Soviet attack on Berlin, NATO's inability to stop it using conventional forces, and so on – that would almost surely have culminated in nuclear war between the superpowers.

comply, but the threatened action may occur by mistake anyway. When such risks exist, the cost of threatened action to oneself becomes an important consideration.

The Cuban missile crisis was replete with such uncertainties. Neither side could be sure of the other's payoffs—that is, of its relative valuations of winning or losing the cold war and the costs of a hot war. Also, the procedures and details of “blockade” and “air strike” were much more complex than the simple phrases suggest, and there were many weak links and random effects between an order in Washington or Moscow and its implementation in the Atlantic Ocean or in Cuba.

Allison's excellent book (1971) brings out all of these complexities and uncertainties. They led him to conclude that the Cuban missile crisis cannot be explained in game-theoretic terms. He considers two alternatives: one explanation based on the fact that bureaucracies have their set rules and procedures; another based on the internal politics of U.S. and Soviet governance and military apparatuses. He concludes that the political explanation is best.

We broadly agree but interpret the Cuban missile crisis differently. It is not that game theory is inadequate for understanding and explaining the crisis; rather, the crisis was not a two-person game—United States versus USSR, or Kennedy versus Khrushchev. Each of these two “sides” was itself a complex coalition of players with differing objectives, information, actions, and means of communication. The players within each side were engaged in other games, and some members were also directly interacting with their counterparts on the other side. In other words, the crisis can be seen as a complex many-person game with alignments into two broad coalitions. Kennedy and Khrushchev can be regarded as the top-level players in this game, but each was subject to constraints of having to deal with others in his own coalition with divergent views and information, and neither had full control over the actions of these others. We argue that this more subtle game-theoretic perspective is not only a good way to look at the crisis, but also essential in understanding how to practice brinkmanship. We begin with some items of evidence that Allison emphasizes, as well as others that emerge from other writings.

First, there are several indications of divisions of opinion on each side. On the U.S. side, as already noted, there were wide differences within ExComm. In addition, Kennedy found it necessary to consult others such as former President Eisenhower and leading members of Congress. Some of them had very different views; for example, Senator William Fulbright said in a private meeting that the blockade “seems to me the worst alternative” (May and Zelikow

(eds) 1997, p. 271). The media and the political opposition would not give the President unquestioning support for too long either. Kennedy could not have continued on a moderate course if the opinion among his advisers and the public became decisively hawkish.

Individual people also shifted positions in the course of the two weeks. For example, McNamara was at first quite dovish, arguing that the missiles in Cuba were not a significant increase in the Soviet threat (May and Zelikow (eds) 1997, p. 89) and favoring blockade and negotiations (May and Zelikow (eds) 1997, p.191), but ended up more hawkish, claiming that Khrushchev's conciliatory letter of Friday, October 26, was "full of holes" (May and Zelikow (eds) 1997, pp. 495, 585) and urging an invasion (May and Zelikow (eds) 1997, p. 537). Most important, the U.S. military chiefs always advocated a far more aggressive response. Even after the crisis was over and everyone thought the United States had won a major round in the cold war, Air Force General Curtis LeMay remained dissatisfied and wanted action: "We lost! We ought to just go in there today and knock 'em off," he said (Allison 1971, p. 206; Reeves 1993, p. 425).

Even though Khrushchev was the dictator of the Soviet Union, he was not in full control of the situation. Differences of opinion on the Soviet side are less well documented, but, for what it is worth, later memoirists have claimed that Khrushchev made the decision to install the missiles in Cuba almost unilaterally, and, when he informed the members of the Presidium, they thought it a reckless gamble (May and Zelikow (eds) 1997, p. 674; Fursenko and Naftali 1997, p. 180). There were limits to how far he could count on the Presidium to rubber-stamp his decisions. Indeed, two years later, the disastrous Cuban adventure was one of the main charges leveled against Khrushchev when the Presidium dismissed him (Fursenko and Naftali 1997, pp. 353–355). It has also been claimed that Khrushchev wanted to defy the U.S. blockade, and only the insistence of First Deputy Premier Anastas Mikoyan led to the cautious response (Kagan 1995, p. 521).

