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Srijit Mishra and Hippu Salk Kristle Nathan

Indira Gandhi Institute of Development Research, Mumbai
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Srijit Mishra and Hippu Salk Kristle Nathan

Indira Gandhi Institute of Development Research (IGIDR)
General Arun Kumar Vaidya Marg
Goregaon (E), Mumbai- 400065, INDIA
Emails: srijit@igidr.ac.in and hnathan@igidr.ac.in

Abstract: *Using Minkowski distance function we propose a class of Human Development Index measures. Special cases of this turn out to be the popularly used linear average method as also a newly proposed displaced ideal method. Two measures of penalty are suggested. One captures the non-uniform attainment across dimensions and the other captures the deviation from the ideal path. A method of adjusting for unequal weights is also provided.*

Key Words: Ideal path, Penalty, Minkowski distance function, Multiple dimensions, Uniform development

JEL Codes: D63, I31, O15

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1. Introduction

The Human Development Index (HDI) is not the same as the larger human development approach where the focus is on enhancing freedoms in all its dimensions. Nevertheless, it has been successful in taking the discourse from a one-dimensional income-based measure to a three-dimensional measure based on education, health and income.[2] The conventional measure reported in the annual *Human Development Reports* is a linear averaging of the three dimensions. We refer to this as HDI_1. In a recent paper, Nathan, Mishra and Reddy (2008) propose an alternative measure by taking the inverse of the Euclidian distance from the ideal and following Zeleny (1974) refer to this as the displaced ideal method, HDI_2. In this paper, we propose a class of measures, which can be adjusted for weights across dimensions, where both the above mentioned methods turn out to be special cases of the normalized Minkowski distance function.[3] Keeping the notion of uniform progress across all dimensions in mind, as it emphasises on the intrinsic importance of each dimension, we suggest a measure of position penalty to capture deviation from this uniformity. The path joining any given position with the ideal point gives the ideal path, which should also serve as a basis for signalling the future course of action. Deviation from this ideal path is captured through a second measure, which we refer to as path penalty. An empirical example for selected countries is given.

2. The Measure

We propose an α-class of measures

$$M_\alpha = 1 - D_{\alpha I} \quad (1)$$

where

[2] For some discussion on methodology and measurement, birth, critique and evolution of the HDI see Anand and Sen (2003), Haq (2003), Jehan (2003) and Rawworth and Stewart (2003) among others.
[3] In a recent paper Subramanian (2004) has used the Minkowski distance function to the Foster, Greer and Thorbecke (1984) class of poverty measures. Mishra (2005) has also used it in a discussion on secluded and proximate illiteracy.

$$D_{\alpha I}=(1/n\sum(1-x_j)^{\alpha})^{1/\alpha} \qquad ;j=1,\ldots,n \qquad (2)$$

is the normalized Minkowski distance function of order α calculated from the ideal, *I*, where x_j referrers to the normalized indices for *n* dimensions such that at the ideal $x_j=1\ \forall\ j$. In HDI calculations, there are three dimensions; namely, health, education and income.

If $\alpha=1$ and $n=3$ then

$$M_1=(x_1+x_2+x_3)/3 \qquad (3)$$

which is the same as HDI_1. Similarly, if $\alpha=2$ and $n=3$ then

$$M_2=1-\sqrt{((1/3)((1-x_1)^2+(1-x_2)^2+(1-x_3)^2))} \qquad (4)$$

where $\sqrt{((1-x_1)^2+(1-x_2)^2+(1-x_3)^2)}$ is the Euclidian distance from the ideal and dividing with $\sqrt{3}$ normalizes it in the three-dimensional space and then subtracting this from unity gives the inverse of the shortfall from the ideal. This is the same as HDI_2. In Figure 1, which depicts a two-dimensional situation, it is represented as $1-D_{2I}$. Higher orders of α can give some further measures of HDI.

