Negotiating Constitution for Political Unions

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Abstract

This paper provides a cradle-to-grave model for political union between two unequally endowed states. We introduce negotiated, contested, and time-consistent contested constitutions to address various classes of merger problems. Merger agreement is shown to be path dependent and, in some cases, time inconsistent. The possibility of contest constrains the set of mutually agreeable tax rates and provides stability to a constitution. Demographic heterogeneity constrains the set of mutually agreeable mergers. Rent extracted by technologically advanced province for transferring technology to the backward province in a union is shown to be increasing in complexity of technology but bounded from above. The model can also support the possibility of historical cycles of political geography. The main contribution of this paper is to highlight the role of technology gap and unequal distribution of resources in all the above cases.

Key words: Bargaining, Constitution, Contest, Political Union

JEL Code(s): C72, C78, D02, D72, D74, F51, K39

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1.0 Introduction

Constitution is a commitment device which helps groups with conflicting interests within a state to commit to a scheme for distribution of power, which is underwritten by the actual distribution of means to power within the polity. A constitution can also be treated as a convention, a self-enforcing equilibrium under which everyone is better-off by conforming as long as all others do so.² Once the underlying distribution of means to power in the polity changes some players would be better-off by deviating from the behaviour prescribed by the convention and others would be obliged to follow the lead. A constitution is thus unlikely to outlive the socio-economic conditions under which it is a stable equilibrium and should thus provide for its own amendment. There are only a few occasions when a state might face constitutional quandaries. The first and foremost occasion comes when a state faces major political changes, e.g., regime change (abolition of monarchy, independence from foreign rule, and transition to/from communism/military dictatorship), territorial change (partition, union with another state), etc. The other occasion when even an otherwise stable state might face constitutional crisis is when the socio-economic structure of the polity changes, e.g., discovery of valuable mineral resources in some province, introduction of new technology benefiting only an ethnic and/or territorial fraction of the state, demographic change in favour of some group, etc. Unfavorable outcome in international wars might also lead to a constitutional crisis. In some countries constitutions have been helpful in handling such developments (e.g., Belgium, Canada, India, Spain, United Kingdom, etc) whereas in others constitutions have been part of the problem (e.g., Lebanon, Pakistan, etc).

We are interested in the case involving political union between two independent states. Incidentally over the last two centuries disagreement over distribution of political power and/or resources torpedoed almost every non-coercive attempt to form a union out of sovereign states in the Third World. In this paper we will try to address the question of optimal constitutional design in the run-up to formation of political unions. In the process we will examine issues like path dependence, time inconsistency, and relation between constitution and contest.

² A self-enforcing equilibrium ‘is called a convention only if one can imagine a different rule that would also be self-enforcing’ (Little 2002: 94-95).
Before proceeding it would be worthwhile to see how this paper relates to existing research. To be precise this paper relates to five lines of research. First of all it relates to the literature on game-theoretic modeling of constitutions. There is a very large literature focusing on economic implications, especially for fiscal policy, of constitutional design. See, for instance, Persson and Tabellini (2003). We are, however, only indirectly concerned with such issues. In spirit this paper relates to Grossman (2002) who models a self-enforcing constitution that provides an alternative to civil war in an established state. Our analysis, even though it applies to trans-national context, is in agreement with the central insight in Grossman (2002), namely, constitution can check conflicts if and only if expected gains from outright victory are not too large. Its subject matter is closer to Bordignon and Brusco (2001) who also present a cradle-to-grave model for union between two states. However, our model differs in significant respects from their model, which is primarily concerned with optimal secession rule. They provide a two-period, two-state (both states identical ex ante) model of a union, which is expected to deliver trans-national public goods in future that cannot be produced within individual states. In contrast the driving force for union in our case is the need to share different kinds of technologies and matching surplus factor inputs.\(^3\)

Secondly, it adds a new perspective to the issue of make-up and break-up of states. The model presented is fashioned so as to rule out merger due to conventional reasons, e.g., trade, economies of scale, etc, and focuses on the twin issues of technology transfer and resource mobility. To the best of our knowledge these issues have not been discussed in the relevant literature in the general framework presented here.\(^4\) Thirdly, the paper relates to work on relationship between bargaining and contest. This paper differs from Anbarci et al (2002) and Skaperdas (2006) who explore how outcome varies as bargaining norm changes for a given type of contest. We are concerned with how outcome changes with the type of contest for a given bargaining norm. Fourthly, this paper contributes to the literature on impact of demographic heterogeneity on political processes. More specifically we show that ethnic and

\(^3\) Chen and Ordeshook (1994) also model constitutional secession rules but for a three province union. Inman and Rubinfeld (2008) model a constitution, which supports transition to democracy in South Africa.

\(^4\) Alesina and Spolaore (2005) provide an extensive review of literature on make-up and break-up of states, which more or less begins with Friedman (1979). Noteworthy among models for specialized mergers is Chang (1995) who models a monetary union. In a paper close to our concern Yarborough and Yarborough (1994) argue that changes in state boundaries are required when cross-border contracts are not credible. But they do not provide a formal model. In a critical review, Young (2004) highlights the widely shared shortcomings of this literature. Also see Herbst (2000) in this regard.
economic heterogeneities severely constrain the set of mergers. Finally our model is in agreement with one of the insights provided in Bulow and Rogoff (1989), namely, there are limits to sovereign debt recovery.

The rest of the paper is organized as follows. Section 2 introduces the setting in two steps. First we define a constitutional agreement and identify its independent components and then introduce the socio-economic setting within which the independent components are determined. Section 3 introduces the benchmark model, with homogeneous population. In Section 4 we discuss the impact of demographic heterogeneity, savings, asymmetric information, etc on constitutional agreements. Section 5 concludes with a summary and a discussion on practical relevance of this model, as part of which we examine the experience of a few post-1945 cases of political unification. Unless otherwise specified please refer Appendix I for proofs.

2.0 Background

Consider two independent states, State A and State B, unequally endowed with human and material resources and production technologies specified in Section 2.2 (infra), exploring the possibility of merging to form a union. After merger erstwhile independent states become provinces of a union. Within the union each province as a whole would like to safeguard its interests. For this purpose they negotiate a constitution, which is the primary concern of this paper. Within a union unequally-endowed states can resolve to jointly manage technology and resources (labour and capital). In case of technology this implies that the better technology is implemented in backward province whereas in case of resources this implies resource mobility, which eliminates resource scarcities and surpluses within the union. These policies enhance the economic product of the union. If both labour and capital are mobile then mobility of one of the two would be redundant in the process of matching excess demand and supply for factor inputs. So without loss of generality we can work with the assumption that only one of the resources, say labour, is mobile. Our conclusions remain unchanged if the possibility of mobility of labour and capital are interchanged. Interchanging labels of the two

---


6 For importance of resource, especially labour, mobility and technology transfer for political unions see case studies in Bookman (1993) and Horowitz (2000 [1985]).
resources in the model would suffice. In our model identity of mobile factor becomes important only in presence of ethnic heterogeneity (see Section 4.1, infra).

2.1 Constitutional setting

2.1.1 Typology of Unions

Unions can be classified according to extent and type of merger that brings states together. A merger of states leading to a unitary state is followed by implementation of common tax rate throughout the post-merger state whereas a merger of states leading to a federal state is followed by implementation of different tax rates in different provinces of the post-merger state.\(^7\) If states agree to jointly manage either resources or technology in the union then the merger is referred to as partial merger whereas in a full merger states agree to jointly manage both resources and technology. Similarly, when states agree to jointly manage resources and/or technology in a limited way within the union then the merger is referred to as incomplete merger whereas in a complete merger states agree to jointly manage resources and/or technology to the fullest extent. In the event of merger limited technology transfer implies that advanced state offers advanced technology for a fraction of capital in backward state whereas limited resource mobility is obvious. A partial merger could be incomplete or complete and similarly an incomplete merger could be partial or full. It should be clear that the size of resources at the disposal of the union depends on the nature of merger agreement. Obviously an incomplete, partial merger would result in lesser economic output compared to a complete, full merger.

States can either choose to provide only for technology transfer (labour mobility) within the union under a TT-only (LM-only) merger. These are one issue-one step, i.e. partial, mergers. Alternatively, states can choose to provide for both technology transfer and labour mobility within the union under a full merger. Here states have three options: implement technology transfer and labour mobility simultaneously – TTLM merger, implement labour mobility after

\(^7\) Derivations for federal set-up are cumbersome due to different tax rates in provinces so we will focus on unitary unions in this paper. However, the results hold for an equivalent federal union as well after suitable transformation of the tax function. Details are available from author.
completing technology transfer – TTLM (TT) merger, and implement technology transfer after ensuring labour mobility – TTLM (LM) merger. The last two mergers can be interpreted as multi step-multi issue mergers addressing issues sequentially. TTLM merger is referred to as multi issue-one step merger. Given the large number of possibilities (see Figure 1) it is not difficult to see that the sequence of implementation of various components of a merger agreement will have a significant impact on the structure of union.

Figure 1 about here

2.1.2 Constitution

We propose a constitution addressing the basic structure of a state, which can apply to a union of any number of states. It is only when we set about to find an optimal constitution that we need to restrict the number of states due to modelling constraints. Choice of a two state model, however, needs some justification in addition to the usual quip that ‘Three is a crowd in game theory’. There is indeed empirical support for choosing a two-state framework, namely, for the whole of 20th Century we know of no case of negotiated merger involving more than two sovereign states.\(^8\) With this justification we can work in a two-state framework. Our constitution consists of articles divided into two categories. Part I of the constitution contains the objective of union. Part II deals with instruments for implementing Part I. The articles of the constitution are defined below.

1. Preamble (C0) states the objective of union (type and degree of merger) and the necessary conditions for its existence.

\[
C0 = (CC_n, \theta, SR); \quad CC_n \in CC
\]

\[
CC = \{CC_1 = LM, CC_2 = TT, CC_3 = TTLM, CC_4 = TTLM (TT), CC_5 = TTLM (LM)\}
\]

We will show later in the paper that each element of CC relates to an equilibrium in our model. Constitution helps in implementing one of them. We can therefore refer to the elements of CC as constitutional conventions.

\(^8\) The partially successful attempt to include Southern Yemen in an expanded United Arab Republic (a sovereign state formed after the merger of Egypt and Syria in 1958, which was dissolved in 1961) is the closest one gets to an attempt to achieve a non-coercive merger between three sovereign states. But the actual process of merger unfolded in two stages each involving two states at a time. After the dissolution of the Spanish American Empire at least half a dozen failed attempts were made in the 19th Century to unite different groups of successor states.
Secession Rule (SR) specifies the conditions under which the union can be dissolved. Whenever SR specifies “Exit” the union dissolves peacefully else the secessionist province might be punished.

\[ SR: (g, t) \rightarrow \{(a, b)\mid a \in \{\text{Exit, Continue}\}, b \in \{\text{War, No War}\}\} \tag{2} \]

where \( g \) is additional economic product generated due to union and \( t \) is union tax rate.

The degree of merger is given by \( \theta = (\theta_L, \theta_r) \). \( \theta = (1,1) \) for complete, full mergers; \((0,1)\) or \((1,0)\) for complete, partial mergers; \( \theta = (a,b) \) for incomplete, full mergers; \((0,b)\) or \((a,0)\) for incomplete, partial mergers; \((0,0)\) for no merger, where \( a, b \in (0,1) \). Many countries have central and provincial lists of functions. \( \theta_i = 1 \) could be interpreted as issue \( i \) being on central list. Similarly, \( \theta_i = 0 \) could be interpreted as issue \( i \) being on provincial list because it implies that the union is not concerned with the issue, which is therefore left to respective provinces. Similarly \( \theta_i \in (0,1) \) could be interpreted as the issue being on a concurrent list. Another interpretation of \( \theta \) is as follows. \( \theta_L = 1 \) can be interpreted as individual right to free mobility within the union whereas \( \theta_r = 1 \) can be interpreted as obligation of state to ensure technological improvement for all units of capital in the backward province.

2. (a) Seat Sharing Rule (C1) governs seat sharing in the union legislature. State \( i \) is assigned \( \lambda_i \) fraction of seats in the union legislature.

\[ C1 = \left\{ \lambda_i \mid \lambda_i \in (0,1), \sum_{i \in \{A,B\}} \lambda_i = 1 \right\} \tag{3} \]

2. (b) Revenue Sharing Rule (C1a) governs the rate of taxation, \( t \), and the share of State \( i \) in the tax revenue of union, \( \beta_i \).

\[ C1a = \left\{ (t, \beta_i) \mid \beta_i \in (0,1), \sum_{i \in \{A,B\}} \beta_i = 1, \ t \in \left[ t_{\min}, t_{\max} \right] \right\} \tag{4} \]

On the one hand tax rate cannot exceed a level, \( t_{\max} \), beyond which individuals are denied subsistence. On the other hand the minimum tax rate, \( t_{\min} \), has to be positive otherwise the state cannot function and will thus cease to exist. The cost of functioning could include provision of public goods. However, we are not concerned with the composition of basket of
public goods. The limits to taxation are revealed by nature and apply to all types of states. Later on we will show that an optimal secession rule specifies dissolution of unions that require taxation beyond these limits. Allowing different tax limits for different states will complicate the expressions without adding any new insights. Those not comfortable with $t_{\text{min}}$ and $t_{\text{max}}$ can substitute them with numbers close to 0 and 1, respectively.

3. **Revision Rule** (C2) governs rule-based revision of the initial agreement.

$$C2 = \{ \Psi \mid Z(\Gamma + 1) = \Psi(Z(\Gamma)) \}$$

where $Z \in \{C_{c}, SR, \lambda, t\}$, $Z(\Gamma)$ is the value of $Z$ in period $\Gamma = \Gamma'$, where $\Gamma = 0, 1, 2$.