Various parties on the U.S. side had very different information and a very different understanding of the situation, and at times this led to actions that were inconsistent with the intentions of the leadership or even against their explicit orders. The concept of an "air strike" to destroy the missiles is a good example. The nonmilitary people in ExComm thought this would be very narrowly targeted and would not cause significant Cuban or Soviet casualties, but the Air Force intended a much broader attack. Luckily, this difference came out in the open

early, leading ExComm to decide against an air strike and the President to turn down an appeal by the Air Force (Allison 1971, pp. 123, 209). As for the blockade, the U.S. Navy had set procedures for this action. The political leadership wanted a different and softer process: form the ring closer to Cuba to give the Soviets more time to reconsider, allow obviously nonmilitary cargo ships to pass unchallenged, and cripple but not sink the ships that defy challenge. Despite McNamara's explicit instructions, however, the Navy mostly followed its standard procedures (Allison 1971, pp. 130–132).

There was similar lack of information and communication, as well as weakness of the chain of command and control, on the Soviet side. For example, the construction of the missiles was left to the standard bureaucratic procedures. The Soviets, used to construction of ICBM sites in their own country where they did not face significant risk of air attack, laid out the sites in Cuba in a similar way, where they would have been much more vulnerable.

All these factors made the outcome of any decision by the top-level leaders on each side somewhat unpredictable. This gave rise to a substantial risk of the “threat going wrong.” And this risk was rising as the crisis continued. On the day the blockade went into effect, Kennedy thought that the chances of war were 20% (Dobbs 2008, p. 107); others attribute to him the later estimate “between one out of three and even” (Allison 1971, p. 1).

Brinkmanship can use such uncertainty to strategic advantage. A dire threat of waging nuclear war would be too large for Kennedy to tolerate and therefore not credible to compel Khrushchev; the uncertainty can be used to make the threat probabilistic and credible. As usual Schelling has the perfect statement (1965, pp. 97, 99): “If the brink is clearly marked and provides a firm footing, no loose pebbles underfoot and no gusts of wind to catch one off guard, if each climber is in full control of himself and never gets dizzy, neither can pose any risk to the other by approaching the brink . . . [W]hile either can deliberately jump off, he cannot credibly pretend that he is about to. Any attempt to intimidate or to deter the other climber depends on the threat of slipping or stumbling . . . [O]ne can credibly threaten to fall off accidentally by standing near the brink . . . A response that carries some risk of war . . . through a compounding of actions and reactions, of calculations and miscalculations, of alarms and false alarms . . . can be plausible, even reasonable, at a time when a final, ultimate decision to have a general war would be implausible or unreasonable.”

In effect Kennedy is saying to Khrushchev: “Neither of us wants nuclear war. But I can’t accept the missiles as a *fait accompli*. The quarantine I have set in motion creates a risk of war. You can end it by withdrawing the missiles.” The risk should be high enough that Khrushchev will comply, but low enough that Kennedy can tolerate it.

The difficulty in practicing brinkmanship is to control, within these lower and upper bounds, the probability that the threat goes off if the opponent does not comply. And in the Cuban missile crisis, the risks were rapidly rising out of the control of the top leaders on both sides.

Daniel Ellsberg, who later became famous for leaking the Pentagon Papers, participated as a young member of one of the working groups during the crisis. At that time, he and his immediate superiors estimated the probability of war to be very low: between 1/1000 and 1/100 (Ellsberg 2017, pp. 189, 199). The US had overwhelming superiority in nuclear weapons and missiles, so they thought that, given any degree of rationality, Khrushchev simply had to back down without a significant concession from the US. Ellsberg was astounded to hear much higher estimates of the probability of war offered by more senior people in ExComm, but later realized that they were correct. He and other relatively junior participants had not known to what extent the top people were losing control of the situation. In fact those leaders themselves came to realize the extent of their loss of control very late in the game (Ellsberg 2017, pp. 201-222).

At numerous points—for example, when the U.S. Navy was trying to get the freighter *Marucla* to stop and be boarded—the people involved might have set off an incident with alarming consequences by taking some action in fear of an immediate threat to their own lives. Most dramatically, a Soviet submarine crew, warned to surface when approaching the quarantine line on October 27, considered firing a nuclear-tipped torpedo that it carried onboard (unknown to the U.S. Navy). The firing rule required the approval of three officers, only two of whom agreed. So the third officer may have single-handedly prevented all-out nuclear war.⁶

The U.S. Air Force created even greater dangers. A U-2 plane drifted “accidentally” into Soviet air space and almost caused a serious setback. General Curtis LeMay, acting without the President’s knowledge or authorization, ordered the Strategic Air Command’s nuclear bombers to fly past their “turnaround” points and some distance toward Soviet air space to positions