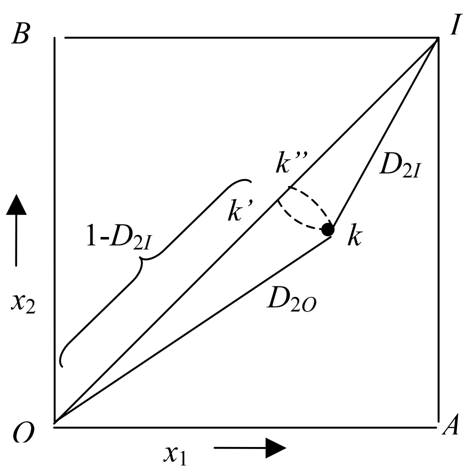

Figure 1: Measurement of Attainment, α=2

At the origin, O, $x_j=0\ \forall\ j$ and the corresponding normalized Minkowski distance can be computed with

$$D_{\alpha O}=(1/n^{1/\alpha})(\sum x_j^{\alpha})^{1/\alpha} \qquad ;j=1,\ldots,n. \tag{5}$$

For $\alpha=1$, $D_{1I}+D_{1O}=1$ and $D_{1O}=\mathrm{HDI}_1$. However, for $\alpha\geq 2$, $D_{\alpha I}+D_{\alpha O}\geq 1$; it is equal to unity on the line of equality only. In other words, $D_{\alpha O}$, as a measure of attainment, could be rewarding movements away from the ideal, $D_{\alpha O}>1-D_{2I}$ in Figure 1.

The discussion so far has assumed equal weight across multiple dimensions, $w_j=1/n$. For unequal weights like the calculation of education index, one of the components used for calculating HDI, is a combination of adult literacy and enrolment ratio, the normalized weight adjusted distance from ideal can be calculated using

$${}_wD_{\alpha I}=(1/(\sum w_j^{\alpha})\sum(w_j(1-x_j))^{\alpha})^{1/\alpha} \qquad ;j=1,\ldots,n. \tag{6}$$

3. Position Penalty: Measure of Deviation from the Line of Equality

Given attainments in the individual dimensions, uniformity across dimensions can be indicated by the mean, $\mu=(\sum x_i)/n$. If we refer to this as the local ideal position then the locus of all such positions is the line of equality, which can be obtained by joining the origin and the ideal in the n-dimensional space. Any deviation from this line would be considered as a move away from uniformity. It not only means that to attain the current position a greater distance was covered than the corresponding ideal position, $Ok>Ok'$, but it also means that a greater distance has to be covered to reach the ideal point, $kI>k''I$ (see Figure 1). To capture this deviation, that is, the excess distance of $k'k''$, we propose a measure of position penalty

$$P_{\alpha}=D_{\alpha I}+D_{\alpha O}-1. \tag{7}$$

Note that $P_{\alpha}\in(0, \max(P_{\alpha}))$. When n is even then $\max(P_{\alpha})=((2^{1-(1/\alpha)})-1)$, but when n is odd then we get this value in the limiting sense only, that is, $\max(P_{\alpha})=(((((n-1)/2)^{(1/\alpha)}+((n+1)/2)^{(1/\alpha)})/(n^{(1/\alpha)}))-1)\rightarrow((2^{1-(1/\alpha)})-1)$ as $n\rightarrow\infty$. For HDI with $n=3$ the upper bound is $((1^{1/\alpha}+2^{1/\alpha})/3^{1/\alpha}\ )-1)$.

It is easy to show that $\max(P_\alpha)$ is an increasing function of α, $\max(P_\alpha)=0$ for $\alpha=1$ and $\max(P_\alpha)=1$ for $\alpha=\infty$. Thus, as α increases from unity to infinity we move from no penalty to full penalty; that is, we move from a measure that allows for perfect substitution to one that allows no substitution across dimensions (Figure 2).[4] This means that an HDI measure calculated on a higher order of α would indicate a greater punishment for non-uniform development.

