
Keshab Bhattarai

Quantitative Approaches to Public Policy – Conference in Honour of Professor T. Krishna Kumar

Held in conjunction with the Fourth Annual International Conference on Public Policy and Management Indian Institute of Management Bangalore (IIMB)

9-12 August 2009

http://www.igidr.ac.in/pdf/publication/PP-062-05.pdf

Keshab Bhattarai
Hull University Business School, Hu6 7RX, UK

August, 2008

Abstract
Impacts of economic policies are evaluated applying econometric and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR and autoregressive models based on quarterly series from 1966:1 to 2007:3 of UK to those from the stochastic general equilibrium models provides insights in objective and subjective analyses of underlying economic processes influenced by public policies. While estimates of econometrics models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solving the stochastic economy are employed to test predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. Thus this paper shows why econometric analyses and general equilibrium modelling should be considered complementary rather than competitive techniques in economic analyses.

Key words: dynamic models, forecasting, general equilibrium

JEL Classification: C6, D9, E6, F41

1 K_R.Bhattarai@hull.ac.uk; Phone: 01482-463207; Fax: 01482-463484.
I. Introduction

Econometric and general equilibrium models have been in use to analyse impacts of micro and macro economic policies on the dynamic prospects of economies (Klien(1971), Lucas(1975), Fair(1984), Cooley (1995), Prescott (1986), Wallis(1989), Hendry(1997), Sargent and Ljungqvists (2000), Wickens(2008)). Advancement in analytical methods, computing technology and enlargement of databases in recent years has made it possible to be more realistic in specifying, estimating or calibrating these models in order to predict the impacts of those policies on growth, investment, redistribution and reallocation of resources.

Impacts of economic policies are evaluated in this paper applying econometric analyses and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR, and autoregressive models based on quarterly series from 1966:1 to 2007:3 of UK to those from the stochastic general equilibrium models has provided insights in objective and subjective understanding of economic processes influenced by public policies. While estimates of econometric models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solutions of the stochastic economy are employed to test predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. After more than 70 years of the Keynesian revolution of 1930s and after more than 30 years of New Classical counter revolution under the market clearing general equilibrium modelling and advancement in corresponding analytical and computation techniques, the major aim of this paper is to show how predications and forecasts made by macroeconometric
and stochastic general equilibrium models can be complementary than competitive in
testing validity of conclusions of each other.

II. Econometric Modelling

In excellent surveys on macroeconometric modelling Wallis (1989) and Pagan
and Wickens (1989) account for contributions to the macroeconometric modelling and
forecasting in the UK since 1969. This development owes to Burns and
Smyth (1978) who have either used simultaneous equation or the time series models
for forecasting (Holly and Weal (2000) report on more recent developments). Various
forecasting groups including the London Business School (LBS), National Institute of
Social and Economic Research (NISER, NIGEM and NIDEM), Liverpool University
Research Group (LPL) and the Cambridge University group (CUBS) were build on
those modelling ideas as the need for model generated forecasts increased economic
decision in the government and the private sector. While DRI, WHARTON or
TAYLOR or the OECD models were popular in the US, multilateral agencies
including the OECD, the World Bank, the IMF, the regional Banks or multinational
companies started making decisions based on their own economic models. Despite
these developments, model based macroeconomic forecasts are often criticised for
large scale prediction errors. Many agree that model based forecasts should be more
accurate than simple and plain extrapolative forecasts and need further improvements
in the procedures of these modelling(Clements and Hendry (2002)). Garratt-Lee-
Pesaran and Shin (2003) shown structural cointegrating VAR approach to
macroecometric modelling that integrates time series analysis with structural features.
The National Institute for Economic and Social Research has advanced techniques to
evaluate role of uncertainties in macro models (Blake and Weale (2003)). This paper
aims to show how predictions, impulse responses and forecasts of econometric models can be tested using corresponding series generated by stochastic dynamic general equilibrium models.

Review of macroeconomic dynamic models broadly into four main categories can be helpful. First the Keynesian IS-LM models for closed or open economies are based on structural equations to explain demand sides of the economy assuming a fixed supply in the short run (Klien (1971), Fair (1984)). When supply shocks, such as the higher oil prices hit economies around the world in 1970s (as now in 2008) scepticism increased on the outcome of the demand determined solutions of the Keynesian model as they were inconsistent with the stagflationary experience of the many advanced economies. Three other alternative models have been proposed to explain the emerging realities of these economies. One approach is to use time series of a particular variable for forecasting for the short run. It is done either by single equation model such as the AR(p), MA(q) ARIMA(p,d,q) and various forms of ARCH-GARCH processes or by multiple equation models such as the vector auto regression (VAR(p)) or structural cointegration VAR (CVAR) models. Another approach is to use small scale macro models with rational expectation or/and micro foundation. Finally there are dynamic general equilibrium models for decentralised markets of infinite horizon with clear focus on the real sides of the economy (Rutherford (1995)). More recently the stochastic dynamic general equilibrium models have been common both in new classical and New Keynesian analysis which explicitly incorporate optimisation by households and firms in applied models.
III. Simultaneous Equation Macro-econometric Model

Analytical structure of the simultaneous system with \( m \) number of endogenous \( y \) variables and \( k \) number of exogenous \( x \) variables and \( e \) vectors of error terms found in the literature can be written as:

\[
\begin{align*}
\mathbf{a}_1 y_{1i} + \mathbf{a}_{12} y_{2i} + \ldots + \mathbf{a}_{1m} y_{mi} + \mathbf{b}_1 x_{1i} + \mathbf{b}_{12} x_{2i} + \ldots + \mathbf{b}_{1k} x_{ki} &= e_{1i} \\
\mathbf{a}_2 y_{1i} + \mathbf{a}_{22} y_{2i} + \ldots + \mathbf{a}_{2m} y_{mi} + \mathbf{b}_2 x_{1i} + \mathbf{b}_{22} x_{2i} + \ldots + \mathbf{b}_{2k} x_{ki} &= e_{2i} \\
\vdots & \quad \vdots \\
\mathbf{a}_m y_{1i} + \mathbf{a}_{m2} y_{2i} + \ldots + \mathbf{a}_{mm} y_{mi} + \mathbf{b}_m x_{1i} + \mathbf{b}_{m2} x_{2i} + \ldots + \mathbf{b}_{mk} x_{ki} &= e_{mi}
\end{align*}
\] (1)

Values of the endogenous variables can be explicitly solved in terms of exogenous variables and economic shocks as:

\[
\begin{bmatrix}
\mathbf{y}_{1i} \\
\mathbf{y}_{2i} \\
\vdots \\
\mathbf{y}_{mi}
\end{bmatrix} = \begin{bmatrix}
\mathbf{a}_1 & \mathbf{a}_{12} & \ldots & \mathbf{a}_{1m} \\
\mathbf{a}_2 & \mathbf{a}_{22} & \ldots & \mathbf{a}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{a}_m & \mathbf{a}_{m2} & \ldots & \mathbf{a}_{mm}
\end{bmatrix}^{-1} \begin{bmatrix}
\mathbf{b}_1 & \mathbf{b}_{12} & \ldots & \mathbf{b}_{1k} \\
\mathbf{b}_2 & \mathbf{b}_{22} & \ldots & \mathbf{b}_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{b}_m & \mathbf{b}_{m2} & \ldots & \mathbf{b}_{mk}
\end{bmatrix} \begin{bmatrix}
\mathbf{x}_{1i} \\
\mathbf{x}_{2i} \\
\vdots \\
\mathbf{x}_{mi}
\end{bmatrix} + \begin{bmatrix}
\mathbf{a}_1 & \mathbf{a}_{12} & \ldots & \mathbf{a}_{1m} \\
\mathbf{a}_2 & \mathbf{a}_{22} & \ldots & \mathbf{a}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{a}_m & \mathbf{a}_{m2} & \ldots & \mathbf{a}_{mm}
\end{bmatrix}^{-1} \begin{bmatrix}
\mathbf{e}_{1i} \\
\mathbf{e}_{2i} \\
\vdots \\
\mathbf{e}_{mi}
\end{bmatrix}
\] (2)

More compactly such system can be represented in matrix notations as:

\[
\mathbf{A} \mathbf{Y}_i + \mathbf{B} \mathbf{X}_i = \mathbf{U}_i
\] (3)

where the error term \( \mathbf{U}_i \) is distributed with zero mean and constant variance and

\[
\begin{bmatrix}
\mathbf{y}_{1i} \\
\mathbf{y}_{2i} \\
\vdots \\
\mathbf{y}_{mi}
\end{bmatrix} = \mathbf{A} \begin{bmatrix}
\mathbf{x}_{1i} \\
\mathbf{x}_{2i} \\
\vdots \\
\mathbf{x}_{mi}
\end{bmatrix} + \mathbf{B} \begin{bmatrix}
\mathbf{e}_{1i} \\
\mathbf{e}_{2i} \\
\vdots \\
\mathbf{e}_{mi}
\end{bmatrix}
\]

This system has explicit solution if \( |A| \) exists, i.e. if \( A \) is non-singular.