4. (a) **Constitution Amendment Rule** (C3) governs the procedure for amending the constitution. This rule is given by $\alpha$, which specifies the quorum to effect a constitutional change, e.g., support of at least one more than $2/3$rd of the legislators is necessary if $\alpha = 2/3$.

$$C3 = \{ \alpha \mid \alpha \in (0.5, 1) \}$$

4. (b) **Inviolability Clause** (C3a) declares if the original scope of merger (C0) is mutable or not. $\text{INV} \in \{0, 1\}$, where C0 is immutable if $\text{INV} = 0$.

We end the introduction to constitution with a few remarks. First, $\text{INV}$ is exogenously fixed. If the union is beneficial to one of the states, which is able to compensate the loser, or both states then $\text{INV} = 0$ does not bind. Else $\text{INV} = 0$ binds because the union cannot be restructured to generate additional surpluses, if possible. Second, a revision rule can only revise certain articles according to a pre-determined formula and is not self-referential while an amendment rule covers all articles of the constitution (unless constrained by $\text{INV}$) in the sense that a group of legislators fulfilling the requirement of quorum can change the entire constitution including the amendment rule itself. Third, we require the constitution amendment rule to meet the following criteria: Stability (should discourage frivolous amendments), Inclusiveness (should not dissuade a weaker state from joining the union), and Flexibility (should be as liberal as possible). Having defined the articles of constitution we can now define a constitution as $C = (C0, C1, C1a, C2, C3, C3a)$. 
Merger negotiations can be decomposed into a number of parts dealing with different components of the constitution, which reminds of multi-issue bargaining. However, we have to adopt a different approach because in our case the issues (articles), as we will show shortly, are not independent. In fact, a closer look at the various articles of constitution would suggest that rather than representing different *pies* these articles address different aspects of the same *pie*. We will isolate the independent articles of the constitution in two steps and argue that the order in which they are taken up does not require any bargaining over agenda. First of all note that even though in principle corresponding to each type of merger we can entertain the possibility of incomplete mergers, i.e. $\theta_r$ and/or $\theta_s < 1$, the following result shows that such mergers, $\theta_r, \theta_s \in [0,1)$, are not possible.\(^9\)

**Proposition 1:** (a) For any type of merger complete merger is strictly preferred to incomplete merger. (b) An incomplete merger of one type is never revealed preferred to a complete merger of another type.

Secondly, we argue that a state’s share in union revenue is non-decreasing in its share of seats in union legislature and that a weaker state would be unwilling to join a union under a relaxed amendment rule.\(^11\) The following result formalizes these points.

**Lemma 1:** For a given equilibrium seat sharing rule, $\lambda_i$ (a) $\beta_i = \lambda_i$ is the equilibrium revenue sharing rule and (b) $\alpha = \max(\lambda_i, \lambda_j)$ is the equilibrium constitutional amendment rule satisfying the requirements of flexibility, stability, and inclusiveness.

Later on we will also show that union tax rate depends on seat sharing rule (**Lemma 7, infra**). So Lemma 1 implies that an agreement on $C1$ also fixes $C1a$ and $C3$. $C3a$ is known to be exogenously fixed. We can now isolate the independent articles of a constitution.

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\(^9\) See Fershtman (1990) for agenda in complete information bargaining games.

\(^10\) It is well known that incomplete mergers are not uncommon, at least in South Asia. In India peripheral provinces, which are substantially different from the “mainstream” in terms of ethnic identity, were incorporated under incomplete and/or partial merger arrangements. In our model incomplete mergers could arise if we introduce demographic heterogeneity (**see Section 4.1, infra**).

\(^11\) In a number of cases Third World minorities were promised equal treatment at the time of independence but later on the majorities easily amended the respective constitutions to the disadvantage of the former.
**Lemma 2:** $CC_n$ and $C1$ are the only independent (sub-) articles in *Part I* and *Part II* of the constitution, respectively.

Note that $C1$ itself depends on $CC_n$ so that ultimately the entire constitution can be reduced to $CC_n$, which is not surprising as the latter is an expression of the objective of merger. But it is *through* $C1$, not directly, that $CC_n$ determines the rest of the constitution ($C1a$, $C2$, and $C3$). (It is not for nothing that seat sharing is the most contentious issue in merger negotiations.) Further it is intuitively obvious that bargaining on agenda is not required since in any case preamble ($CC_n$) has to be fixed before $C1$.

### 2.2 Material setting

#### 2.2.1 Human resources and material resources

Our world consists of two states. $L_i$ and $K_i$, respectively, are the shares of labour and capital held by state $i$. Capital (labour) constrained state’s share in aggregate or worldwide capital (labour) falls short of its share in aggregate labour (capital). It should not be difficult to see that between a pair of states one would be capital constrained while the other would be labour constrained. In other words the upper limit of availability of one of the factors is fixed in each state due to skewed distribution of resources. Relative distribution of resources across states is assumed to be time invariant. Within each state assume equal division of capital and labour resources, available in the state, among all individuals, who are endowed with identical preferences. Depending on merger agreement labour might be mobile. Once labour mobility is allowed it leads to instantaneous and optimal reallocation of labour between states. *As pointed out above all our results hold if capital or both labour and capital are mobile instead of labour* (see Section 2.0, *supra*). We assume that resource mobility does not lead to permanent redistribution of resources so that labour and capital continue to belong to the state of origin irrespective of where they are employed. In Section 4.4 we discuss a plausible rationale for this assumption.

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12 Normalization used in the paper, namely, $K_A + K_B = 1$ and $L_A + L_B = 1$ is without loss of generality. One could as well use $K_A + K_B = K'$ and $L_A + L_B = L'$. 
2.2.2 Technology

Output is governed by the following production function, which applies to both the states:

\[ Q_i = X_i \min(q_L L_i, q_K K_i), \text{ where } X_i \in \{A, B\} \]  \hspace{1cm} (7)

For sake of simplicity assume \( q_L = q_K = 1 \) so that one unit each of labour and capital are required to produce \( X \) units of output.\(^{13}\) \( A \) (\( B \)) represents productivity of technology in State A (B). The state that is initially relatively technologically advanced is also referred to as advanced state while the other is referred to as backward. The same good is produced in both states and its price is normalized to unity.\(^{14}\) We assume that individuals in both states have identical subsistence requirements, which can be fulfilled using the backward technology even at the maximum rate of taxation, \( t^{\text{max}} \).

Technologies differ in terms of time \( (T^*) \) required to transfer them to a new user. Technologies requiring infinite time to transfer \( (T^* \rightarrow \infty) \) are referred to as Type I technologies while the remainder are referred to as Type II technologies. The type of technology is denoted by \( \gamma \in \{I, II\} \). \( T^* \) is in fact a proxy for complexity of technology. Transfer of Type I technology involves only the transfer of use rights while in case of Type II technology transfer implies that after a gestation period the ability of new user to use the technology is no longer dependent on the continued support of the original owner. Note that in case of Type I technologies TTLM (TT) merger is not possible because in such a merger LM has to be implemented after completion of technology transfer, which never happens with this type of technology. So a TTLM (TT) merger with Type I technology is effectively a TT merger. Also note that we assume that productivity in backward province increases to the maximum possible level as soon as agreement for technology transfer takes place even though assimilation/transfer of technology, if at all possible (as in case of Type II technology), takes

\(^{13}\) To use the normalization \( K_A + K_B = K' \), etc instead of \( K_A + K_B = 1 \), etc we need to assume \( K'q_k = L'q_L = 1 \).

\(^{14}\) Without loss of generality we can interpret output as a basket of goods and services with the only restriction that the basket be same across states. We are not concerned with the composition of the basket. Further note that our setting differs from a Heckscher-Ohlin economy. In our case two factors are used to produce one output and states are endowed with different technologies and factors can be mobile.
place over a longer period. The alternate assumption that productivity increases gradually over time only adds complexity to the analysis without affecting any of our results.

2.2.3 Division of economic product

Given production function (Eqn 7 with \( q_L = q_K = 1 \)) labour and capital receive equal returns. In case of excess availability of an input in state \( i \), say labour, some units \((L_i - K_i)\) are left completely unutilised or all units of that input are equally under-utilized, i.e. each unit of labour works at \( K_i / L_i \) fraction of its potential. However, this distinction does not matter for us because we assume that within an independent state or province returns to a factor are divided equally among all the owners. More specifically we do not differentiate between owners of employed and unemployed resources.\(^{15}\) The share of product accruing to capital is divided equally among all capital holders and that accruing to labour is divided equally among all labour owners. So, within a state all individuals, who happen to own equal capital and labour by assumption, receive equal shares from the economic product and are thus equally well-off.

State taxes individuals uniformly and spends the entire revenue on public goods. Pre-merger states can have different tax rates, \( (t_i^{\text{independent}}, t_j^{\text{independent}}) \), but within a unitary union tax rate is same across provinces, \( t_i^{\text{union}} = t_j^{\text{union}} \). Pre-merger tax rate is immaterial since everything is spent within the same political entity. However, post-merger tax rate has implications for redistribution of income across provinces. Note three very important points here. One, we assume that there is no deadweight loss associated with inter-province tax-based transfers. However, all our results can be easily derived even in case of non-zero deadweight losses, i.e. for every unit of transfer from a province only a fraction reaches the recipient province. Two, our results remain unaffected if incomes from labour and capital resources are taxed differently. Three, for simplicity we assume that income generated from employment of resources outside the province of origin is taxed in the home province. Even though the arithmetic becomes complicated the substance of this paper remains unchanged if instead we

\(^{15}\) If agents are uncertain about whose resources are going to be left unused then organization into a cooperative serves as unemployment insurance.
provide for taxation of income generated by mobile resources in the province of employment or partly in the province of employment and partly in the province of origin.

2.2.4 Players

Since each state/province has individuals endowed with identical preferences and material resources we can treat a state/province as an internally undifferentiated monolith. So within an independent state/province aggregate and individual optimization problems are equivalent. In absence of union independent states are the players otherwise provinces of the union are the players. Policy-makers of state/representatives of provinces in union legislature are chosen at random from among their respective populations. Alternatively, all individuals within a state/province contribute equal time to the task of policy making, which can be interpreted as part of the basket of goods and services produced in the concerned state. In either case we do not face principal-agent problem, which also rules out merger driven by empire builders. Finally assume the players to be risk neutral.

2.2.5 Pay-offs

Economic and political (share of seats in legislature) components of pay-off to a state are additive. The pay-off function is given by:

\[ U_i = E_i(Q_i^s, t_i^s) + P_i(\lambda_i^s, Q^s); s \in \{independence, union\} \quad (8) \]

\( Q_i^s \) and \( \lambda_i^s \) are, respectively, state \( i \)'s economic product and share in legislature. \( Q_i^{independence} = Q_i^{independence} \) and \( Q_i^{union} = \sum Q_i^{union} \). Independent states enjoy full control over levying and utilization of taxes due to complete control over legislature, i.e. \( \lambda_i^{independent} = 1 \), but the size of state resources is smaller. The share of provinces in union legislature is necessarily limited, \( \lambda_i^{union} = \lambda_i < 1 \), but the size of state resources is larger than before so that there is a possibility of trade-off between the two components of pay-off. Political pay-off to a province in a union from share of seats in union legislature is equivalent to its share in union taxes and
is given by $\lambda_i^{\text{union}}(tQ)^{\text{union}}$, where $\beta_i^{\text{union}} = \lambda_i^{\text{union}}$ by virtue of Lemma 1 and $(tQ)^{\text{union}}$ is the total tax revenue of union. Economic component of pay-off consists of economic product net of taxation, i.e. $Q_i^{\text{union}}(1 - t_i^{\text{union}})$. The pay-off functions for a state under different conditions are provided below.

$$U_i^{\text{indep}} = (1 - t_i^{\text{indep}})Q_i^{\text{indep}} + t_i^{\text{unindep}} = Q_i^{\text{indep}}$$

$$U_i^{\text{union}} = (1 - t_i^{\text{union}})Q_i^{\text{union}} + \lambda_i^{\text{union}} \sum Q_i^{\text{union}}$$

2.2.6 Post-merger intra-union arrangement

If labour from one province migrates to the province where output is labour constrained the migrants send back remittances proportionate to their labour input while the capital rich province gets additional income due to employment of hitherto unutilized capital. Migrant labour is not discriminated against and is paid as much as the native labour. Relaxing this assumption affects the details but not the substance of the paper. It is trivial to establish that whenever free labour mobility is agreed upon migration takes place only from labour surplus to labour constrained province and that the upper limit to migration is provided by the fraction of surplus labour (which can be loosely referred to as unemployment rate) in labour surplus province. This holds true since labour force and technology are not differentiated by skill levels in our model. If technology transfer has been agreed upon then after merger better technology is implemented in the backward province. Recall that we do not allow permanent resource migration, which would imply irreversible resource transfer and consequent loss of bargaining power of the “donor” state/province. Depending on initial material and technological endowments, i.e. gross economic product, each province, erstwhile state, exercises some control over policy making in the union, which is denoted by $\lambda_i$. $\lambda_i$ might not coincide with population/resource shares in the union.