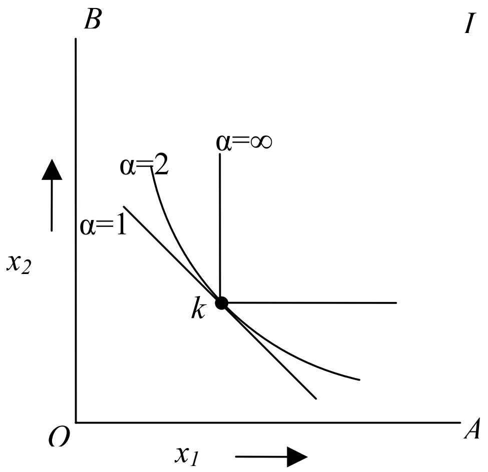

Figure 2: Substitution curves for different *α*

Thus, a measure of normalized positional penalty is

$$_{Nor}P_\alpha = P_\alpha/\max(P_\alpha). \quad (8)$$

Some other measures are discussed in Appendix 1.

4. Path Penalty: Measure of Deviation from Ideal Path

Given a position, the minimum distance for maximum attainment is through the ideal path. Path penalty captures the deviation from this. Unlike position penalty, it involves comparison of the path between two positions with that of the ideal path when computed from the initial position. As shown in Figure 4, from position *k* the movement to the ideal point, *I*, is minimized by the ideal path *kI*. Moving in any other path, say to *l* instead of *l'*, will make the entity cover more distance to reach the ideal point. For such a deviation, a measure of path penalty is

[4] The similarity with constant elasticity of substitution (CES) production function is obvious.

$$Q_{\alpha kl}=(D_{\alpha kl}+D_{\alpha l'I})-D_{\alpha kI}. \qquad (9)$$

The maximum path penalty under equation (9) would be for $Q_{\alpha AI}=(D_{\alpha AO}+D_{\alpha OI})-D_{\alpha AI}=1$, as $D_{\alpha AO}=D_{\alpha AI}$ and $D_{\alpha OI}=1$. Hence, the path penalty measure can be considered as a normalized one, $Q_{\alpha kl}\equiv {}_{Nor}Q_{\alpha kl}$.

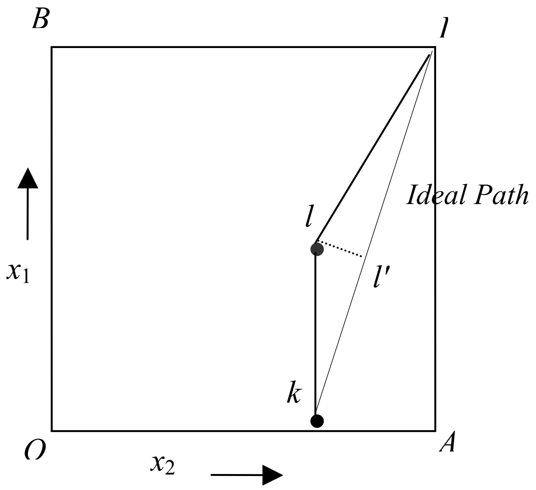

Figure 3: Path penalty

A second measure of path penalty is discussed in Appendix 2.

4. An Empirical Example

We make use of indicator values with regard to the dimensions of education, health and income to compute HDI_α (α=1,2,3), respective ranks, R_α, across 177 countries, change in ranks, and for higher order HDIs the deviation from the line of equality, ${}_{Nor}P_\alpha$, and the deviation from the ideal path, ${}_{Nor}Q_\alpha$. The reference year is 2004 and for ${}_{Nor}Q_\alpha$ it also uses 2000 as the base year. Our calculations for HDI_2 and HDI_3, as compared to HDI_1, indicate that the number of high human development countries ($HDI_\alpha \geq 0.8$) reduces from 63 to 56, the number of medium human development countries ($HDI_\alpha \geq 0.5$, but less than 0.8) increases from 83 to 84 and the number of low human development countries ($HDI_\alpha < 0.5$) increases from 31 to 37 countries.