⁶ This story became public in a conference held in Havana, Cuba, in October 2002, to mark the 40th anniversary of the missile crisis. See Sullivan (2002). A participant there, Vadim Orlov, who was a member of the Soviet submarine crew, identified the officer who refused to fire the torpedo as Vasili Arkhipov, who died in 1999. See also Ellsberg (2017), pp. 216-217.

where they would be detected by Soviet radar. Fortunately, the Soviets responded calmly; Khrushchev merely protested to Kennedy (Rhodes 1995, pp. 573–75).⁷

On Saturday, October 27, Castro ordered his anti-aircraft forces to fire on all U.S. planes overflying Cuba and refused the Soviet ambassador's request to rescind the order (Kagan 1995, p. 544). On the same day an overflying U.S. U-2 plane was shot down by a Soviet surface-to-air missile; a lower-level local commander interpreted his orders more broadly than Moscow had intended (Kagan 1995, p. 537; May and Zelikow (eds) 1997, p. 682).⁸

**** FIGURE 2 ABOUT HERE ****

Each day the crisis continued, the probability of nuclear war was increasing, and the top leaders were losing control over it. Combine this rising risk of disaster with each side's uncertainty about the other's payoffs, and we have a game of dynamic or real-time chicken, also called the war of attrition in game theory literature. Instead of the two teenagers of the Chicken "story" driving their cars toward each other while the risk of a collision increases, testing each other's bravery (or foolhardiness) in deciding when to swerve, we have leaders of the two major powers of the day, testing each other's resolve as the risk of armageddon increases.

Therefore we need a model with two key features: (1) Each side's payoffs are private information. (2) The risk of catastrophe increases over time, and cannot be controlled by either party except by conceding.⁹

3: Brinkmanship – A Game Of Real-Time Chicken

The war of attrition is usually modeled as a game in continuous time (e.g. Nalebuff and Riley 1985, Bliss and Nalebuff 1984, Bulow and Klemperer 1999). We develop a discrete-time

⁷ LeMay, renowned for his extreme views and his constant chewing of large unlit cigars, is supposed to be the original inspiration, in the 1963 movie *Dr. Strangelove*, for General Jack D. Ripper, who orders his bomber wing to launch an unprovoked attack on the Soviet Union.

⁸ The wreckage is displayed in the theme park in Havana; see Figure 2.

⁹ Many game-theoretic models of the crisis exist in the literature. Some use solution concepts other than Nash, for example metagames (Howard, 1971) and theory of moves (Brams (2001). For overviews and critiques of these, see O'Neill (1994, pp.997-9) and Zagare (2014).

version, both to simplify numerical calculations and to fit the story, so the game has thirteen steps, one for each day of the crisis.¹⁰

In the standard game of chicken, each player has the binary choice between Concede (Chicken) and Defy (Tough). If both choose the latter, they crash. The payoff matrix is shown in Table 1. The payoffs are ordered $W > 0 > -L > -C$.

**** Table 1 about here ****

The dynamic version of the game takes place in multiple steps, which we label $n = 1, 2, \dots, N$. At step n , if one or both players concede the game ends, with payoffs as shown in Table 1. If neither player concedes, the mutual catastrophe (with payoffs $-C$ each) strikes with probability p_n , where

$$0 < p_1 < p_2 < \dots < p_{N-1} < p_N = 1; \quad (1)$$

in that event the game ends at step n . With probability $(1 - p_n)$ the game goes on to step $(n + 1)$.¹¹ In conformity with the evidence, detailed in Section 2, that the players had very poor control over the level of risk at any point, we take these probabilities to be exogenous. The only endogeneity is whether the players will choose to continue the game to the next higher level of risk. But some control over the probabilities themselves may be worth exploring in future work.

We assume that the value of winning W and the cost of losing L are fixed and common knowledge. Each player's cost of catastrophe C is uniformly distributed over the interval $[0, C_{\max}]$. All payoffs accrue when the game ends (either by a concession or by the catastrophe), with no discounting. Since a player would never concede if his realized C is less than L , the implied probability that each side is so hard-line is now L/C_{\max} ; this will guide our choice of numbers in the numerical solution.¹²

A player at the soft extreme of the distribution (with a high cost of catastrophe C close to C_{\max}) would concede quickly. Each day, as the probability of disaster rises, only the successively

¹⁰ Hendricks and Wilson (1985) is a rare discrete-time model, but with complete information, infinite time horizon and discounting.