Having described at length the political (constitution) and the economic primitives of the model we can now proceed to examine the actual process of merger. But before that few words on what happens in the event of lack of consensus.
2.2.7 Contest

In case the two sides fail to settle issues via negotiation they choose between contest and status quo. The latter is obvious. For the former we assume the following contest success function after Skaperdas (1996):

\[ p_i(e_i, e_j) = f(e_i) / \sum_{j \neq i} f(e_j) \]

where \( p_i \) is the probability state \( i \) emerges successful in a contest with state \( j \), \( r \geq 0 \) measures the effectiveness of additional effort in a contest, and \( e_i \) denotes the resources state \( i \) can mobilize for a contest. In the event of contest states incur a fixed cost \( C_i \) irrespective of the outcome.\(^{16} \) \( F \) captures the advantages of contest strategy. In case state \( i \) strikes first \( F_i > 0 \), which brings down the cost of contest to state \( i \). \( F_i = 0 \) when both attack simultaneously. Also \( F_i > 0 \Rightarrow F_j < 0 \), i.e. when state \( i \) strikes first then state \( j \) incurs additional costs for being defensive, which in other words adds to the cost of contest incurred by state \( j \). A non-zero first strike advantage ensures contest is never a strictly dominated strategy. Assume \( |F_i| < C_i \) holds for both states. In our model the process of negotiation is assumed to be costless relative to contest. There are two categories/technologies of contest, \( \kappa \in \{I, II\} \).

1. Category I: Winner extracts entire additional economic production possible due to a merger, which leaves the loser with its No Merger economic product.
2. Category II: Winner implements most favourable tax rate within the set of mutually agreeable bargaining solutions for constitutional union. (This type of contest can also be interpreted as an electoral contest over tax policy.)

Cost of contest and first strike advantage increase with the degree of appropriation involved. Under an interesting possibility ignored here a state could indulge in destruction or permanent appropriation of a counterpart’s productive resource base to increase its bargaining power as in Skaperdas (2006). This is tantamount to assuming that the cost of permanent appropriation of resources is prohibitive, i.e. \( C_i (permanent) > \ell_{max} \cdot \max(A, B) \cdot \min(L_i, K_i) \). It is indeed

\(^{16}\) We ignore the possibility that the cost of contest to a state could be endogenous insofar as it depends on economic conditions. Higher productivity means more efficient war making, which saves cost but it also means higher opportunity cost. Higher economic product likewise means that cost of contest as a fraction of whole economy might be smaller but in absolute terms it might be higher compared to a state with lesser economic output. Another source of endogeneity could result from loss of productive assets in a contest.
very easy to overlook the fact that without savings or reasonably large wealth endowment costly wars cannot be financed (for more on savings see Section 4.3, infra).\textsuperscript{17} Alternatively, the presumption could be that the present distribution of resources has been arrived at after all possibilities of appropriation of factors of production have been exhausted.

### 3.0 Constitutions

Agreement on independent articles, namely, type of merger ($CC_n$) and seat sharing rule ($\lambda$), is the primary concern of negotiation in the run-up to merger. The ultimate objective of the negotiation is not to actually divide the unionwide economic product. Rather the intention is to use the shares obtained from a hypothetical partition of the unionwide economic product generated by a given type of merger to arrive at $\lambda$, the share of seats in the union legislature, which in turn determines union tax revenue sharing. $\lambda$ derived below have to be interpreted in this spirit. Ideally a full and complete merger leading to simultaneous technology transfer and labour mobility (TTLM) should be the preferred outcome under which total product is maximized. However, the two sides can choose from among several different kinds of complete mergers, including both partial and full types. Recall that by virtue of Proposition 1 incomplete mergers are never contracted. Given the large number of ways of achieving merger we will briefly discuss the issue of choice of type of merger and conflict resolution technology and then introduce the actual negotiation process. A plausible solution could be that we allow nature to choose the set of desirable mergers for each state, which is made known to both states.\textsuperscript{18}

Let $\mathcal{I}(i)$ denote the set of desirable types of merger chosen by nature for State $i$. At the beginning of period $\Gamma = 0$ nature chooses $N = (K_i, L_i, A, B, \gamma, \kappa, \mathcal{I}(i), \mathcal{I}(j), INV)$, i.e. material endowments, type of technology and technological gap, type of contest, set of desirable mergers, and whether preamble is mutable or not. $\mathcal{I}(i) \cap \mathcal{I}(j)$ is the set of mutually agreeable merger types. States decide the type of merger to be negotiated through a normal form game

\textsuperscript{17} Colomer (2007: 35) draws attention to the fact that some of the 20th Century wars cost more than 100% GDP to the participants. Incidentally all these wars involved efforts to change existing borders. In other words it is easier to snatch part of current output than to appropriate productive resources permanently.

\textsuperscript{18} Earlier we saw that TTLM (TT) mergers are not possible with Type I technologies (see Section 2.2.2, supra). Hence by fixing type of technology nature can constrain the set of desirable mergers. Historical experience can also play the role of nature. A state that had a bad experience with labour mobility in past ignores mergers involving labour mobility thereafter. However, to account for such possibilities we need to introduce incomplete information and history (prior to the onset of present round of merger negotiations) into the model. Alternatively, we can constrain the set of desirable mergers by introducing demographic heterogeneity as in Section 4.1 (infra).
in which each state has \(|\mathcal{T}(i) \cap \mathcal{T}(j)|\) strategies. If \(|\mathcal{T}(i) \cap \mathcal{T}(j)| = 1\) then the outcome is unique else the normal form game resembles a pure coordination game and has \(|\mathcal{T}(i) \cap \mathcal{T}(j)| > 1\) pure strategy Nash equilibria and no mixed strategy equilibria. States can choose a conflict resolution technology from \(\mathcal{CRT} = \{\text{status quo, contest, negotiate}\}\) to resolve the conflict.

3.1 Game of Merger, \(\mathcal{M}\)

We model the game of merger with rational players and complete information using an alternating offers bargaining model with endogenous inside options and contest as outside option. Pay-off functions, rationality of each player, and all relevant parameters of the model are common knowledge. In such a model if outside option of both players is dominated by outcome generated by bargaining outcome based on inside option then outside option has no impact on solution of the problem. If, however, outside option dominates the bargaining outcome for even one player it dictates the solution to the game as if inside options were not there (Muthoo 1999: Section 6.4). In other words we have two separable regimes, under the first bargaining dominates \(\mathcal{M}\). In the second contest dictates the solution \(\mathcal{CN}\). In the latter there are two structurally identical sub-cases depending on whether contest dominates negotiation for state \(i\) or \(j\). We can therefore model the problem dichotomously.

3.1.1 Negotiation Game, \(\mathcal{N}\)

We use the alternating offers bargaining model with endogenous inside options\(^{19}\) to model the process of bargaining. The ‘pie’ under negotiation is the present discounted value of the total economic product of the union, 

\[
Q^{\text{Union}}/r = Q(CC_n)/r = \sum_n Q^n/r,
\]

where \(n\) signifies type of merger. In each period of disagreement the inside options of the states are determined endogenously. Endogeneity results from the fact that the inside options depend on the choice made by players within the process of bargaining through a non-cooperative game. Gains from merger are the present discounted values of difference between the total economic product of the union and the sum of inside options. We assume that both players have identical discount rates \((r_a = r_b = r)\), so their evaluations of gains from a given type of merger

\(^{19}\) Busch and Wen (1995) extend Rubinstein’s (1982) alternating offers model to account for endogenous inside options. We follow the slightly modified version of their model presented in Muthoo (1999).
are identical. Discount factors $\left( \delta_A \neq \delta_B \right)$, applicable to bargaining, differ due to different response times, $\Delta_A \neq \Delta_B$. $\Delta_i$ has two interpretations. One, it is the time, $\Delta_i$, which state $i$ takes to make an offer after rejecting an offer of state $j$. Two, it is the time, $\Delta_{ij}$, after which state $j$ allows state $i$ to make a counter-offer after the latter rejects an offer of the former. Ideally $\Delta_i = \max(\Delta_{ii}, \Delta_{ij})$. We will assume $\Delta_{ii} < \Delta_{ij}$ so that $\Delta_i = \Delta_{ij}$.

Disagreement game, $\Theta(n)$

The actions of players when in disagreement in the process of negotiation are captured by a disagreement game, which is represented by a simultaneous move, one-shot game of complete information, $\Theta(n) = \left[ \left( U_i \times S_i(n) \right) \right]_{i \in [A,B]}, U_i : \Pi_{i \in [A,B]} S_i(n) \to \Re$ and $S_i(n)$ is set of strategies available to state $i$, identical for both states, in the event of disagreement in the course of negotiation for $n$-type merger. $\hat{\Theta}(n)$ is the set of Nash equilibria of $\Theta(n)$. In the event of disagreement the two sides choose actions in the disagreement game according to which they enjoy interim benefits till they reconvene for negotiations.

In one issue-one step (partial) mergers, TT or LM, the two sides choose to remain independent in case of disagreement over merger. In case of multi issue-multi step mergers, TTLM (LM) or TTLM (TT), in the event of disagreement in each step the two sides have only one option. If there is a disagreement during the first step the two sides can only choose not to merge (NM) whereas if there is a disagreement in latter steps the two sides can only choose not to proceed with further unification (SQ). In other words multi-step mergers are structurally akin to one issue-one step mergers. In multi issue-one step mergers, TTLM, in the event of disagreement over full, complete merger the two sides can choose from among three different options – no merger (NM), merger with only technology transfer (TT), and merger with only labour mobility (LM). TTLM merger cannot be reduced to a one issue-one step merger or a combination of two such mergers.

$S(n) = \{NM\}$ in case of one issue-one step mergers whereas in case of multi issue-one step mergers $S(n) = \{NM, LM, TT\}$. The case involving $|S(n)| = 1$ is trivial. It is to cases involving $|S(n)| > 1$ that we turn next. Disagreement game with multiple strategies, as in case of TTLM
mergers, can be depicted using a normal form game shown in Figure 2. For description of pay-offs see Appendix II.

Figure 2 about here

A number of cases can be obtained for interaction of two states depending on different combinations of technology and resource distribution. Table 1 summarizes the entire range of possibilities. The disagreement game shown in Figure 2 is provided only as an aid to understanding. Otherwise the following analysis relates to the general case introduced in Section 2.2 (supra).

Table 1 about here

Negotiation

With rational players and complete information alternating offers bargaining with endogenous inside options yields an agreement in the initial period with $|\Theta|$ stationary subgame perfect equilibria (SSPE), where $|\Theta|$ is the number of Nash equilibria of $\Theta$ (Muthoo 1999). For each essentially one issue-one step merger there is one SSPE corresponding to one Nash equilibrium of $\Theta$, namely, (NM, NM). In case of multi issue-one step merger there are four SSPEs corresponding to the four Nash equilibria of $\Theta$, namely, (LM, LM), (TT, TT), (NM, TT) and (NM, NM). The last two equilibria are not only pay-off equivalent but also identical in terms of outcome, namely, complete independence in absence of agreement on merger. Therefore, we consider only (LM, LM), (TT, TT), and (NM, NM) and the corresponding sub-types of merger are referred to as TTLM-LM, TTLM-TT, and TTLM-NM. The following result provides the equilibrium seat sharing rule. The proof follows in straightforward manner from a slight modification of similar results available in literature (Busch and Wen 1995, Muthoo 1999) for the case $(r_i \neq r_j)$ to our case where $(r_i = r_j = r)$ and $(\Delta_i \neq \Delta_j)$.

**Lemma 3:** In the limit as $\Delta_i \to 0$ and $\Delta_j \to 0$ (but $\Delta_i \neq \Delta_j$) the unique SSPE seat sharing rule for states $i, j \in \{A, B\}$ is given by $(\lambda_i (m, n | \Theta(n)))_{1-\lambda_i (m, n | \Theta(n))}$, where $d_i(m, n)$ is the pay-off accruing to State $i$ as inside option in the $m$th Nash equilibrium of $\Theta(n)$, the disagreement game for $n$-type merger, $\eta_i$ is bargaining power of State $i$, and $g(m, n)$ is the
rate of gain from $n$-type merger when strategies corresponding to the $m$th Nash equilibrium of $\Theta(n)$ are chosen in the event of disagreement.

\[
\lambda_i(m,n|\Theta(n)) = \left(\frac{d_i(m,n)}{r} + \eta_i \frac{g(m,n)}{r}\right)O(CC_n) \quad (12)
\]

\[
g(m,n) = O(CC_n) - d_i(m,n) - d_j(m,n)
\]

\[
\eta_i = \frac{\Delta_i}{(\Delta_i + \Delta_j)}
\]

\[
n \in \{1, 2, 3\}, \ m \in \{1, \ldots, \hat{\Theta}(n)\} \text{ and } i \neq j
\]

Note a few things here. One, since we are interested in structurally independent types of mergers we restrict $n$ such that $n \in \{1, 2, 3\}$ in Eqn 1. Two, using the limiting solution of the negotiation model saves us from making unreasonable assumption about the identity of the first mover in the negotiation game. Three, $\lambda_i(m,n|\Theta(n))$ is equivalent to the asymmetric Nash bargaining solution to our problem. Four, our model does not have mixed strategy SSPEs or non-Markov perfect equilibria.\(^{20}\) Four, we can arrive at (12) using a more rigorous model in which state $i$ has capacity to cause additional delay of $\Delta_i - \Delta_j$ before the next round of bargaining whenever its offer is rejected, where $\Delta = \Delta_i = \Delta_j$. We can show that in a model with inside options given that state $j$ delays whenever state $i$’s offer is rejected state $i$ also resorts to delay tactics if and only if certain conditions are fulfilled and that these conditions necessarily hold for both states in the limiting case.\(^{21}\) Finally, we can say that $\Delta_i$, the time for which state $i$ is forced to wait before making its offer after rejecting state $j$’s offer, is non-decreasing in the resources state $j$ can generate from inside option. Since, the only difference between our players is their resource and technological endowments, i.e. economic product, relative bargaining power has to depend on relative economic product. We assume a linear dependence between the inside option of state $i$ and the delay state $i$ can cause before the next round of bargaining whenever state $j$ rejects its offer.

\[
\forall i \neq j \quad \Delta_j(n,m) = a \cdot d_i(n,m) \quad (13)
\]

so that relative bargaining power of state $i$ is given by

\(^{20}\) The model has infinite Markov perfect equilibria, many of them pay-off equivalent, in which agreement is reached in the initial period. Markov SPEs do not add much to our understanding of the present problem other than drawing attention to the fact that there is no limit to variation in equilibrium power sharing agreements.

