Understanding Fundamentalist Belief Through Bayesian Updating

Srijit Mishra

Indira Gandhi Institute of Development Research,
General AK Vaidya Marg (Film City Road),
Goregaon (East),
Mumbai-400 065
INDIA
Email: srijit@igidr.ac.in and srijimishra@hotmail.com
Phone: +91-22-2840 0919/20/21
Fax: +91-22-2840 2752

Abstract:
Using Bayesian updating to deterministic priors persistence of fundamentalist belief like those in the mind of a terrorist is explained. Under such belief system if conditional evidence is diametrically opposite and also deterministic then a process of change will set in and in the present war against terrorism this can be effectively done through Islamic religious authorities. In situations where interaction is the basis, self-defeating scenarios can be avoided by giving space to others’. Thus, in the political sphere one has to be accommodative about the concerns of Middle East, this will also make things easier for intervention through Islam.

Key words: Bayesian updating, fundamentalist belief, interaction, variation.

JEL Codes: A13, C11

December 2002

Acknowledgements:
The author thanks the students and apologies to them for experimenting the idea with them while teaching B08: Statistical Prerequisites during August-December 2001 at GIPE, Pune. Some of the ideas mentioned in the present paper have been sketched in “Exploring Bayes’ Rule: Some Commonsense Intuitions and a Dynamic Interpretation.” Discussions with Santanu Gupta, MP Khare, Arup Maharatna and Ajit Sinha during the formative stages and comments from Sushanta Mallick and Diganta Mukherjee on “Exploring …” were helpful.
Understanding Fundamentalist Belief Through Bayesian Updating

Srijit Mishra

“PROBABILITY DOES NOT EXIST.”
de Finetti (1970, p. x)

1 Introduction
Bayesian interpretation of probability is epistemic – a degree of belief, as against the frequentist tradition where it is a limiting ratio of a repeatable phenomena (Howie, 2002, p.1). In situations where it is difficult or impossible to obtain repeatable events like the occurrence of terrorist strike tomorrow the probability formed will be a belief about something that one is uncertain about (von Plato, 1994, p. 240). This belief in a Bayesian sense is based on prior information and conditional evidence; or, statistically one calculates posteriors that are reciprocal to priors and conditional evidence (Zellner, 1987, p. 208). In other words, prior information based hypothesis and conditional evidence can through Bayes’ rule update the prior to posterior hypothesis such that the posterior can be a new prior, which with further conditional evidence will lead to subsequent updating (Lange, 1999, p. 294). The present paper seeks to understand fundamentalist belief such as those by the perpetrators of September 11, 2001 terrorist attack through Bayesian interpretation. It considers the nature of fundamentalist belief to have a prior hypothesis that is deterministic – total disbelief, zero or total conviction, unity, and as a result posteriors will be equal to priors, and hence, deterministic – no amount of updating is going to change a fundamentalist belief. Does this mean that once a fundamentalist, always a fundamentalist? Two possible suggestions are given and a combination of these will be relevant in our present concern against terrorism.

Explaining the persistence of a fundamentalist belief is a definite strength of Bayesian analysis, as the framework is meant to analyze something that one is uncertain about. However, if there exists sufficient scientific evidence that is also deterministic but contrary to prior then there is room for posterior to be different. An illustration is given with Kepler’s use of Brahe’s data to discover planetary laws of motion that conclusively proved Copernicus’ heliocentric view (sun at the center) as against Church backed geocentric view (earth at the center) of Brahe and thus dispelled the then prevailing popular belief (Guidry, 2002A, 2002B & 2002C).

1 Usage of frequentist probability, in Baccini’s (2001) understanding of Edgeworth and Venn, cannot be of much help for choice under uncertainty; however, subjective probability in a Bayesian sense, as conceptualised by Frank Ramsey, has done remarkably well as a theory of rational decisions that led Davidson (1996) to even suggests such an approach for a theory of truth. Independent of the philosophical underpinnings of frequentist of Bayesian approach, Beebe & Papineau (1997) suggest that a prudentially correct choice is that of relative probability - subjective probability relative to knowledge set-up, as it will help one to act on, given that one wants such-and-such results.

