IGIDR Proceedings/Project Reports Series PP-062-05

Econometric and Stochastic General Equilibrium Models for Evaluation of Macro Economic Policies

Keshab Bhattarai

Quantitative Approaches to Public Policy – Conference in Honour of Professor T. Krishna Kumar

Held in conjunction with the
Fourth Annual International Conference on Public Policy and Management
Indian Institute of Management Bangalore (IIMB)

9-12 August 2009

School of Business and Management
Queen Mary, University of London
London, United Kingdom

Indira Gandhi Institute of
Development Research
Mumbai, India

Centre for Public Policy
Indian Institute of Management
Bangalore, India

http://www.igidr.ac.in/pdf/publication/PP-062-05.pdf

Econometric and Stochastic General Equilibrium Models for Evaluation of Macro Economic Policies

Keshab Bhattarai[1]
Hull University Business School, Hu6 7RX, UK

August, 2008

Abstract

Impacts of economic policies are evaluated applying econometric and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR and autoregressive models based on quarterly series from 1966:1 to 2007:3 of UK to those from the stochastic general equilibrium models provides insights in objective and subjective analyses of underlying economic processes influenced by public policies. While estimates of econometrics models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solving the stochastic economy are employed to test predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. Thus this paper shows why econometric analyses and general equilibrium modelling should be considered complementary rather than competitive techniques in economic analyses.

Key words: dynamic models, forecasting, general equilibrium

JEL Classification: C6, D9, E6, F41

[1] K.R.Bhattarai@hull.ac.uk; Phone: 01482-463207; Fax: 01482-463484.

I. Introduction

Econometric and general equilibrium models have been in use to analyse impacts of micro and macro economic policies on the dynamic prospects of economies (Klien(1971), Lucas(1975), Fair(1984), Cooley (1995), Prescott (1986), Wallis(1989), Hendry(1997), Sargent and Ljungqvists (2000), Wickens(2008)). Advancement in analytical methods, computing technology and enlargement of databases in recent years has made it possible to be more realistic in specifying, estimating or calibrating these models in order to predict the impacts of those policies on growth, investment, redistribution and reallocation of resources.

Impacts of economic policies are evaluated in this paper applying econometric analyses and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR, and autoregressive models based on quarterly series from 1966:1 to 2007:3 of UK to those from the stochastic general equilibrium models has provided insights in objective and subjective understanding of economic processes influenced by public policies. While estimates of econometric models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solutions of the stochastic economy are employed to test predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. After more than 70 years of the Keynesian revolution of 1930s and after more than 30 years of New Classical counter revolution under the market clearing general equilibrium modelling and advancement in corresponding analytical and computation techniques, the major aim of this paper is to show how predications and forecasts made by macroeconometric and stochastic general equilibrium models can be complementary than competitive in testing validity of conclusions of each other.

II. Econometric Modelling

In excellent surveys on macroeconometric modelling Wallis (1989) and Pagan and Wickens (1989) account for contributions to the macroeconometric modelling and forecasting in the UK since 1969. This development owes to Burns and Michell(1946), Cairncross (1969), Klein (1971), Sims (1980), Hendry(1974), Ash and Smyth (1978) who have either used simultaneous equation or the time series models for forecasting (Holly and Weal (2000) report on more recent developments). Various forecasting groups including the London Business School (LBS), National Institute of Social and Economic Research (NISER, NIGEM and NIDEM), Liverpool University Research Group (LPL) and the Cambridge University group (CUBS) were build on those modelling ideas as the need for model generated forecasts increased economic decision in the government and the private sector. While DRI, WHARTON or TAYLOR or the OECD models were popular in the US, multilateral agencies including the OECD, the World Bank, the IMF, the regional Banks or multinational companies started making decisions based on their own economic models. Despite these developments, model based macroeconomic forecasts are often criticised for large scale prediction errors. Many agree that model based forecasts should be more accurate than simple and plain extrapolative forecasts and need further improvements in the procedures of these modelling(Clements and Hendry (2002)). Garratt-Lee-Pesaran and Shin (2003) shown structural cointegrating VAR approach to macroecometric modelling that integrates time series analysis with structural features. The National Institute for Economic and Social Research has advanced techniques to evaluate role of uncertainties in macro models (Blake and Weale (2003)). This paper aims to show how predictions, impulse responses and forecasts of econometric models can be tested using corresponding series generated by stochastic dynamic general equilibrium models.

Review of macroeconomic dynamic models broadly into four main categories can be helpful. First the Keynesian IS-LM models for closed or open economies are based on structural equations to explain demand sides of the economy assuming a fixed supply in the short run (Klien (1971), Fair (1984)). When supply shocks, such as the higher oil prices hit economies around the world in 1970s (as now in 2008) scepticism increased on the outcome of the demand determined solutions of the Keynesian model as they were inconsistent with the stagflationary experience of the many advanced economies. Three other alternative models have been proposed to explain the emerging realities of these economies. One approach is to use time series of a particular variable for forecasting for the short run. It is done either by single equation model such as the AR(p), MA(q) ARIMA(p,d,q) and various forms of ARCH-GARCH processes or by multiple equation models such as the vector auto regression (VAR(p)) or structural cointegration VAR (CVAR) models. Another approach is to use small scale macro models with rational expectation or/and micro foundation. Finally there are dynamic general equilibrium models for decentralised markets of infinite horizon with clear focus on the real sides of the economy (Rutherford (1995)). More recently the stochastic dynamic general equilibrium models have been common both in new classical and New Keynesian analysis which explicitly incorporate optimisation by households and firms in applied models.

III. Simultaneous Equation Macro-econometric Model

Analytical structure of the simultaneous system with **m** number of endogenous ***y*** variables and **k** number of exogenous ***x*** variables and **e** vectors of error terms found in the literature can be written as:

$$a_{11}y_{1i} + a_{12}y_{2i} + .. + a_{1m}y_{mi} + b_{11}x_{1i} + b_{12}x_{2i} + .. + b_{1k}x_{ki} = e_{1i}$$

$$a_{21}y_{1i} + a_{22}y_{2i} + .. + a_{2m}y_{mi} + b_{21}x_{1i} + b_{22}x_{2i} + .. + b_{2k}x_{ki} = e_{2i}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_{m1}y_{1i} + a_{m2}y_{2i} + .. + a_{mm}y_{mi} + b_{m1}x_{1i} + b_{m2}x_{2i} + .. + b_{mk}x_{ki} = e_{mi} \qquad (1)$$

Values of the endogenous variables can be explicitly solved in terms of exogenous variables and economic shocks as:

$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ . \\ . \\ y_{mi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & .. & a_{1m} \\ a_{21} & a_{22} & .. & a_{2m} \\ & & & \\ & & & \\ a_{m1} & a_{m2} & .. & a_{mm} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} & .. & b_{1k} \\ b_{21} & b_{22} & .. & b_{2k} \\ & & & \\ & & & \\ b_{m1} & b_{m2} & .. & b_{mk} \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \\ . \\ . \\ x_{mi} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & .. & a_{1m} \\ a_{21} & a_{22} & .. & a_{2m} \\ & & & \\ & & & \\ a_{m1} & a_{m2} & .. & a_{mm} \end{bmatrix}^{-1} \begin{bmatrix} e_{1i} \\ e_{2i} \\ \\ \\ e_{mi} \end{bmatrix} \qquad (2)$$

More compactly such system can be represented in matrix notations as:

$$AY_i + BX_i = U_i \qquad (3)$$

where the error term U_i is distributed with zero mean and constant variance and

$$Y_i = \begin{bmatrix} y_{1i} \\ y_{2i} \\ . \\ . \\ y_{mi} \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & .. & a_{1m} \\ a_{21} & a_{22} & .. & a_{2m} \\ & & & \\ & & & \\ a_{m1} & a_{m2} & .. & a_{mm} \end{bmatrix} \quad X_i = \begin{bmatrix} x_{1i} \\ x_{2i} \\ . \\ . \\ x_{mi} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & .. & b_{1k} \\ b_{21} & b_{22} & .. & b_{2k} \\ & & & \\ & & & \\ b_{m1} & b_{m2} & .. & b_{mk} \end{bmatrix} \quad U_i = \begin{bmatrix} e_{1i} \\ e_{2i} \\ \\ \\ e_{mi} \end{bmatrix}.$$

This system has explicit solution if |A| exists, i.e. if *A* is non-singular.

Since the application of the OLS in each of the above equations generates inconsistent estimates of the structural parameters, a number of estimation techniques have been developed over years in estimating simultaneous equation models. Broadly they fall between single equation techniques such as the indirect least square or two stage least square (2SLS) or multiple equation methods such as 3SLS (Pindyck and Robinfeld(1998), Hendry (1997)). Advanced software such as the PC-Give-OX, Microfit, Eviews, Limdep, RATS or Shazam have built-in-routines for estimating such system.

Two steps need to be performed before a simultaneous model is estimated. First, model equations need to be identified in order to be able to retrieve the structural coefficients, $a_{i,j}$ and $b_{i,j}$ from the reduced form of the model. The rank and order conditions are used to identify individual equations as discussed below. Secondly the exogenous variables, which mostly depend upon policy decisions, need to be predicted before forecasting the values of endogenous variables.

The reduced form of this system is $Y_i = -A^{-1}BX_i + A^{-1}U_i$ in which the impacts of changes in X_i exogenous or policy variables is given by the multiplier term $-A^{-1}B$ with the model based forecast being, $\hat{Y}_i = -A^{-1}BX_i$. A model that has the least variance of the forecast error $Var\left(Y_i - \hat{Y}_i\right) = A^{-2}\sigma_u^2$ is the best model.

Explicitly illustration can be made with a simultaneous equation model consisting to two equations:

$$\begin{bmatrix} y_{1i} \\ y_{2i} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} e_{1i} \\ e_{2i} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} y_{1i} \\ y_{2i} \end{bmatrix} = \frac{1}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + \frac{1}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} e_{1i} \\ e_{2i} \end{bmatrix} \quad (5)$$

The reduced form of the system is

$$y_{1i} = \frac{\left(a_{22}b_{11} - a_{12}b_{21}\right)}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} x_{1i} + \frac{\left(a_{22}b_{12} - a_{12}b_{22}\right)}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} x_{2i} + \frac{a_{22}}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} e_{1i} - \frac{a_{12}}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} e_{2i} \quad (6)$$

$$y_{2i} = \frac{\left(-a_{21}b_{11} + a_{11}b_{21}\right)}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} x_{1i} + \frac{\left(-a_{21}b_{12} + a_{11}b_{22}\right)}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} x_{2i} - \frac{a_{21}}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} e_{1i} + \frac{a_{11}}{\left(a_{11}a_{22} - a_{12}a_{21}\right)} e_{2i} \quad (7)$$

Application of the OLS technique to individual equation (6) or (7) generates biased and inconsistent results because explanatory variables are linked to error terms which violate fundamental assumptions underlying BLU properties of the OLS estimators. This short coming is rectified by full information likelihood method or the generalised least square methods.

Consider an illustrative numerical example for a small Keynesian macro economic model with output, consumption and tax revenue, Y_t, C_t, T_t as three endogenous variables and investment, pubic spending and exports, I_0, G_0, and X_0 as three exogenous variables.