¹¹ $p_N = 1$ is needed to ensure that the game ends at step N and to start the solution by backward induction.

¹² We have assumed W , L , and C_{\max} to be the same for the two sides in absence of any better information; asymmetries may be worth exploring in future work.

tougher players would remain. Therefore we stipulate an equilibrium characterized by a sequence of thresholds

$$(\mathbf{C}_{\max} \geq) \mathbf{C}_1 \geq \mathbf{C}_2 \geq \dots \geq \mathbf{C}_N (\geq L), \quad (2)$$

such that at step n a player will concede if and only if his C exceeds \mathbf{C}_n . Then at this step only those players with C in the interval $[\mathbf{0}, \mathbf{C}_{n-1}]$ remain, among whom those in the interval $[\mathbf{C}_n, \mathbf{C}_{n-1}]$ concede. So we can write the probability that a random remaining player at this step concedes as

$$\mathbf{q}_n = (\mathbf{C}_{n-1} - \mathbf{C}_n) / \mathbf{C}_{n-1}, \quad (3)$$

and then

$$\mathbf{1} - \mathbf{q}_n = \mathbf{C}_n / \mathbf{C}_{n-1}. \quad (4)$$

Begin the backward induction at step N . Here the game is sure to end one way or another, so the payoff matrix is as given by Table 1. The Row player knows his own C , but regards the Column player as a random draw from the interval $[\mathbf{0}, \mathbf{C}_{N-1}]$, and the probability that Column concedes as \mathbf{q}_N . Row's expected payoffs from his two choices are:

$$\text{Concede: } \mathbf{q}_N \cdot \mathbf{0} + (\mathbf{1} - \mathbf{q}_N)(-L); \text{ Defy: } \mathbf{q}_N \cdot W + (\mathbf{1} - \mathbf{q}_N)(-C)$$

Therefore Row should concede if

$$-(\mathbf{1} - \mathbf{q}_N)L > \mathbf{q}_N W - (\mathbf{1} - \mathbf{q}_N)C,$$

or

$$C > L + \frac{W \mathbf{q}_N}{\mathbf{1} - \mathbf{q}_N} = L + \frac{W (\mathbf{C}_{N-1} - \mathbf{C}_N)}{\mathbf{C}_N}$$

using (3) and (4). The right hand side of this then defines the concession threshold for step N :

$$\mathbf{C}_N = L + \frac{W (\mathbf{C}_{N-1} - \mathbf{C}_N)}{\mathbf{C}_N}$$

Write this as

$$\mathbf{C}_{N-1} - \mathbf{C}_N = (\mathbf{C}_N - L) \mathbf{C}_N / W. \quad (5)$$

If $\mathbf{C}_{N-1} > L$, the right hand side of (5) increases starting at $\mathbf{0}$ as \mathbf{C}_N increases over its range $[L, \mathbf{C}_{N-1}]$, and the right hand side decreases ending at $\mathbf{0}$ over the same range. Therefore (5)

defines a unique interior solution for C_N given C_{N-1} . As written, (5) gives a form more useful for the backward iteration needed in our numerical solution.¹³

Turn to the general step $n < N$. Here the game may continue to the next step. The payoffs for the Row player given his C are as shown in Table 2, where $V_{n+1}(C)$ denotes the continuation payoff for the game at step $(n + 1)$.

**** Table 2 about here ****

At this step, if $C_{n-1} > C_{n+1}$, we can have three types of outcomes:¹⁴

(a) Interior solution, $C_{n-1} > C_n > C_{n+1}$

Reasoning as before, the Row player should concede at this step if

$$q_n \cdot 0 + (1 - q_n)(-L) > q_n W + (1 - q_n) [p_n(-C) + (1 - p_n) V_{n+1}(C)]$$

or

$$p_n C > L + \frac{W q_n}{1 - q_n} + (1 - p_n) V_{n+1}(C).$$

Equality in this defines the threshold C_n at this step.

A player on the threshold of conceding at step n , if he were to defy, would in our stipulated equilibrium be sure to concede at step $(n + 1)$. Therefore the continuation payoff must be what he would get by conceding then, namely $(1 - q_{n+1})(-L)$. Using (4), we have

$$V_{n+1}(C_n) = -L C_{n+1}/C_n$$

Therefore

$$p_n C_n = L + \frac{C_{n-1} - C_n}{C_n} W - (1 - p_n) \frac{C_{n+1}}{C_n} L$$

or

¹³ If $C_{N-1} = L$, we have $C_N = L$ also. (Everyone who is ever going to concede has already conceded.)