\(^{21}\) See Avery and Zemsky (1994) and Muthoo (1999) for bargaining with delays.
\[ \eta_i(n,m) = \frac{\Delta_j}{\Delta_i + \Delta_j} = \frac{d_i(n,m)}{d_i(n,m) + d_j(n,m)}, \text{ where } \sum_i \eta_i = 1 \text{ and } \eta_i \epsilon (0,1) \] (14)

The next result, which relates seat sharing and relative bargaining power, follows directly from (12) – (14).

**Lema 4:** \( \eta_i(n,m) = \lambda_j(n,m) \ \forall n, m(n) \).

**Tax Adjustment**

Now we will address the issue of omission of tax adjustments in \( \Theta \). Pay-offs in \( \Theta \) need to be adjusted for tax transfers within the *ad hoc* union whenever disagreement game supports the possibility of partial mergers, \( |S_i(n)| > 1 \). This is required only in case of multi issue-one step TTLM mergers so we will confine ourselves to disagreement games of the type shown in Figure 2. Using (8) – (10) we can arrive at tax adjusted pay-offs. The corresponding tax adjusted disagreement game is denoted by \( \Theta' \). The next result establishes the equivalence of \( \Theta' \) (Figure 3) and a reduced form disagreement game, \( \Theta'' \) (Figure 4).

**Lema 5:** \( \forall \ t \epsilon (0,1) \ \Theta' \), the tax adjusted disagreement game, is strategically equivalent to \( \Theta'' \), the reduced form disagreement game so that \( \hat{\Theta}' = \hat{\Theta}'' \).

**Corollary 1:** \( \hat{\Theta}' = \hat{\Theta}'' = \hat{\Theta} \setminus (NM, TT) \), where \( (NM, TT) \) is equivalent to \( (NM, NM) \) in terms of merger outcome, namely, *no merger*, as well as pay-offs.

The following result establishes equivalence between equilibrium seat sharing determined using tax adjusted and unadjusted disagreement games. It follows from Lemma 4 and 5 and Corollary 1.

**Lema 6:** \( \lambda_j(m,n | \Theta') = \lambda_j(m,n | \Theta'') = \lambda_j(m,n | \Theta) \ \forall t \).
The next result formalizes the idea that the net recipient of transfers within the union would prefer higher union tax rate and vice versa.

**Lemma 7:** Unequally endowed states have opposed preferences over union tax rate with state $i$ favouring higher taxes if and only if $\lambda_i^n \geq E_i^n \left( E_i^n + E_j^n \right)$ or $\lambda_i^n g^n - x_i^n \geq 0$, where $E_i^n$ represents economic product of state $i$ after $n$-type merger, $x_i^n$ represents additional product generated within province $i$ (erstwhile state $i$) after merger, and $g^n = \sum x_i^n$ represents the corresponding total gain from merger.

### 3.1.2 Contested Negotiations, CN

As suggested above states can use one of the following CRTs, namely, *status quo, contest*, and *negotiate*, to resolve the conflict. We have already derived results for negotiations without contests while the case involving *status quo* as a strategy is trivial. In fact, it was subsumed as a disagreement strategy, namely, No Merger, within the negotiation game. Since contest cannot be subsumed as an inside option we introduce it as an outside option in the negotiation game. We assume that the winner of the contest imposes its favourite solution, which varies with the category of contest under consideration. $SQ_i$ denotes the returns from maintaining status quo. The expressions for total gains from merger and returns to state $i$ from negotiations and contest, respectively, are given below where we omit the superscript ($n$) for type of merger and assume without loss of generality that $\lambda_i g > x_i$. Note that $g$ captures the overall hike in economic output due to a given type of a merger, which is different from what individual provinces gain from merger.

\begin{align*}
\text{Gains from merger: } g &= x_i + x_j \\
\text{Negotiation: } U_i &= (1-t)E_i + t\lambda_i \left( E_i + E_j \right) = E_i + t \left( \lambda_i g - x_i \right) \\
\text{Contest (Type I): } R_i' &= p_i \left( E_i + x_j \right) \left( 1 - p_i \right) SQ_i - \left( C_i - F_i \right) = p_i g + SQ_i - \left( C_i - F_i \right) \\
\text{Contest (Type II): } R_i'' &= p_i U_i \left( t^\text{max} \right) \left( 1 - p_i \right) U_i \left( t^\text{min} \right) - \left( C_i - F_i \right)
\end{align*}

\[^{22}\text{See Muthoo (1999) for difference between inside and outside options.}\]
Or \( R''_i = E_i + \rho(\lambda_i g - x_i) - (C_i - F_i) \)

where \( \rho = p_i t^\text{max} + (1 - p_i) t^\text{min} \in (t^\text{min}, t^\text{max}) \) and use is made of Lemma 7 following which we can say that if \( \lambda_i g > x_i \) then state \( i \) prefers \( t^\text{max} \) and state \( j \) prefers \( t^\text{min} \). In the following result we show the limits to possibility of contest. In the proof of this proposition we specify conditions under which different conflict resolution strategies dominate.

**PROPOSITION 2**: For a given type of merger contest cannot dominate negotiate for both states simultaneously.

The following is immediately apparent from the proof of Proposition 2.

**COROLLARY 2**: Set of feasible bargains is never empty in case of Category II contests, which essentially involve contests to relocate along the bargaining frontier.

In the next proposition we identify the set of CRTs, seat sharing rule, and tax rate under different conditions. Recall that \( \mathcal{I}(i) \) is the set of desirable types of merger chosen by nature for State \( i \) and note the following tie-breaking property.

**PROPERTY 1**: (a) When indifferent between independence and union a state chooses the former. (b) When indifferent between contest and negotiation a state chooses the latter.

**PROPOSITION 3**: Given Property 1 and \( |F_i| < C_i \)

1. **Sovereign Peace**: If \( |\mathcal{I}(l)| = 0, l \in \{i, j\} \), in equilibrium there is no conflict, and hence no contest, between states. Conflict resolution technology is given by \( \mathcal{CRT} = \{\text{status quo}\} \).

   The states remain independent and \( \lambda_i^{\text{independent}} = 1 \). States are free to choose tax rates, \( t \in [t^\text{min}, t^\text{max}] \).

2. **Negotiated/Contested Peace**: If \( |\mathcal{I}(i) \cap \mathcal{I}(j)| \geq 1 \) in equilibrium there is conflict between the states. For sufficiently large gains and/or small cost of contest status quo is strictly dominated and \( \mathcal{CRT} = \{\text{contest, negotiate}\} \). \( n \)-type merger takes place in the initial period, where \( CC_n \in \mathcal{I}(i) \cap \mathcal{I}(j) \).
a. *Negotiated Constitutional Union*: *negotiate* dominates *contest* for both states. 
\[ \lambda_i^* = \lambda_i^*(m | T'(n)) \]  
is given by Lemma 6. Post-merger tax rate is given by 
\[ t^* = \left[ \frac{p_i g - x_i - (C_i - F_i)}{\lambda_i g - x_i}, \frac{p_i g - x_i + (C_j - F_j)}{\lambda_i g - x_j} \right] \cap [t^\min, t^\max] \]

b. *Type I Contest*: 
\[ t^* = \left[ \frac{p_i g - x_i - (C_i - F_i)}{\lambda_i g - x_i}, \frac{p_i g - x_i + (C_j - F_j)}{\lambda_i g - x_j} \right] \cap [t^\min, t^\max] \]

c. *Type II Contest*: 
\[ t^* = \left[ \frac{p_i g - x_i - (C_i - F_i)}{\lambda_i g - x_i}, \frac{p_i g - x_i + (C_j - F_j)}{\lambda_i g - x_j} \right] \cap [t^\min, t^\max] \]

b. *Contested Constitutional Union*: *negotiate* dominates *contest* for only one of the states, say, \( j \). 
\[ \lambda_i^*(m | T'(n)), \kappa = j \] is given by (17). Post-merger tax rate is given by 
\[ t^* = t^\max \text{ if } \lambda_i^* g > x_i, \text{ else } t^* = t^\min. \]

3. *Mutual War*: If \( |T(i) \cap T(j)| = 0 \) and \( |T(i)| \neq 0 \) for at least one state there is conflict between the states. 
\[ \mathcal{CRT} = \{\text{status quo, contest}\} \] iff cost of contest is sufficiently high else \( \mathcal{CRT} = \{\text{contest}\} \). Contest takes place in every period with success being determined by contest success function, Eqn (11).

In words whenever a merger is mutually desirable and gains are modest relative to costs of contest states opt for the purely negotiated solution. Otherwise state for which contest is the dominant conflict resolution strategy dictates the solution. Note that in many cases the exact value of tax rate remains indeterminate even after introducing the possibility of contest. The following corollary should be apparent from Proposition 3.

**COROLLARY 3**: (a) The set of all possible pure strategy equilibria of Game of Merger, \( \mathcal{M} \), consists of 
\[ 1 + |\mathcal{g}| + \sum_n \sum_m |\mathcal{\Theta}^*(n, m)| \] equilibria corresponding to different combinations of merger types and contest categories, where \( \mathcal{g} \) is the set of mergers for which *negotiate* dominates *contest* for only one of the states and gains from merger are not identical. \( n \) and \( m \) refer to the type of merger and disagreement strategy, respectively. (b) The equilibria of Game

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23 An important point bears noting here. Assume for simplicity that quantity of public goods varies directly with total tax collection. What if the people of the province, which is a net donor, demand that quantity of public goods in their province should not fall below pre-merger level? This would enter the merger negotiation as an exogeneous constraint on seat sharing rule, which in turn governs revenue sharing, but one that does not affect the marginal calculus. Adding this constraint to Proposition 3 would only complicate expressions without providing any new insight. By the way this constraint is not as restrictive as it seems at first. The reason being that the output increases after merger so that even after transfers to other province there are additional resources at the disposal of the province. Furthermore whether migrant labour force enjoys public goods in province of employment or not does not affect the equilibria in Proposition 3.
of Merger, $\mathcal{M}$, are path dependent, $\lambda_\phi(m | \Theta'(n)) = f(P)$, where $P = (n, m(n))$ is the path of merger.

Path of merger refers to the choice of type of merger and the ad hoc arrangement, if any, during the period of disagreement during negotiation for the merger. Different values of $\lambda_\phi(m, n)$ relate to different merger types. The shares of the states in the union vary with type of equilibria even though the type of merger might be same. In other words we have shown that for the same initial distribution of resources and technology gap states end up with different shares in a union depending on the steps followed to implement merger, i.e. path of merger. We provide a numerical example below to illustrate this point for equilibria of $\mathcal{M}$.

**Numerical Example**

Consider the following values for endowment parameters in the example discussed in Appendix II. Technology: $A = 2B = 2$, Resources: $K_A = 0.70$, $K_B = 0.30$, $L_A = 0.80$, and $L_B = 0.20$. In Table 2 we provide share of state B in union legislature for above endowment parameters and different types of mergers. Irrespective of the route chosen backward state’s share in policy-making is far from the ideal distribution of power based on share in economic product of the union. So why does a backward state choose to merge? One reason why backward state might agree for such a merger is that complete, full merger (TTLM), in the present example, leads to 150% hike in its absolute product whereas the corresponding figure for State A is merely 7.14%. In this example the advanced state is driven towards merger by greater share of an expanded tax base whereas higher income drives the backward state to merge. Disproportionate share gained by advanced state, which entitles it to transfer some of state B’s newly generated wealth to its regions via taxation, can be interpreted as the price for technology transfer paid by the backward state.

Table 2 about here

3.1.3 Time Inconsistency

Time inconsistency in behaviour refers to mismatch between preferred ex post and optimal ex ante strategies. In the next result we show the time inconsistency of constitutional equilibria
involving Type II technology, which can be transferred over a finite time. The intuition behind the result is that once technology transfer is over the erstwhile backward state would seek revision of existing arrangement, something not accounted for in Proposition 3. Time inconsistency influences outcomes by affecting the disagreement game. For instance, in case of TTLM mergers the number of possible routes to merger and associated SSPEs are reduced since TT is no longer a strategy in the redefined disagreement game after technology transfer.

**PROPOSITION 4:** If a merger involves technology transfer then irrespective of the type of merger the equilibria supported by Proposition 3 are time consistent iff merger involves Type I technology.

This result implies that whenever Type II technology is involved in merger initial agreement has to be revised at $T^*$ because the initial arrangement is no longer sub-game perfect in all periods. In this case only an arrangement with a time-consistent revision and secession rule can be sub-game perfect. Following Proposition 4 it is easy to see that for Type I technology revision and secession rules are trivial. Further irrespective of type of technology deriving the equilibrium revenue sharing, $\beta_i^* = \lambda_i^*$, and amendment, $\alpha^* = \max(\lambda_i^*, \lambda_j^*)$, rules from seat sharing rule, $\lambda^*$, is straightforward. We, therefore, turn our attention to revision and secession rules, which brings us face to face with the problem of time inconsistency in merger agreements. As noted above only Type II technologies, which can be transferred over a finite period, are associated with time inconsistency. However, since in most of the cases technology is Type II it is, indeed, surprising that TT mergers have not been uncommon in the 20th Century. In other words, TT merger equilibrium is not superfluous. We now provide optimal secession and revision rules taking into account the problem of time inconsistency. Here it bears noting that time inconsistency problem is not faced in TTLM (TT) mergers because in such mergers final shares are based on post-technology transfer bargaining powers. So in the following propositions the problem of time inconsistency involves TT, TTLM, and TTLM (LM) mergers.