2 In a Bayesian framework prior probability is a degree of belief about the universe, and hence, any evidence from the universe is considered as conditional to the priors. It is for this that we use the term conditional evidence as against evidence; one can also refer to conditional evidence as sample proportions. Eckhardt (1997, pp. 247-248) points out that any evidence that is not connected to priors or that is outside the domain of the universe will not be of help in Bayesian updating.
Alternatively, if one has to retain the uncertain nature of outcomes then it is proposed that the values of zero and unity remain out of bounds. Thus, ensuring scope for revision. It is based on the premise that no knowledge is final; there is always scope for change. Interestingly, such an approach is consistent with multiplicity of views, or debates arising thereof. In fact, it is these that can also ensure change – say, emergence of a new middle position that could not be visualized earlier.

After this introductory section, section 2 introduces some notations and explains Bayes’ rule. Section 3 explains impossibility of updating fundamentalist belief. Section 4 shows how under special situations there is room for fundamentalist belief to change. Section 5 discusses a possible scope for Bayesian updating that accommodates multiple views. Concluding discussions are in section 6.

2 Notations and Bayes’ Rule

Prior information based hypothesis, $h$, conditional evidence, $e|h$, and posterior hypothesis, $h|e$, have their complementary denoted with a prefix Not, $\sim$, and probability distributions, $p(\cdot)$, such that through axiom of certainty

\[ \sum p(\cdot) = 1 \]

Further, as per Bayes’ rule

\[ p(h|e) = \frac{p(e|h)p(h)}{[p(e|h)p(h) + p(e|\sim h)p(\sim h)]} \]

If we introduce the notion of time, $t=1, \ldots, T$, to incorporate a series of conditional evidences such that at each $t$ equations (1) and (2) hold, but in addition to this one further assumes that posterior at time $t$ is prior at time $t+1$.

\[ p(h|e)_t = p(h)_{t+1} \]

3 Persistence of Fundamentalist Belief

One considers belief to be persistent if the posterior is the same as the prior, or, in other words conditional evidence has no impact on changing the belief. This happens in two situations: when conditional evidences of mutually exclusive scenarios are equally likely; in equation (2) this can be the case when $p(e|h) = p(e|\sim h) = 0.5$. This is so because conditional evidences give equal weight to all possible scenarios and as a result the weights that matter are that of the priors only. For instance, if after scientifically examining shape, size and structure of a coin the priors that the toss of a particular type of coin will be heads is 0.8 and that of tails is 0.2. However, after ten throws one has conditional evidence that getting heads or tails is equally likely. Calculating posteriors using Bayes’ rule will show that they will be the same as priors: heads 0.8 and tails 0.2. Now, let us go to the second situation, as it is relevant for the present discussion.

The second situation where posterior is equal to prior is when prior is based on fundamentalist belief, that is, prior is deterministic and takes the values of zero or unity.

---

There is no scope for doubt – there can be no intermediary position. If the hypothesis has prior as zero, \( p(h) = 0 \), then the numerator in equation (2) because of the second term is zero, and hence, \( p(h|e) = 0 \). Similarly, if hypothesis has prior as unity, \( p(h) = 1 \), then from equation (1) it follows that \( p(-h) = 0 \) and as a result the second term in the denominator of equation (2) is zero, note that the remaining part of denominator is equal to the numerator, thus, \( p(h|e) = 1 \). In both the cases when prior is a fundamentalist belief then the posterior also retains the fundamentalist belief.

While deriving the above result a basic tenet of Bayesian inference has been violated, that is, probabilities are subjective, and hence, cannot be deterministic. This is a very valid point, but it turns out that this violation is our strength. What it implies is that if one begins with a fundamentalist belief or one is dogmatic then there is no scope for revision. In other words, if one believes that a particular thing is true then any amount of conditional evidence that is probabilistic in nature will not cut much ice. This has two implications: first, if truth has been ascertained and the prior is deterministic then looking for further conditional evidences to prove or disprove can be a futile exercise that should be avoided; second, if one is dogmatic then it will act as hindrance and all conditional evidences become redundant, thus, growth of knowledge is the casualty. Either way the case for deterministic prior is best done away with, as taken up in section 5, but for the present purpose, sections 3 and 4, it is these implications that are relevant.

3.1 Two Examples

We will discuss the persistence of fundamentalist belief with two examples. The first one is taken from Tycho Brahe’s observations on planetary movements. Brahe (1546-1601) who...