¹⁴ If $C_{n-1} = C_{n+1}$, then $C_{n-1} = C_n = C_{n+1}$, a combination of cases (b) and (c) below.

$$C_{n-1} - C_n = \frac{1}{W} \{ p_n [C_n^2 - C_{n+1}L] - L (C_n - C_{n+1}) \}. \quad (6-a)$$

(b) Upper boundary solution, $C_{n-1} = C_n > C_{n+1}$

Here we have no concessions at n , so for all C_n in the range $[C_{n+1}, C_{n-1}]$,

$$q_n \cdot 0 + (1 - q_n)(-L) \leq q_n W + (1 - q_n) [p_n(-C) + (1 - p_n) V_{n+1}(C)]$$

and $q_n = 0$. Also C_n will concede at step $(n + 1)$, so $V_{n+1}(C_n) = -(1 - q_{n+1})L$. Using (4), the above inequality becomes

$$-L \leq -p_n C_n - (1 - p_n) \frac{C_{n+1}}{C_n} L$$

or

$$p_n [C_n^2 - C_{n+1}L] - L (C_n - C_{n+1}) \leq 0.$$

Write this in a form comparable to (6-a) above:

$$C_{n-1} - C_n = 0 \geq p_n [C_n^2 - C_{n+1}L] - L (C_n - C_{n+1}). \quad (6-b)$$

(iii) Lower boundary solution, $C_{n-1} > C_n = C_{n+1}$

Here all the $C \in [C_{n+1}, C_{n-1}]$ concede, so for all of them,

$$q_n \cdot 0 + (1 - q_n)(-L) \geq q_n W + (1 - q_n) [p_n(-C) + (1 - p_n) V_{n+1}(C)]$$

In particular, C_{n+1} concedes. Also $q_{n+1} = 0$, therefore $V_{n+1}(C_{n+1}) = -L$. Then

$$\frac{C_n}{C_{n-1}} (-L) \geq \frac{C_{n-1} - C_n}{C_{n-1}} W - \frac{C_n}{C_{n-1}} [p_n C + (1 - p_n) L]$$

or

$$C_{n-1} - C_n \leq \frac{1}{W} \{ p_n [C_n^2 - C_n L] \}. \quad (6-c)$$

The right hand side of this is just that of (6-a) with $C_n = C_{n+1}$. Therefore given (6-a), in the present case (6-c) is also true (as an equality), and there is no need to impose it separately.

We can now combine all of (6-a,b,c) into one:

$$C_{n-1} - C_n = \max \left[0, \frac{1}{W} \{ p_n [C_n^2 - C_{n+1}L] - L (C_n - C_{n+1}) \} \right]. \quad (6)$$

Equations (5) and (6) equip us for numerical solution. We start with a trial value of C_N slightly above L , use (5) to calculate C_{N-1} , and then work backward using (6) to calculate C_{N-2}, \dots, C_1 . Finally, formally for $n = 1$, (6) gives us a value for C_0 , which should equal C_{\max} . We adjust the trial value of C_N until this is achieved.

4. Numerical solution of the Cuban missile crisis “game”

We solve the above model numerically, using parameter values in the context of the Cuban missile crisis. We choose $W = 1$, $L = 1$ and $C_{\max} = 10$. This makes $L/C_{\max} = 0.1$, i. e. 10% of the types on each side will never concede. This is just a guess, but perhaps reasonable given the magnitude of the catastrophe.¹⁵ We tried values of C_{\max} ranging from 5 to 20 and the results do not change substantially. The sequence of probabilities of disaster is also a guess but an informed one, starting very low at 1% on the first day (when there was some sentiment for immediate action in ExComm), conforming in the mid-range to Kennedy’s estimates of 20% or more cited above, and rising sharply on Friday and Saturday October 26 and 27 because of the loss of control also discussed above; the numbers are shown in Table 3. On Sunday October 28, the risk rises to 1.000 because if Khrushchev had not conceded that day (he actually did that morning), US invasion plans would have gone into effect.

Table 3 also shows the results of our calculations. For easy reference, we include a brief summary of major events of each day in the second column.¹⁶ The third column shows the rising

¹⁵ We experimented with a range of values of C_{\max} and the other parameters, and found broadly similar results. Readers may wish to conduct their own experiments; that is easy to do using a spreadsheet.