Results for some selected countries are given in Table 1. Botswana and Swaziland, both from Southern part of Sub-Saharan Africa reeling under a human immunodeficiency virus/acquired immune deficiency syndrome (HIV/AIDS) epidemic, are penalized for their poor performance in longevity and income dimensions. Kazakhstan, as also some other countries of the erstwhile Soviet Union, is also not doing well in the longevity and income domain. The intriguing part is the downfall of Ireland and United States who despite high income and educational attainment have a very poor health record and have been rightly penalized. In contrast, the uniform development across dimensions has rewarded Israel and Italy. Besides Israel, some medium human development Middle East countries like Saudi Arabia, The Islamic Republic of Iran and Turkey have also done reasonably well.

Table 1: Different Measures of HDI, Their Ranks, and Penalties in Selected Countries, 2004

Country Name	HDI_1	HDI_2	HDI_3	R_1	R_2	R_3	R_1-R_2	R_1-R_3	${}_{Nor}P_2$	${}_{Nor}P_3$	${}_{Nor}Q_2$	${}_{Nor}Q_3$
Botswana	0.570	0.485	0.414	131	148	159	-17	-28	0.3992	0.4074	0.0767	0.0968
Swaziland	0.500	0.429	0.365	146	161	168	-15	-22	0.3823	0.3874	0.0620	0.0765
Kazakhstan	0.774	0.735	0.716	79	96	99	-17	-20	0.1235	0.1185	0.0083	0.0123
Ireland	0.956	0.932	0.918	4	14	20	-10	-16	0.0646	0.0685	0.0051	0.0069
United States	0.948	0.926	0.913	8	19	23	-11	-15	0.0590	0.0628	0.0015	0.0016
Israel	0.927	0.926	0.924	23	20	14	3	9	0.0039	0.0051	0.0038	0.0037
Italy	0.940	0.936	0.934	17	9	8	8	9	0.0037	0.0042	0.0024	0.0046
Saudi Arabia	0.777	0.779	0.781	76	68	62	8	14	0.0088	0.0082	0.0003	0.0004
Iran	0.746	0.747	0.747	96	85	79	11	17	0.0024	0.0032	0.0002	0.0005
Turkey	0.757	0.756	0.754	92	80	74	12	18	0.0132	0.0175	0.0002	0.0002

Note and Source: HDI_α is the Human Development Index computed with Minkowski distance function of order α where R_α are their respective ranks across 177 countries, P_α is the penalty depicting deviation from the line of equality and Q_α is the penalty depicting deviation from the ideal path. For higher order HDIs, the education index computed using adult literacy and gross enrolment was also based on the appropriate weighted Minkowski distance function. The reference year is 2004 and for Q_α it also uses 2000 as the base year. Calculations are based on comparable time series data obtained from Human Development Report Office through personal communication.

5. Concluding Remarks

We have used the Minkowski distance function to propose a class of Human Development Index (HDI) measures, which can also be adjusted for weights. Special cases of this turn out to be the popularly used linear average method as also a newly proposed displaced ideal (Euclidian) method. Two measures of penalty are also suggested. Keeping the intrinsic importance of each dimension in mind, one measure of penalty captures the deviation from uniform development across dimensions. This increases as the order of the distance function increases. The linear average method, which is the lowest order of the Minkowski distance function, does not provide any signal for future course of action. As against this, higher order distance functions do indicate an ideal path for obtaining a higher value of HDI. A second measure of penalty indicates deviation from the ideal path. Of course, this ideal path is merely technical and future research has to incorporate cost and other aspects relevant for public policy. An empirical example using 2004 data indicates how countries like Botswana and Swaziland in Southern Africa, Kazakhstan from the erstwhile Soviet Union, Ireland and the United States are penalized for non-uniform development whereas Italy, Israel and other Middle East countries like Iran, Saudi Arabia and Turkey are rewarded for uniform development. The class of measures can also be used to calculate the Gender Development Index (GDI) or any composite index weighted across multiple dimensions. The distance from the ideal can also be used, as suggested by Kumar, Holla and Guha (2008), to obtain a single measure for multiple deprivations in the consumption of education, health and other necessities calculated through an Engel curve analysis.