Consumption function: $$C_t = c_0 + \alpha(Y_t - T_t) + \varepsilon_{1t} \quad (8)$$

Taxation function: $$T_t = tY_t + \varepsilon_{2t} \quad (9)$$

National income identity $$Y_t = C_t + I_0 + G_0 + NX_0 \quad (10)$$

This model contains three parameters c_0, α, and t, representing an autonomous consumption, marginal propensity to consume and the tax rate. Model is solved first finding its reduced form. The endogenous variables can be expressed in terms of the model parameters to be estimated and exogenous variables, which are assumed to be known to the modellers by substituting equations (8) and (9) into (10) as:

$$Y_t = \frac{c_0}{(1-\alpha+\alpha\cdot t)} + \frac{I_0}{(1-\alpha+\alpha\cdot t)} + \frac{G_0}{(1-\alpha+\alpha\cdot t)} + \frac{NX_0}{(1-\alpha+\alpha\cdot t)} + V_y \quad (11)$$

The solution of output function can then be substitute in the revenue and consumption functions to derive reduced form of consumption and revenue as:

$$C_t = \frac{c_0}{(1-\alpha+\alpha\cdot t)} + \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)} I_0 + \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)} G_0 + \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)} NX_0 + \alpha(1-t)\cdot V_2 \quad (12)$$

$$T_t = \frac{t\cdot c_0}{(1-\alpha+\alpha\cdot t)} + \frac{t\cdot I_0}{(1-\alpha+\alpha\cdot t)} + \frac{t\cdot G_0}{(1-\alpha+\alpha\cdot t)} + \frac{t\cdot NX_0}{(1-\alpha+\alpha\cdot t)} + t\cdot V_1 \quad (13)$$

These equations are more compactly written in terms of reduced form parameters as:

$$C_t = \Pi_{1,0} + \Pi_{1,1} I_0 + \Pi_{1,2} G_0 + \Pi_{1,3} X_0 + v_{1,t}$$

$$T_t = \Pi_{2,0} + \Pi_{2,1} I_0 + \Pi_{2,2} G_0 + \Pi_{2,3} X_0 + v_{2,t}$$

where the reduced form parameters are defined as:

$$\Pi_{1,0} = \frac{c_0}{(1-\alpha+\alpha\cdot t)}; \Pi_{1,1} = \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)}; \Pi_{1,2} = \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)}; \Pi_{1,3} = \frac{\alpha(1-t)}{(1-\alpha+\alpha\cdot t)};$$

$$\Pi_{2,0} = \frac{t}{(1-\alpha+\alpha\cdot t)}; \Pi_{2,1} = \frac{t}{(1-\alpha+\alpha\cdot t)}; \Pi_{2,2} = \frac{t}{(1-\alpha+\alpha\cdot t)}; \Pi_{1,3} = \frac{t}{(1-\alpha+\alpha\cdot t)}$$

These model parameters can be estimated by using the 3SLS routine in the PcGive (Doornik and Hendry (2003)) with the quarterly macro time series of the UK for quarters1966:1 to 2007:3.

Table 1
Equations for Private Consumption (C)

	Coefficient	Std.Error	t-value	t-prob
Investment	1.4493	0.1375	10.5000	0.0000
GovCons	3.0285	0.1807	16.8000	0.0000
Trdbal	-0.3036	0.1603	-1.8900	0.0600
Constant	-43631.7000	4135.0000	-10.6000	0.0000

Table 2
Equations for Revenue (T)

	Coefficient	Std.Error	t-value	t-prob
Investment	1.0606	0.1189	8.9200	0.0000
GovCons	1.9503	0.1563	12.5000	0.0000
Trdbal	0.5033	0.1386	3.6300	0.0000
Constant	-15578.1000	3575.0000	-4.3600	0.0000

$$\Pi_{1,0} = -43631.7; \Pi_{1,1} = 1.4493; \Pi_{1,2} = 3.0285; \Pi_{1,3} = -0.3036;$$

$$\Pi_{2,0} = -15578.1; \Pi_{2,1} = 1.0606; \Pi_{2,2} = 1.9053; \Pi_{1,3} = 0.5033$$

All above parameters are statistically significant.

Table 3
Correlation structure among model variables

	GDP_MP	Private cons	Revenue	Gov Cons	Trade bal	Investment
GDP_MP	1.0000	0.9972	0.9383	0.9653	-0.8264	0.9570
Priv Cons	0.9972	1.0000	0.9219	0.9568	-0.8481	0.9506
Revenue	0.9383	0.9219	1.0000	0.9397	-0.7685	0.9189
Gov Cons	0.9653	0.9568	0.9397	1.0000	-0.7803	0.8851
Trade bal	-0.8264	-0.8481	-0.7685	-0.7803	1.0000	-0.8743
Investment	0.9570	0.9506	0.9189	0.8851	-0.8743	1.0000

Structural coefficients of the model, c_0, α, and t can be retrieved from the above estimates of reduced form parameters.

By observing, $\Pi_{1,0} = \frac{c_0}{(1-\alpha+\alpha\cdot t)}$ and $\Pi_{2,0} = \frac{t}{(1-\alpha+\alpha\cdot t)}$, the tax rate parameter can be estimated as $t = \frac{\Pi_{2,0}}{\Pi_{1,0}} = \frac{-15578.1}{-43631.7} = 0.357$ which is the average tax rate in the UK. Given this tax rate now it is possible to retrieve the marginal propensity to consume α parameter as $\frac{\Pi_{1,2}}{\Pi_{2,2}} = \frac{\alpha(1-t)}{t} = \frac{1.447}{1.061} = 1.367$ or $\alpha(1-t) = 1.367t$

$\alpha = 1.367\frac{t}{(1-t)} = 1.367\frac{0.357}{(1-0.357)} = 0.759$ which is a reasonable value for the marginal propensity to consume. The estimates of α, and t can be used with mean values of income and consumption in the consumption function to derive the estimated value of the autonomous consumption.

$$\overline{C} = c_0 + (\alpha - t)\overline{Y}$$

$$c_0 = \overline{C} - (\alpha - t)\overline{Y} = 96668 - (0.759 - 0.357)\times 154450 = 34579.1 \qquad (15)$$

Table 4
Descriptive statistics of model variables

	GDP_MP	Priv_cons	Revenu	GovCons	Trdbal	Investment
Mean	154450	96668	75976	32709	-3334.6	27759
Variance	42012	31511	18316	5525.6	5973.7	9354.3

Thus the complete estimated structural model is:

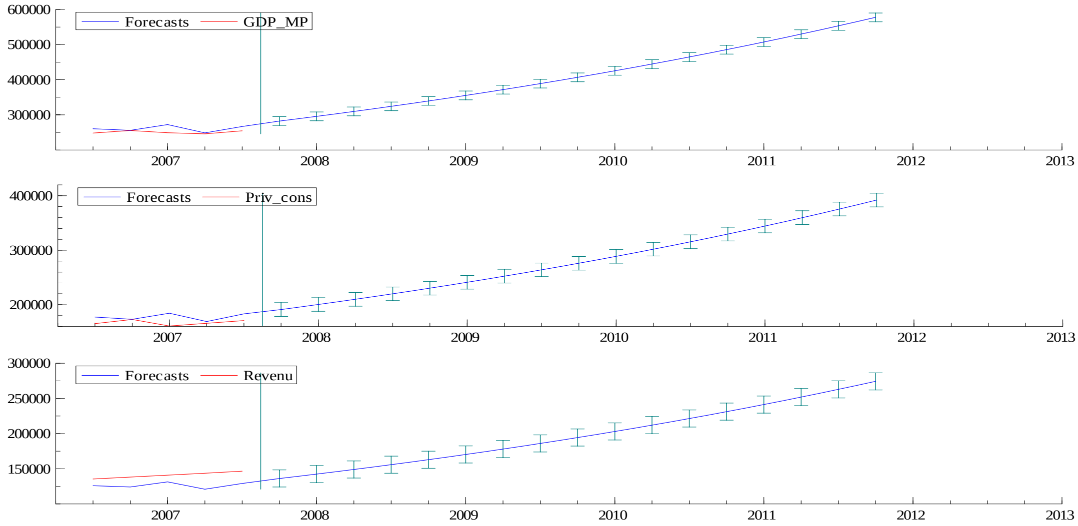
Consumption function: $C_t = 34579.1 + 0.759(Y_t - T_t)$

Taxation function: $T_t = 0.357Y_t$

National income identity $Y_t = C_t + I_0 + G_0 + NX_0$

Now policy scenarios can be considered depending on one's belief about how exogenous variables I_0, G_0, and X_0 are likely to change in the model horizon.

Figure 1
Forecast of Quarterly Income, Consumption and Revenue up to 2012 in UK



This model is made a bit complicated in the next section by introducing imports, investment and money market equations and followed by discussion of rank and order conditions for identification of each equation included in the model.

IV. Identification and Estimation of a Simultaneous Equation Model

This section provides some technical details on how to identify each equation in a traditional Keynesian Macroecnometric Model (IS-LM model).

Consumption function: $$C_t = \beta_0 + \beta_1 (Y_t - T_t) + \beta_2 X_t + \varepsilon_{1t} \quad (16)$$

Taxation function: $$T_t = t_0 + t_1 Y_t + t_2 M_t + t_3 G_t + \varepsilon_{2t} \quad (17)$$

Import function: $$M_t = m_0 + m_1 Y_t + m_2 R_t + m_3 T_t + \varepsilon_{3t} \quad (18)$$

Investment function $$I_t = \mu_0 + \mu_1 R_t + \phi \Delta Y_{t-1} + \varepsilon_{4t} \quad (19)$$

Money market (LM curve): $$\left(\frac{MM_t}{P_t} \right) = b_0 + b_1 Y_t - b_2 R_t + \varepsilon_{6t}$$

Money market (LM curve): $R_t = \frac{b_0}{b_2} - \frac{1}{b_2}\left(\frac{MM_t}{P_t}\right) + \frac{b_1}{b_2}Y_t + \varepsilon_{6t}$ (20)

National income identity $Y_t = C_t + I_t + G_t + X_t - M_t$ (21)

where Y_t, C_t, M_t, I_t, R_t, T_t are six endogenous variables representing total output, consumption, imports, investment, interest rate and taxes respectively and $\Delta Y_{t-1}, G_t, \frac{MM_t}{P}$ and X_t are predetermined or exogenous variables representing change in income in the previous period (Δ denotes a change in the variable), government spending, real money balances and exports. Each equation in such a simultaneous equation model need to satisfy order and rank conditions of identification to be able to retrieve the structural parameters $\beta_0, \beta_1, \beta_2$, t_0, t_1, $t_2, t_3, m_0, m_1, m_2, m_3$, μ_0, μ_1, ϕ, b_0, b_1, b_2 from the estimates of the reduced form parameter of the model from the time series data on endogenous and exogenous variables. The order conditions for an equation included in the model is given by $K - k \geq m - 1$, where M is number of endogenous variables, K is number of exogenous variables including the intercept; *m* the number of endogenous variable in an equation; *k* the number of exogenous variables in an equation.

Each above equations are identifies by the order conditions. For instance, with nine exogenous variables in the model including the intercept term the consumption function has only two exogenous variables; $K - k = 9 - 2 = 7 \geq M - 1 = 6 - 1 = 5$. All other equations similarly satisfy order conditions, which is a necessary but not sufficient condition for identification. Each equation is identified by the rank condition when a rank of the coefficients of the matrix of dimension of (M-1)×(M-1) exists for that equation in a model with M endogenous variables. This matrix is formed from the coefficients in model for both endogenous and exogenous variables excluded from that particular equation but included in other equations of the model.

The rank condition, $\rho(A) \geq (M-1)\times(M-1)$, used to find out whether a particular equation is identified involves following steps: .

1. Write down the system in the tabular form.
2. Strike out all coefficients in the row corresponding to the equation to be identified.
3. Strike out the columns corresponding to non-zero coefficients in that particular equation.
4. Form matrix from the remaining coefficients. It will contain only the coefficients of the variables included in the system but not in the equation under consideration. From these coefficients form all possible A matrices of order *M*-1 and ascertain that determinant of order *M*-1 exist for this system. If at least one of these determinants is non-zero then that equation is identified.