**PROPOSITION 5:** Given complete information and Property 1 states desirous of forging a union under the shadow of contest commit to $SR^*$, which is the unique sub-game perfect *Secession Rule:*
(1) Type I Technology and pre-technology transfer Type II Technology: \( SR^*(g \leq 0 \text{ or } t \notin [t_{\min}, t_{\max}]) = \text{(Exit, No War)} \) and \( SR^*(g > 0 \text{ and } t \in [t_{\min}, t_{\max}]) \in \{(\text{Continue, No War}), (\text{Exit, War})\} \). For Category I and II contests \( SR^* \) is sub-game perfect iff 
\[ \rho \left( \lambda_i g - x_i \right) \left( C_i - F_i \right) - x_i \forall l \in \{i, j\}, \text{ respectively.} \]

(2) Post-technology transfer Type II Technology: \( T^* \) is the time required for technology transfer

(a) \( \mathcal{I}(i) = \mathcal{I}(j) = \mathcal{I}(i) \cap \mathcal{I}(j) = \{\text{TT}\}: SR^* (g, t|\Gamma \geq T*) = \{\text{Exit, No War}\} \) and Proposition 3 (case 1) applies (dissolution followed by peaceful coexistence),

(b) \( \mathcal{I}(i) \cap \mathcal{I}(j) = \{\text{TT}\} \) and \( |\mathcal{I}(i)| \text{ and/or } |\mathcal{I}(j)| \geq 1: SR^* (g, t|\Gamma \geq T*) = \{\text{Exit, No War}\} \) and Proposition 3 (case 3) applies (dissolution followed by contests),

(c) \( |\mathcal{I}(i) \cap \mathcal{I}(j)| \geq 1 \) and INV = 0: \( SR^* (g, t|\Gamma \geq T*) = \{\text{Exit, No War}\} \) and Proposition 3 (case 2) applies (dissolution followed by renegotiation), and

(d) \( |\mathcal{I}(i) \cap \mathcal{I}(j)| \geq 1 \) and INV = 1: Case (1) above applies (constitutional accommodation).

The secession rule is optimal in the sense that as long as union generates additional benefits from synergy none of the province secedes otherwise the rule allows for peaceful dissolution. By peaceful we mean that there is no contest related to existing merger arrangement. But this does not rule out later contests. Further even though in equilibrium there is no secession still if it so happens that one of the states has an infinitesimal tendency to stray from the equilibrium in some period then the other state would punish the deviant as long as conditions specified in Proposition 5 are satisfied for both states. If, however, the conditions are fulfilled only for one of the states, say \( j \), then when state \( i \) secedes in contravention of secession rule then state \( i \) has to face contest immediately in the same period but not vice versa. But if neither of the provinces can fulfill the requirement to punish the other province in case the latter deviates from agreement then our constitution does not have the capacity to restrain defections. In other words technology of contest provides limited internal capacity to the constitution to check non-equilibrium secessionist behaviour, if any. Finally we state the optimal revision rule.

**Proposition 6:** Given complete information states desirous of forging a union under the shadow of contest commit to \( C2^* \), which is the unique sub-game perfect Revision Rule, where \( Z \in \{CC_{\gamma}, SR, \lambda, t\} \) and \( Z(\Gamma = \Gamma) = Z(\Gamma') \), where \( \Gamma = 0, 1, 2, 3 \ldots \)

1) \( C2^*(\Gamma < T^*, \gamma \in \{I, II\}) \) and \( C2^*(\Gamma \geq T^*, \gamma = I) : Z(\Gamma + 1) = \Psi(Z(\Gamma)) = Z(0) \)
2) $C_2^* (\Gamma \geq T^*, \gamma = II)$:

a. Proposition 5 (case 2a-2c): $Z(\Gamma) = \phi$.

b. Proposition 5 (case 2d): $Z(\Gamma + 1) = \Psi(Z(\Gamma)) = Z(\Gamma = T^*)$, where $CC_n(\Gamma \geq T^*) 

\in \mathcal{E}(i) \cap \mathcal{E}(j) \setminus \{CC_n(\Gamma < T^*)\}$, which in turn determines rest of the articles through $\lambda(\Gamma \geq T^*)$ following Proposition 3 (case 2).

The important point about Proposition 5 and 6 is that irrespective of type of technology neither of them is time-inconsistent, which means that we are not using time-inconsistent revision and secession rules to resolve the time inconsistency of other articles. To summarize our findings so far note that whenever possible in equilibrium merger takes place in the initial period itself24 without any contest under the *time consistent contested constitution*, $C^* = (C_0^* - Proposition 1 and 5, C_1^* - Proposition 3, C_1a^* - Proposition 3, C_2^* - Proposition 6, C_3^* - Proposition 3) and there is no secession in contravention to secession rule.

One last point before we close this section. Despite complete information and threat of contest there are limits to what an advanced state can extract for transferring technology. In the next proposition we specify the upper limit in case of TT mergers. Results for other types of mergers involving technology transfer can be obtained similarly.

**Proposition 7:** The fraction of additional product generated due to technology transfer in a TT merger, which advanced state, say $i$, can extract through tax transfers is bounded from above by $\lambda_i(\Gamma = 0) t_{\text{max}} < 1$.

### 4.0 Extensions and Qualifications

#### 4.1 Demographic heterogeneity

We suggested above that demographic heterogeneity could play a role like nature or history in constraining the set of desirable mergers (see Footnote 18). Heterogeneity can be of different types, say, ethnic, economic, or both. Economic heterogeneity can result from unequal material and/or technological endowments. Another way of looking at heterogeneity is to

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24 With some modifications to this section we can also construct equilibria which support delayed mergers.
differentiate between inter-state and intra-state heterogeneity. Figure 5 summarizes the entire range of possibilities in this regard. Type I (inter-state economic) heterogeneity has been addressed in the benchmark model in Section 3. In case of Type VI heterogeneity there is no incentive for merger so we will not discuss this case here. Note that in our discussion below we assume intra-group homogeneity, absence of cross-border collaboration between groups, and absence of possibility of contests. The last assumption restricts us to the negotiation model presented in Section 3.1.1 \(^{25}\). In addition we make a very restrictive assumption regarding intra-state ethnic heterogeneity, namely, the same ethnic distribution applies to both states.

Figure 5 about here

4.1.1 *Type II (intra-state and inter-state economic) heterogeneity*

Assume intra-state economic heterogeneity in the form of population divided into two classes, namely, labour and capital. In absence of resource mobility return to a factor in a state is inversely proportional to its availability in that state. The exact relationship is immaterial as long as it implies inverse proportionality so that owners of scarce resource earn scarcity rents. The identity of scarce resource is immaterial. All we need is scarcity of a resource and a group that can extract rents. Disproportionate division of the product captures the relative powers of the two groups and is also the source of conflict between them, which can be eliminated only when resource mobility is permitted. The group holding scarce input has a vested interest in maintaining the asymmetry. Consequently owners of scarce resource have a disproportionate power if every merger proposal has to be cleared by each group within a state before it is negotiated with other state. The game for choice of merger transforms into a nested or two-level game with the first round being played within each state followed by another round between states in case of intra-state agreement in favour of inter-state merger negotiation.

The key insights from the above model are as follows. Mergers involving resource mobility are never agreed to domestically as long as each economic interest group has a veto within a state. The veto of the group having excess resources is, in fact, redundant because in equilibrium it does not use its veto. The owners of scarce resource in a backward state would

\^{25} Details of the extension are available from author. Here we will discuss the intuition behind the model and key insights.
support a full merger with technology transfer and resource mobility only if technology gap is sufficiently large. Otherwise this group would agree to technology transfer and limited resource mobility, i.e. an incomplete merger. In the limiting case it would veto any merger that includes more than technology transfer. In this latter case if there is an apprehension that the union might impose resource mobility in future even agreeing to merger involving only technology transfer becomes a dominated strategy for owners of scarce resources. In an advanced state the owners of scarce resource veto every type of merger proposal. In other words a merger which is Pareto optimal for the states as a whole might not be implemented once we introduce even minimal economic heterogeneity in the population of at least one state, with each interest group having a veto over choice of merger options. More importantly this result is irrespective of the identity of mobile resource.

4.1.2 Type III (intra-state ethnic and inter-state economic) heterogeneity

Type III heterogeneity is akin to Type I (inter-state economic heterogeneity) because in our model individuals within a state have equal labour and capital resources. So there is no economic dependence on co-citizens, including those belonging to other ethnic group. The issue of negative returns from working with ethnic others arises only when one has to collaborate with capital or labour resources of others.

4.1.3 Type IV (inter-state ethnic and economic) heterogeneity

Assume that states are ethnically different. For a while assume that only labour is mobile. In addition to earning returns to their respective unemployed factor inputs immigrant labour from labour surplus province and owners of excess capital in capital surplus province both suffer some disutility from engaging with ethnic strangers. For sufficiently large ethnic antipathy labour mobility becomes a dominated strategy for at least one of the states. It is in such cases that the identity of mobile resources matters in our model because states could bargain on a third dimension namely capital mobility (CM). Now TTLMCM is full merger while TTLM, TTTCM, LMCM, LM, CM, and TT are partial mergers. Disagreement strategies now include CM, in addition to TT, LM, and NM. Since our analysis holds for the case
involving \( n \)-types of mergers and \( m(n) \) disagreement strategies we can very easily introduce capital mobility into the model without affecting the structure of the model.

4.1.4 Type V (intra-state ethnic and economic and inter-state economic) heterogeneity

Introducing (perfectly) correlated intra-state economic and ethnic heterogeneities (Type Va) does not lead to new outcomes because this case is structurally akin to Type II (intra-state economic) heterogeneity. Further the case involving uncorrelated intra-state economic and ethnic heterogeneities (Type Vb) can also be reduced to Type II heterogeneity if we recognize that without being tied to systematic economic differences ethnicity does not have any influence in our model.

4.1.5 Discussion

The first casualty in all cases above, in particular in Section 4.1.1 and 4.1.3 \( (supra) \), is labour mobility, which becomes less favoured once we introduce heterogeneity. With introduction of heterogeneity the model can support incomplete mergers contrary to Proposition 1 \( (supra) \). In case of inter-state ethnic heterogeneity capital mobility is never a dominated strategy even though labour mobility might be whereas in case of intra-state economic heterogeneity any kind of resource mobility could be dominated. To conclude in our model intra-state ethnic heterogeneity and inter-state economic heterogeneity are not restrictive while inter-state ethnic heterogeneity and intra-state economic heterogeneity can severely constrain the set of desirable mergers. Of the two the latter is more restrictive. Also note that unequal treatment of erstwhile foreign labour in a prospective union has non-trivial consequences in presence of ethnic heterogeneity. For instance, labourers from a labour surplus state could accept lesser wages, but more than their inside options, and become more acceptable in an ethnically different province.

Possible extensions not yet implemented are as follows. One, interacting demographic heterogeneity with labour force and/or technology differentiated by skill levels; intuitively the key change because of this would be that unemployment rate would no longer provide an
upper limit to resource migration. Two, interacting divergent institutional preferences (unitary/federal, majoritarian/proportional, official language and religion, etc) with heterogeneity; here the problem is especially acute with time inconsistent mergers, where the prospective recipient of technology might accept certain repugnant institutions as a temporary expedient. In both the cases the set of mutually desirable mergers would be constrained. In fact, lexicographical preferences, with institutions being prior, can completely rule out merger. Three, replacing the veto with a policy contest between labour and capital within states. In this case the only difference is that with a positive probability the veto of owners of scarce resources is void, which yields two structurally independent cases corresponding to Section 3.0 and Section 4.1.1. Last but not the least are issues like possibility of cross-border class and/or ethnic collaboration and contests.

4.2 Role of information

Whenever nature does not reveal $N$ (see Section 3.0, supra) fully to at least one of the states there is asymmetric information and consequently there would be incentives to misrepresent the same under certain conditions. States could potentially use private information to seek larger share in union. However, even if a state were to gain some advantage in this way the other state would figure out the discrepancy without much delay after formation of the union and seek revision through C2, C3, or outright contest. While misrepresenting private information to its advantage a state has to take into account such future setbacks. So the choice to misrepresent has to be dynamically consistent and efficient. Asymmetric information could lead to more contests after formation of unions in case of pooling equilibria.

4.3 Role of Saving

Introducing common saving rates across states does not affect relative bargaining power in negotiations and rate of success in contests over time. It, however, makes expensive contests affordable and necessitates inclusion of more categories of contest. Introducing more

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26 This might not be possible always. Numerous instances of stalled or rigged census exercises across the globe are a case in point (Horowitz 2000 [1985]). Likewise it might not be feasible to carry out adequate survey of capital resources in troubled provinces of a union.
destructive or appropriate types of contest raises a number of questions. What if a backward state emerges winner in a contest and manages to capture some labour and capital of advanced state. Has the backward state managed to appropriate the advanced technology as well? Even if we were to ignore this savings might help a backward state to buy advanced technology through lump sum transfer. However, the amassed wealth can itself be targeted by the advanced state undercutting the backward state’s ability to follow such a course of action. Advanced state’s action would be governed by two motivations, namely, greed and fear. The former is obvious while the latter arises because if the advanced state does not save as much as backward state it might end up losing a future contest when the backward state would be able to organize more resources for contest. Possibly in equilibrium we would be back to the situation where both sides agree to a merger in initial period leading to payment through tax transfers rather than outright purchase at some future date. Unequal savings rates are though more problematic even under complete information because this new feature destabilizes the basic process of bargaining itself by changing relative bargaining power with time in a non-trivial manner.