Since the application of the OLS in each of the above equations generates inconsistent estimates of the structural parameters, a number of estimation techniques have been developed over years in estimating simultaneous equation models. Broadly
they fall between single equation techniques such as the indirect least square or two
stage least square (2SLS) or multiple equation methods such as 3SLS (Pindyck and
Robinfeld(1998), Hendry (1997)). Advanced software such as the PC-Give-OX, Microfit,
Eviews, Limdep, RATS or Shazam have built-in-routines for estimating such system.

Two steps need to be performed before a simultaneous model is estimated. First, model equations need to be identified in order to be able to retrieve the
structural coefficients, $a_{i,j}$ and $b_{i,j}$, from the reduced form of the model. The rank and
order conditions are used to identify individual equations as discussed below.
Secondly the exogenous variables, which mostly depend upon policy decisions, need
to be predicted before forecasting the values of endogenous variables.

The reduced form of this system is $Y_i = -A^{-1}BX_i + A^{-1}U_i$ in which the
impacts of changes in $X_i$ exogenous or policy variables is given by the multiplier
term $-A^{-1}B$ with the model based forecast being, $\hat{Y}_i = -A^{-1}BX_i$. A model that has
the least variance of the forecast error $Var(\hat{Y}_i - Y_i) = A^{-2}\sigma^2_u$ is the best model.

Explicitly illustration can be made with a simultaneous equation model
consisting to two equations:

$$
\begin{bmatrix}
    y_{1i} \\
    y_{2i}
\end{bmatrix} = 
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix} 
\begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22}
\end{bmatrix} 
\begin{bmatrix}
    x_{1i} \\
    x_{2i}
\end{bmatrix} + 
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix} 
\begin{bmatrix}
    \varepsilon_{1i} \\
    \varepsilon_{2i}
\end{bmatrix}
\tag{4}
$$

$$
\begin{bmatrix}
    y_{1i} \\
    y_{2i}
\end{bmatrix} = 
\frac{1}{(a_{11}a_{22} - a_{12}a_{21})} 
\begin{bmatrix}
    a_{22} & -a_{12} \\
    -a_{21} & a_{11}
\end{bmatrix} 
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix} 
\begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22}
\end{bmatrix} 
\begin{bmatrix}
    x_{1i} \\
    x_{2i}
\end{bmatrix} + 
\frac{1}{(a_{11}a_{22} - a_{12}a_{21})} 
\begin{bmatrix}
    a_{22} & -a_{12} \\
    -a_{21} & a_{11}
\end{bmatrix} 
\begin{bmatrix}
    \varepsilon_{1i} \\
    \varepsilon_{2i}
\end{bmatrix}
\tag{5}
$$

The reduced form of the system is

$$
y_{1i} = \frac{(a_{22}b_{11} - a_{12}b_{21})}{(a_{11}a_{22} - a_{12}a_{21})} x_{1i} + \frac{(a_{22}b_{12} - a_{12}b_{22})}{(a_{12}a_{22} - a_{12}a_{21})} x_{2i} + \frac{a_{22}}{(a_{11}a_{22} - a_{12}a_{21})} \varepsilon_{1i} - \frac{a_{12}}{(a_{11}a_{22} - a_{12}a_{21})} \varepsilon_{2i}
\tag{6}
$$

$$
y_{2i} = \frac{(-a_{21}b_{11} + a_{11}b_{21})}{(a_{11}a_{22} - a_{12}a_{21})} x_{1i} + \frac{(-a_{21}b_{12} + a_{11}b_{22})}{(a_{12}a_{22} - a_{12}a_{21})} x_{2i} - \frac{a_{21}}{(a_{11}a_{22} - a_{12}a_{21})} \varepsilon_{1i} + \frac{a_{11}}{(a_{11}a_{22} - a_{12}a_{21})} \varepsilon_{2i}
\tag{7}
$$
Application of the OLS technique to individual equation (6) or (7) generates biased and inconsistent results because explanatory variables are linked to error terms which violate fundamental assumptions underlying BLU properties of the OLS estimators. This shortcoming is rectified by full information likelihood method or the generalised least square methods.

Consider an illustrative numerical example for a small Keynesian macroeconomic model with output, consumption and tax revenue, $Y_t$, $C_t$, $T_t$ as three endogenous variables and investment, public spending and exports, $I_0$, $G_0$, and $X_0$ as three exogenous variables.

Consumption function: \[ C_t = c_0 + \alpha(Y_t - T_t) + \epsilon_{it} \] (8)

Taxation function: \[ T_t = tY_t + \epsilon_{2t} \] (9)

National income identity \[ Y_t = C_t + I_0 + G_0 + NX_0 \] (10)

This model contains three parameters $c_0$, $\alpha$, and $t$, representing an autonomous consumption, marginal propensity to consume and the tax rate. Model is solved first finding its reduced form. The endogenous variables can be expressed in terms of the model parameters to be estimated and exogenous variables, which are assumed to be known to the modellers by substituting equations (8) and (9) into (10) as:

\[ Y_t = \frac{c_0}{1 - \alpha + \alpha \cdot t} + \frac{I_0}{(1 - \alpha + \alpha \cdot t)} + \frac{G_0}{(1 - \alpha + \alpha \cdot t)} + \frac{NX_0}{(1 - \alpha + \alpha \cdot t)} + V_y \] (11)

The solution of output function can then be substitute in the revenue and consumption functions to derive reduced form of consumption and revenue as:

\[ C_t = \frac{c_0}{1 - \alpha + \alpha \cdot t} + \frac{\alpha(t)}{1 - \alpha + \alpha \cdot t} I_0 + \frac{\alpha(1-t)}{1 - \alpha + \alpha \cdot t} G_0 + \frac{\alpha(1-t)}{1 - \alpha + \alpha \cdot t} NX_0 + \alpha(1-t) \cdot V_z \] (12)

\[ T_t = \frac{t \cdot c_0}{1 - \alpha + \alpha \cdot t} + \frac{t \cdot I_0}{1 - \alpha + \alpha \cdot t} + \frac{t \cdot G_0}{1 - \alpha + \alpha \cdot t} + \frac{t \cdot NX_0}{1 - \alpha + \alpha \cdot t} + t \cdot V_1 \] (13)

These equations are more compactly written in terms of reduced form parameters as:

\[ C_t = \Pi_{1,0} + \Pi_{1,1} I_0 + \Pi_{1,2} G_0 + \Pi_{1,3} X_0 + v_{1,t} \]
\[ T_t = \Pi_{2,0} + \Pi_{2,1} I_0 + \Pi_{2,2} G_0 + \Pi_{2,3} X_0 + \nu_{2,t} \]

where the reduced form parameters are defined as:

\[ \Pi_{1,0} = \frac{c_0}{(1 - \alpha + \alpha \cdot t)}; \Pi_{1,1} = \frac{\alpha(1 - t)}{(1 - \alpha + \alpha \cdot t)}; \Pi_{1,2} = \frac{\alpha(1 - t)}{(1 - \alpha + \alpha \cdot t)}; \Pi_{1,3} = \frac{\alpha(1 - t)}{(1 - \alpha + \alpha \cdot t)}; \]

\[ \Pi_{2,0} = \frac{t}{(1 - \alpha + \alpha \cdot t)}; \Pi_{2,1} = \frac{t}{(1 - \alpha + \alpha \cdot t)}; \Pi_{2,2} = \frac{t}{(1 - \alpha + \alpha \cdot t)}; \Pi_{1,3} = \frac{t}{(1 - \alpha + \alpha \cdot t)} \]

These model parameters can be estimated by using the 3SLS routine in the PcGive (Doornik and Hendry (2003)) with the quarterly macro time series of the UK for quarters 1966:1 to 2007:3.

### Table 1

**Equations for Private Consumption (C)**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>1.4493</td>
<td>0.1375</td>
<td>10.5000</td>
</tr>
<tr>
<td>GovCons</td>
<td>3.0285</td>
<td>0.1807</td>
<td>16.8000</td>
</tr>
<tr>
<td>Trdbal</td>
<td>-0.3036</td>
<td>0.1603</td>
<td>-1.8900</td>
</tr>
<tr>
<td>Constant</td>
<td>-43631.7000</td>
<td>4135.0000</td>
<td>-10.6000</td>
</tr>
</tbody>
</table>

### Table 2

**Equations for Revenue (T)**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>1.0606</td>
<td>0.1189</td>
<td>8.9200</td>
</tr>
<tr>
<td>GovCons</td>
<td>1.9503</td>
<td>0.1563</td>
<td>12.5000</td>
</tr>
<tr>
<td>Trdbal</td>
<td>0.5033</td>
<td>0.1386</td>
<td>3.6300</td>
</tr>
<tr>
<td>Constant</td>
<td>-15578.1000</td>
<td>3575.0000</td>
<td>-4.3600</td>
</tr>
</tbody>
</table>

\[ \Pi_{1,0} = -43631.7; \Pi_{1,1} = 1.4493; \Pi_{1,2} = 3.0285; \Pi_{1,3} = -0.3036; \]

\[ \Pi_{2,0} = -15578.1; \Pi_{2,1} = 1.0606; \Pi_{2,2} = 1.9053; \Pi_{1,3} = 0.5033 \]

All above parameters are statistically significant.