Table 5
Table of Coefficients in a Macro Econometric Model

	Constant	Y_t	C_t	M_t	I_t	R_t	T_t	G_t	X_t	MM_t	ΔY_{t-1}
C_t	$-\beta_0$	$-\beta_1$	1	0	0	0	β_1	0	$-\beta_2$	0	0
T_t	$-t_0$	$-t_1$	0	$-t_2$	0	0	1	$-t_3$	0	0	0
M_t	$-m_0$	$-m_1$	0	1	0	$-m_2$	$-m_3$	0	0	0	0
I_t	$-\mu_0$	0	0	0	1	$-\mu_1$	0	0	0	0	$-\phi$
R_t	$-\frac{b_0}{b_2}$	$-\frac{b_1}{b_2}$	0	0	0	1	0	0	0	$\frac{1}{b_2}$	0
Y_t	0	1	-1	1	-1	0	0	-1	-1	0	0

Summary of the order and rank conditions of identification:

1. If $K-k > m-1$ and the rank of the $\rho(A)$ is *M*-1 then the equation is over-identified.
2. If $K-k = m-1$ and the rank of the $\rho(A)$ is *M*-1 then the equation is exactly identified.
3. If $K-k \geq m-1$ and the rank of the $\rho(A)$ is less than *M*-1 then the equation is under identified.
4. If $K-k \leq m-1$ the structural equation is unidentified.

If the rank of the matrix with remaining coefficients $\rho(A)$ equals less than *M*-1, the corresponding equation is not identified and the model breaks down. Over-identification is less serious problem than under identification.

Identification for each equation can be examined by the rank condition as following:

consumption function: $A_1 = \begin{bmatrix} -t_2 & -t_3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi \\ 0 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$ ➔

$$|A_C| = \frac{1}{b_2}\phi m_2 t_2 \text{ ➔ } \rho(A_1) = 4\,. \tag{22}$$

It is obvious that there exists at least on non-singular matrix of order *M*-1.

Tax function: $A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -m_2 & 0 & 0 & 0 \\ 0 & -\mu_1 & 1 & 0 & -\phi \\ 0 & 1 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$

$$|A_T| = -\frac{1}{b_2}m_2 \text{ ➔ } \rho(A_1) = 4 \tag{23}$$

Import function: $A_1 = \begin{bmatrix} 1 & 0 & -\beta_2 & 0 & 0 \\ 0 & -t_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\phi \\ 0 & 0 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$ ➔ $A_1 = \begin{bmatrix} 0 & -\beta_2 & 0 & 0 \\ -t_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi \\ 0 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$

$|A_M| = -\frac{1}{b_2}\phi t_2 \beta_2$ ➔ $\rho(A_1) = 4$

Investment function: $A_1 = \begin{bmatrix} -\beta_1 & 0 & -\beta_1 & 0 \\ -t_1 & -t_2 & 1 & 0 \\ -m_1 & 1 & -m_3 & 0 \\ -\frac{b_1}{b_2} & 0 & 0 & \frac{1}{b_2} \end{bmatrix}$ ➔

$$|A_1| = -\beta_1 t_2 m_3 \frac{1}{b_2} + \beta_1 t_2 m_1 \frac{1}{b_2} \text{ ➔ } \rho(A_1) = 4 \tag{24}$$

It is very easy to identify the interest rate function.

Interest rate function: $A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -t_2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -\phi \end{bmatrix}$

$$|A_1| = t_2\phi \Rightarrow \rho(A_1) = 4 \quad (25)$$

Thus all equations in the model are identified. This model then can be represented in the reduced form as:

$$C_t = \Pi_{1,0} + \Pi_{1,1}G_t + \Pi_{1,2}X_t + \Pi_{1,3}MM_t + \Pi_{1,4}\Delta Y_{t-1} + v_{1,t} \quad (26)$$

$$M_t = \Pi_{2,0} + \Pi_{2,1}G_t + \Pi_{2,2}X_t + \Pi_{2,3}MM_t + \Pi_{2,4}\Delta Y_{t-1} + v_{2,t} \quad (27)$$

$$I_t = \Pi_{3,0} + \Pi_{3,1}G_t + \Pi_{3,2}X_t + \Pi_{3,3}MM_t + \Pi_{3,4}\Delta Y_{t-1} + v_{3,t} \quad (28)$$

$$R_t = \Pi_{4,0} + \Pi_{4,1}G_t + \Pi_{4,2}X_t + \Pi_{4,3}MM_t + \Pi_{4,4}\Delta Y_{t-1} + v_{4,t} \quad (29)$$

$$T_t = \Pi_{5,0} + \Pi_{5,1}G_t + \Pi_{5,2}X_t + \Pi_{5,3}MM_t + \Pi_{5,4}\Delta Y_{t-1} + v_{5,t} \quad (30)$$

Here $v_{1,t}$ to $v_{5,t}$ are composite normally distributed random error terms. The generalised least square estimation requires estimation procedure involves first estimations each equation by the OLS and to estimate a covariance matrix of the system as:

$$\mathrm{cov}[\hat{v}\hat{v}'] = \begin{bmatrix} \mathrm{var}(\hat{v}_1) & \mathrm{cov}(\hat{v}_1\hat{v}_2) & \mathrm{cov}(\hat{v}_1\hat{v}_3) & . & \mathrm{cov}(\hat{v}_1\hat{v}_5) \\ \mathrm{cov}(\hat{v}_2\hat{v}_1) & \mathrm{var}(\hat{v}_2) & \mathrm{cov}(\hat{v}_2\hat{v}_3) & . & \mathrm{cov}(\hat{v}_2\hat{v}_5) \\ \mathrm{cov}(\hat{v}_3\hat{v}_1) & \mathrm{cov}(\hat{v}_3\hat{v}_2) & \mathrm{var}(\hat{v}_3) & . & \mathrm{cov}(\hat{v}_3\hat{v}_5) \\ . & . & . & . & . \\ \mathrm{cov}(\hat{v}_5\hat{v}_1) & \mathrm{cov}(\hat{v}_5\hat{v}_2) & \mathrm{cov}(\hat{v}_5\hat{v}_3) & . & \mathrm{var}(\hat{v}_v) \end{bmatrix} = \Omega \quad (31)$$

where each of the cells in the matrix have $T\times T$ dimension. Thus the covariance matrix Ω has $5T\times 5T$ dimension. Using the theorem in matrix algebra Ω can be decomposed into two parts as:

$$P'P = \Omega^{-1} \quad (32)$$

Use this partition of Ω to transform the original model as

$$PY = PX\beta + Pe \quad (33)$$

$$Y^* = X^*\beta + e^* \quad (34)$$

$$\beta_{GLS} = (X^{*}{}'X^*)^{-1}X^{*}{}'Y^* = (X'P'PX)^{-1}X'P'PY \Rightarrow \beta_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \quad (35)$$

The model parameters are estimated using annual time series data for the UK economy from 1966:1 to 2006:1 using the routines in the PcGive and are presented in Table 6. Historical simulations, shown in Figure 2(a), illustrate how well the model is able to track endogenous variables in the economy.

Table 6
Estimation of Macro Econometric Model

Private consumption function				Investment function			
	Coefficient	Std.Error	t-prob		Coefficient	Std.Error	t-prob
GovCons	1.4333	0.1834	0.0000	GovCons	-0.4132	0.1334	0.0010
Exports	0.1666	0.1016	0.1420	Exports	0.3261	0.0738	0.0000
M4	0.0539	0.0051	0.0000	M4	0.0120	0.0037	0.0010
DGDP_MP	0.5150	0.0544	0.0000	DGDP_MP	0.0026	0.0396	0.9460
Constant	21992.8000	4586.0000	0.0000	Constant	23849.2000	3334.0000	0.0000
Imports function				Revenue function			
	Coefficient	Std.Error	t-prob		Coefficient	Std.Error	t-prob
GovCons	-0.3082	0.1280	0.0030	GovCons	1.90979	0.2761	0
Exports	0.7810	0.0709	0.0000	Exports	0.59504	0.1529	0.001
M4	0.0303	0.0035	0.0000	M4	-0.0136389	0.007605	0.076
DGDP_MP	-0.0147	0.0380	0.7410	DGDP_MP	-0.297555	0.08191	0.001
Constant	10318.8000	3200.0000	0.0000	Constant	-4378.42	6903	0.528
Treasury bills function							
	Coefficient	Std.Error	t-prob				
GovCons	0.0009	0.0001	0.0000				
Exports	-0.0001	0.0001	0.3720				
M4	0.0000	0.0000	0.0000				
DGDP_MP	0.0000	0.0000	0.5870				
Constant	-13.4783	2.7680	0.0000				

Most of these estimates are consistent to underlying Keynesian economic theory (Mankiw(1989), Minford and Peel (2002)). Increase in money supply, government spending and exports raise consumption, investment and output. The tax revenue rises with higher government spending and greater money supply. Intuitively imports depend on exports and supply of money but its volume diminishes with government spending. Increased money supply reduces the interest rate. Government spending rises with the level of income. Higher income implies higher tax income and more public spending. Imports, however, reduce the amount of public spending. Imports represent leak out of the income from the economic system. Higher amount of imports implies less revenue for the government and less spending.

Figure 2 (a)
Macroeconomic Forecast of the UK Economy

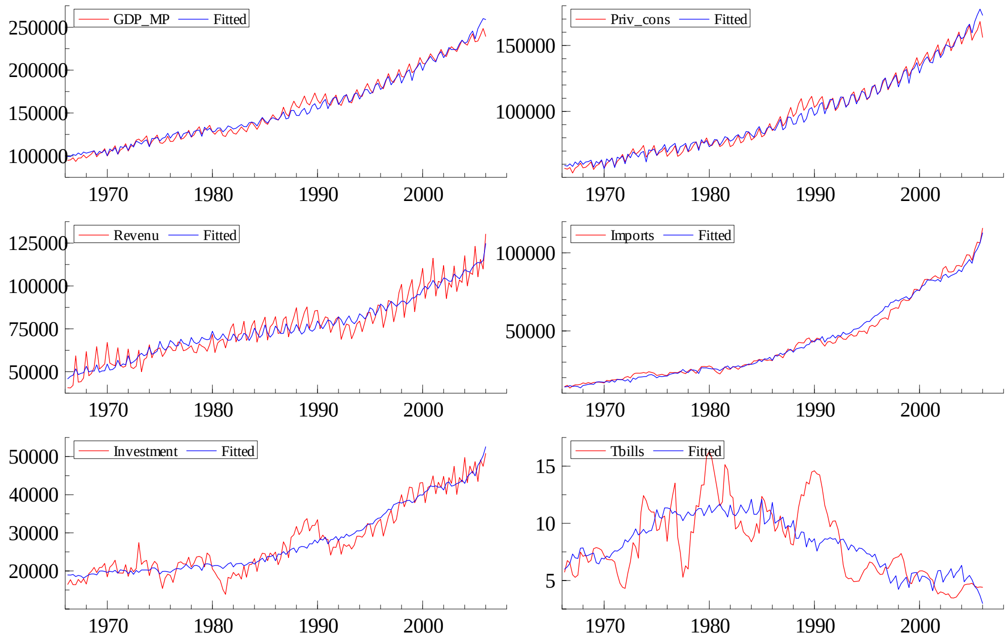
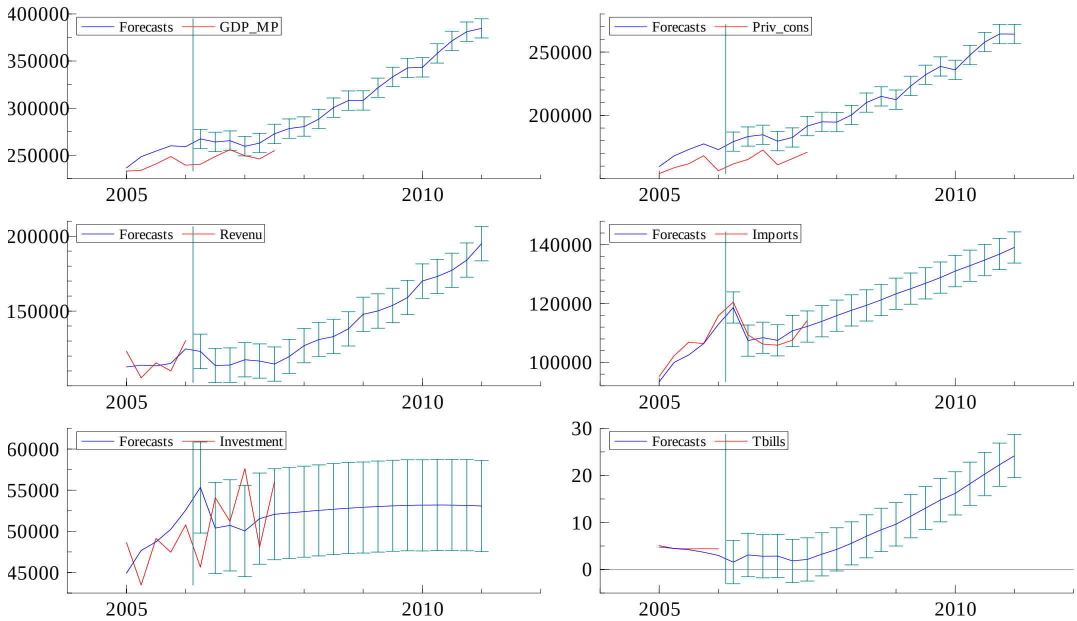