4.4 Time inconsistency and permanent resource mobility

Irreversible resource mobility changes the disagreement game. For this very reason a state that is going to lose bargaining power should not agree for permanent migration of its surplus resources. Consider two cases: (a) labour surplus backward state and (b) labour surplus advanced state. Recall that individuals own both labour and capital endowments. In the latter case moving to backward state with capital is not profitable. So surplus labour of an advanced state would relocate to the backward, labour deficient state while retaining capital in home state where productivity is higher and capital is scarce. In the former case, labour surplus backward state, migrants could profit from migrating along with their share in capital whereas in case of sufficiently high technology gap even migration sans capital is beneficial. In this case to rule out the possibility of permanent migration with or without capital it is sufficient (for the purpose of Section 3.0) to assume that states do not allow permanent transfer of capital resources and the technology gap between the two states falls below a certain threshold so that it does not make sense to migrate permanently leaving behind income from capital. However, further work is required to establish the conditions under which a state would agree to irreversible resource transfer.
5.0 Concluding Remarks

5.1 Recapitulation

This paper provides a model for optimal design of constitution for political union between unequally endowed states. We propose a constitution and then find the conditions under which negotiation equilibrium exists for a merger leading to a unitary state. We introduced a contested constitution to take into account the changes in negotiated solution under the threat of contest. The impact of contest is twofold. Firstly, it constrains the set of optimal taxes. Secondly, and more importantly, it increases the stability of constitution by making the punishment strategy in secession rule credible. The equilibrium in case of contested constitutional agreement is shown to be path dependent and, in some cases, time inconsistent. To address the latter issue we introduced a time consistent contested constitution. We also show that the rents extracted by the advanced state in the union for transferring technology to the backward state are increasing in complexity of technology and that there are limits to such rents. We then show that demographic heterogeneity constrains the set of mutually desirable mergers. Finally we draw attention to a few unresolved issues, e.g., impact of savings and permanent resource migration on inter-state mergers.

5.2 Historical cycles of political geography

In addition to the above our model supports the possibility of cycles in state size driven solely by technology and material endowments. In our model in certain cases unions dissolve after completion of technology transfer (see Proposition 5, supra). However, merger can take place again in future if technology levels diverge again. This logic applies to any permanently transferable resource. (Currently in our model certain kinds of technology are the only permanently transferable resources.) In other words historical cycles in state size, i.e. political geography, might be driven among other things by rate of diffusion of technology and other resources, including capital and labour. In our model discord is limited while diffusion is in progress and after that cooperation is limited.
5.3 Empirical evidence

We will now look at the experience of a few post-1945 political unions in light of the above discussion. Table 3 lists a few examples discussed below. It bears noting that we are drawing attention to association between different types of merger in our model and these cases without denying the significance, or even primacy, of other factors, which might have governed the actual outcomes.

Table 3 about here

Case I: Let us begin with the ethnic minority, peripheral provinces of India and China, which in principle were united with the rest of the state on lines of a TT (technology transfer only) merger. The centre committed itself to uplifting the economic status of the region, i.e. transferring technology, while allowing these provinces to decide when they would like to extend the merger to include labour mobility. However, for all practical purposes secession is ruled out due to the overwhelming dominance enjoyed by the centre in these countries. But in both the countries the provinces merged under TT merger scheme have proved to be restive, which relates to the time inconsistency problems in such mergers pointed above. There have been two kinds of problems. One, in each of these cases the two sides disagree over the right to secede. The most well-known example of this is the 9th point of the famous Naga-Akbar Hydari Accord (1947).

Period of Agreement – The Governor of Assam as the Agent of the Government of the Indian Union will have a special responsibility for a period of 10 years to ensure the observance of the agreement, at the end of this period the Naga Council will be asked whether they require the above agreement to be extended for a further period or a new agreement regarding the future of Naga people arrived at.

The Naga National Council, a “separatist” underground outfit, interpreted this point as supportive of secession after 10 years while the Government of India read it as a provision for amending the Accord in light of experience. No wonder the accord was still born (See Kumar 2005 for details of this case). Divergent evaluation of progress made due to merger has been another source of disagreement between the two sides in these cases. On the one hand the provincial leaders claim that the actual development falls far short of initial promises and/or that “outsiders” have appropriated the gains, if any. On the other the centre negates these claims but at the same time one can figure out that it is also concerned that rapid development would hand over more resources to “secessionists”. Post-Cold War India and China have
handled this latter problem in their own ways. Incidentally Indian provinces incorporated under TTLM (LM) and TTLM merger types of arrangements have not seen similar developments.

Case II: Next consider the German re-unification. The re-unification was carried out under a TTLM (technology transfer and labour mobility) merger scheme, which according to our model implies lesser bargaining power for capital constrained, backward East Germany. It is indeed well-known that West Germany dominated the entire process of merger ranging from choice of constitution to fate of ailing industries of the East. In India Sikkim was merged under TTLM scheme and it is well known that it enjoys lesser bargaining power vis-à-vis Nagaland, etc, which were incorporated under TT merger. This disparity between Sikkim and Nagaland is in line with our model.

Case III: The South Korean approach to re-unification rests on two assumptions. One, a gradual movement towards merger will help the North to catch up so that the cost of re-unification to South Koreans is minimized. Two, as North Korea recovers economically it would be more confident about its ability to cope with life after re-unification. It would not be sacrilegious to label this approach as a step towards de facto TT merger. Our analysis suggests that in the process North Korea will improve its bargaining power, which seems to be the implicit policy of South Koreans, who think that a weak, and therefore paranoid, North Korea will be unpredictable.

Case IV: Finally recall that our model suggests that the union is stable only if the province opposed to secession can credibly threaten to punish the potentially secessionist province. The most convincing empirical support for this comes from the erstwhile USSR. When Ukraine and the Baltic republics wanted to secede in the 1940s and 1950s the USSR survived due to Russian willingness to contest the demand for secession. But the USSR could not survive the Russian attempt to secede in the late 1980s. See, for instance, Hale (2005) in this regard who argues that a state cannot survive secessionist attempt by the core province(s), where the latter is the preponderant sub-state entity.
Appendices

Appendix I: Proofs of results

Proof of Proposition 1: It is advisable to go through this proof after reading Sections 2 and 3.
(a) \( \theta_r \in [0,1] \) and \( \theta_s \in [0,1] \) denote, respectively, fraction of capital in backward province for which advanced technology is provided and fraction of unemployed labour force of labour surplus province permitted to work in labour deficient province. Pay-off to state \( i \) from \( n \)-type merger is given by the following expression, where \( M^n_l \) is economic product of state \( l \) under \( n \)-type merger and \( l \in \{i, j\} \):

\[
U^n_i = M^n_i (1-t) + t \lambda_i \left( M^n_i + M^n_j \right) = M^n_i + t \left( \lambda_i \left( M^n_i + M^n_j \right) - M^n_i \right)
\]

We know that \( SQ_l \leq M^n_i \quad \forall l \in \{i, j\} \) and \( \forall n \), where \( SQ_l \) is pay-off to state \( l \) from No Merger. We can substitute \( SQ_i + x^n_i \) for \( M^n_i \), where \( x^n_i \geq 0 \) is the additional economic product of state \( l \) in case of \( n \)-type merger and \( x^n_i > 0 \) for at least one \( l \).

\[
U^n_i = (SQ_i + x^n_i)(1-t) + t \lambda_i \left( SQ_i + x^n_i + SQ_j + x^n_j \right) = (1-t)SQ_i + t \lambda_i \left( SQ_i + SQ_j \right) + \left( t \lambda_i \left( x^n_i + x^n_j \right) + x^n_i (1-t) \right)
\]

First consider an incomplete LM only merger \((n = 1)\). Excess labour from one state works with excess capital of the other and the economic product is divided equally between the two. Thus, additional economic product accruing to each state when labour mobility is unlimited is given by \( x^1_i = x^1_j \). When only \( \theta_L \) fraction of labour is allowed mobility then the additional economic product accruing to each state is given by \( \theta_L x^1_i = \theta_L x^1_j \). The pay-off to state \( i \) and \( j \) in this case is given by the following, both strictly increasing in \( \theta_L \).

\[
U^1_i = (1-t)SQ_i + t \lambda_i \left( SQ_i + SQ_j \right) + \left( t \lambda_i \left( \theta_L x^1_i + \theta_L x^1_j \right) + \theta_L x^1_i (1-t) \right)
\]

\[
U^1_j = (1-t)SQ_j + t \lambda_j \left( SQ_i + SQ_j \right) + \left( t \lambda_j \left( \theta_L x^1_i + \theta_L x^1_j \right) + \theta_L x^1_j (1-t) \right)
\]

Now consider an incomplete TT only merger \((n = 2)\). Let \( i \) be the advanced state. Since in this type of merger the advanced state \( i \) merely transfers technology there is no increase in its
economic product, $x_i^2 = 0$, even though it benefits from tax based transfers. The pay-off to state $i$ and $j$ in this case is given by the following both of which are strictly increasing in $\theta_r$.

$$U_i^2 = (1-t)S_Q + t\lambda_i(S_Q + S_Q) + \left(t\lambda_i(x_i^2)\right)$$

$$U_j^2 = (1-t)S_Q + t\lambda_j(S_Q + S_Q) + \left(t\lambda_j(x_j^2) + \theta_r x_j^2(1-t)\right)$$

Now it is easy to show that pay-off to state from multi-issue mergers is also strictly increasing in both $\theta_r$ and $\theta_L$. Note that we have not assumed any particular distribution of resources and the assumption of state $i$ being advanced is without loss of generality. It remains to be pointed out that in absence of any other constraints, which is the case in our model so far (Sections 2 and 3), both states irrespective of their technological and material endowments strictly prefer a complete merger ($\theta_r = 1$ and/or $\theta_L = 1$) to incomplete merger ($\theta_r \in [0, 1]$ and/or $\theta_L \in [0, 1]$) $\forall n$, which establishes the result. $QED$

(b) Let us assume the contrary to be true. If so then an incomplete merger of $n_i$-type would be preferred to a complete merger of $n_j$-type. However, if an incomplete merger of $n_i$-type is preferred we know from the first part of this proposition that then a complete merger of $n_i$-type would be strictly preferred to both a complete merger of $n_j$-type as well as an incomplete merger of $n_i$-type. So the two states would conclude a merger agreement for a complete merger of $n_i$-type rather than an incomplete merger of $n_i$-type, which leads to the claim made in the proposition. $QED$

**Proof of Lemma 1:** (a) Suppose $\beta_i = \lambda_i$ is not the equilibrium. Instead assume $\beta_i > \lambda_i$ to be the equilibrium. $\beta_i > \lambda_i \Rightarrow \beta_j = 1-\beta_i < \lambda_j = 1-\lambda_i$. However, $\lambda_i$ is by definition the seat sharing equilibrium and is therefore reflective of the relative bargaining power of the states. Cognizant of its real bargaining power State $j$ would renegotiate and achieve $\beta_j \geq \lambda_j \Rightarrow \beta_j = 1-\beta_j \leq \lambda_i$. Similarly, if $\beta_i < \lambda_i$ then State $i$ would renegotiate and achieve $\beta_i \geq \lambda_i$. Combining the two we conclude that in equilibrium $\beta_i \neq \lambda_i$ cannot be true. $QED$

(b) Suppose $\alpha$ is not the equilibrium. Instead assume $\alpha^* = \min(\lambda_i, \lambda_j)$ to be the equilibrium. Since the two states are unequally endowed $\lambda_i \neq \lambda_j$, so that $\sum_i \lambda_i = 1 \Rightarrow \lambda_i \neq 0.5$
In this case $\alpha'$ is more relaxed than simple majority voting rule and violates the requirement of stability because partisans of both provinces can unilaterally amend the constitution. Next assume $\alpha' \in \left[ \min(\lambda_i, \lambda_j), \max(\lambda_i, \lambda_j) \right)$ to be the equilibrium. If this is true then immediately after merger State $i$ with higher $\lambda$ can amend the constitution, including C1 where $\lambda$ is specified, to its advantage. This contradicts the fact that $\lambda$ is by definition equilibrium seat sharing rule. Here $\alpha'$ violates the requirement of inclusiveness because a priori the weaker state would find it sub-optimal to join the union because it is completely defenceless against moves to reduce its seat and revenue share in union. The weaker state would therefore not join the union the moment $\alpha'$ is known to be the initial amendment rule.