### Table 3

**Correlation structure among model variables**

<table>
<thead>
<tr>
<th></th>
<th>GDP_MP</th>
<th>Private cons</th>
<th>Revenue</th>
<th>Gov Cons</th>
<th>Trade bal</th>
<th>Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP_MP</td>
<td>1.0000</td>
<td>0.9972</td>
<td>0.9383</td>
<td>0.9653</td>
<td>-0.8264</td>
<td>0.9570</td>
</tr>
<tr>
<td>Priv Cons</td>
<td>0.9972</td>
<td>1.0000</td>
<td>0.9219</td>
<td>0.9568</td>
<td>-0.8481</td>
<td>0.9506</td>
</tr>
<tr>
<td>Revenue</td>
<td>0.9383</td>
<td>0.9219</td>
<td>1.0000</td>
<td>0.9397</td>
<td>-0.7685</td>
<td>0.9189</td>
</tr>
<tr>
<td>Gov Cons</td>
<td>0.9653</td>
<td>0.9568</td>
<td>0.9397</td>
<td>1.0000</td>
<td>-0.7803</td>
<td>0.8851</td>
</tr>
<tr>
<td>Trade bal</td>
<td>-0.8264</td>
<td>-0.8481</td>
<td>-0.7685</td>
<td>-0.7803</td>
<td>1.0000</td>
<td>-0.8743</td>
</tr>
<tr>
<td>Investment</td>
<td>0.9570</td>
<td>0.9506</td>
<td>0.9189</td>
<td>0.8851</td>
<td>-0.8743</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Structural coefficients of the model, $c_0$, $\alpha$, and $t$ can be retrieved from the above estimates of reduced form parameters.

By observing, $\Pi_{1,0} = \frac{c_0}{(1 - \alpha + \alpha \cdot t)}$ and $\Pi_{2,0} = \frac{t}{(1 - \alpha + \alpha \cdot t)}$, the tax rate parameter can be estimated as $t = \frac{\Pi_{2,0}}{\Pi_{1,0}} = \frac{-15578.1}{-43631.7} = 0.357$ which is the average tax rate in the UK. Given this tax rate now it is possible to retrieve the marginal propensity to consume $\alpha$ parameter as $\Pi_{1,2} = \frac{\alpha(1-t)}{t} = \frac{1.447}{1.061} = 1.367$ or $\alpha(1-t) = 1.367t$

$\alpha = 1.367 \frac{t}{(1 - t)} = 1.367 \frac{0.357}{(1 - 0.357)} = 0.759$ which is a reasonable value for the marginal propensity to consume. The estimates of $\alpha$, and $t$ can be used with mean values of income and consumption in the consumption function to derive the estimated value of the autonomous consumption.

$\overline{C} = c_0 + (\alpha - t)\overline{FY}$

$c_0 = \overline{C} - (\alpha - t)\overline{FY} = 96668 - (0.759 - 0.357)\times154450 = 34579.1$ \hspace{1cm} (15)

<table>
<thead>
<tr>
<th></th>
<th>GDP_M</th>
<th>Priv_co</th>
<th>Reve</th>
<th>GovCo</th>
<th>Trdbal</th>
<th>Investm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>154450</td>
<td>96668</td>
<td>75976</td>
<td>32709</td>
<td>-3334.6</td>
<td>27759</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>42012</td>
<td>31511</td>
<td>18316</td>
<td>5525.6</td>
<td>5973.7</td>
<td>9354.3</td>
</tr>
</tbody>
</table>

Thus the complete estimated structural model is:

Consumption function: $C_t = 34579.1 + 0.759(Y_t - T_t)$

Taxation function: $T_t = 0.357Y_t$

National income identity $Y_t = C_t + I_0 + G_0 + NX_0$
Now policy scenarios can be considered depending on one’s belief about how exogenous variables \( I_0, G_0, \) and \( X_0 \) are likely to change in the model horizon.

This model is made a bit complicated in the next section by introducing imports, investment and money market equations and followed by discussion of rank and order conditions for identification of each equation included in the model.

**IV. Identification and Estimation of a Simultaneous Equation Model**

This section provides some technical details on how to identify each equation in a traditional Keynesian Macroeconometric Model (IS-LM model).

Consumption function:  
\[
C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2X_t + \varepsilon_{1t}
\]  \( (16) \)

Taxation function:  
\[
T_t = t_0 + t_1Y_t + t_2M_t + t_3G_t + \varepsilon_{2t}
\]  \( (17) \)

Import function:  
\[
M_t = m_0 + m_1Y_t + m_2R_t + m_3T_t + \varepsilon_{3t}
\]  \( (18) \)

Investment function  
\[
I_t = \mu_0 + \mu_1R_t + \phi_1\Delta Y_{t-1} + \varepsilon_{4t}
\]  \( (19) \)

Money market (LM curve):  
\[
\begin{align*}
\frac{MM_t}{P_t} &= b_0 + b_1Y_t - b_2R_t + \varepsilon_{6t}
\end{align*}
\]
Money market (LM curve): 
\[ R_t = \frac{b_0}{b_2} - \frac{1}{b_2} \left( \frac{MM_t}{P_t} \right) + \frac{b_1}{b_2} Y_t + \epsilon_{et} \] (20) 

National income identity 
\[ Y_t = C_t + I_t + G_t + X_t - M_t \] (21) 

where \( Y_t, C_t, M_t, I_t, R_t, T_t \) are six endogenous variables representing total output, consumption, imports, investment, interest rate and taxes respectively and \( \Delta Y_{t-1}, G_t, \frac{MM_t}{P} \) and \( X_t \) are predetermined or exogenous variables representing change in income in the previous period (\( \Delta \) denotes a change in the variable), government spending, real money balances and exports. Each equation in such a simultaneous equation model need to satisfy order and rank conditions of identification to be able to retrieve the structural parameters \( \beta_0, \beta_1, \beta_2, \theta_0, \theta_1, \theta_2, \theta_3, m_0, m_1, m_2, m_3, \mu_0, \mu_1, \phi, b_0, b_1, b_2 \) from the estimates of the reduced form parameter of the model from the time series data on endogenous and exogenous variables. The order conditions for an equation included in the model is given by 
\[ K - k \geq m - 1 \], where \( M \) is number of endogenous variables, \( K \) is number of exogenous variables including the intercept; \( m \) the number of endogenous variable in an equation; \( k \) the number of exogenous variables in an equation.

Each above equations are identifies by the order conditions. For instance, with nine exogenous variables in the model including the intercept term the consumption function has only two exogenous variables; \( K - k = 9 - 2 = 7 \geq M - 1 = 6 - 1 = 5 \). All other equations similarly satisfy order conditions, which is a necessary but not sufficient condition for identification. Each equation is identified by the rank condition when a rank of the coefficients of the matrix of dimension of \((M-1) \times (M-1)\) exists for that equation in a model with \( M \) endogenous variables. This matrix is formed from the coefficients in model for both endogenous and exogenous variables excluded from that particular equation but included in other equations of the model.
The rank condition, \( \rho(A) \geq (M - 1) \times (M - 1) \), used to find out whether a particular equation is identified involves following steps:

1. Write down the system in the tabular form.
2. Strike out all coefficients in the row corresponding to the equation to be identified.
3. Strike out the columns corresponding to non-zero coefficients in that particular equation.
4. Form matrix from the remaining coefficients. It will contain only the coefficients of the variables included in the system but not in the equation under consideration. From these coefficients form all possible A matrices of order M-1 and ascertain that determinant of order M-1 exist for this system. If at least one of these determinants is non-zero then that equation is identified.

### Table 5

Table of Coefficients in a Macro Econometric Model

<table>
<thead>
<tr>
<th></th>
<th>( C_t )</th>
<th>( Y_t )</th>
<th>( M_t )</th>
<th>( I_t )</th>
<th>( R_t )</th>
<th>( T_t )</th>
<th>( G_t )</th>
<th>( X_t )</th>
<th>( MM_t )</th>
<th>( \Delta Y_{t-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_t )</td>
<td>-( \beta_0 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \beta_1 )</td>
<td>0</td>
<td>-( \beta_2 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( T_t )</td>
<td>-( t_0 )</td>
<td>-( t_1 )</td>
<td>0</td>
<td>-( t_2 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-( t_3 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( M_t )</td>
<td>-( m_0 )</td>
<td>-( m_1 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-( m_2 )</td>
<td>-( m_3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( I_t )</td>
<td>-( \mu_0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-( \mu_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( R_t )</td>
<td>-( b_0 )</td>
<td>-( b_1 )</td>
<td>( b_0 )</td>
<td>( b_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Y_t )</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Summary of the order and rank conditions of identification:

1. If \( K - k > m - 1 \) and the rank of the \( \rho(A) \) is M-1 then the equation is over-identified.
2. If \( K - k = m - 1 \) and the rank of the \( \rho(A) \) is M-1 then the equation is exactly identified.
3. If \( K - k \geq m - 1 \) and the rank of the \( \rho(A) \) is less than M-1 then the equation is under identified.
4. If \( K - k \leq m - 1 \) the structural equation is unidentified.