“... was both a theorist and an observer. As a theorist, he believed to his last day in the year 1601 that the planet went round the sun and that the sun and the stars went round the earth as the fixed centre of the universe. As an observer, he made with infinite patience and integrity thousands of records of the stars and planets; upon these records Kepler, in due course, based his laws and brought the truth to light” (Beveridge, 1937, p. 478).

The point that one would like to make here is that Tycho Brahe was believer of the geocentric universe of Aristotle and Ptolemy till his death. But, despite being wrong in his theories he was honest and careful in his observations (Beveridge, 1937, p. 479). His observations in the pre-telescopic era were remarkable and probably made him ask relevant questions and subsequently proposed an alternative that was in between Ptolemaic and Copernican model but geocentric that was most widely accepted. However, he erred while rejecting Nicolai Copernicus’ (1473-1543) heliocentric system published in *On the Revolutions of the Heavenly Bodies* when he lay on his deathbed; the work was dedicated to the Pope but did not find favor with the Church (Guidry, 2002A & 2002B). It is possible that Brahe’s belief in the geocentric system prevented him from rigorously pursuing the observations that he collected, and thus, could not gain insights from the data. Interestingly enough, Johannes Kepler (1571-1630) who believed in the Copernican system used Brahe’s data to obtain the results that he did, we will elaborate on this in section 4.1.

Our second example is based on the recent happening of September 11, 2001 at World Trade Center, New York. One argues that perpetrators of this and other terrorist attack have a fundamentalist belief, that is, they have totalistic ideology (Morgan, 2001B). “The thinking of the terrorist evidently shows some of the cognitive faults or distortions observed in others...
who engage in violent acts, either solely as individuals or as members of a group. These include overgeneralization - that is, the supposed sins of the Enemy may spread to encompass the entire population. Also, they show dichotomous thinking ... either totally good or totally bad. Finally, they demonstrate tunnel vision - once they are engaged in their holy mission (e.g., jihad), the only objective they can envision is the destruction of the target” (Beck, c.2001/02).

The perpetrators consider themselves as victims and the enemy as oppressor and they are fighting for a good cause against evil for which they can even lay down their lives. Justification in the total conviction and belief might be political or guided by a particular interpretation of religion. Similarly, based on the fact that all the terrorist involved in September 11, 2001 are Muslims and that they take inspiration from a particular interpretation of Islam does not mean that Islam as a religion is responsible for the emergence of terrorism (Morgan, 2001A), in fact, as we will elaborate in the next section, Islam can be of help in changing the mind of the terrorist. However, existence of a fundamentalist belief means that prior is deterministic (say, unity), and hence, further arguments or conditional evidences are redundant and will have no impact on posterior. This takes us to the question posed in the first paragraph: Does this mean that once a fundamentalist, always a fundamentalist? The answer is No. Two different proposals emerge from some further discussion of Bayesian updating. A combination of these two, along with other proposals not mentioned here, will be relevant in our present concern against terrorism.

4. A Way Out: Conditional Evidence is Complementary to Prior

In the discussion in section 3, as per Bayesian tradition, it was implicitly assumed that conditional evidence is not deterministic, 0<p(e|h)<1, it can not take the value of zero or unity. However, for the discussion in this section we waive that assumption and consider a situation where p(e|h) is not only deterministic but also diametrically opposite to p(h), that is, if p(h)=1 then p(e|h)=0; and if p(h)=0 then p(e|h)=1. Now, if one puts these values in equation (2) then the denominator is zero, and hence, p(h|e)=undefined.4

One can interpret this result to explain situations where scientific experiments give results that dispel conventional belief. An undefined posterior could mean that things are not clear and it can be such that posteriors can have any value between zero and unity, 0≤p(h|e)≤1. Or, in other words it could mean that some people still believe the conventional orthodoxy, others go by the new result and yet others whose degree of belief is somewhere in between. This interpretation is like Nash’s mass-action interpretation (Kuhn et al., 1995). Further, from the mass-action interpretation it can be said that if there are a series of other experiments that also conclusively go against the conventional orthodoxy then there will be a shift in the posterior probability distribution towards the new finding: p(h|e)→ p(e|h), as t→T>2; this should happen at t=2 because as per mass-action interpretation p(h|e)>0 at t=1. However, in real life people may consider a series of different experiments as a single conditional evidence, and hence, there will be a time gap when the new idea is accepted across the whole population.5

4 It is undefined because it is in a position of conflict like Nelkin’s (2000) two standpoints where, on the one hand, one believes in laws of nature and in that sense is deterministic, and on the other hand, one also believes in freedom and in that sense is for free choice.