¹⁶ JFK is of course President John F. Kennedy, RFK is his brother Robert, and NSK is Nikita Sergeivich Khrushchev.

probabilities of disaster. Columns 4-7 show the results of the calculations. In column 4 we have the threshold levels of C such that a player with C above it would concede at that stage.

We emphasize that the numbers we put into the model are guesses, albeit informed by the narration of events. Therefore the numbers that emerge in the solution must be regarded as only rough indicators of the reality of the situation, and subject to much discussion and correction. Nonetheless, they do suggest some tentative inferences.

First, we see why Khrushchev may have embarked on this adventure at all. He clearly put significant value on the prestige that a victory in this confrontation would yield to the USSR (and to him as its leader) in other communist countries, in the third world, and in many countries of the Western bloc as well. It would also strengthen his position in other confrontations with the US, especially over Berlin. So he may have had a low C relative to L . Combine this with the likelihood that he thought Kennedy to be weak (having a high C), based on his assessment formed during their meeting in Vienna in 1961, and during the Bay of Pigs episode the same year.

Secondly, we must infer that both sides in fact must have been very hard-line (low C) to have lasted as long as they did without conceding. Even if we accept October 22 as the date when the Soviet Presidium decided to concede, it implies a numerical value of C about 4.3, i.e. closer to L (=1) than C_{\max} (=10). And the U.S. never seriously considered conceding (accepting the missiles as a fait accompli). However, it is also possible that both sides underestimated the risk of war (i.e. the numbers in the p_n sequence) for quite a while, and only the near-collisions that occurred in the last two days of the crisis brought home to them the facts of how far they had lost control of the situation.

**** TABLE 3 ABOUT HERE ****

Column 5 shows that the probability of concession does not rise monotonically; its behavior depends on how tough are the remaining types. In fact in the last two days it falls sharply as only the toughest players are left: ones with C below 1.8, and of those the ones with C below 1 are never going to concede. In Column 6, the conditional probability of disaster on the Saturday is high enough (near 50%) that McNamara's fear as he left the White House that beautiful Fall evening "that might be the last sunset I saw" (Ellsberg 2017, p. 201) is

understandable. And in Column 7, the cumulative probability of disaster over the whole duration of the crisis becomes high enough (near 60%) that we should indeed be thankful that it did not end in a nuclear disaster.

5. Practicing Brinkmanship

In the Cuban missile crisis, the very features that make it inaccurate to regard it as a two-person game make it easier to practice brinkmanship. The blockade was a relatively small action, unlikely to start a nuclear war at once. But once Kennedy set the blockade in motion, its operation, escalation, and other features were not totally under his control. So Kennedy was not saying to Khrushchev, “If you defy me (cross a sharp brink), I will coolly and deliberately launch a nuclear war that will destroy both our peoples.” Rather, he was implicitly saying, “The wheels of the blockade have started to turn and are gathering their own momentum. The longer you defy me, the more likely it is that some operating procedure will slip up, the domestic political pressure will rise to a point where I must give in to the hawks, or some military guy will run amok. We are on a slippery slope of a gradually steepening brink. I may be unable to prevent nuclear war, no matter how much I may regret it at that point. Only you can now defuse the tension by complying with my demand to withdraw the missiles.” And Khrushchev was making similar implicit statements to Kennedy until he did concede.

We believe that this perspective gives a much better and deeper understanding of the crisis than can most analyses based on simple threats. It tells us why the risk of war played such an important role in all discussions. It even makes Allison’s compelling arguments about bureaucratic procedures and internal divisions on both sides an integral part of the picture: these features allow the top-level players on both sides credibly to lose some control—that is, to practice brinkmanship.

One important condition remains to be discussed. Every threat must have an associated implicit assurance—namely, that the bad consequence will not take place if your opponent complies with your wishes (Schelling 1960, pp. 48-50). The same is required for brinkmanship. If, as you are increasing the level of risk, your opponent does comply, you must be able to “go into reverse” very fast—begin reducing the risk immediately and quite quickly remove it from the picture. Otherwise, the opponent would not gain anything by compliance. This may have

been a problem in the Cuban missile crisis. If the Soviets feared that Kennedy could not control hawks such as LeMay (“We ought to just go in there today and knock ’em off”), they would gain nothing by giving in.