If the rank of the matrix with remaining coefficients \( \rho(A) \) equals less than M-1, the corresponding equation is not identified and the model breaks down. Over-identification is less serious problem than under identification.

Identification for each equation can be examined by the rank condition as following:
consumption function: $A_i = \begin{bmatrix} -t_2 & -t_3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi \\ 0 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$

$|A_c| = \frac{1}{b_2} \phi m_2 t_2 \Rightarrow \rho(A_i) = 4$. \quad (22)

It is obvious that there exists at least on non-singular matrix of order $M-1$.

Tax function: $A_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -m_2 & 0 & 0 & 0 \\ 0 & -\mu_i & 1 & 0 & -\phi \\ 0 & 1 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$

$|A_t| = -\frac{1}{b_2} m_2 \Rightarrow \rho(A_i) = 4$. \quad (23)

Import function: $A_i = \begin{bmatrix} 1 & 0 & -\beta_2 & 0 & 0 \\ 0 & -t_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\phi \\ 0 & 0 & 0 & \frac{1}{b_2} & 0 \end{bmatrix}$

$|A_{m}| = -\frac{1}{b_2} \phi t_2 \beta_2 \Rightarrow \rho(A_i) = 4$

Investment function: $A_i = \begin{bmatrix} -\beta_i & 0 & -\beta_i & 0 \\ -t_i & -t_2 & 1 & 0 \\ -m_i & 1 & -m_3 & 0 \\ -\frac{b_1}{b_2} & 0 & 0 & \frac{1}{b_2} \end{bmatrix}$

$|A_i| = -\beta_i t_2 m_3 \frac{1}{b_2} + \beta_i t_2 m_1 \frac{1}{b_2} \Rightarrow \rho(A_i) = 4$. \quad (24)

It is very easy to identify the interest rate function.

Interest rate function: $A_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -t_2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -\phi \end{bmatrix}$
\[ |A_t| = t_2 \phi \Rightarrow \rho(A_t) = 4 \]  

Thus all equations in the model are identified. This model then can be represented in the reduced form as:

\[
C_t = \Pi_{1,0} + \Pi_{1,1} G_t + \Pi_{1,2} X_t + \Pi_{1,3} M_t + \Pi_{1,4} \Delta Y_{t-1} + v_{1,t}
\]  

\[
M_t = \Pi_{2,0} + \Pi_{2,1} G_t + \Pi_{2,2} X_t + \Pi_{2,3} M_t + \Pi_{2,4} \Delta Y_{t-1} + v_{2,t}
\]  

\[
I_t = \Pi_{3,0} + \Pi_{3,1} G_t + \Pi_{3,2} X_t + \Pi_{3,3} M_t + \Pi_{3,4} \Delta Y_{t-1} + v_{3,t}
\]  

\[
R_t = \Pi_{4,0} + \Pi_{4,1} G_t + \Pi_{4,2} X_t + \Pi_{4,3} M_t + \Pi_{4,4} \Delta Y_{t-1} + v_{4,t}
\]  

\[
T_t = \Pi_{5,0} + \Pi_{5,1} G_t + \Pi_{5,2} X_t + \Pi_{5,3} M_t + \Pi_{5,4} \Delta Y_{t-1} + v_{5,t}
\]  

Here \( v_{1,t} \) to \( v_{5,t} \) are composite normally distributed random error terms. The generalised least square estimation requires estimation procedure involves first estimations each equation by the OLS and to estimate a covariance matrix of the system as:

\[
\text{cov}[\hat{v}] = \begin{bmatrix}
\text{var}(\hat{v}_1) & \text{cov}(\hat{v}_1, \hat{v}_2) & \text{cov}(\hat{v}_1, \hat{v}_3) & \cdots & \text{cov}(\hat{v}_1, \hat{v}_5)

\text{cov}(\hat{v}_2, \hat{v}_1) & \text{var}(\hat{v}_2) & \text{cov}(\hat{v}_2, \hat{v}_3) & \cdots & \text{cov}(\hat{v}_2, \hat{v}_5)

\text{cov}(\hat{v}_3, \hat{v}_1) & \text{cov}(\hat{v}_3, \hat{v}_2) & \text{var}(\hat{v}_3) & \cdots & \text{cov}(\hat{v}_3, \hat{v}_5)

\vdots & \vdots & \vdots & \ddots & \vdots

\text{cov}(\hat{v}_5, \hat{v}_1) & \text{cov}(\hat{v}_5, \hat{v}_2) & \text{cov}(\hat{v}_5, \hat{v}_3) & \cdots & \text{var}(\hat{v}_5)
\end{bmatrix} = \Omega
\]  

where each of the cells in the matrix have \( T \times T \) dimension. Thus the covariance matrix \( \Omega \) has \( 5T \times 5T \) dimension. Using the theorem in matrix algebra \( \Omega \) can be decomposed into two parts as:

\[
P^T P = \Omega^{-1}
\]  

Use this partition of \( \Omega \) to transform the original model as:

\[
P Y = PX \beta + Pe
\]  

\[
Y^* = X^* \beta + e^*
\]  

\[
\beta_{GLS} = (X^{**}X^*)^{-1}X^{**}Y^* = (X^T P^T P X)^{-1}X^T P^T Y \Rightarrow \beta_{GLS} = (X^T \Omega^{-1} X)^{-1}X^T \Omega^{-1} Y
\]
The model parameters are estimated using annual time series data for the UK economy from 1966:1 to 2006:1 using the routines in the PcGive and are presented in Table 6. Historical simulations, shown in Figure 2(a), illustrate how well the model is able to track endogenous variables in the economy.

<table>
<thead>
<tr>
<th>Private consumption function</th>
<th>Investment function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>Std.Error</td>
</tr>
<tr>
<td>GovCons</td>
<td>1.4333</td>
</tr>
<tr>
<td>Exports</td>
<td>0.1666</td>
</tr>
<tr>
<td>M4</td>
<td>0.0539</td>
</tr>
<tr>
<td>DGDP_MP</td>
<td>0.5150</td>
</tr>
<tr>
<td>Constant</td>
<td>21992.8000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Imports function</th>
<th>Revenue function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>Std.Error</td>
</tr>
<tr>
<td>GovCons</td>
<td>-0.3082</td>
</tr>
<tr>
<td>Exports</td>
<td>0.7810</td>
</tr>
<tr>
<td>M4</td>
<td>0.0303</td>
</tr>
<tr>
<td>DGDP_MP</td>
<td>-0.0147</td>
</tr>
<tr>
<td>Constant</td>
<td>10318.8000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treasury bills function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
</tr>
<tr>
<td>GovCons</td>
</tr>
<tr>
<td>Exports</td>
</tr>
<tr>
<td>M4</td>
</tr>
<tr>
<td>DGDP_MP</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

Most of these estimates are consistent to underlying Keynesian economic theory (Mankiw(1989), Minford and Peel (2002)). Increase in money supply, government spending and exports raise consumption, investment and output. The tax revenue rises with higher government spending and greater money supply. Intuitively imports depend on exports and supply of money but its volume diminishes with government spending. Increased money supply reduces the interest rate. Government spending rises with the level of income. Higher income implies higher tax income and more public spending. Imports, however, reduce the amount of public spending. Imports represent leak out of the income from the economic system. Higher amount of imports implies less revenue for the government and less spending.
Imports vary positively with money supply but negatively with the government spending. Interest rate function has positive and significant relation with income and money supply but negative relation with the interest rate. The coefficients of the
investment function have expected signs and significant except for the change in income. Forecasts of endogenous variables are presented with their confidence interval in Figure 2(b). Under this modelling paradigm a model with the minimum forecast error is the best model (Clement (1995) and Hendry (1997)). Despite good fits and forecasts this forecast assumes that the estimated parameters remain constant over the forecast horizon. It is equivalent to assuming that behaviour of households and firms do not alter their choices following changes in economic policies. This is Lucas critique (1976) which is remedied by introducing the rational expectation in the model (Wallis (1980)) where anticipated policies can have significantly different impact than unanticipated policies. Since forming rational expectations is very difficult other macroeconomists have tended to pay more attention just to the time series properties of variables to forecast their future values rather than using the structural parameters as estimated above under the “let data speak for itself” paradigm of time series modelling presented in the next section.