Figure 2 (b)
Macroeconomic Forecast of the UK Economy



Imports vary positively with money supply but negatively with the government spending. Interest rate function has positive and significant relation with income and money supply but negative relation with the interest rate. The coefficients of the investment function have expected signs and significant except for the change in income. Forecasts of endogenous variables are presented with their confidence interval in Figure 2(b). Under this modelling paradigm a model with the minimum forecast error is the best model (Clement (1995) and Hendry (1997)). Despite good fits and forecasts this forecast assumes that the estimated parameters remain constant over the forecast horizon. It is equivalent to assuming that behaviour of households and firms do not alter their choices following changes in economic policies. This is Lucas critique (1976) which is remedied by introducing the rational expectation in the model (Wallis (1980)) where anticipated policies can have significantly different impact than unanticipated policies. Since forming rational expectations is very difficult other macroeconomists have tended to pay more attention just to the time series properties of variables to forecast their future values rather than using the structural parameters as estimated above under the "let data speak for itself" paradigm of time series modelling presented in the next section.

V. Time Series Models and Forecasts

Broadly there are two "a-theoretical" approaches of time series modelling. The Box-Jenkins (1970) approach of forecasting involves constructing an AR, MA, ARMA or ARIMA models to predict the future values of an economic variable in terms of its current and past values. Time series modellers interpret the coefficients of an ARIMA equation to be a sort of the reduced form of a SEM model although the ARMA model does not have structural features that are found in the SEM models. An ARMA(p, q) model is specified as:

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + .. + a_p y_{t-p} + e_1 + b_1 e_{t-1} + b_1 e_{t-1} + .. + b_q e_1 \quad (36)$$

Each of the coefficients in $a_1..a_p$ and $a_1..a_q$ in this polynomial represents the turning points of a particular time series. Stationary series are convergent to their steady state values and non-stationary series are divergent. Unit root and cointegration tests are carried out to ensure stationarity of these series before their use in analysis.

Predictions and forecasts of quarterly growth rate of UK generated by an ARMA model are given in Figure 3 and 4. An ARMA(8,4) model fits quite well with the data and is the best based on the AIC criteria. This tracks growth well (Figure 3) and generates forecasts for growth in coming quarters (Figure 4). Economy peaks up in the first quarter, dips down in the second and then starts to bounce back in the third quarter reaching to another peak in the fourth quarter. The cycle continues in this manner.

Figure 3
Fit of quarterly growth rates of output by ARMA(8,4)

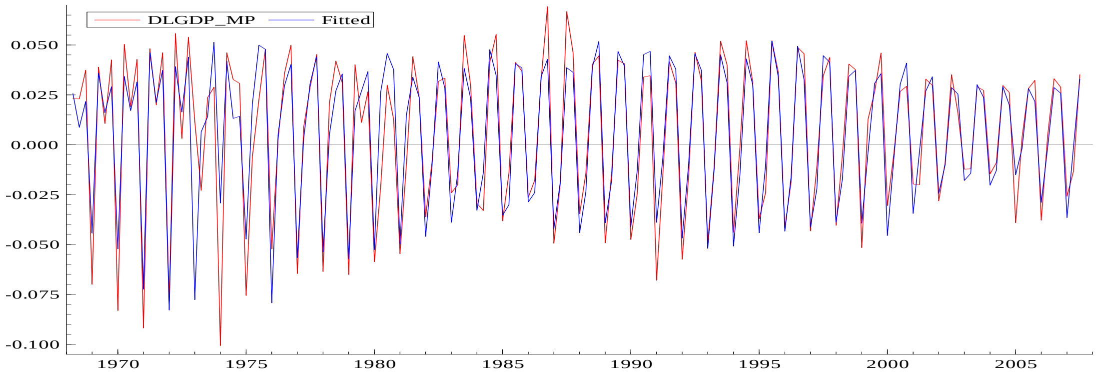


Figure 4
Forecast of quarterly growth rates of output by ARMA(8,4)

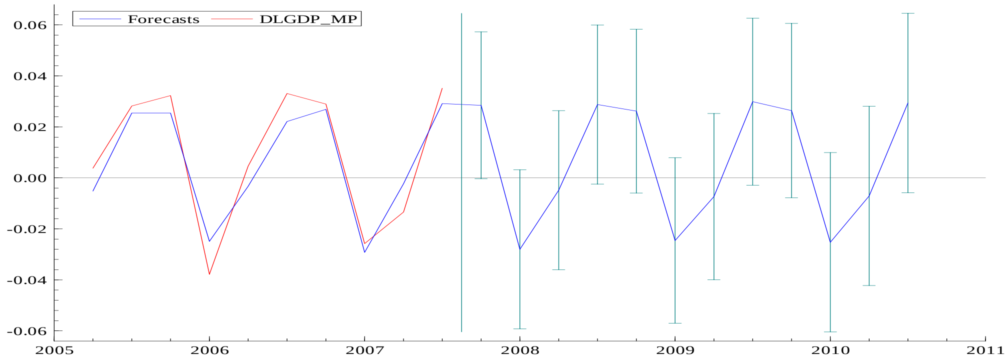


Table7

Above fit is based on estimation of Quarterly Growth Rate by ARMA(8,4) Model

	Coefficient	Std.Error	t-prob
AR-1	-0.7750	0.3164	0.0150
AR-2	-0.9745	0.3663	0.0090
AR-3	-0.7685	0.3943	0.0530
AR-4	0.0832	0.3682	0.8210
AR-5	0.1140	0.1643	0.4890
AR-6	0.2869	0.1423	0.0450
AR-7	0.1013	0.1255	0.4210
AR-8	0.2177	0.1276	0.0900
MA-1	0.3615	0.3189	0.2590
MA-2	0.6549	0.2606	0.0130
MA-3	0.4084	0.2574	0.1150
MA-4	-0.1000	0.2330	0.6690
Constant	0.0059	0.0010	0.0000

Various forms of ARCH-GARCH models also can produce such predictions and forecasts (Nelson and. Plosser (1982). Kocherlakota and Yi (1996) and Holland and Scott (1998)).

VI. Test of Cointegration and VAR Modelling

Only stationary variables can be used in regression to generate meaningful results, regression between two non-stationary variables generates a spurious relation (Phillips (1987)). Cointegration test was the rectification for this suggested by Eagle-Granger (1987) which states that non-stationary variables can be included in the model only when they have long run cointegration relationship that makes them move together. The cointegration test process involves three steps. First, the order of integration of each variable is determined by DF or ADF tests for unit root. Determining whether estimated errors of the model follow a unit root is checked by testing unit root hypothesis, $H_0 : a_1 = 0$. If this is rejected the errors, $\Delta e_t = a_1 \Delta e_{t-1} + \varepsilon_t$ are not co-integrated. The relevant series are then transformed by taking the first or higher order differences or by logs to make them stationary which then are used in the model.

Major disadvantage of a single equation time series model is its inability to illustrate the interdependency and simultaneity among the sets of economic variables since the causality runs only from the dependent to independent variables. Vector autoregression (VAR) model, another popular time series method originated in Sims' (1982) unrestricted VAR model takes the lagged values of endogenous variables and exogenous variables. If there are n endogenous variables, y_1 , y_2 .. y_n and m exogenous variables x_1 , x_2 .. x_m then the a standard VAR model is specified as:

$$y_{1,t} = a_{10} + \sum_{j=1}^{P} a_{1,1j} y_{1,t-j} + .. + \sum_{j=1}^{P} a_{1,n,j} y_{n,t-j} + \sum_{j=1}^{r} b_{11j} x_{1,t-j} + .. + \sum_{j=1}^{P} b_{1mj} x_{m,t-j} + e_{1t}$$

.

$$y_{n,t} = a_{n0} + \sum_{j=1}^{P} a_{n1j} y_{1,t-j} + .. + \sum_{j=1}^{P} a_{nnj} y_{n,t-j} + \sum_{j=1}^{r} b_{11j} x_{1,t-j} + .. + \sum_{j=1}^{P} b_{1mj} x_{m,t-j} + e_{nt} \quad (37)$$

Estimated model is used mainly for forecasting or for impulse response analyses Selection of endogenous and exogenous variables are guided by minimisation of the sum of the square of errors using mean absolute error (MAE), root of mean square errors (RMSE) and with lags of the variables to be chosen in a particular equation based on certain criteria such as the Akaike information criterion (AIC).

Again like the simultaneous equation system the basic assumptions of the model can be illustrated by using a two variable model:

$$y_{1,t} = a_{10} + a_{11} y_{1,t-1} + a_{12} y_{2,t-2} + b_{11} x_{1,t-1} + b_{12} x_{1,t-2} + e_{1t} \quad (38)$$

$$y_{2,t} = a_{20} + a_{21} y_{1,t-1} + a_{22} y_{2,t-2} + b_{21} x_{1,t-1} + b_{22} x_{2,t-2} + e_{2t} \quad (39)$$

in matrix notation:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (40)$$

The steady state of the time series y_t and x_t are given by

$$\overline{Y} = (I-A)^{-1}BX + (I-A)^{-1}U$$

$$\overline{X} = (I-B)^{-1}AY + (I-B)^{-1}U \qquad (41)$$

The first term $(I-A)^{-1}BX$ represents long run impacts of changes in exogenous variables in endogenous variables and the second term represents impacts of shocks. More important aspect of VAR analysis is innovation accounting, to see how the time path of a variable changes above its steady state value if there is either a policy shock in the second equation at time T is given by:

$$\begin{bmatrix} y_{1,T} \\ y_{2,T} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ then } \begin{bmatrix} y_{1,T+1} \\ y_{2,T+1} \end{bmatrix} = (I-A)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$\begin{bmatrix} y_{1,T+2} \\ y_{2,T+2} \end{bmatrix} = (I-A)^{-1}Y_1 = (I-A)^{-1}(I-A)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \qquad (42)$$

This process can continue successively for periods greater that *T*+1.