Appendix 1: Two Other Measures of Position Penalty

A second measure of position penalty is to take the deviation from the line of equality as a proportion of the maximum possible deviation for that mean, μ. In Figure 4, it can be denoted by $kk'/k'k''$ and in n-dimensional space this normalized measure is

$$_{Nor1}G_\alpha = G_\alpha / \max(G_{\alpha|\mu}) \tag{10}$$

where $G_\alpha = ((\sum|x_i - \mu|^\alpha)/n)^{(1/\alpha)}$, $\max(G_{\alpha|\mu}) = ((r|1-\mu|^\alpha + |\sum x_i - r - \mu|^\alpha + (n-r-1)\mu^\alpha)/n)^{(1/\alpha)}$, $\mu = (\sum x_i)/n$, and r is the greatest integer less than or equal to $\sum x_i$. Proof for $\max(G_{\alpha|\mu})$ is that given μ the most non uniform distribution will correspond to minimum dimensions having maximum attainment or maximum dimensions getting zero attainments.

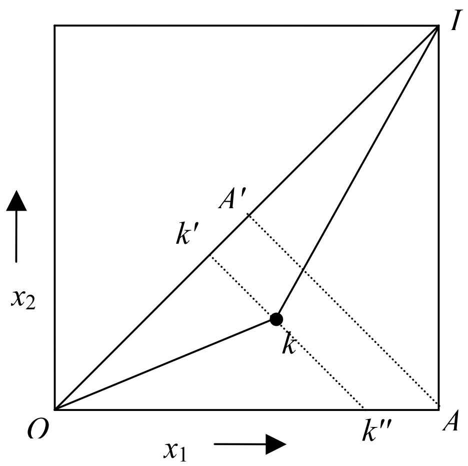

Figure 4: Deviation from the line of equality

A third measure is to take the deviation from the line of equality as a proportion of the maximum possible deviation. In Figure 4, it can be denoted by *kk'*/*AA'*, which is equivalent to $\Delta OkI/\Delta OAI$ and its similarity to the Gini coefficient, a popular measure of inequality, is obvious. In *n*-dimensional space this normalized measure of distance is

$$_{Nor2}G_{\alpha}=G_{\alpha}/\max(G_{\alpha}) \qquad (11)$$

where G_{α} is as in equation 8 and $\max(G_{\alpha})$ is 1/2 when *n* is even and $((((n\text{-}1)(n+1)^{\alpha}+(n\text{-}1)^{\alpha}(n+1))/2n)^{(1/\alpha)})/2n$ when *n* is odd; in the limiting case as $n\rightarrow\infty$ both values coincide.

Appendix 2: A Second Measure of Path Penalty

A second measure of path penalty captures the deviation by calculating the distance between the new position and a corresponding position in the ideal path, $D_{ll'}$ (see Figure 3). Note that point *l'* cuts the line *kI* in proportion to the distances $D_{\alpha kl'}$ and $D_{\alpha Il'}$. And hence, the coordinate values will follow the same proportion. Now, given the first position, $k=(x_{1k},x_{2k},\ldots,x_{nk})$, and the subsequent position, $l=(x_{1l},x_{2l},\ldots,x_{nl})$, the normalized expression of the second measure of path penalty is

$$_{Nor}H_{\alpha kl} = (\sum|x_{ik}+(1\text{-}\ x_{ik})(D_{\alpha kI}\text{-}\ D_{\alpha Il})/D_{\alpha kI}\text{-}x_{il}|^{\alpha})^{1/\alpha}/2n^{1/\alpha}) \qquad (12)$$

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