Thus the set of equilibrium amendment rules satisfying inclusiveness is given by $C3^{Incl} = \{ \alpha \mid \alpha \geq \max(\lambda_i, \lambda_j) \}$. Under such an amendment rule the stronger province of the union would need the support of at least one legislator from the weaker province to successfully implement an amendment advantageous to it. Any legislator from weaker province who votes for the amendment does so if and only if the change is beneficial for him. Since individuals in each state are identical (see Section 2.2) it ensures that such a change would be beneficial for all others in his province as well. Further, it is easy to see that only amendment rules that satisfy inclusiveness also satisfy the requirement of stability. So, $C3^{Incl} = C3^{Stable}$. To account for flexibility we choose the most liberal amendment rule. Thus in equilibrium constitutional amendment rule is given by $C3^* = \min (C3^{Incl} \cap C3^{Stable}) = \max(\lambda_i, \lambda_j)$. $QED$

Proof of Lemma 2: SR depends on two factors: the stream of additional economic product generated by virtue of merger ($CC_n$, C1 and C1a) and the ability of union to restructure via C2 and C3 in face of crises. But we have shown in Lemma 1 that C1a and C3 are pegged to agreement on C1. Further revision rule C2 depends on C1 since the extent of revision required depends on the initial arrangement (C1, C1a, C3, SR, and CC_n). Now it is easy to see that CC_n and C1 are the only independent (sub-) articles, respectively, in Part I and II of constitution. $QED$
**Proof of Lemma 5**: The result is trivially true for $|S_i(n)| = 1$. For $|S_i(n)| > 1$ let $V = \Theta(n) \setminus \{NM, NM\}$, i.e. there are $|V| = |\Theta(n)| - 1$ equilibria of disagreement game which are different from (NM, NM) and support some kind of partial merger. Key to establishing the equivalence is that pay-off in $\Theta'$ corresponding to $v$th disagreement equilibrium is a weighted average of two quantities with one of them necessarily greater than the (NM, NM) pay-off while the other is greater than or equal to the latter depending on the strategy chosen. Recall that there is one tax rate across the unitary union, say, $t$. The equivalence suggested in the lemma follows if the following requirement is satisfied $\forall l \in \{i, j\}$ and $\forall v$, where revenue share $\beta_i = \lambda_i$, vide Lemma 1.

$$SQ_i < M_i^v (1-t) + t\lambda_i (M_i^v + M_{[v,j]}^v)$$

where $SQ_i$ is No Merger disagreement pay-off of state $l$ and $M_i^v$ is the economic product of state $l$ under $v$th equilibrium of disagreement game, $\Theta'$, where $v \in \{1, 2, \ldots |V|\}$. We know that the following always holds, $SQ_i \leq M_i^v \ \forall l \in \{i, j\}$ and $\forall v$. We can substitute for $M_i^v$ with $SQ_i + x_i^v$, where $x_i^v \geq 0$ is the additional economic product of state $l$ and $x_i^v > 0$ for at least one $l$. The above condition can now be recast as follows:

$$SQ_i < (SQ_i + x_i^v) (1-t) + t\lambda_i (SQ_i + x_i^v + SQ_{[v,j]}^v + x_{[v,j]}^v)$$

This last condition can be recast by substituting $\lambda_i = SQ_i / (SQ_i + SQ_{[v,j]}^v)$

$$0 < x_i^v (1-t) + t\lambda_i (x_i^v + x_{[v,j]}^v)$$

which is true for $\forall t \in (0,1)$, $\forall l$, and $\forall v$. Hence, all equilibria of normal form disagreement game $\Theta'(n)$ lie along the diagonal. The justification for the substitution $\lambda_i = SQ_i / (SQ_i + SQ_{[v,j]}^v)$, which is not immediately apparent, would bear some elaboration. The $v$th equilibrium of disagreement game corresponds to either TT or LM merger. The substitution for $\lambda_i$ suggested is actually the corresponding $\lambda_i$ for TT or LM merger derived using Lemma 3. And here it is very important to note that we are not proceeding along a circular loop. For TT and LM mergers $|S_i(n)| = 1$ so the tax adjustment is trivial and tax adjusted game is identical, and not merely strategically equivalent, to the unadjusted game as indicated at the very beginning of this proof. So when we carry out the above substitution we
are not using unadjusted seat shares to show equivalence between adjusted and unadjusted disagreement games.

To obtain $\Theta''$ we need to add an infinitesimal quantity $\varepsilon(>0)$ to the pay-off of State $A$ corresponding to the equilibrium (TT, TT) in $\Theta$ to preserve strict preferences in $\Theta'$. (Without this addition we get an additional, but pay-off and also outcome equivalent, equilibrium.) With this adjustment it is easy to see that the strategic choices available to players in disagreement games $\Theta'$ and $\Theta''$ are identical and lead to identical equilibria. QED

**Proof of Lemma 7:** In case of unequal endowments $\lambda_i \neq \lambda_j$ follows from Lemma 4. The total pay-off to state $i$ from $n$-type merger is given by the following expression:

$$U_i^n = E_i^n(1-t) + t\lambda_i(1+E_i^n + E_j^n) = E_i^n + t(\lambda_i(E_i^n + E_j^n) - E_i^n)$$

which is strictly non-decreasing in $t$ if and only if $\lambda_i^n \geq E_i^n / (E_i^n + E_j^n)$, the condition stated in the lemma, holds. Equivalence between this condition and $\lambda_i^n g^n - x_i^n \geq 0$ arises from the following substitutions $E_i^n = d_i^n + x_i^n$ and $\lambda_i^n = d_i^n / (d_i^n + d_j^n)$. $d_i^n$ is the disagreement pay-off. For such a state its contribution to union tax revenue is less than what it gets back and thus it prefers highest possible tax rate and likewise we can argue the other way round. QED

**Proof of Proposition 2:** Recall that $|F_l| < C_i$, where $l \in \{i, j\}$. Assume without loss of generality that $\lambda_i g - x_i > 0$. Nature chooses a contest category for both the states. We will first determine the conditions under which certain CRT(s) dominate other CRTs for a state for a given type of merger. Using (15) - (18) it is easy to show that the following hold:27

a) **Status Quo** is strictly dominated by **Contest** for state $l$ for $\kappa = I$ and $II$ if $p_i > (C_i - F_i)$ and $p(\lambda_i g - x_i) + x_i > (C_i - F_i)$, respectively.

b) **Negotiate** always dominates **Status Quo**.

27 Note that in each case possibility of contest increases with increase in size of potential surplus generated through cooperation or increase in probability of success of one of the states.
c) *Negotiate* dominates *Contest* for state $i$, state $j$, and both if $t(i) \neq \phi$, $t(j) \neq \phi$, and $t(i) \cap t(j) \neq \phi$, respectively.

**Type I contest:**

\[
\begin{align*}
    t(i) &= \left\{ t \mid t \geq t_i \quad \& \quad t_i \leq t_{\max} \right\}, \text{ where } t_i \left( \lambda_i g - x_i \right) = p_i g - \left( C_i - F_i \right) - x_i \\
    t(j) &= \left\{ t \mid t \leq t_j \quad \& \quad t_j \geq t_{\min} \right\}, \text{ where } t_j \left( \lambda_i g - x_i \right) = p_j g + \left( C_j - F_j \right) - x_j
\end{align*}
\]

**Type II contest:**

\[
\begin{align*}
    t(i) &= \left\{ t \mid t \geq t_i \quad \& \quad t_i \leq t_{\max} \right\}, \text{ where } t_i = \rho - \left( C_i - F_i \right) / \left( \lambda_i g - x_i \right) \\
    t(j) &= \left\{ t \mid t \leq t_j \quad \& \quad t_j \geq t_{\min} \right\}, \text{ where } t_j = \rho + \left( C_j - F_j \right) / \left( \lambda_i g - x_i \right)
\end{align*}
\]

Now we proceed in two steps. (1) First we establish that $t(i) \cap t(j) = \phi$ is impossible for Category II contests. (2) Then we show that for Category I contests $t(i) \neq \phi$ and $t(j) \neq \phi$ cannot be simultaneously true.

(1) For Category II contests $t(i) \cap t(j) = \phi$ is possible only if one of following holds: a) \[ \rho + \frac{C_j - F_j}{\lambda_i g - x_i} < \rho - \frac{C_i - F_i}{\lambda_i g - x_i}, \] b) \[ t_{\max} < \rho - \frac{C_i - F_i}{\lambda_i g - x_i}, \text{ or } \] c) \[ \rho + \frac{C_j - F_j}{\lambda_i g - x_i} < t_{\min}. \] Given that \[ \rho \in [t_{\min}, t_{\max}] \] is always true none of the above hold. Therefore *contest* is always dominated by *negotiate* for both states for Category II contests.

(2) For Category I contests $t(i) \cap t(j) = \phi$ is possible only if one of following holds: a) \[ \frac{p_i g - \left( C_i - F_i \right)}{\lambda_i g - x_i} - x_i > \frac{p_j g + \left( C_j - F_j \right)}{\lambda_i g - x_i}, \] b) \[ \frac{p_i g + \left( C_j - F_j \right)}{\lambda_i g - x_i} - x_i < t_{\min}, \text{ or } \] c) \[ \frac{p_i g - \left( C_i - F_i \right)}{\lambda_i g - x_i} - x_i > t_{\max}. \] Now assume that *contest* dominates *negotiate* for state $i$, which implies the following \[ t \left( \lambda_i g - x_i \right) < p_i g - \left( C_i - F_i \right) - x_i. \] Contrary to our assertion further assume that the same is true about the state $j$ as well. This implies \[ t \left( \lambda_i g - x_i \right) < p_j g - \left( C_j - F_j \right) - x_j. \] Combining the two we obtain \[ 0 > \sum_i \left( C_i - F_i \right), \] which cannot be true. This implies that *contest* cannot dominate *negotiate* for both states.
simultaneously. Further it is easy to see that a) does not hold under any condition and combining b) and c) results in an impossibility \((t_{\text{max}} < t_{\text{min}})\), which leads to the claim that for this category of contests contest dominates negotiate at most for one of the sides.

\[ QED \]

**Proof of Proposition 3**: 1) This case is straightforward. Here according to nature’s draw neither is interested in any kind of merger.

2) First note that by virtue of Proposition 2 we do not have a case where Negotiate is dominated by Contest for both states.

a) We are in regime \(\mathcal{N}\) since neither state’s outside option is more attractive than their respective inside options. It is as if contests are unknown. Equilibrium seat sharing rule \(\lambda_i\) is given by Lemma 6 while tax rate is determined using \(\lambda_i\) subject to \(t(i) \cap t(j) \neq \emptyset\) (see proof of Proposition 2, supra).

b) In this case outside option of one state (here \(i\)) dominates the strategic interaction so that the other state (here \(j\)) makes an offer so as to make the former indifferent between contested and negotiated solutions or accepts a corresponding offer. Now by virtue of Property 1 (b) state \(i\) settles for negotiated solution dictated by its outside option. Note that \(\sum_{j} E_j^n\) for Category I \((\kappa = I)\) contests as long as \(\text{FC}_{\text{Gp}} - > \text{In}_{mij} = \Theta \neq \lambda_{\text{I}}\) in case negotiate dominates contest only for state \(j\).

3) State with \(|\pi(l)| \neq 0\) will precipitate contest every period because the other state is unwilling to consider any type of merger. Given that first strike advantage is non-zero contest is never strictly dominated. So the other state would at least try to avoid loss due to defensive contest strategy and will resist every period. \[ QED \]

**Proof of Proposition 4**: After technology transfer is complete disagreement game changes in favour of erstwhile backward state, say \(j\), which implies \(\eta_j(\Gamma \geq T^*) > \eta_j(\Gamma < T^*)\). Assume that contrary to the claim in the proposition equilibria described in Proposition 3 are also time
consistent for Type II technology. If true this implies that erstwhile backward state chooses not to seek revision of sharing rules. In other words it prefers to continue with the existing arrangement despite the fact that its winning probabilities and bargaining power have increased. But the pay-off function (see Eqn 10) is strictly increasing in shares in legislature seats and revenue, and therefore the economic product, accruing to a state, which brings us to a contradiction. Therefore, after completion of technology transfer the initial agreement has to be revised. If not the challenger withdraws from the union and in a fresh round of merger negotiation/contest it emerges better-off. Hence the claim in the proposition that constitution for mergers not involving technology transfers at all or involving Type I technology are trivially time-consistent.

**QED**

**Proof of Proposition 5:** (1) With Type I technology post-merger gains from merger, \( g \), are never zero so if a merger agreement takes place it remains in place till perpetuity. It is easy to verify that agreements characterized by Proposition 3 are self-enforcing since neither state has an incentive to withdraw from the union. From Proposition 4 we know that with Type II technologies equilibria described in Proposition 3 are not sub-game perfect because the underlying disagreement game changes starting from \( \Gamma = T' + T^* \), where \( T' \) is the period in which merger agreement is arrived at (= 0 in our case), which will be addressed later.

Assume \( \lambda, g - x > 0 \), i.e. \( i \) is the net recipient of transfers, without loss of generality. There are only two possibilities. One, \( g = x_i + x_j > 0 \) and \( x_i > 0 \) \( \forall i \in \{i,j\} \). Province \( j \) retains \( x_j - t(\lambda, g - x_i) > 0 \) of the additional product generated within its domain even after tax based transfers to the other province. So as long as \( g > 0 \) neither \( i \) nor \( j \) has any incentive to leave the union. Two, \( g = x_j > 0 \) and \( x_i = 0 \). Share of province \( i \) in additional product generated is \( t\lambda, x_j > 0 \) while province \( j \) retains \( x_j + t(\lambda, x_j - x_j) = x_j - t\lambda, x_j > 0 \). Once again, as long as \( g > 0 \), neither has any incentive to leave the union. If tax rate falls within the specified range the union is viable. So whenever SR does not permit secession none of the states has an incentive to secede.