5 This is similar to stating, as Kandori, Mailath and Rob (1993, p. 31) point out, that uncertainties can lead to inertia.
Let us elaborate on this with our two examples of Tycho Brahe and perpetrators of September 11, 2001 discussed in section 3.1.

4.1 The Two Examples Again

Tycho Brahe had appointed Johannes Kepler and the two had differences of opinion because the latter believed in the Copernican system. Brahe’s distrust of Kepler and fear that the latter might use his data led him to restrict Kepler to work on the troublesome data of planet Mars. It so happened that it is this data that helped Kepler derive the three planetary laws of motion that conclusively proved the heliocentric system (Guidry, 2002C). Thus, Kepler’s result in a Bayesian sense is conditional evidence that is diametrically opposite to the prior of geocentric system. For the larger population, this would have made the posteriors undefined, that is, some people would accept Kepler’s result, others would have continued with the old orthodox view and yet others who would have some degree of belief in between the two. In the initial days larger number of people would have either believed or their degree of belief would have been closer to accepting the geocentric system. However, as more and more scientists would have confirmed Kepler’s results then there was a shift to the heliocentric system and today we hardly have anybody who will have the belief that the universe is geocentric.

In a similar vein one can argue that if the perpetrators of terrorism are conclusively shown that their thinking is wrong then contradictory claims would make it chaotic, but would initiate a process of change. One feels that this can be done through Islam. As mentioned earlier, the perpetrators of September 11, 2001 and organizations like Al Qaida are guided by a particular interpretation of Islam. Some of their fatwas mentioned in Morgan (2002) are as follows: “All these crimes and sins committed by Americans are a clear declaration of war on God, his messenger, and Muslims.” “The ruling to kill the Americans and their allies – civilian and military – is an individual duty for every Muslim, who can do it in any country it is possible to do it.” “To comply with God’s order to kill Americans, plunder their money…We call on all Muslim. ... clerics, youths and soldiers to launch the raid on Satan’s US troops and the Devil’s supporters allying with them.” “Terrorizing the American occupiers is a religious and logical obligation.”

In contrast to the fatwas Morgan (2001B) gives some quotations from the Qur’an that are as follows: “If you’ve killed one innocent person it’s as if you’ve killed all of humanity.” “Among the major sins of Islam is suicide. These are listed as ‘enormities’ in Reliance of the Traveller, a manual of Sharia in the tradition of Imam Shafi’i.” “Whoever commits suicide with piece of iron will be punished with the same piece of iron in the Hell Fire…He who commits suicide by throttling shall keep on throttling himself in the Hell Fire (forever) and he who commits suicide by stabbing himself shall keep on stabbing himself in the Hell-Fire. (Narrated Jundab the Prophet Sahih Bukhari 2.445 and 2.446).” “Nor take life - which Allah has made sacred - except for just cause. And if anyone is slain wrongfully, we have given his heir authority (to demand retaliation or to forgive): but let him not exceed bounds in the matter of taking life (Qur’an, 17:33).” “Fight in the cause of Allah with those who fight with you, and do not exceed the limits, surely Allah does not love those who exceed the limits (Qur’an, 2:190-192).” “We must treat the enemy civilians and the innocents with kindness.”

It is appropriate to consider that the above quotations from Qur’an can act as perfect counter to the Al Qaida’s interpretation of Islam. An effort in this direction from the religious authorities of Islam would be effective because the perpetrators belief is identified with a
fight for their religious cause. It is only through more authentic and authoritative interpretations of Islam that one can replace the erroneous, regressive and conservative interpretation of organizations like Al Qaida. This is similar to the Church’s role in replacing the once popular geocentric view with that of the heliocentric view.