V. Time Series Models and Forecasts

Broadly there are two “a-theoretical” approaches of time series modelling. The Box-Jenkins (1970) approach of forecasting involves constructing an AR, MA, ARMA or ARIMA models to predict the future values of an economic variable in terms of its current and past values. Time series modellers interpret the coefficients of an ARIMA equation to be a sort of the reduced form of a SEM model although the ARMA model does not have structural features that are found in the SEM models. An ARMA(p, q) model is specified as:

\[ y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_p y_{t-p} + e_t + b_1 e_{t-1} + b_2 e_{t-2} + \ldots + b_q e_{t-q} \quad (36) \]
Each of the coefficients in $a_1 \ldots a_p$ and $a_1 \ldots a_q$ in this polynomial represents the turning points of a particular time series. Stationary series are convergent to their steady state values and non-stationary series are divergent. Unit root and cointegration tests are carried out to ensure stationarity of these series before their use in analysis.

Predictions and forecasts of quarterly growth rate of UK generated by an ARMA model are given in Figure 3 and 4. An ARMA(8,4) model fits quite well with the data and is the best based on the AIC criteria. This tracks growth well (Figure 3) and generates forecasts for growth in coming quarters (Figure 4). Economy peaks up in the first quarter, dips down in the second and then starts to bounce back in the third quarter reaching to another peak in the fourth quarter. The cycle continues in this manner.

![Figure 3](image1)

*Figure 3*
Fit of quarterly growth rates of output by ARMA(8,4)

![Figure 4](image2)

*Figure 4*
Forecast of quarterly growth rates of output by ARMA(8,4)
Table 7
Above fit is based on estimation of Quarterly Growth Rate by ARMA(8,4) Model

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR-1</td>
<td>-0.7750</td>
<td>0.3164</td>
<td>0.0150</td>
</tr>
<tr>
<td>AR-2</td>
<td>-0.9745</td>
<td>0.3663</td>
<td>0.0090</td>
</tr>
<tr>
<td>AR-3</td>
<td>-0.7685</td>
<td>0.3943</td>
<td>0.0530</td>
</tr>
<tr>
<td>AR-4</td>
<td>0.0832</td>
<td>0.3682</td>
<td>0.8210</td>
</tr>
<tr>
<td>AR-5</td>
<td>0.1140</td>
<td>0.1643</td>
<td>0.4890</td>
</tr>
<tr>
<td>AR-6</td>
<td>0.2869</td>
<td>0.1423</td>
<td>0.0450</td>
</tr>
<tr>
<td>AR-7</td>
<td>0.1013</td>
<td>0.1255</td>
<td>0.4210</td>
</tr>
<tr>
<td>AR-8</td>
<td>0.2177</td>
<td>0.1276</td>
<td>0.0900</td>
</tr>
<tr>
<td>MA-1</td>
<td>0.3615</td>
<td>0.3189</td>
<td>0.2590</td>
</tr>
<tr>
<td>MA-2</td>
<td>0.6549</td>
<td>0.2606</td>
<td>0.0130</td>
</tr>
<tr>
<td>MA-3</td>
<td>0.4084</td>
<td>0.2574</td>
<td>0.1150</td>
</tr>
<tr>
<td>MA-4</td>
<td>-0.1000</td>
<td>0.2330</td>
<td>0.6690</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0059</td>
<td>0.0010</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Various forms of ARCH-GARCH models also can produce such predictions and forecasts (Nelson and Plosser (1982), Kocherlakota and Yi (1996) and Holland and Scott (1998)).

VI. Test of Cointegration and VAR Modelling

Only stationary variables can be used in regression to generate meaningful results, regression between two non-stationary variables generates a spurious relation (Phillips (1987)). Cointegration test was the rectification for this suggested by Eagle-Granger (1987) which states that non-stationary variables can be included in the model only when they have long run cointegration relationship that makes them move together. The cointegration test process involves three steps. First, the order of integration of each variable is determined by DF or ADF tests for unit root. Determining whether estimated errors of the model follow a unit root is checked by testing unit root hypothesis, $H_0 : a_1 = 0$. If this is rejected the errors, $\Delta e_t = a_1 \Delta e_{t-1} + \varepsilon_t$ are not co-integrated. The relevant series are then transformed by taking the first or higher order differences or by logs to make them stationary which then are used in the model.
Major disadvantage of a single equation time series model is its inability to illustrate the interdependency and simultaneity among the sets of economic variables since the causality runs only from the dependent to independent variables. Vector autoregression (VAR) model, another popular time series method originated in Sims’ (1982) unrestricted VAR model takes the lagged values of endogenous variables and exogenous variables. If there are $n$ endogenous variables, $y_1, y_2, \ldots, y_n$ and $m$ exogenous variables $x_1, x_2, \ldots, x_m$ then the a standard VAR model is specified as:

$$y_{1,t} = a_{10} + \sum_{j=1}^{p} a_{1,j} y_{1,t-j} + \ldots + \sum_{j=1}^{p} a_{1,n,j} y_{n,t-j} + \sum_{j=1}^{r} b_{11,j} x_{1,t-j} + \ldots + \sum_{j=1}^{r} b_{1m,j} x_{m,t-j} + e_{1t}$$

$$y_{nt} = a_{n0} + \sum_{j=1}^{p} a_{n,j} y_{1,t-j} + \ldots + \sum_{j=1}^{p} a_{n,n,j} y_{n,t-j} + \sum_{j=1}^{r} b_{n1,j} x_{1,t-j} + \ldots + \sum_{j=1}^{r} b_{nm,j} x_{m,t-j} + e_{nt}$$

Estimated model is used mainly for forecasting or for impulse response analyses. Selection of endogenous and exogenous variables are guided by minimisation of the sum of the square of errors using mean absolute error (MAE), root of mean square errors (RMSE) and with lags of the variables to be chosen in a particular equation based on certain criteria such as the Akaike information criterion (AIC).

Again like the simultaneous equation system the basic assumptions of the model can be illustrated by using a two variable model:

$$y_{1,t} = a_{10} + a_{11} y_{1,t-1} + a_{12} y_{2,t-2} + b_{11} x_{1,t-1} + b_{12} x_{1,t-2} + e_{1t}$$

$$y_{2,t} = a_{20} + a_{21} y_{1,t-1} + a_{22} y_{2,t-2} + b_{21} x_{1,t-1} + b_{22} x_{2,t-2} + e_{2t}$$

in matrix notation:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

The steady state of the time series $y_i$ and $x_i$ are given by
\[
\bar{Y} = (I - A)^{-1}BX + (I - A)^{-1}U
\]

\[
\bar{X} = (I - B)^{-1}AY + (I - B)^{-1}U
\]

(41)

The first term \((I - A)^{-1}BX\) represents long run impacts of changes in exogenous variables in endogenous variables and the second term represents impacts of shocks. More important aspect of VAR analysis is innovation accounting, to see how the time path of a variable changes above its steady state value if there is either a policy shock in the second equation at time \(T\) is given by:

\[
\begin{bmatrix}
y_{1,T+1} \\
y_{2,T+1}
\end{bmatrix}
= (I - A)^{-1}
\begin{bmatrix}
y_{1,T} \\
y_{2,T}
\end{bmatrix}
\]

then

\[
\begin{bmatrix}
y_{1,T+2} \\
y_{2,T+2}
\end{bmatrix}
= (I - A)^{-1}Y_1 = (I - A)^{-1}(I - A)^{-1}
\begin{bmatrix}
o \\
1
\end{bmatrix}
\]

(42)

This process can continue successively for periods greater that \(T+1\).

The single equation cointegration test procedure is extended to multivariate case by Johansen (1988) which involves examination of significant canonical correlations between two sets of variables. Cointegration vectors are found for each significant eigen values. In case of two variables the test of cointegration involves examining the existence of linear dependence among variables as:

\[
\begin{bmatrix}
a_{11} & a_{12} & b_{11} & b_{12} \\
a_{21} & a_{22} & b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
y_{1,t-1} \\
y_{2,t-2} \\
x_{1,t-1} \\
x_{2,t-2}
\end{bmatrix}
= 0
\]

(43)

In this case a cointegrating vector exists if the coefficient matrix \(\beta = \begin{bmatrix} a_{11} & a_{12} & b_{11} & b_{12} \\ a_{21} & a_{22} & b_{21} & b_{22} \end{bmatrix}\) has a rank of order of two, when the canonical correlations are significant and at least one eigen value is greater than 1. In
\[ x_t = A_t x_{t-1} + \varepsilon_t \] where \( x_t \) is an \( n \times 1 \) vector of endogenous variables and \( A_t \) is an \( n \times n \) matrix of parameters the difference form equation is written as:

\[ \Delta x_t = \left( A_t - I \right) x_{t-1} + \varepsilon_t = \pi x_{t-1} + \varepsilon_t. \]  

where \( I \) is an \( n \times n \) identity matrix. If the rank(\( \pi \)) = 0, there is no linearly stationary relationship among \( x_t \) the variables, sequence \( x_t \) are non stationary and follows unit root process, in contrast if rank(\( \pi \)) = n, there is a linear dependence among \( n \) variables, they are cointegrated.