In case of deviation by province, say, \( i \), we need to check if the punishment strategy prescribed in SR is optimal. Province \( j \) would contest an attempt to secede by province \( i \) iff the cost of contest is sufficiently low. Parallel requirement emerges for province \( i \) as well.
Finally, note that as long as the cost of contest is positive whenever \( g \leq 0 \) contest is a dominated strategy. The result stated in the proposition now follows. \(^{28}\) \( QED \)

(2) With Type II technologies for \( \Gamma \in [I, T^*] \) the above applies. Otherwise if union cannot reinvent itself and/or we have \( g = 0 \) dissolution is optimal and players are free to engage in accordance with Proposition 3. (a) Potential \( g = 0 \) so after dissolution contest is strictly dominated strategy and peaceful co-existence as sovereign states is optimal under Proposition 3 (case 1). (b) Potential \( g > 0 \) for at least one state and the sets of desirable mergers for the states are mutually exclusive so at least one state will contest after dissolution and the other has to respond as defending or not contesting is a strictly dominated strategy due to non-zero first strike advantage. So in this case dissolution is followed by contests as per Proposition 3 (case 3). Success in contest is determined by Eqn (11). (c) Union is dissolved due to inflexibility of initial agreement (INV = 0). But potential \( g > 0 \) for both states and the post-dissolution sets of desirable mergers are not mutually exclusive. So renegotiation takes place as per Proposition 3 (case 2). (d) Initial agreement is flexible (INV=1). Potential \( g > 0 \) for both states and the post-technology transfer sets of desirable mergers are not mutually exclusive so constitutional accommodation is possible through revision of the initial agreement. Fresh shares are determined as per Proposition 3 (case 2), and any deviation is punished as per secession rule (Case 1 above). \( QED \)

**Proof of Proposition 6**: The intuition is simple. Even after technological convergence states need each other to harness excess factor inputs so that rational players in complete information set up would avoid loss of benefits from cooperation and instantly adjust the agreement as and when relative bargaining power changes. (1) Following Proposition 4 agreement is time-consistent for the cases \( (\Gamma < T^*, \gamma \in \{I, II\}) \) and \( (\Gamma \geq T^*, \gamma = I) \), which implies that the optimal revision rule specified is unique sub-game perfect revision rule because \( \eta_j^\Gamma = \eta_j^{\Gamma=0} \ \forall \Gamma > 0 \). (2) Regarding the other case \( (\Gamma \geq T^*, \gamma = II) \) first note that it is easy to see that in case union is dissolved the specified revision rule is the unique sub-game perfect rule. Otherwise merger type is revised to \( CC_n^{\Gamma \geq T^*} \in (T(i) \cap T(j)) \backslash CC_n^{\Gamma=T^*} \) and Part (1) of this proposition applies thereafter. \( QED \)

\(^{28}\) It can be shown that in our framework if \( g = 0 \) for whatever reasons, exogenous or endogenous, then a merger cannot be salvaged through federalization of the state.
Proof of Proposition 7: The advanced state, say \( i \), can at most reap
\[
\Pi = \min(K_j, L_j)(X_i - X_j) \sum_{\Gamma=0}^{T^*} \mu^\Gamma \lambda_i(\Gamma) \cdot t(\Gamma)
\]
for transferring technology to the backward state under TT merger, where \( \mu \) is the discount factor. Following Propositions 3 - 6 we can say that \( \max t(\Gamma) \leq t^{\max} \) and \( \lambda_i(\Gamma) = \lambda_i(\Gamma = 0) \) for \( \Gamma \in [0, T^*] \). So \( \partial \Pi / \partial T^* > 0 \), that is the more complex a technology (larger \( T^* \)) higher the technology rents accruing to the advanced state. \( T^* \rightarrow \infty \) relates to the most complex technology. The upper limit specified in the proposition is given by
\[
\lim_{T^* \rightarrow \infty} \left( \Pi \left( \min(K_j, L_j)(X_i - X_j) \sum_{\Gamma=0}^{T^*} \mu^\Gamma \right) \right).
\]
QED

Appendix II: Description of Disagreement Game (Fig 1)

Assume that the initial distribution of capital and labour is such that production in State A is capital constrained whereas the production in State B is labour constrained. In a labour constrained State B by definition \( K_B > L_B \). Given that \( K_A + K_B = 1 \) and \( L_A + L_B = 1 \) hold this implies \( K_A < L_A \). Further assume \( A > B \), which implies technological preponderance of State A. So we are dealing with an advanced capital constrained State A facing a backward labour constrained State B. Recall our production function \( X_i \min(L_j, K_j) \), where \( X_i \in \{A, B\} \), and that we divide economic product equally between labour and capital inputs and also that migrant labour is not discriminated against.

In case states adopt different disagreement strategies or both adopt No Merger (NM) as their disagreement strategy they obtain their respective No Merger/status quo pay-offs. Under No Merger State A’s pay-off is \( A K_A : 0.5AK_A \) from use of its capital and another \( 0.5AK_A \) from commensurate use of its labour because it is capital constrained. Likewise State B’s pay-off is \( B L_B : 0.5BL_B \) from use of its labour and another \( 0.5BL_B \) from commensurate use of its capital because it is labour constrained.

If labour mobility (LM) is the disagreement strategy of both states then State A gets additional income, \( 0.5B(L_A - K_A) \), due to employment of previously unemployed share of its population whereas State B gets additional income due to utilization of its excess capital. Pay-offs to
State A and B are $AK_A + 0.5B(L_A - K_A)$ and $0.5B(K_B + L_B)$, respectively. If technology transfer (TT) is the disagreement strategy of both states then State B obtains additional income, $(A - B)L_B$, due to improved productivity. State A receives only its No Merger pay-off, $AK_A$, as its unemployed labour is not permitted to work in State B state. The corresponding pay-off for State B is $AL_B$.

Finally, if agreement is reached upon both technology transfer and labour mobility, i.e. a TTLM merger is agreed upon, then State B obtains additional income due to improved productivity and full utilization of its capital within its state whereas State A gets additional income due to full utilization of its labour force. Pay-offs to State A and B are $0.5A(L_A + K_A)$ and $0.5A(L_B + K_B)$, respectively. To obtain disagreement games for one issue one step mergers (e.g., TT merger and LM merger) ignore the strategies TT and LM in Fig 1, which leaves behind a 1x1 matrix where both players have only one strategy, namely, NM.
### Tables

**Table 1: Distribution of Material and Technological Endowments**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Material Endowments</th>
<th>Technology Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State A L_A</td>
<td>State B L_B</td>
</tr>
<tr>
<td>I: Resource constraint, unequal technology</td>
<td>K_A ≠ K_B</td>
<td>A/B ≠ 1</td>
</tr>
<tr>
<td>II: Resource sufficiency, unequal technology</td>
<td>K_A=K_B</td>
<td>A/B ≠ 1</td>
</tr>
<tr>
<td>III: Resource constraint, identical technology</td>
<td>K_A ≠ K_B</td>
<td>A/B = 1</td>
</tr>
<tr>
<td>IV: Resource sufficiency, identical technology</td>
<td>K_A=K_B</td>
<td>A/B = 1</td>
</tr>
</tbody>
</table>

**Table 2: Numerical Example (Capital Constrained State A vs. Labour Constrained State B)**

Technology: A = 2B = 2, Resources: K_A = 0.70, K_B = 0.30, L_A = 0.80, L_B = 0.20

<table>
<thead>
<tr>
<th>Type of Merger</th>
<th>Share of State B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in union legislature</td>
</tr>
<tr>
<td><strong>Partial mergers</strong></td>
<td></td>
</tr>
<tr>
<td>Technology Transfer, TT</td>
<td>0.125</td>
</tr>
<tr>
<td>Labour Mobility, LM</td>
<td>0.125</td>
</tr>
<tr>
<td><strong>Full mergers</strong></td>
<td></td>
</tr>
<tr>
<td>Technology Transfer and Labour Mobility</td>
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<tr>
<td>(disagreement strategy, no merger), TTLM-NM</td>
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</tr>
<tr>
<td>Technology Transfer and Labour Mobility</td>
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</tr>
<tr>
<td>(disagreement strategy, LM merger), TTLM-LM</td>
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<tr>
<td>Technology Transfer and Labour Mobility</td>
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</tr>
<tr>
<td>(disagreement strategy, TT merger), TTLM-TT</td>
<td></td>
</tr>
<tr>
<td><strong>Full mergers</strong></td>
<td></td>
</tr>
<tr>
<td>Technology Transfer after Labour Mobility,</td>
<td>0.147</td>
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<tr>
<td>TTLM(LM)</td>
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<tr>
<td>Technology Transfer followed by Labour Mobility,</td>
<td>0.222</td>
</tr>
<tr>
<td>TTLM(TT)</td>
<td></td>
</tr>
</tbody>
</table>

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29 Example discussed at length in Appendix II belongs to Case I. All other cases are special cases of Case I.
Table 3: *Historical Cases of Political Union*

<table>
<thead>
<tr>
<th>Type of Merger</th>
<th>Historical example(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT</td>
<td>Princely states and tribal areas in North West and North East India (late 1940s), Western peripheral provinces (Tibet and Xinjiang) of China (late 1940s)</td>
</tr>
<tr>
<td>LM</td>
<td>North and South Yemen (1990)</td>
</tr>
<tr>
<td>TTLM (LM)</td>
<td>Princely states in mainland India (late 1940s)</td>
</tr>
<tr>
<td>TTLM (TT)</td>
<td>Korean approach to merger (1990 onwards)</td>
</tr>
<tr>
<td>TTLM</td>
<td>East and West Germany (1990), Sikkim and India (1975)</td>
</tr>
</tbody>
</table>
**Figures**

**Figure 1: Typology of Mergers**

<table>
<thead>
<tr>
<th>Extent of resource sharing</th>
<th>Resources covered under merger agreement</th>
<th>Labour Mobility LM</th>
<th>Technology Transfer TT</th>
<th>Labour Mobility and Technology Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>Labour Mobility $\theta_L = 1$</td>
<td>Partial, complete</td>
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<td>Full, complete</td>
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<tr>
<td></td>
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<td></td>
<td>Labour Mobility and Technology Transfer $\theta_L, \theta_T = 1$</td>
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<td>NA</td>
<td>Full, complete</td>
</tr>
<tr>
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<td>Partial, incomplete</td>
<td>NA</td>
<td>Full, incomplete</td>
</tr>
<tr>
<td></td>
<td>Technology Transfer $\theta_T \in (0,1)$</td>
<td>NA</td>
<td>Partial, incomplete</td>
<td>Full, incomplete</td>
</tr>
<tr>
<td></td>
<td>Labour Mobility and Technology Transfer $\theta_L, \theta_T \in (0,1)$</td>
<td>NA</td>
<td>NA</td>
<td>Full, incomplete</td>
</tr>
</tbody>
</table>
**Figure 2: Disagreement Game, Θ**

<table>
<thead>
<tr>
<th>State B</th>
<th>LM</th>
<th>TT</th>
<th>NM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM</td>
<td>((AK_A + 0.5B(L_A - K_A), 0.5B(K_B + L_B))^{30})</td>
<td>((AK_A, BL_B))</td>
<td>((AK_A, BL_B))</td>
</tr>
<tr>
<td>TT</td>
<td>((AK_A, BL_B))</td>
<td>((AK_A, AL_B))</td>
<td>((AK_A, BL_B))</td>
</tr>
<tr>
<td>NM</td>
<td>((AK_A, BL_B))</td>
<td>((AK_A, BL_B))</td>
<td>((AK_A, BL_B))</td>
</tr>
</tbody>
</table>

In Figure 2 - 4 the underlined entries are pure strategy Nash equilibria.
Figure 3: Tax Adjusted Disagreement Game, $\Theta'$

<table>
<thead>
<tr>
<th>State B</th>
<th>LM</th>
<th>TT</th>
<th>NM</th>
</tr>
</thead>
<tbody>
<tr>
<td>State A</td>
<td>$(AK_A + 0.5B(L_A - K_A))(1 - t^{LM}) + \lambda_A t^{LM} (AK_A + BK_B)$</td>
<td>$(AK_A, BL_B)$</td>
<td>$(AK_A, BL_B)$</td>
</tr>
<tr>
<td>LM</td>
<td>$0.5B(K_B + L_B)(1 - t^{LM}) + \lambda_B t^{LM} (AK_A + BK_B)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT</td>
<td>$(AK_A, BL_B)$</td>
<td>$(AK_A, BL_B)$</td>
<td>$(AK_A, BL_B)$</td>
</tr>
<tr>
<td>NM</td>
<td>$(AK_A, BL_B)$</td>
<td>$(AK_A, BL_B)$</td>
<td>$(AK_A, BL_B)$</td>
</tr>
</tbody>
</table>

No merger pay-off: $\left( AK_A (1-t^A) + AK_A t^A \right), \left( BL_B (1-t^B) + BL_B t^B \right) = (AK_A, BL_B)$
**Figure 4:** Reduced Form Disagreement Game, $\Theta''$

<table>
<thead>
<tr>
<th>State A</th>
<th>State B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>LM</strong></td>
</tr>
<tr>
<td><strong>LM</strong></td>
<td>$(AK_A + 0.5B(L_A - K_A), 0.5B(K_B + L_B))$</td>
</tr>
<tr>
<td><strong>TT</strong></td>
<td>$(AK_A, BL_B)$</td>
</tr>
<tr>
<td><strong>NM</strong></td>
<td>$(AK_A, BL_B)$</td>
</tr>
</tbody>
</table>

$\varepsilon > 0$
Figure 5: Types of Demographic Heterogeneity

- Inter-state Economic Heterogeneity
  - (Type I)
  - Intra-state Economic
    - (Type II)
  - Intra-state Ethnic
    - (Type III)
  - Inter-state Ethnic
    - (Type IV)
  - Intra-state Economic and Ethnic
    - (Type V)
      - Correlated
        - (Type Va)
      - Uncorrelated
        - (Type Vb)
  - Intra-state Economic and Inter-state Ethnic
    - (Type VI)
References