The single equation cointegration test procedure is extended to multivariate case by Johansen (1988) which involves examination of significant canonical correlations between two sets of variables. Cointegration vectors are found for each significant eigen values. In case of two variables the test of cointegration involves examining the existence of linear dependence among variables as:

$$\begin{bmatrix} a_{11} & a_{12}\, b_{11} & b_{12} \\ a_{21} & a_{22}\, b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} y_{1t-1} \\ y_{2t-2} \\ x_{1t-1} \\ x_{2t-2} \end{bmatrix} = 0 \qquad (43)$$

In this case a cointegrating vector exists if the coefficient matrix $\beta = \begin{bmatrix} a_{11} & a_{12}\, b_{11} & b_{12} \\ a_{21} & a_{22}\, b_{21} & b_{22} \end{bmatrix}$ has a rank of order of two, when the canonical correlations are significant and at least one eigen value is greater than 1. In

$x_t = A_1 x_{t-1} + \varepsilon_t$ where x_t is n×1 vector of endogenous variables and A_1 is $n \times n$ matrix of parameters the difference form equation is written as:

$$\Delta x_t = (A_1 - I)x_{t-1} + \varepsilon_t = \pi x_{t-1} + \varepsilon_t \,. \tag{44}$$

where I is $n \times n$ identity matrix. If the $rank(\pi) = 0$, there is no linearly stationary relationship among x_t the variables, sequence x_t are non stationary and follows unit root process, in contrast if $rank(\pi) = n$, there is a linear dependence among n variables, they are cointegrated.

This can be better illustrated by a first order VAR model of y_t and x_t as following:

$$y_t = a_{11} y_{1t-1} + a_{12} x_{t-1} + e_{1t} \tag{45}$$

$$x_t = a_{21} y_{1t-1} + a_{22} x_{1,t-1} + e_{2t} \tag{46}$$

$$y_t = a_{11} L y_t + a_{12} L x_t + e_{1t}$$

$$x_t = a_{21} L y_t + a_{22} L x_t + e_{2t}$$

$$(1 - a_{11} L) y_t - a_{12} L x_t = e_{1t}$$

$$-a_{21} L y_t + (1 - a_{22} L) x_t = e_{2t}$$

$$\begin{bmatrix} (1 - a_{11} L) & -a_{12} L \\ -a_{21} L & (1 - a_{22} L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}; \quad \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} (1 - a_{11} L) & -a_{12} L \\ -a_{21} L & (1 - a_{22} L) \end{bmatrix}^{-1} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{47}$$

Impulse response of shocks on endogenous variables takes the form:

$$y_t = \frac{(1 - a_{22} L) e_{1t} + a_{12} L e_{2t}}{(1 - a_{11} L)(1 - a_{22} L) - a_{12} a_{21} L^2} \tag{48}$$

$$x_t = \frac{(1 - a_{11} L) e_{2t} + a_{21} L e_{1t}}{(1 - a_{11} L)(1 - a_{22} L) - a_{12} a_{21} L^2} \tag{49}$$

Unit root in this VAR model implies $(1 - a_{11} L)(1 - a_{22} L) - a_{12} a_{21} L^2 = 0$

$$(1 - a_{11} L - a_{22} L + a_{11} a_{22} L) - a_{12} a_{21} L^2 = 0$$

or $(a_{11}a_{22} - a_{12}a_{21})L^2 - (a_{11} + a_{22})L + 1 = 0$

Defining $\lambda = \frac{1}{L}$ and re-arranging the terms: $\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0$

This quadratic equation has two solutions (as many roots as many equations)

$$\lambda_{1,}\lambda_2 = \frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2} \qquad (50)$$

If roots λ_1 λ_2 lie within the unit circle each variable is stationary and it is cointegrated for order 1, C(1,1). If both roots λ_1 λ_2 lie outside the unit circles y_t and x_t processes are explosive and cannot be cointegrated of order 1, y_t and x_t are explosive, if $a_{12} = a_{21} = 0$ and $a_{11} = a_{22} = 1$ then $\lambda_1 = 1$ $\lambda_2 = 1$ then two variables evolve without any long run relationship; y_t and x_t have cointegration of order 1, C(1,1) only if one of the roots λ_1 λ_2 is unity and another is less than unity (Elders (1995:6)).

Since the absence of an economic theory in unrestricted VAR no clear criteria exist on choice of variables to be included in a VAR model. Similar problems remain in interpreting the its results. A structural VAR model rectifies this short coming by imposing restrictions on model parameters based on economic theory. As Garratt, Lee , Pesaran and Shin (2003) suggested there can be $m\frac{(m-1)}{2}$ such restrictions in a model with *m* equations. As shown in equation (41) the basic long run elation among variables in the VAR model is decomposed into short run adjustment α and long run relations, β.

$$\Pi - I = \begin{bmatrix} 0.440 & -0.013 & -0.000 \\ 1.0130 & -0.598 & 0.336 \\ 0.759 & -0.097 & -177 \end{bmatrix} = \begin{bmatrix} 0.048 & -0.000 & 0.001 \\ 2.149 & -0.032 & 0.004 \\ 0.408 & 0.010 & 0.001 \end{bmatrix} \begin{bmatrix} 1.000 & -0.263 & -0.072 \\ 33.745 & 1.000 & 14.335 \\ 2.098 & 0.093 & 1.000 \end{bmatrix} = \alpha\beta'$$

Order of cointegration in vectors show the long run relationship in a VAR model is tested by using trace of the eigen values or maximum of the eigen values.

Figure 5
Tracking Revenue Spending and Transfers in UK by a VAR Model

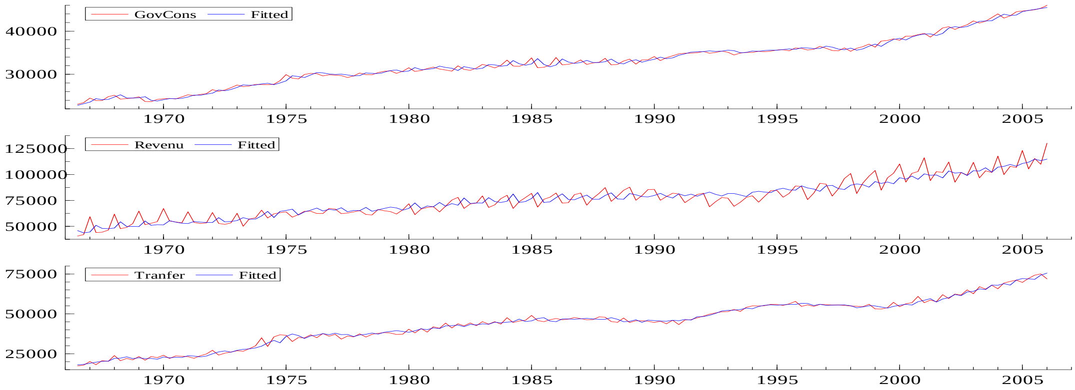
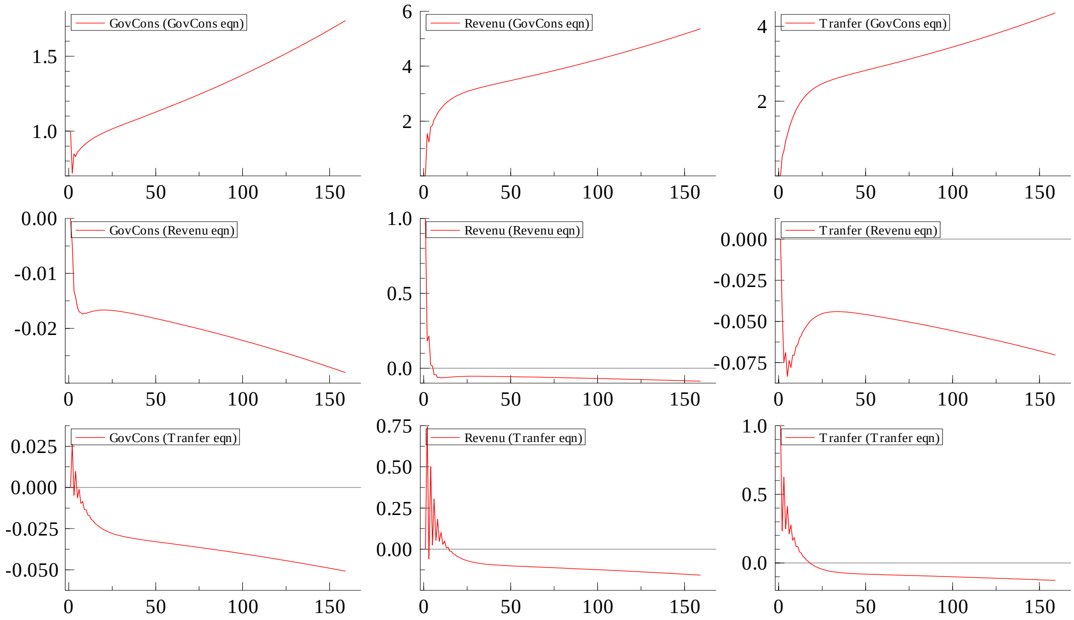


Table 8:Test of Cointegration

Rank	Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max test (T-nm)
0	63.34 [0.000]**	43.93 [0.000]**	60.95 [0.000]**	42.28 [0.000]**
1	19.40 [0.011]*	19.11 [0.007]**	18.67 [0.015]*	18.39 [0.009]**
2	0.29 [0.590]	0.29 [0.590]	0.28 [0.597]	0.28 [0.597]

Figure 6
Impulse Response Analsys of Revenue Spending and Transfers in UK



Econometric models reviewed thus far relied on time series data for estimation of behavioural parameters have very little in explaining the optimisation aspect of economic agents and in analysing the market mechanism. In fact an economic model should be able to explain how these series can be generated from the decision process of consumers and producers in the economy. How solutions of stochastic dynamic general equilibrium model can provide such series is illustrated in the next section.

VII. Stochastic Dynamic General Equilibrium Model

Stochastic dynamic general equilibrium model for small open economy and global economy are considered in this part to show how economic time series are generated by dynamic optimisation process of households, firms, government and traders. Small open economy model is applied to UK and followed by three country global economy consisting of Euro area, UK and the US (further extension of Bhattarai(2008)). Generic modelling structure is explained first followed by some discussion of frequency distributions and impulse response analyses comparables to the ones conducted above based on model generated time series of variables included in this DSGE model.

Household utility function contains goods produced at home, imported and leisure. Government uses taxes on consumption, imported goods and labour income. With the Cobb-Douglas production function, the household problem can be stated as:

$$\text{Max } U_0^i = \sum_{t=0}^{\infty} \theta^t \left(C_t^{\alpha} M_t^{\beta} l_t^{\gamma} \right) \quad \text{where } \alpha + \beta + \gamma = 1; 0 < \alpha, \beta, \gamma < 1; 0 < \theta < 1 \quad (5.1)$$

Subject to life time budget constraint

$$\sum_{t=0}^{\infty} \left[P_{i,t} (1 + tc_i) C_{i,t} + P_{j,t} (1 + tm_i) M_{i,t} + w_{i,t} (1 - tw_i) l_{i,t} \right] \leq \sum_{t=0}^{\infty} \left[w_{i,t} (1 - tw_i) \overline{L}_{i,t} + r_{i,t} (1 - tk_i) K_{i,t} \right] \quad (5.2)$$

Firms maximise profit in each period

$$\text{Max} \quad \Pi_{i,t} = P_{i,t} Y_{i,t} - r_{i,t} K_{i,t} - w_{i,t} LS_{i,t} \quad (5.3)$$

Subject to

$$Y_{i,t} = A_i K_{i,t}^{\eta_i} L_{i,t}^{(1-\eta_i)} \tag{5.4}$$

$$I_{i,t} = K_{i,t} - (1-\delta)K_{i,t-1} \tag{5.5}$$

Productivity shock $A_{i,s}$ is heterogeneous across countries generated randomly with a constant mean $\bar{A}_i$ and variance $\sigma_{A_i}^2$.