This can be better illustrated by a first order VAR model of \( y_t \) and \( x_t \) as following:

\[ y_t = a_{11} y_{t-1} + a_{12} x_{t-1} + \varepsilon_{yt} \tag{45} \]

\[ x_t = a_{21} y_{t-1} + a_{22} x_{t-1} + \varepsilon_{xt} \tag{46} \]

\[ y_t = a_{11} Ly_t + a_{12} Lx_t + \varepsilon_{yt} \]

\[ x_t = a_{21} Ly_t + a_{22} Lx_t + \varepsilon_{xt} \]

\[ (1-a_{11}L)y_t - a_{12}Lx_t = \varepsilon_{yt} \]

\[ -a_{21}Ly_t + (1-a_{22}L)x_t = \varepsilon_{xt} \]

\[ \begin{bmatrix} (1-a_{11}L) & -a_{12}L \\ -a_{21}L & (1-a_{22}L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \]

\[ \begin{bmatrix} (1-a_{11}L) & -a_{12}L \\ -a_{21}L & (1-a_{22}L) \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \]

Impulse response of shocks on endogenous variables takes the form:

\[ y_t = \frac{(1-a_{22}L)e_{yt} + a_{12}L e_{xt}}{(1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2} \tag{48} \]

\[ x_t = \frac{(1-a_{11}L)e_{xt} + a_{21}L e_{yt}}{(1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2} \tag{49} \]

Unit root in this VAR model implies \((1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2 = 0\)

\((1-a_{11}L-a_{22}L+a_{11}a_{22}L) - a_{12}a_{21}L^2 = 0\)
or \((a_{11}a_{22} - a_{12}a_{21})L^2 - (a_{11} + a_{22})L + 1 = 0\)

Defining \(\lambda = \frac{1}{L}\) and re-arranging the terms: \(\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0\)

This quadratic equation has two solutions (as many roots as many equations)

\[
\lambda_1, \lambda_2 = \frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}
\]

(50)

If roots \(\lambda_1, \lambda_2\) lie within the unit circle each variable is stationary and it is cointegrated for order 1, C(1,1). If both roots \(\lambda_1, \lambda_2\) lie outside the unit circles \(y_t\) and \(x_t\) processes are explosive and cannot be cointegrated of order 1, \(y_t\) and \(x_t\) are explosive, if 
\[a_{12} = a_{21} = 0\] and \(a_{11} = a_{22} = 1\) then \(\lambda_1 = 1, \lambda_2 = 1\) then two variables evolve without any long run relationship; \(y_t\) and \(x_t\) have cointegration of order 1, C(1,1) only if one of the roots \(\lambda_1, \lambda_2\) is unity and another is less than unity (Elders (1995:6)).

Since the absence of an economic theory in unrestricted VAR no clear criteria exist on choice of variables to be included in a VAR model. Similar problems remain in interpreting the its results. A structural VAR model rectifies this short coming by imposing restrictions on model parameters based on economic theory. As Garratt, Lee, Pesaran and Shin (2003) suggested there can be \(m(m-1)/2\) such restrictions in a model with \(m\) equations. As shown in equation (41) the basic long run elation among variables in the VAR model is decomposed into short run adjustment \(\alpha\) and long run relations, \(\beta\).

\[
\Pi - I = \begin{bmatrix}
0.440 & -0.013 & -0.000 \\
1.0130 & -0.598 & 0.336 \\
0.759 & -0.097 & -177
\end{bmatrix} = \begin{bmatrix}
0.048 & -0.000 & 0.001 \\
2.149 & -0.032 & 0.004 \\
0.408 & 0.010 & 0.001
\end{bmatrix} \begin{bmatrix}
1.000 & -0.263 & -0.072 \\
33.745 & 1.000 & 14.335 \\
2.098 & 0.093 & 1.000
\end{bmatrix} = \alpha\beta^t
\]

Order of cointegration in vectors show the long run relationship in a VAR model is tested by using trace of the eigen values or maximum of the eigen values.
Econometric models reviewed thus far relied on time series data for estimation of behavioural parameters have very little in explaining the optimisation aspect of economic agents and in analysing the market mechanism. In fact an economic model
should be able to explain how these series can be generated from the decision process of consumers and producers in the economy. How solutions of stochastic dynamic general equilibrium model can provide such series is illustrated in the next section.

VII. Stochastic Dynamic General Equilibrium Model

Stochastic dynamic general equilibrium model for small open economy and global economy are considered in this part to show how economic time series are generated by dynamic optimisation process of households, firms, government and traders. Small open economy model is applied to UK and followed by three country global economy consisting of Euro area, UK and the US (further extension of Bhattarai(2008)). Generic modelling structure is explained first followed by some discussion of frequency distributions and impulse response analyses comparables to the ones conducted above based on model generated time series of variables included in this DSGE model.

Household utility function contains goods produced at home, imported and leisure. Government uses taxes on consumption, imported goods and labour income. With the Cobb-Douglas production function, the household problem can be stated as:

$$\text{Max } U_0 = \sum_{t=0}^{\infty} \theta^t \left( C_t^\alpha M_t^\beta I_t^\gamma \right) \quad \text{where } 1 = \alpha + \beta + \gamma = 1; 0 < \alpha, \beta, \gamma < 1; 0 < \theta < 1 \quad (5.1)$$

Subject to life time budget constraint

$$\sum_{i=0}^{\infty} \left[ P_{i,t} (1 + t) C_{i,t} + P_{j,t} (1 + t m_{i,j}) M_{i,j} + w_{i,j} (1 - t w_{i,j}) I_{i,t} \right] \leq \sum_{i=0}^{\infty} \left[ w_{i,j} (1 - t w_{i,j}) I_{i,t} + r_{i,j} (1 - t k_{i,j}) K_{i,t} \right] \quad (5.2)$$

Firms maximise profit in each period

$$\text{Max } \Pi_{i,t} = P_{i,t} Y_{i,t} - r_{i,t} K_{i,t} - w_{i,t} L S_{i,t} \quad (5.3)$$

Subject to
\[
Y_{i,t} = A_i K_{i,t}^{a_0} L_{i,t}^{(1-a_0)}
\]

\[
I_{i,t} = K_{i,t} - (1-\delta)K_{i,t-1}
\]

Productivity shock \( A_{i,t} \) is heterogeneous across countries generated randomly with a constant mean \( \bar{A}_i \) and variance \( \sigma^2_{A_i} \).

**Government Sector:**

\[
R_{i,t} = P_{i,t} c_t C_{i,t} + m_{i,t} P_{j,i} M_{i,t} + t w_{i,t} L S_{i,t} + t_r K_{i,t} \leq G_{i,t}
\]

Market clearing

\[
Y_{i,t} = C_{i,t} + I_{i,t} + X_{i,t} + G_{i,t}
\]

There can be two different ways of trade balance

**Period by period trade balance:** \( M_{i,t} = X_{i,t} \)

\[
\sum_{i=0}^{\infty} \theta^i \left( M_{i,t} - X_{i,t} \right) = \sum_{i=0}^{\infty} \theta^i \left( TB_{i,t} \right)
\]

\[
(S_{i,t} - I_{i,t}) + (X_{i,t} - M_{i,t}) = 0
\]

\[
(L_{i,t} - I_{i,t}) = L S_{i,t}
\]

Prices from the inter temporal arbitrage condition

\[
P_{i,t} = \frac{P_{i,t+1}}{1 + r_{i,t}}
\]

Exchange rates:

\[
E_{i,t} = \frac{P_{i,t}}{P_{j,t}}
\]

A competitive economy is the sequence of prices \( P_{i,t}, P_{j,t}, r_{i,t}, r_{j,t}, w_{i,t}, w_{j,t}, E_{i,t}, E_{j,t} \) and public policy \( tc_{i,t}, tc_{j,t}, tm_{i,t}, tm_{j,t}, tw_{l,t}, tr_{i,t}, tr_{j,t} \), in which allocation of \( C_{i,t}, M_{i,t}, l_{i,t}, C_{j,t}, M_{j,t}, l_{j,t} \) maximise the lifetime utility of households \( U_{i0} \) and \( U_{j0} \) and \( LS_{j,t}, K_{j,t}, LS_{j,t}, K_{j,t} \) that maximise firms profit and
government expenditures are \( G_{i,t} \), \( G_{j,t} \) are compatible with the government revenue \( R_{i,t} \), \( R_{j,t} \) and exports \( X_{i,t} \), \( X_{j,t} \) are compatible with imports \( M_{i,t} \), \( M_{j,t} \).