However, this approach should be considered only after addressing some of the other conditions that make Al Qaida’s fatwas widely acceptable. Here one is referring to the political dimension or the reference in the fatwas to the act of Americans and its allies. Religion is invoked to fight against some political injustice. In other words, the fatwas and preaching of Qur’an that look contradictory can be held together, as Martin (2002) while analysing consensus and tightness of beliefs in communities of smaller groups (5-40 members) suggests, through the same domains of authority via webs of implications. Thus, if the fight is for a religious cause (jihad) then dying for this cause (not suicide) and in the process killing the enemy becomes acceptable. It may be reiterated that the identification of the enemy is not just religious, but it is also political.

The relevance of a political dimension comes out clearly in Wanniski (2001) who gives an extract of Ramzi Ahmed Yousef statement to court in his trial for bombing the World Trade Centre on February 26, 1993 where six people died, Yousef was sentenced for life imprisonment on January 8, 1998. Yousef’s statement refers to American action in bombing Japan, war with Vietnam, and more particularly its embargo on Iraq and Cuba. In this context he has no regrets in being called a terrorist. He says: “Yes, I am a terrorist and I am proud of it. And I support terrorism so long as it was against the United States Government and against Israel, because you are more than terrorists; you are the one who invented terrorism and using it every day. You are butchers, liars and hypocrites.” It is this political dimension that will be taken up in the next section by putting a restriction that prior cannot be deterministic.

5 Prior cannot be Deterministic: Giving Space to Others
In section 3 the discussion more or less assumed that probability can be deterministic – zero or unity, somewhat contrary to Bayesian philosophy that is based on subjective probability. We now consider that prior, as also conditional evidence, and posterior can be infinitesimally close but will not be equal to zero or unity,

\[0+\varepsilon \leq p(\cdot) \leq 1-\varepsilon\]

where \(\varepsilon\) is infinitesimally small, but greater than zero, or, one can also further restrict the boundary such that \(\varepsilon\) can be little larger, \(0<\varepsilon<0.5\).

Now, if one gets conditional evidence that is contrary to prior then a series of such experiments will bring out a change in posterior in the direction of conditional evidence but can never attain a value that is zero or unity. In other words, if \(p(h) > 0.5\) and \(p(e|h) > 0.5\) then \(p(h|e) \rightarrow 1\) as \(t \rightarrow T = \infty\), note that by equation (4) \(p(h|e) \neq 1\) indicating that the posterior tends towards unity in a limiting sense but it will never be equal to it.

---

6 For deviation of probability from being deterministic by infinitesimal values and its relevance to calculation of posteriors see Arló-Costa (2001).
In equation (4), particularly when $\varepsilon$ is not infinitesimally small the conditional evidence will also be constrained by the limits possible to posterior, or in other words the limits to posterior is itself a part of the prior information. Such restriction of equations (4), for the context under discussion, is akin to acceptance of multiple views – they cannot be dismissed outright.

To elaborate, if Bayesian updating is like a Darwinian evolutionary process of natural selection then principle of heredity (off springs have to resemble their parents more than other members of their species) and the principle of the struggle for existence (better-adapted individuals have more number of off springs) can be explained by replication dynamics. This might ignore the principle of variation or diversity in the population, a necessary prerequisite in the process of natural selection. In other words, if interaction between sub-groups is necessary then a process where all the other sub-groups perish can be self-defeating. Thus, continuity requires that there should be perturbation at some period that at least prevents some, if not all, sub-groups from not perishing in the process of interaction.

Accepting multiple views warrants that we give space to others. In the present concern against terrorism it means understanding the concerns of Middle East. This also means appreciating their point of view or grievances. For instance, the impact of depleted Uranium used in Gulf War of 1991 on the rise of cancer and birth defects in Iraq (Johnson, 2002).