Government Sector:

$$R_{i.t} = P_{i,t} tc_i C_{i,t} + tm_i P_{j,t} M_{i,t} + tw_i LS_{i,t} + tr_i K_{i,t} \leq G_{i,t} \tag{5.6}$$

Market clearing

$$Y_{i,t} = C_{i,t} + I_{i,t} + X_{i,t} + G_{i,t} \tag{5.7}$$

There can be two different ways of trade balance

Period by period trade balance: $M_{i,t} = X_{i,t}$ (5.8)

$$\sum_{t=0}^{\infty} \theta^t \left(M_{i,t} - X_{i,t}\right) = \sum_{t=0}^{\infty} \theta^t \left(TB_{i,t}\right) \tag{5.9}$$

$$\left(S_{i,t} - I_{i,t}\right) + \left(X_{i,t} - M_{i,t}\right) = 0 \tag{5.10}$$

$$\left(\bar{L}_{i,t} - I_{i,t}\right) = LS_{i,t} \tag{5.11}$$

Prices from the inter temporal arbitrage condition

$$P_{i,t} = \frac{P_{i,t+1}}{1 + r_{i,t}} \tag{5.12}$$

Exchange rates: $E_{i,t} = \dfrac{P_{i,t}}{P_{j,t}}$ (5.13)

A competitive economy is the sequence of prices $P_{i,t}$ $P_{j,t}$ $r_{i,t}$ $r_{j,t}$ $w_{i,t}$ $w_{j,t}$ $E_{i,t}$ $E_{j,t}$ and public policy $tc_{i,t}$ $tc_{j,t}$ $tm_{i,t}$ $tm_{j,t}$ $tw_{i,t}$ $tw_{j,t}$ $tr_{i,t}$ $tr_{j,t}$ in which allocation of $C_{i,t}$ $M_{j,t}$ $l_{i,t}$, $C_{j,t}$ $M_{i,t}$ $l_{j,t}$ maximise the lifetime utility of households U_0^i and U_0^j and $LS_{i,t}$ $K_{i,t}$, $LS_{j,t}$ $K_{j,t}$ that maximise firms profit and government expenditures are $G_{i,t}\ G_{j,t}$ are compatible with the government revenue $R_{i,t}\ R_{j,t}$ and exports $X_{i,t}\ X_{j,t}$ are compatible with imports $M_{i,t}, M_{i,t}$.

The infinite horizon problem is analytically intractable. Such problems are solved using the first order inter temporal optimisation for any two time intervals with generalisation that solutions that satisfy any two periods can be extended to any other periods. First order conditions for households for two periods are:

$$C_t: \qquad \alpha_i \theta^t \left(C_t^{\alpha_i - 1} M_t^{\beta_i} l_t^{\gamma_i}\right) = \lambda_t P_{i,t}\left(1 + tc_i\right) \tag{5.14}$$

$$C_{t+1}: \qquad \alpha_i \theta^{t+1} \left(C_{t+1}^{\alpha_i - 1} M_{t+1}^{\beta_i} l_{t+1}^{\gamma_i}\right) = \lambda_t P_{i,t+1}\left(1 + tc_i\right) \tag{5.15}$$

$$M_t: \qquad \beta_i \theta^t \left(C_t^{\alpha_i} M_t^{\beta_i - 1} l_t^{\gamma_i}\right) = \lambda_t P_{j,t}\left(1 + tm_i\right) \tag{5.16}$$

$$M_{t+1}: \qquad \beta_i \theta^{t+1} \left(C_{t+1}^{\alpha_i} M_{t+1}^{\beta_i - 1} l_{t+1}^{\gamma_i}\right) = \lambda_t P_{j,t+1}\left(1 + tm_i\right) \tag{5.17}$$

$$l_t: \qquad \gamma_i \theta^t \left(C_t^{\alpha_i} M_t^{\beta_i} l_t^{\gamma_i - 1}\right) = \lambda_t w_{i,t}\left(1 - tw_i\right) \tag{5.18}$$

$$l_{t+1}: \qquad \gamma_i \theta^{t+1} \left(C_{t+1}^{\alpha_i} M_{t+1}^{\beta_i} l_{t+1}^{\gamma_i - 1}\right) = \lambda_t w_{i,t+1}\left(1 + tw_i\right) \tag{5.19}$$

$$\lambda_t: \ P_{i,t}\left(1 + tc_i\right)C_{i,t} + P_{j,t}\left(1 + tm_i\right)M_{i,t} + w_{i,t}\left(1 - tw_i\right)l_{i,t} = w_{i,t}\left(1 - tw_i\right)\overline{L}_{i,t} + r_{i,t}\left(1 - tk_i\right)K_{i,t} \tag{5.20}$$

$$\lambda_{t+1}: \ P_{i,t+1}\left(1 + tc_i\right)C_{i,t+1} + P_{j,t+1}\left(1 + tm_i\right)M_{i,t+1} + w_{i,t+1}\left(1 - tw_i\right)l_{i,t+1} = w_{i,t+1}\left(1 - tw_i\right)\overline{L}_{i,t+1} + r_{i,t+1}\left(1 - tk_i\right)K_{i,t+1} \tag{5.21}$$

Above first order conditions can be simplified in terms of Euler equations as:

$$\frac{C_{i,t}}{C_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{(\alpha_i - 1)} \left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{\beta_i} \left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i} = \frac{P_{i,t}}{P_{j,t}} \tag{5.22}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i} \left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i - 1)} \left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i} = \frac{P_{j,t}}{P_{j,t+1}} \tag{5.23}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i} \left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i - 1)} \left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{(\gamma_i - 1)} = \frac{w_{i,t}}{w_{i,t+1}} \tag{5.24}$$

$$\frac{C_{i,t+1}}{M_{i,t+1}}: \qquad \frac{\alpha_i}{\beta_i}\left(\frac{M_{i,t+1}}{C_{i,t+1}}\right) = \frac{P_{i,t+1}\left(1 + tc_i\right)}{P_{j,t+1}\left(1 + tm_i\right)} \tag{5.25}$$

$\frac{l_{i,t+1}}{M_{i,t+1}}$: $$\frac{\alpha_i}{\gamma_i}\left(\frac{l_{i,t+1}}{C_{i,t+1}}\right)=\frac{P_{i,t+1}(1+tc_i)}{w_{i,t+1}(1+tw_i)} \tag{5.26}$$

$\frac{M_{i,t+1}}{l_{i,t+1}}$: $$\frac{\beta_i}{\gamma_i}\left(\frac{l_{i,t+1}}{M_{i,t+1}}\right)=\frac{P_{j,t+1}(1+tm_i)}{w_{i,t+1}(1+tw_i)} \tag{5.27}$$

Similarly the first order conditions for firms are:

$$\Pi_{i,t}=P_{i,t}A_{i,s}K_{i,t}^{\eta_i}L_{i,t}^{(1-\eta_i)}-r_{i,t}K_{i,t}-w_{i,t}LS_{i,t} \tag{5.28}$$

$$K_{i,t}:\ \eta_{i,t}A_{i,s}P_{i,t}K_{i,t}^{\eta_i-1}L_{i,t}^{(1-\eta_i)}=r_{i,t}\ \text{ or }\ \frac{\eta_{i,t}P_{i,t}Y_{i,t}}{K_{i,t}}=r_{i,t} \tag{5.29}$$

$$K_{j,t}:\ \eta_{j,t}P_{j,t}A_{i,s}K_{j,t}^{\eta_i-1}L_{j,t}^{(1-\eta_i)}=r_{j,t}\ \text{ or }\ \frac{\eta_{j,t}P_{j,t}Y_{j,t}}{K_{j,t}}=r_{j,t} \tag{5.30}$$

$$L_{i,t}:\ (1-\eta_{i,t})P_{i,t}A_{i,s}K_{i,t}^{\eta_i-1}L_{i,t}^{-\eta_i}=w_{i,t}\ \text{ or }\ \frac{(1-\eta_{i,t})P_{i,t}Y_{i,t}}{L_{i,t}}=w_{i,t} \tag{5.31}$$

$$L_{j,t}:\ (1-\eta_{j,t})P_{j,t}A_{i,s}K_{j,t}^{\eta_i-1}L_{j,t}^{-\eta_i}=w_{j,t}\ \text{ or }\ \frac{(1-\eta_{j,t})P_{j,t}Y_{j,t}}{L_{j,t}}=w_{j,t} \tag{5.32}$$

Initial condition $K_{i,0}$ $K_{j,0}$ and (5.33)

Terminal conditions $I_{i,T}=(g+\delta)K_{i,T-1}; I_{j,T}=(g+\delta)K_{j,T-1}$. (5.34)

Whether the wages rates and the interest rates are same or differ from one country to another partly depends upon the mobility of factors and partly to the tariff rates across countries. If labour and capital are perfectly mobile then the ratios of use of labour and capital across two countries depend on ratios of production.

$$\frac{\eta_{j,t}P_{j,t}Y_{j,t}}{\eta_{i,t}P_{i,t}Y_{i,t}}\frac{K_{i,t}}{K_{j,t}}=\frac{r_{j,t}}{r_{i,t}} \tag{5.35}$$

$$\frac{(1-\eta_{j,t})P_{j,t}Y_{j,t}}{(1-\eta_{i,t})P_{i,t}Y_{i,t}}\frac{L_{i,t}}{L_{j,t}}=\frac{w_{j,t}}{w_{i,t}} \tag{5.36}$$

The exchange rate between two countries should be compatible with goods, labour and capital markets.

$$E_{j,t} = \frac{P_{j,t}}{P_{i,t}} = \frac{r_{j,t}}{r_{i,t}} \frac{K_{j,t}}{K_{i,t}} \frac{\eta_{i,t} Y_{i,t}}{\eta_{j,t} Y_{j,t}} = \frac{(1-\eta_{i,t}) Y_{i,t}}{(1-\eta_{j,t}) Y_{j,t}} \frac{L_{j,t}}{L_{i,t}} \frac{w_{j,t}}{w_{i,t}} = \frac{\alpha_i}{\beta_i} \frac{M_{i,t}}{C_{i,,t}} \frac{(1+tm_i)}{(1+tc_i)} \qquad (5.37)$$

These analytical results are significantly different than found in the literature (Dornbusch (1976), Taylor (1995)). For empirical implementation the infinite horizon problem is reduced to finite horizon by fixing the terminal period to be some T in the far distance in the future. Similarly the labour endowment in each period $\overline{L}_{i,t}$ and $\overline{L}_{j,t}$ are taken as given as are the model parameters α , β , γ and θ and the policy parameters $tc_{i,t}$, $tc_{j,t}$, $tm_{i,t}$, $tm_{j,t}$, $tw_{i,t}$, $tw_{j,t}$, $tr_{i,t}$ and $tr_{j,t}$. Labour markets and trade aspects are simplified further (see Whalley(1975), Cooley and LeRoy(1985), Perroni (1995), Rutherford (1995) Sargent and Ljungqvists (2000), Chari, Kehoe and McGrattan(2007)for more elaboration on structural DGE models).

VIII. Stochastic Small Open Economy and Global Economy Models

Simplified versions of stochastic dynamic general equilibrium models in line of Ramsey (1928) Cass (1965), Lucas (1975) and Quah (1995), Dixon and Rankin (1994) presented above for a small open economy and global economy is implemented to study the time series properties of major macro economic variables such as consumption, output, investment, exports, imports and government spending. Open economy SDGE was solved for 100 years. Model generated series are presented in Figure 7 and correlations from the solutions are reported in Table 9. The major factor underlying these series is technological shocks whose distribution is given in Figure 8. This shock affects the production of output, income and consumption of households and the government. These correlation coefficients between capital, output, private consumption, investment and public consumption and exports for one particular technology shock z1 are similar to what one would find in the actual time series.