The infinite horizon problem is analytically intractable. Such problems are solved using the first order inter temporal optimisation for any two time intervals with generalisation that solutions that satisfy any two periods can be extended to any other periods. First order conditions for households for two periods are:

\[
C_i : \quad \alpha_t \theta^t \left( C_{i,t} \right)^{a_t - 1} M_i^{\beta_t} l_i^{\gamma_t} = \bar{\lambda}_t P_{i,t} (1 + t_c) \tag{5.14}
\]

\[
C_{i,t+1} : \quad \alpha_t \theta^{t+1} \left( C_{i,t+1} \right)^{a_t - 1} M_i^{\beta_t} l_i^{\gamma_t} = \bar{\lambda}_t P_{i,t+1} (1 + t_c) \tag{5.15}
\]

\[
M_i : \quad \beta_t \theta^t \left( C_{i,t} \right)^{a_t - 1} M_i^{\beta_t - 1} l_i^{\gamma_t - 1} = \bar{\lambda}_t P_{j,t} (1 + t_m) \tag{5.16}
\]

\[
M_{i,t+1} : \quad \beta_t \theta^{t+1} \left( C_{i,t+1} \right)^{a_t - 1} M_i^{\beta_t - 1} l_i^{\gamma_t - 1} = \bar{\lambda}_t P_{j,t+1} (1 + t_m) \tag{5.17}
\]

\[
l_i : \quad \gamma_t \theta^t \left( C_{i,t} \right)^{a_t - 1} M_i^{\beta_t} l_i^{\gamma_t - 1} = \bar{\lambda}_t w_{i,t} (1 - t_w) \tag{5.18}
\]

\[
l_{i,t+1} : \quad \gamma_t \theta^{t+1} \left( C_{i,t+1} \right)^{a_t - 1} M_i^{\beta_t} l_i^{\gamma_t - 1} = \bar{\lambda}_t w_{i,t+1} (1 + t_w) \tag{5.19}
\]

\[
\bar{\lambda}_t : \quad P_{i,t} (1 + t_c) C_{i,t} + P_{j,t+1} (1 + t_m) M_i + w_{i,t} (1 - t_w) y_{i,t} = w_{i,t} (1 - t_w) \bar{L}_{i,t} + r_{i,t} (1 - t_k) K_{i,t} \tag{5.20}
\]

\[
\bar{\lambda}_{i,t+1} : \quad P_{i,t+1} (1 + t_c) C_{i,t+1} + P_{j,t+1} (1 + t_m) M_i + w_{i,t+1} (1 - t_w) y_{i,t+1} = w_{i,t+1} (1 - t_w) \bar{L}_{i,t+1} + r_{i,t+1} (1 - t_k) K_{i,t+1} \tag{5.21}
\]

Above first order conditions can be simplified in terms of Euler equations as:

\[
\frac{C_{i,t}}{C_{i,t+1}} : \quad \frac{1}{\theta} \left( \frac{C_{i,t}}{C_{i,t+1}} \right)^{a_t - 1} \left( \frac{M_{i,t}}{M_{i,t+1}} \right)^{\beta_t} \left( \frac{l_{i,t}}{l_{i,t+1}} \right)^{\gamma_t} = \frac{P_{i,t}}{P_{j,t}} \tag{5.22}
\]

\[
\frac{M_{i,t}}{M_{i,t+1}} : \quad \frac{1}{\theta} \left( \frac{C_{i,t}}{C_{i,t+1}} \right)^{a_t} \left( \frac{M_{i,t}}{M_{i,t+1}} \right)^{\beta_t - 1} \left( \frac{l_{i,t}}{l_{i,t+1}} \right)^{\gamma_t - 1} = \frac{P_{j,t}}{P_{j,t+1}} \tag{5.23}
\]

\[
\frac{M_{i,t}}{M_{i,t+1}} : \quad \frac{1}{\theta} \left( \frac{C_{i,t}}{C_{i,t+1}} \right)^{a_t} \left( \frac{M_{i,t}}{M_{i,t+1}} \right)^{\beta_t - 1} \left( \frac{l_{i,t}}{l_{i,t+1}} \right)^{\gamma_t - 1} = \frac{w_{i,t}}{w_{i,t+1}} \tag{5.24}
\]

\[
\frac{C_{i,t+1}}{M_{i,t+1}} : \quad \frac{\alpha_t}{\beta_t} \left( \frac{M_{i,t+1}}{C_{i,t+1}} \right) = \frac{P_{i,t+1}}{P_{j,t+1}} \frac{(1 + t_c)}{(1 + t_m)} \tag{5.25}
\]
\[
\frac{I_{i,t+1}}{M_{i,t+1}} = \frac{\alpha_i}{\gamma_i} \left( \frac{I_{i,t+1}}{C_{i,t+1}} \right) = \frac{P_{i,t+1}(1+tc_i)}{w_{i,t+1}(1+tw_i)} \tag{5.26}
\]

\[
\frac{M_{j,t+1}}{I_{j,t+1}} = \frac{\beta_j}{\gamma_j} \left( \frac{M_{j,t+1}}{I_{j,t+1}} \right) = \frac{P_{j,t+1}(1+tm_j)}{w_{j,t+1}(1+tw_j)} \tag{5.27}
\]

Similarly the first order conditions for firms are:

\[
\Pi_{j,t} = P_i A_{i,t} K_{j,t}^{\eta_i} L_{i,t}^{(1-\eta_i)} - r_{j,t} K_{j,t} - w_{j,t} L_{i,t} \tag{5.28}
\]

\[
K_{j,t}: \eta_{j,t} A_{j,t} P_{j,t} K_{j,t}^{\eta_{j,t} - 1} L_{i,t}^{(1-\eta_{j,t})} = r_{j,t} \quad \text{or} \quad \frac{\eta_{j,t} P_{j,t} Y_{j,t}}{K_{j,t}} = r_{j,t} \tag{5.29}
\]

\[
K_{j,t}: \eta_{j,t} A_{j,t} P_{j,t} K_{j,t}^{\eta_{j,t} - 1} L_{i,t}^{(1-\eta_{j,t})} = r_{j,t} \quad \text{or} \quad \frac{\eta_{j,t} P_{j,t} Y_{j,t}}{K_{j,t}} = r_{j,t} \tag{5.30}
\]

\[
L_{j,t}: (1-\eta_{j,t}) P_{j,t} A_{j,t} K_{j,t}^{\eta_{j,t} - 1} L_{j,t}^{(1-\eta_{j,t})} = w_{j,t} \quad \text{or} \quad \frac{(1-\eta_{j,t}) P_{j,t} Y_{j,t}}{L_{j,t}} = w_{j,t} \tag{5.31}
\]

\[
L_{j,t}: (1-\eta_{j,t}) P_{j,t} A_{j,t} K_{j,t}^{\eta_{j,t} - 1} L_{j,t}^{(1-\eta_{j,t})} = w_{j,t} \quad \text{or} \quad \frac{(1-\eta_{j,t}) P_{j,t} Y_{j,t}}{L_{j,t}} = w_{j,t} \tag{5.32}
\]

Initial condition \( K_{i,0} \) \( K_{j,0} \) and

Terminal conditions \( I_{i,T} = (g + \delta) K_{i,T-1} \); \( I_{j,T} = (g + \delta) K_{j,T-1} \). \tag{5.34}

Whether the wages rates and the interest rates are same or differ from one country to another partly depends upon the mobility of factors and partly to the tariff rates across countries. If labour and capital are perfectly mobile then the ratios of use of labour and capital across two countries depend on ratios of production.

\[
\frac{\eta_{j,t} P_{j,t} Y_{j,t}}{\eta_{j,t} P_{j,t} Y_{j,t} K_{j,t}} = \frac{r_{j,t}}{r_{j,t}} \tag{5.35}
\]

\[
\frac{(1-\eta_{j,t}) P_{j,t} Y_{j,t}}{(1-\eta_{j,t}) P_{j,t} Y_{j,t} L_{j,t}} = \frac{w_{j,t}}{w_{j,t}} \tag{5.36}
\]

The exchange rate between two countries should be compatible with goods, labour and capital markets.
These analytical results are significantly different than found in the literature (Dornbusch (1976), Taylor (1995)). For empirical implementation the infinite horizon problem is reduced to finite horizon by fixing the terminal period to be some $T$ in the far distance in the future. Similarly the labour endowment in each period $L_t$ and $L_j$ are taken as given as are the model parameters $\alpha$, $\beta$, $\gamma$ and $\theta$ and the policy parameters $tjc_t$, $tjc_j$, $tm_{ij}$, $tm_{ij}$, $tw_{ij}$, $tw_{ij}$, $tr_{ij}$ and $tr_{ij}$. Labour markets and trade aspects are simplified further (see Whalley(1975), Cooley and LeRoy(1985), Perroni (1995), Rutherford (1995) Sargent and Ljungqvists (2000), Chari, Kehoe and McGrattan(2007)for more elaboration on structural DGE models).