---

7 This can be because of perturbation or random disturbances. We consider this randomness, following Kumar (2000), to be an outcome of the privileged information of various agents including that of nature.
8 Replicator dynamics has been explained in Börgers (1996, p. 1382), as "a gradual process in which strategies which perform worse than the average become less frequent and strategies which perform better than the average become more frequent."
9 Mukherjee, White and Wuyts (1998, p. 3) quote Gould (1996, p. 40) to suggest that: “we are still suffering from a legacy as old as Plato, a tendency to abstract a single ideal or average as the ‘essence’ of a system, and to devalue or ignore, variation among the individuals that constitute the whole population.”
10 This interpretation of the principles of natural selection involving heredity, struggle for existence, and variation has been based on an understanding of Gould (1991) and Visvanathan (1996).
11 Krieger (1995) suggests that the probability of such self-defeating scenarios (that is, doom) is either zero or unity through an application of Kolmogorov probability theory for scenarios with lower tails in their distribution. There are instances where birth rank is used as Bayesian conditional to show that the probability of doomsday is zero, but Eckhardt (1997) shows that it is an improper use of evidence as conditional evidence (also see note 2 above).
12 This can be because of randomness arising out of privileged information (see note 7). In a Darwinian sense this can be as a result of mutation and it is this that ensures variation (see discussions that led to notes 9 and 10). Thus, it is not necessary for $p(e|h) > 0.5$, it can also be the case that $p(e|h) < 0.5$. By restricting the boundary of $p(\cdot)$, as in equation (4), one ensures a minimum variation necessary for interaction and such restrictions, one feels, will also constrain the Kandori, Mailath & Rob (1993) long-run equilibria.
13 On denial of clemency to Mir Aimal Kasi one lost another opportunity of showing concern and winning a million hearts in the fight against terrorism. Note that Mr Kasi, a Pakistani citizen, was executed with lethal injection on November 14, 2002 for having killed two
This not only requires an initiation of dialogue and conversation at multiple levels such as government-to-government and people-to-people but an involvement in positive action. Further, note that conversations also have an ability to change beliefs and a reliance on conversational stance necessitates that people consider each other as responsible and free believers and desirers (Pettit & Smith, 1996). Moves in this direction will give a lot of strength to the Islamic religious authorities in countering the wrong invoking of Islam in the fatwas. We have further elaborated on this in the conclusions.

6 Conclusions
In this paper we try to understand fundamentalist belief through Bayesian updating, that is, when priors are deterministic. Doing away with a basic Bayesian philosophy of subjective probability turns out to be the strength in the present discussion because it can also explain persistence of fundamentalist belief. This result is used to explain why Brahe fails to go beyond geocentric consideration whereas Kepler using Brahe’s data conclusively proves the heliocentric view. The role of belief including experiences and views of correct scientific inquiry has also influenced research in probability: Sir Ronald A. Fisher concerned with reduction and evaluation of experimental data believed and developed tools and methods as a frequentist, whereas Sir Harold Jeffreys who tried to calculate the internal structure of the earth was aware that conclusions will always be uncertain made important contributions as a Bayesian (Howie, 2002). Belief (not necessarily fundamentalist) but at least in a subjective sense has also had an impact on many great scientists (Press and Tanur, 2001).

As Kepler’s result led to a shift in popular belief from geocentric to heliocentric view of universe, one feels that the involvement of Islamic religious authorities can act as an effective counter to the terrorist organizations who issue fatwas by invoking Islam. However, to create ground for the effective use of religion one has to take care of another condition that affects the fundamentalist mind – the political dimension in the fatwas.

To analyse this we make use of the conventional understanding of subjective probability, that is, it lies between zero and unity. One can also suggest a higher lower bound and a lower upper bound depending upon the context and this can be considered as part of the prior. This can be interpreted in social situations as one where multiple scenarios co-exist and form the basis for interaction. The role of interaction can be gauged from the fact that Hindu-Muslim conflicts in India are prevalent in those cities where associational interactions between them are absent (Varshney, 2002).

Having identified the relevance of interaction in the political dimension it suggests that the world ought to be accommodative and appreciative about the views of Middle East. If Islam has to play a positive role in countering the role for fatwa then conditions from outside should make it convenient. This requires a respect and appreciation for the other viewpoints. This approach, however, does not deny difference of opinion but rather consider this necessary for discussions and debates, a basic requirement in a civil society.

We do agree that the above two approaches proposed to be used in conjunction with each other do not exhaust all possible suggestions of tackling terrorism. This is consistent to our discussions using Bayesian updating in two ways. First, we are not dogmatic of our proposals

Central Intelligence Agency (CIA) employees and wounding three others on January 5 1993 in front of CIA headquarters Virginia (Glod & Weiss, 2002).
and agree that these can only form a small part in our concern against terrorism. Second, by considering the existence of other possible methods we leave scope for multiple views. In fact, to tackle such a serious global concern the fight has to be in multiple directions.

References:


Beveridge, Sir W. (1937). The Place of the Social Science in Human Knowledge, Politica. 2(9), 459-479.