Figure 7

Macro time series from open economy stochastic dynamic general equilibrium model

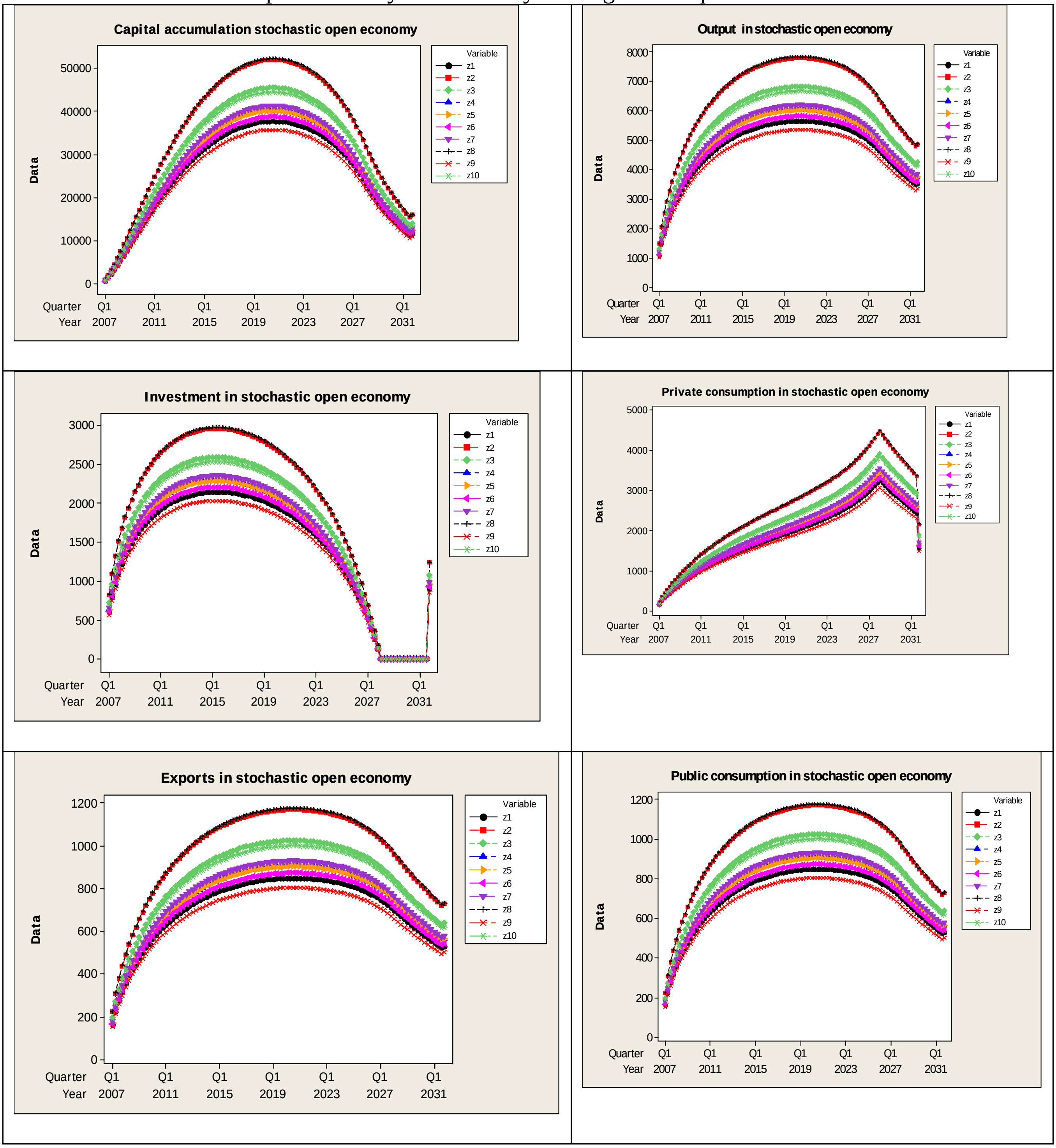
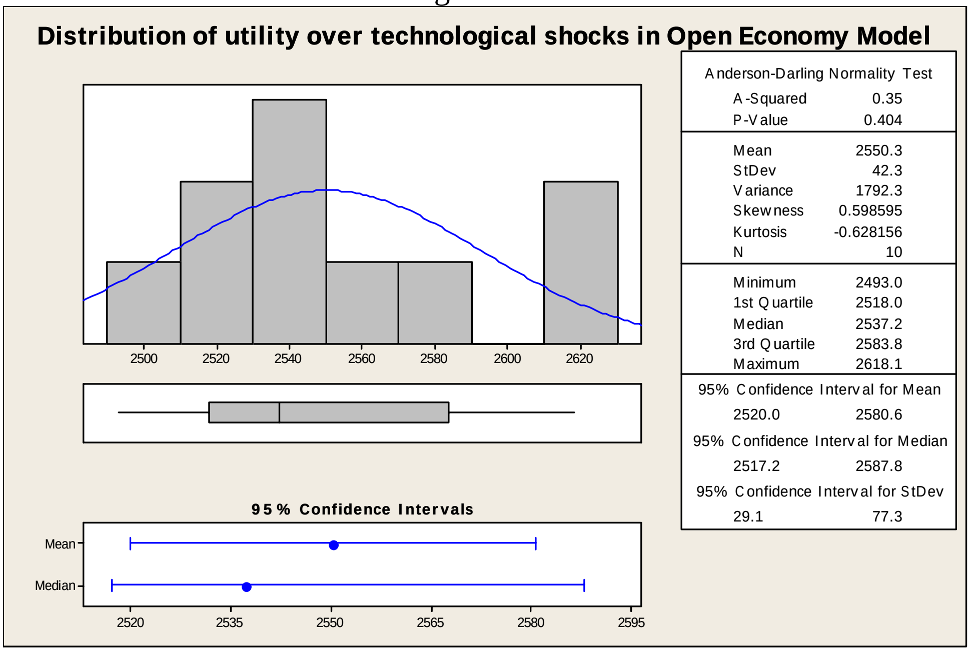


Table 9

Correlation matrix among model generated series for SDGE-SOE model

	k_z1	q_z1	C_z1	i_z1	x_z1
k_z1	1.00	0.97	0.43	0.47	0.97
q_z1	0.97	1.00	0.50	0.43	1.00
c_z1	0.43	0.50	1.00	-0.57	0.50
I_z1	0.47	0.43	-0.57	1.00	0.43
x_z1	0.97	1.00	0.50	0.43	1.00

Figure 8



Model results are used in cointegration test of long run relationship in Table 10 and for VAR impulse response analyses as given in Figure 9.

Table 10

VAR cointegration tests for Series from SDGE-SOE Model

Rank	Trace test	[Prob]	Max test	[Prob]	Trace test	(T-nm)	Max test	(T-nm)
0	240.09	[0.000]**	124.51	[0.000]**	215.59	[0.000]**	111.8	[0.000]**
1	115.58	[0.000]**	57.99	[0.000]**	103.79	[0.000]**	52.07	[0.000]**
2	57.6	[0.000]**	35.32	[0.000]**	51.72	[0.000]**	31.71	[0.001]**
3	22.28	[0.003]**	21.85	[0.002]**	20.01	[0.009]**	19.62	[0.005]**
4	0.43	[0.511]	0.43	[0.511]	0.39	[0.533]	0.39	[0.533]

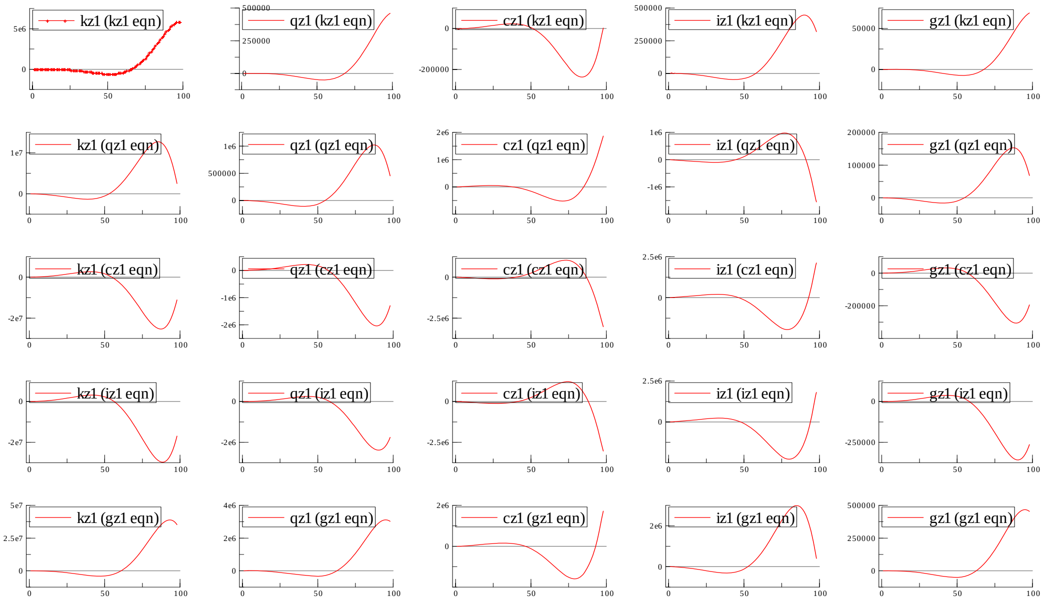
Table 11

Parameters of the small open economy and global economy Models

	Euro area	UK	USA
Depreciation rate (d)	0.05	0.03	0.02
Share of labour (a)	0.60	0.62	0.65
Discount factor (b)	0.98	0.95	0.90
Growth rate of labour (gr)	0.03	0.03	0.03
Endowment (L)	6886.00	1685.00	11265.00
Export share (xr)	0.05	-0.02	-0.03
Net public consumption (rv)	0.20	0.17	0.15

Then global economy stochastic dynamic general equilibrium model is solved for 100 years and many other horizons using a standard non-linear optimisation routines in GAMS (1998) with parameters in Table 11.

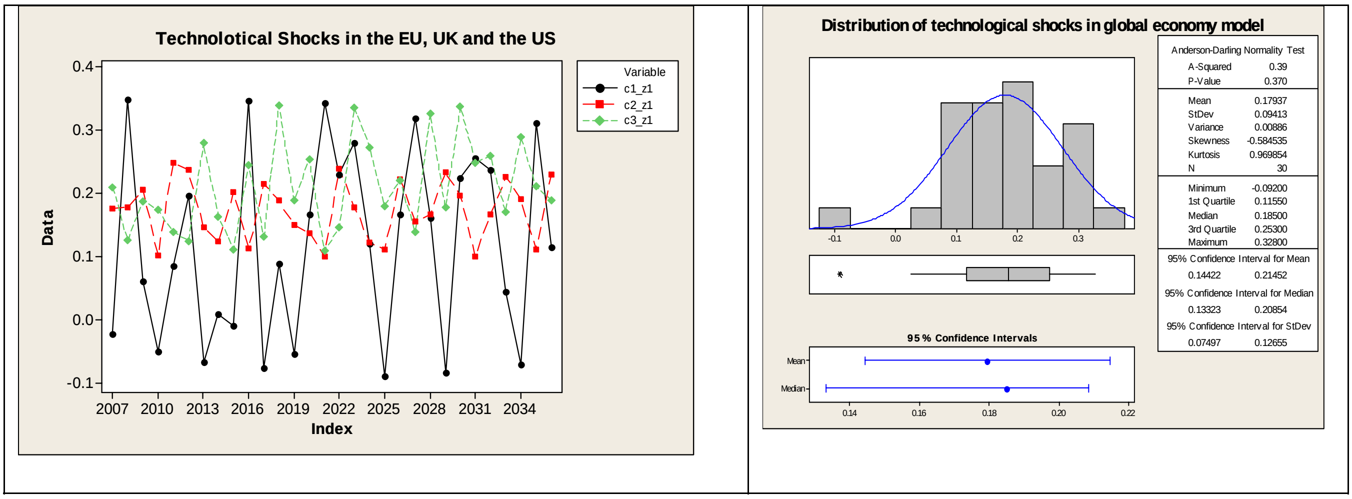
Figure 9
Impulse responses in SDGE-SOE Model



This DSGE model generates distributions of consumption, investment, capital stock, exports, imports, exchange rate and output for various technologies. A sample of time series resulting from the solution of global economy version of this model is given in Figure 10. Each economy here has been subject to technological shocks in each period. For simplicity it is assumed that the technology is randomly generated in ten different levels which is not known to consumers and producers in the economy before its realisation. These technological shocks affect the productivity and hence income and consumption profiles of households. They respond to these shocks and take account of all these possible shocks while maximizing their expected utility over the life time. The distribution of each model variable follows from these stochastic shocks.

This model mimics the time series properties of actual economies. Ratios of consumption, investment, exports and imports to the GDP and the utility of households are computed and compared. Results of the stochastic model with its whole state space are massive and only a tiny sample of model output can be reported in this section. Economies with more restrictive trade policy and experiencing greater technological shock end up losing in terms of welfare gains to households.

Figure 10
Nature of technology shocks dynamic general equilibrium model of global economy



Economic prospects are influenced by the subjective discount factors. This is found by comparing sensitivity of life time utility to beta values of 0.9, 0.95, 0.95 for the EU, UK and the US in Table 12 to 0.95, 0.95 and 0.90 values in Table 13. Even a slight change in these discount factors reverses the structure of the time series and the welfare results in such dynamic economy.

.Table 12
Impact of stochastic technology on welfare of households (beta 0.9, 0.95, 0.95)

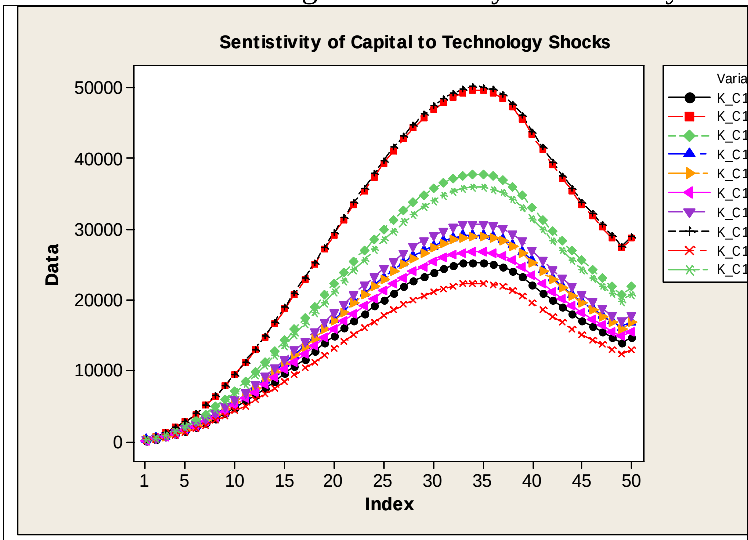
	u_z1	U_z2	u_z3	u_z4	U_z5	u_z6	u_z7	U_z8	u_z9	u_z10
Euro area	1660	1783	1733	1686	1684	1671	1696	1785	1637	1724
UK	1019	1000	1018	1009	979	1003	980	985	1004	994
USA	994	994	981	982	1007	1020	987	1011	1017	991

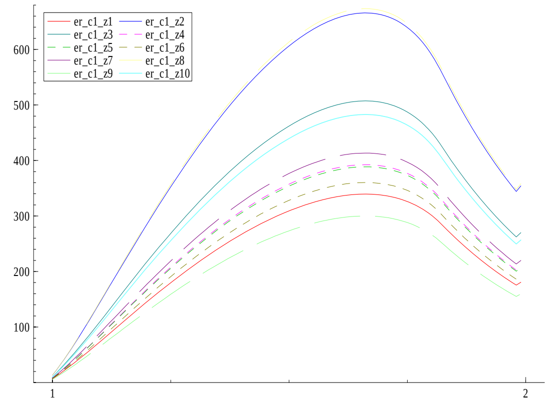
Table 13
Impact of stochastic technology on welfare of households(beta 0.9, 0.95, 0.90)

	u_z1	U_z2	u_z3	u_z4	U_z5	U_z6	u_z7	u_z8	u_z9	U_z10
Euro area	1660	1783	1733	1686	1684	1671	1696	1785	1637	1724
UK	1019	1000	1018	1009	979	1003	980	985	1004	994
USA	6810	6807	6725	6732	6892	6974	6762	6918	6956	6790

Figure 10

Time series from global economy stochastic dynamic general equilibrium model





Exchange rates for different technologies

Fiscal or trade policies that affect the nature of technological shocks and the subjective discount factors of individuals were likely to have very large impacts. Current set up of the model retains net government expenses (that equals revenue net of transfer) and the export ratio as policy variables in the model. Both tax and trade policies influence the stochastic process of the economy.

IX. Conclusion

Dynamic economic modelling using econometric analysis and general equilibrium models generate scenarios to assess evolution of an economy. Structural parameters estimated from actual time series data in econometric models to make predications about the likely impacts of economic policies in a given horizon. These models, however, do not focus enough on the optimising behaviour of households and firms. This shortcoming in analyses is complemented by decentralised stochastic general equilibrium models that generate time series upon which various predictions of econometric analyses can be tested. It has been illustrated here how econometric and general equilibrium models of a dynamic economy can be complementary to each other.

Impacts of economic policies are evaluated applying econometric analyses and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR and autoregressive models based on quarterly series 1966:1 to 2007 of UK to those from the stochastic general equilibrium models has provided insights in objective and subjective analyses of underlying economic processes influenced by public policies. While estimates of econometrics models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solving the stochastic economy are employed to test the predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. This paper has shown why econometric analyses and general equilibrium modelling should be considered complementary rather than competitive techniques in economic analyses.

VII. References:

Ash J.C.K and Smyth (1973) Forecasting the United Kingdom Economy, Saxon House.

Bhattarai K (2008) Economic Theory and Models: Derivations, Computations and Applications for Policy Analyses, *Serials*, New Delhi.

Bhattarai, K. (2008) Openness and economic growth, paper presented in the 6th International Conference on Public Economic Theory, Seoul, South Korea, July.

Blake A. P., M.R. Weale and G. Young (1998) Optimal Monetary Policy, *National Institute Economic Review*, April, 164:100-109.

Box G. E. P. and G. M. Jenkins (1970) Time series analysis, forecasting and control, Holden Day, San Francisco.

Burns, A and W. Michell, (1946), Measuring Business Cycles NBER, New York.

Cairncross A (1969) Economic Forecasting, *Economic Journal*, 79:797-812.

Clement M.P. (1995) Rationality and role of judgement in macroeconomic forecasting, *Economic Journal.* 105:429:410-420.

Chari V.V., P. Kehoe and E.R. McGrattan (2007) Business Cycle Accounting, *Econometrica*, 75:3:781-836.

Cooley Thomas F (1995) *Frontiers of Business Cycle Research*, Princeton.

Cooley and Thomas F. and S.F. LeRoy (1985) Atheoretical Macroeconometrics, *Journal of Monetary Economics*, North Holland 16: 283-308.

Dickey D.A. and W.A. Fuller (1979) Distribution of the Estimator for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association,* 74:427-431.

Dickey D.A. and W. A. Fuller (1981) Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root, *Econometrica*, 49:4:1057-1071.

Dixon H. and N. Rankin (1994) Imperfect competition and fiscal multiplier: Survey Oxford Economic Papers, 46:171-99.

Dornbusch R. (1976) Expectations and Exchange Rate Dynamics, *Journal of Political Economy,* 84:.6:1161-1176.

Engle R E and C.W.J. Granger (1987) Co-integration and Error Correction: Representation, Estimation and Testing. *Econometrica,* 55: 2-251-276.

Enders W. (1995) Applied Econometric Time Series, John Wiley and Sons.

Fair R.C.(1984) Specification, Estimation, and Analysis of Macroeconomic Models, Harvard.

Cass, D. (1965) Optimum Growth in Aggregative Model of Capital Accumulation, *Review of Economic Studies*, 32:233-240.

Doornik J.A and D.F. Hendry (2003) Econometric Modelling Using PCGive Volumes I, and II Timberlake Consultant Ltd, London.

Fair R C (1984) Specification, estimation, and analysis of macroeconometric models, Harvard.

GAMS Corporation (1998) GAMS User's Guide, Washington, DC.

Garratt A., K. Lee, M.H. Pesaran and Y. Shin (2000) A Structural Cointegration VAR Approach to Macroeconometric Modelling, *Economic Journal* in Holly S and M Weale Eds. Econometric Modelling: Techniques and Applications, the Cambridge University Press.

Holland A. and A. Scott (1998) Determinants of UK Business Cycle, *Economic Journal*, 08:449:1067-1092.

Hendry D.F. (1974) Stochastic specification in aggregate demand model of the United Kingdom, Econometrica, 42:559-78.

Johansen S. (1988) Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control* 12 : 231-254, North Holland.

Kocherlakota N. R. and K. M Yi (1996) A simple time series tests of endogenous vs. exogenous growth models : An application to the United States, Review of Economic Studies, 78:1:126-134.

Klien L (1971) Forecasting and policy evaluation using large scale econometric models, in *Frontiers of quantitative economics,* ed. M.D. Intriligator, North Holland.

Lucas R.E. (1975) An equilibrium model of business cycle, Journal of Political Economy, 83:1113-44.

Mankiw N.G. (1989) Real Business Cycles: A New Keynesian Perspective, *Journal of Economic Perspectives* 3:3: 19-90.

Minford P. and D. Peel (2002) Advanced Macroeconomics: A Primer, Edward Elgar Publishing.

Nelson C. R. and C. I. Plosser (1982) Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications, *Journal of Monetary Economics*, 10,139-162.

Pagan A. and M. Wickens (1989) A Survey of Some Recent Econometric Methods, *Economic Journal*, 99:962-1025.

Perroni, C. (1995), Assessing the Dynamic Efficiency Gains of Tax Reform When Human Capital is Endogenous, *International Economic Review* 36, 907-925.

Phillips P.C.B. (1987) Time Series Regression with an Unit Root, *Econometrica*, 55:2:277-301.

Prescott, E.C. (1986), Theory Ahead of Business Cycle Measurement, *Federal Reserve Bank of Minneapolis*, Quarterly Review; Fall.

Pindyck R.S and Robinfeld D.L. (1998) Econometric Models and Economic Forecasts, 4th edition, McGraw Hill.

Quah, D.T., (1995), Business Cycle Empirics: Calibration and Estimation, *Economic Journal* 105 (November) 1594-1596

Ramsey, F.P. (1928) A Mathematical Theory of Saving, *Economic Journal* 38:543-559.

Sargent T.J.and L. Ljungqvists (2000) Recursive Macroeconomic Theory, MIT Press.

Sims C. A. (1980) Macroeconomics and Reality, *Econometrica*, 48:1: 1-48.

Taylor M.P. (1995) The Economics of Exchange Rates, *Journal of Economic Literature*, 33:1:13-47.

Wallis K.F. (1989) Macroeconomic Forecasting: A Survey, *Economic Journal*, 99, March, pp 28-61.

Wallis K.F. (1980) Econometric Implications of the Rational Expectations Hypothesis, *Econometrica*, 48:1, pp, 48-71.

Whalley J. (1975) A General Equilibrium Assessment of the 1973 United Kingdom tax reform, *Economica*, 42, 139-161.

Wickens M. (2008) Macroeconomic Theory: A Dynamic General Equilibrium Approach, Princeton University, Press.