To reemphasize and sum up, brinkmanship is the strategy of exposing your rival and yourself to a gradually increasing risk of mutual harm. The actual occurrence of the harmful outcome is not totally within the threatener’s control. However, the loss of control must itself be controlled; the probability of disaster should remain within certain bounds that make the risk acceptable to the threatener. This is difficult to achieve, and in the last couple of days of Cuban missile crisis the situation came close to becoming totally uncontrolled for the two principals.

We now recapitulate some important lessons from the general theory, and from the case study of the Cuban missile crisis, considering both the successes and the failures or dangers found there:

1. Start small and safe.
2. Raise the risks gradually.
3. As this process continues, update your estimates of the opponent’s payoffs. It appears that in the Cuban missile crisis both sides initially greatly underestimated the opponent’s toughness, and updated their estimates in the light of continued defiance but almost too late.
4. Retain enough control over the situation to control the risk of catastrophe – high enough to gain the opponent’s compliance and low enough to be tolerable to you.
5. Remain alert for signs that the situation is getting out of your control, and be ready to reassert control and de-escalate. In the Cuban crisis Khrushchev’s realization of this came almost too late; it is not clear whether the US leadership ever fully realized it; certainly not the military commanders.¹⁷

A game of brinkmanship can end in one of three ways: with success for one side and loss for the other, or in mutual disaster. Fortunately the Cuban missile crisis did not end disastrously; if it had, none of us would be here to analyze it as a case study!

¹⁷ Ellsberg (2017, especially chs. 3, 19) argues that both sides still have too much delegation of control – figuratively “too many fingers on the nuclear button” – leading to loss of control and creation of a de facto “doomsday machine”.

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Figure 1: The missiles on display
(Photo credit: Avinash Dixit)



Figure 2: Wreckage of shot-down US U-2
(Photo credit: Avinash Dixit)

		Column	
		Concede	Defy
Row	Concede	0, 0	-L, W
	Defy	W, -L	-C, -C

Table 1: Standard Chicken payoff matrix

		Column	
		Concede	Defy
Row	Concede	0	-L
	Defy	W	$-p_n C + (1 - p_n) V_{n+1}(C)$

Table 2: Payoffs of Row player at step $n < N$

Date (October 1962)	Major Events	Probability of disaster at this step if neither concedes	Concession threshold	Probability that each player concedes at this step	Probability of disaster at this step (conditional on it being reached)	Cumulative probability of disaster up to and including this step
15	U2 photos show MRBM missile sites					
16	JFK briefed, convenes ExComm	0.010	9.852	0.015	0.010	0.010
17	New overflight, evidence of IRBMs	0.025	9.120	0.074	0.021	0.030
18	ExComm continues meetings. Differences of opinion, changes of mind. Options debated.	0.040	7.971	0.126	0.031	0.055
19		0.050	6.853	0.140	0.037	0.080
20		0.060	5.919	0.136	0.045	0.109
21	ExComm majority for blockade/quarantine	0.075	5.054	0.146	0.055	0.144
22	JFK addresses nation. Soviets deny, bluster. Their Presidium meets in secret and decides they will eventually withdraw but will first explore best possible deals in exchange.	0.100	4.329	0.143	0.073	0.186
23	UN Security Council meeting	0.150	3.533	0.184	0.100	0.240
24	Blockade goes into effect. Some Soviet ships approaching quarantine line stop or reverse, others continue.	0.200	2.889	0.182	0.134	0.297
25	U2 overflights increase. "Marcula" pursued.	0.300	2.330	0.194	0.195	0.371
26	NSK's conciliatory letter (asking No invasion promise). "Marcula" boarded. Castro authorizes anti-aircraft fire on low-flying US reconnaissance flights. Soviet junior officer in Cuba nearly fires missile at overflying U2 while boss is away from desk. Attitudes harden in ExComm.	0.500	1.801	0.227	0.299	0.456
27	NSK's's hardline letter (Cuba/Turkey swap), JFK independently offers this as secret deal in letter RFK hands over to Dobrynin. Overflying U2 shot down. Soviet sub crew considers firing nuclear torpedo; but fails in required unanimity of 3 officers. ExComm feeling: "We haven't got but one more day." US air strike planned for Mon. Oct. 29.	0.750	1.416	0.214	0.464	0.531
28	NSK speech withdrawing missiles	1.000	1.190	0.160	0.706	0.583

Table 3: Numerical solution of the Thirteen Days “game”