**VIII. Stochastic Small Open Economy and Global Economy Models**

Simplified versions of stochastic dynamic general equilibrium models in line of Ramsey (1928) Cass (1965), Lucas (1975) and Quah (1995), Dixon and Rankin (1994) presented above for a small open economy and global economy is implemented to study the time series properties of major macro economic variables such as consumption, output, investment, exports, imports and government spending. Open economy SDGE was solved for 100 years. Model generated series are presented in Figure 7 and correlations from the solutions are reported in Table 9. The major factor underlying these series is technological shocks whose distribution is given in Figure 8. This shock affects the production of output, income and consumption of households and the government. These correlation coefficients between capital, output, private consumption, investment and public consumption and exports for one particular technology shock $z_1$ are similar to what one would find in the actual time series.

\[
E_{j,t} = \frac{P_{j,t}}{P_{i,t}} = \frac{r_{i,t} K_{i,t} \eta_{j,t} Y_{j,t}}{r_{i,t} K_{i,t} \eta_{j,t} Y_{j,t}} = \frac{(1-\eta_{j,t})Y_{i,t} L_{j,t} w_{i,t}}{(1-\eta_{j,t})Y_{i,t} L_{j,t} w_{i,t}} = \frac{\alpha_j M_{i,t} (1 + tm_t)}{\beta_i C_{j,t} (1 + tc_t)} \tag{5.37}
\]
Figure 7
Macro time series from open economy stochastic dynamic general equilibrium model

Table 9
Correlation matrix among model generated series for SDGE-SOE model

<table>
<thead>
<tr>
<th></th>
<th>k_z1</th>
<th>q_z1</th>
<th>C_z1</th>
<th>i_z1</th>
<th>x_z1</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_z1</td>
<td>1.00</td>
<td>0.97</td>
<td>0.43</td>
<td>0.47</td>
<td>0.97</td>
</tr>
<tr>
<td>q_z1</td>
<td>0.97</td>
<td>1.00</td>
<td>0.50</td>
<td>0.47</td>
<td>0.43</td>
</tr>
<tr>
<td>C_z1</td>
<td>0.43</td>
<td>0.50</td>
<td>1.00</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>i_z1</td>
<td>0.47</td>
<td>0.43</td>
<td>-0.57</td>
<td>1.00</td>
<td>0.43</td>
</tr>
<tr>
<td>x_z1</td>
<td>0.97</td>
<td>1.00</td>
<td>0.50</td>
<td>0.43</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Model results are used in cointegration test of long run relationship in Table 10 and for VAR impulse response analyses as given in Figure 9.

Table 10
VAR cointegration tests for Series from SDGE-SOE Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>240.09 [0.000]**</td>
<td>124.51 [0.000]**</td>
<td>215.59 [0.000]**</td>
<td>111.8 [0.000]**</td>
</tr>
<tr>
<td>1</td>
<td>115.58 [0.000]**</td>
<td>57.99 [0.000]**</td>
<td>103.79 [0.000]**</td>
<td>52.07 [0.000]**</td>
</tr>
<tr>
<td>2</td>
<td>57.6 [0.000]**</td>
<td>35.32 [0.000]**</td>
<td>51.72 [0.000]**</td>
<td>31.71 [0.001]**</td>
</tr>
<tr>
<td>3</td>
<td>22.28 [0.003]**</td>
<td>21.85 [0.002]**</td>
<td>20.01 [0.009]**</td>
<td>19.62 [0.005]**</td>
</tr>
<tr>
<td>4</td>
<td>0.43 [0.511]</td>
<td>0.43 [0.511]</td>
<td>0.39 [0.533]</td>
<td>0.39 [0.533]</td>
</tr>
</tbody>
</table>

Table 11
Parameters of the small open economy and global economy Models

<table>
<thead>
<tr>
<th></th>
<th>Euro area</th>
<th>UK</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation rate (d)</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Share of labour (a)</td>
<td>0.60</td>
<td>0.62</td>
<td>0.65</td>
</tr>
<tr>
<td>Discount factor (b)</td>
<td>0.98</td>
<td>0.95</td>
<td>0.90</td>
</tr>
<tr>
<td>Growth rate of labour (gr)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Endowment (L)</td>
<td>6886.00</td>
<td>1685.00</td>
<td>11265.00</td>
</tr>
<tr>
<td>Export share (xr)</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>Net public consumption (rv)</td>
<td>0.20</td>
<td>0.17</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Then global economy stochastic dynamic general equilibrium model is solved for 100 years and many other horizons using a standard non-linear optimisation routines in GAMS (1998) with parameters in Table 11.
This DSGE model generates distributions of consumption, investment, capital stock, exports, imports, exchange rate and output for various technologies. A sample of time series resulting from the solution of global economy version of this model is given in Figure 10. Each economy here has been subject to technological shocks in each period. For simplicity it is assumed that the technology is randomly generated in ten different levels which is not known to consumers and producers in the economy before its realisation. These technological shocks affect the productivity and hence income and consumption profiles of households. They respond to these shocks and take account of all these possible shocks while maximizing their expected utility over the life time. The distribution of each model variable follows from these stochastic shocks.

This model mimics the time series properties of actual economies. Ratios of consumption, investment, exports and imports to the GDP and the utility of households are computed and compared. Results of the stochastic model with its whole state space are massive and only a tiny sample of model output can be reported.
in this section. Economies with more restrictive trade policy and experiencing greater technological shock end up losing in terms of welfare gains to households.

Figure 10
Nature of technology shocks dynamic general equilibrium model of global economy

Economic prospects are influenced by the subjective discount factors. This is found by comparing sensitivity of life time utility to beta values of 0.9, 0.95, 0.95 for the EU, UK and the US in Table 12 to 0.95, 0.95 and 0.90 values in Table 13. Even a slight change in these discount factors reverses the structure of the time series and the welfare results in such dynamic economy.

Table 12
Impact of stochastic technology on welfare of households (beta 0.9, 0.95, 0.95)

<table>
<thead>
<tr>
<th></th>
<th>u_z1</th>
<th>U_z2</th>
<th>u_z3</th>
<th>u_z4</th>
<th>U_z5</th>
<th>u_z6</th>
<th>U_z7</th>
<th>u_z8</th>
<th>u_z9</th>
<th>U_z10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro area</td>
<td>1660</td>
<td>1783</td>
<td>1733</td>
<td>1686</td>
<td>1684</td>
<td>1671</td>
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<td>1785</td>
<td>1637</td>
<td>1724</td>
</tr>
<tr>
<td>UK</td>
<td>1019</td>
<td>1000</td>
<td>1018</td>
<td>1009</td>
<td>979</td>
<td>1003</td>
<td>980</td>
<td>985</td>
<td>1004</td>
<td>994</td>
</tr>
<tr>
<td>USA</td>
<td>994</td>
<td>994</td>
<td>981</td>
<td>982</td>
<td>1007</td>
<td>1020</td>
<td>987</td>
<td>1011</td>
<td>1017</td>
<td>991</td>
</tr>
</tbody>
</table>

Table 13
Impact of stochastic technology on welfare of households (beta 0.9, 0.95, 0.90)

<table>
<thead>
<tr>
<th></th>
<th>u_z1</th>
<th>U_z2</th>
<th>u_z3</th>
<th>u_z4</th>
<th>U_z5</th>
<th>u_z6</th>
<th>U_z7</th>
<th>u_z8</th>
<th>u_z9</th>
<th>U_z10</th>
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</thead>
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<tr>
<td>Euro area</td>
<td>1660</td>
<td>1783</td>
<td>1733</td>
<td>1686</td>
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Fiscal or trade policies that affect the nature of technological shocks and the subjective discount factors of individuals were likely to have very large impacts. Current set up of the model retains net government expenses (that equals revenue net of transfer) and the export ratio as policy variables in the model. Both tax and trade policies influence the stochastic process of the economy.

**IX. Conclusion**

Dynamic economic modelling using econometric analysis and general equilibrium models generate scenarios to assess evolution of an economy. Structural parameters estimated from actual time series data in econometric models to make predications about the likely impacts of economic policies in a given horizon. These models, however, do not focus enough on the optimising behaviour of households and firms. This shortcoming in analyses is complemented by decentralised stochastic general equilibrium models that generate time series upon which various predictions of econometric analyses can be tested. It has been illustrated here how econometric and general equilibrium models of a dynamic economy can be complementary to each other.
Impacts of economic policies are evaluated applying econometric analyses and stochastic dynamic general equilibrium models for growing economies. Comparing analyses of economic structure and forecasts generated from simultaneous equation, VAR and autoregressive models based on quarterly series 1966:1 to 2007 of UK to those from the stochastic general equilibrium models has provided insights in objective and subjective analyses of underlying economic processes influenced by public policies. While estimates of econometrics models are used in objective formulation of the stochastic dynamic general equilibrium models, the time series of macro variables generated by solving the stochastic economy are employed to test the predictions of econometric analyses by calibrating ratios, variances, covariance and correlations for scientific analyses of economic policy. This paper has shown why econometric analyses and general equilibrium modelling should be considered complementary rather than competitive techniques in economic analyses.

VII. References:


