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Abstract

This paper investigates the effects of cross-ownership on optimal privatization, and vice-versa, in mixed duopoly. It shows that cross-ownership is profitable to the private firm only if the level of privatization of the public firm is sufficiently high. In equilibrium, cross-ownership does not take place even if there is partial privatization. However, the possibility of cross-ownership significantly limits the socially optimal level of privatization in most of the situations. Moreover, it demonstrates that full nationalization is socially optimal, in case of sufficiently convex identical cost functions and homogeneous goods. These results have strong implications to both divestment and competition policies.

Keywords:

Cross-ownership, mixed duopoly, partial privatization, product differentiation

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1 Introduction

Empirical evidence shows that many developing as well as developed economies are partially privatizing state-owned firms across several industries since 1980s (Megginson and Netter, 2001; Maw, 2002). In most of these industries (partially privatized) state-owned firms and private firms coexist and compete with each other in product market. It is also well documented that there are evidences of cross-ownership among firms, which apparently compete with each other, in many industries.[1] Interestingly, incidence of cross-ownership is not limited to private firms only. There are evidences of cross-ownership in mixed oligopolistic industries as well. For example, in January 2011, the state-controlled Russian oil company OJSC Rosneft signed an agreement with a private international oil firm BP Plc to exchange shares between the two companies (The Economist, 2011a).[2]

A firm may acquire its rival's stock, which gives it a share in the rival's profit but not necessarily in the rival's decision-making power, in order to dampen product market competition, gain access to rival's know-how, create synergy and diversify portfolio (Gilo et al., 2006; Alley, 1997; Macho-Stadler and Verdier, 1991; Reynolds and Snapp, 1986). Thus, the possibility of post-privatization cross-ownership between a public and a private firm may have significant bearing to the prospect of privatization of the public firm. On the other hand, a firm's decision to acquire a stake in its rival firm is likely to depend on the objective of the rival firm, i.e., on the extent of profit orientation of the rival firm. Therefore,

[1]See, for example, Kester (1992), Hansen and Lott (1996), Alley (1997), Dietzenbacher et al. (2000), Gilo et al. (2006), Khanna and Thomas (2009), and the references therein.

[2]We note here that this deal was blocked by one of the BPs existing partners, AAR. Subsequently, state-controlled Rosneft entered in a strategic cooperation agreement on 30th August 2011 with ExxonMobil, an American private company, which allows Rosneft to gain equity interest in ExxonMobil (The Economist, 2011b).

it is important to examine whether the government as a social planner should privatize the public firm, when the possibility of cross-ownership exists. What is the socially optimal level of privatization in the presence of cross-ownership? Is it profitable for a private firm to acquire a stake in its rival partially privatized firm? If yes, what proportion of the privatized firm should the rival private firm acquire? This paper attempts to answer these questions.

Recently, a large strand of literature has developed, which examines strategic interaction between public and private firms often with the objective of determining the socially optimal level of privatization. It is argued that, when firms produce homogeneous goods with decreasing returns to scale (DRS) technology and compete in terms of quantities, mixed oligopoly consisting of a public firm and one or more private firms leads to lower social welfare compared to that in oligopoly with only private firms (deFraja and Delbono, 1989). However, full privatization of the public firm is also not desirable from social welfare point of view. Matsumura (1998) demonstrates that the inefficiency in mixed oligopoly can be mitigated by partially privatizing the public firm, and partial privatization is socially optimal. This partial privatization result remains valid in case of differentiated products mixed oligopoly with constant returns to scale (CRS) technology as well (Fujiwara, 2007). Other mixed oligopoly models offer useful insights to understand a variety of issues related to privatization.[3] However, the issue of cross-ownership has not received much attention in this strand of literature so far.

On the other hand, existing studies on cross-ownership consider oligopoly with only profit maximizing private firms and disregard mixed oligopoly market structure, in spite of empirical evidences of cross-ownership between public and private firms. To the best of our knowledge, this is among the first papers to investigate (a) the implications of cross-

[3] Implications of entry regulation (Matsumura and Kanda, 2005; Brando and Castro, 2007), competition with foreign firm (Pal and White, 2003; Ohnishi, 2010b; Wang and Chen, 2011), managerial delegation (Barros, 1995; White, 2001), sequential move (Pal, 1998; Matsumura, 2003; Brcena-Ruiz, 2007; Brcena-Ruiz and Garzon, 2010), research and development (Poyago-Theotoky, 1998; Matsumura and Matsushima, 2004; Heywood and Ye, 2009), so on so forth.

ownership on socially optimal level of privatization and (b) effects of mixed oligopoly on equilibrium cross-ownership.

It is easy to observe that, in Cournot duopoly framework with profit maximizing firms, a firm has unilateral incentive to have as high stake as possible in its rival firm. The reason being higher cross-ownership makes the participating firm less aggressive in the product market and, thus, softens the product market competition, which results in higher profit of the rival firm and the share of the rival's profit that accrues to the participating firm overcompensates for reduction in its own profit for being less aggressive. This is true even if the rival firm is relatively less efficient. However, it is not clear whether it is incentive compatible for a profit maximizing firm to have any stake in its partially privatized rival firm, because the latter is not totally profit oriented.

Considering a sequential move game in the context of a mixed duopoly model this paper examines the effects of cross-ownership on socially optimal level of privatization, and vice-versa, in three scenarios: (a) firms produce homogeneous goods using identical DRS technologies, (b) public firm is relatively less efficient and (c) differentiated products mixed duopoly. It demonstrates that the level of privatization of the public firm has important consequences on cross-ownership. Unlike as in case of private duopoly, it is optimal for the private firm to have any stake in the partially privatized rival firm only if the level of privatization is more than a threshold level. Moreover, the level of privatization needs to be very high to induce the private firm to fully own the privatized portion of the public firm.

This paper also demonstrates that that the possibility of cross-ownership in post-privatization regime limits the prospect of privatization in most of the situations. When there is possibility of cross-ownership, the socially optimal level of privatization is less than that in case of no cross-ownership; unless the cost function is almost linear, the efficiency gap is very low, and products are close substitutes, respectively, in the first, second and third scenario. Moreover, it shows that full privatization turns out to be socially optimal in case of homogeneous products, if the cost function is sufficiently convex. These are new results.

We note here that there are few studies on merger incentives in mixed oligopoly. These papers examine the merger possibility in two alternative scenarios: (a) merger among private firms in presence of a public firm in the market (Heywood and McGinty, 2011)and (b) merger between a public firm and private firm(s) Barcena-Ruiz and Garzon (2003); Kamijo and Nakamura (2009). The second group of studies are somewhat related to the present analysis, since cross-ownership and merger has similar effects on product market competition. However, this paper differs from these studies in the following way. First, unlike as in case of merger, this paper considers non-cooperative game between the firms. Second, in this paper the level of cross-ownership and level of privatization are endogenously determined in the model, while in merger analysis the governments stake in the merged firm is assumed to be exogenously determined. Third, this paper deals with the issue of possibility of tacit collusion through cross-ownership.

The rest of the paper is organized as follows. The next section explains the model. Section 3 analyzes the case of homogeneous products mixed duopoly. It also examines the implications of efficiency gap between the public and private firms. Section 4 examines the interdependence of cross-ownership and privatization when products are differentiated. Section 5 concludes.

2 The model

We consider that there are two firms, firm 1 and firm 2, in an industry. Firms are engaged in Cournot type quantity competition in the product market.[4] For simplicity, we assume

[4]Note that, when firms are engaged in Bertrand type price competition, the intensity of product market competition is sufficiently high to ensure that decrease in consumer surplus due to an increase in level of privatization more than offset rise in industry profit. This is true irrespective of the demand and cost conditions. As a result, privatization is not socially desirable under price competition (Anderson et al., 1997; Roy-Chowdhury, 2009; Sanjo, 2009; Ohnishi, 2010a), except in case of complementary goods (Ohnishi, 2011). We mention here that, if firms produce homogeneous goods with DRS technologies, there are multiple pure strategy Nash equilibrium under price competition (Dastidar, 1995; Hoernig, 2002; Hirata and Matsumura, 2010). Nonetheless, it is fairly intuitive that the 'no privatization' result holds

that the demand function of firm i is given by $p_i = A - q_i - \gamma\, q_j$ $(i,\ j\ = 1,\ 2,\ i \neq j)$, where p_i and q_i denote the price and quantity demanded, respectively, of the product of firm i. A is the demand intercept and the parameter γ $(0 < \gamma \leq 1)$ denotes the degree of product differentiation.[5] Clearly, higher value of γ implies lower degree of product differentiation, and $\gamma = 1$ corresponds to the case of perfectly substitutes products.

The cost function of firm i is given by $C_i = c_i q_i + \frac{d}{2} q_i^2$, $i = 1, 2$; where c_1, c_2 and d are cost parameters. We assume that $c_1 \geq c_2 \geq 0$ and $d \geq 0$. That is, either the two firms are equally efficient or firm 1 is less efficient than firm 2. And, production technologies exhibit either decreasing-returns-to-scale (DRS) or constant-returns-to-scale (CRS).

We assume that firm 1 is a public firm, i.e. owned by the government, and firm 2 is a profit maximizing private firm. That is, if the two firms differ in terms of production efficiency, the public firm is assumed to be less efficient.[6]

The government's objective is to maximize social welfare (SW), which is the sum of the consumers' surplus (CS) and profits (π). In notations, $SW = CS + \pi_1 + \pi_2$, where $CS = \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2)$ and $\pi_i = p_i q_i - C_i$.[7] The government decides on the level of privatization θ $(0 \leq \theta \leq 1)$ of firm 1 in order to maximize social welfare. Higher value of θ denotes higher level of privatization and $\theta = 1$ $(\theta = 0)$ corresponds to the case of full privatization (full nationalization) of firm 1.

We consider that a fully privatized firm maximizes its profit, whereas a fully nationalized firm maximizes social welfare. The level of privatization (θ) determines the power of the private partner(s) in bargaining with the public sector over the payoff, as in Matsumura

true, at least in limiting sense, in that case also. Therefore, the case of price competition does not appear to be interesting in the present context.

[5]The underlying utility function of the representative consumer is $U = Aq_1 + Aq_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$, where m is the quantity of the numeraire good. This specification of the representative consumer's utility function is similar to that of Singh and Vives (1984).

[6]It is often argued that public firms are less efficient than the private firms. However, the empirical evidence on relative (in)efficiency of public firms is mixed (see, for example, Das and Ray (2010)). Therefore, it seems to be important to analyze both the scenarios: (a) public and private firms are equally efficient ($c_1 = c_2$) and (b) the public firm is less efficient than the private firm ($c_1 > c_2$).

[7]$CS = [Aq_1 + Aq_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2)] - (A - q_1 - \gamma\, q_2)q_1 - (A - q_2 - \gamma\, q_1)q_2 = \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2)$.

(1998).[8] Note that existing institutional factors of the economy play a crucial role in determining the objectives of fully nationalized firms as well as of partially privatized firms. Without any loss of generality, the objective function of firm 1 can be considered as the weighted average of its own profit and the sum of consumer surplus and producer surplus, $O_1 = \theta\pi_1 + (1-\theta)[SW]$.[9]

If firm 1 is privatized, firm 2 decides how much stake it should have in firm 1. Needless to mention here that firm 2 can have a stake in firm 1 only if the government decides to at least partially privatize firm 1. Suppose that firm 2 decides to own s $(0 \leq s \leq 1)$ proportion of the privatized share (θ) of firm 1, where $s = 0$ ($s = 1$) corresponds to the extreme situation of 'no cross-ownership' ('maximum possible cross-ownership'). We refer to s as the cross-ownership parameter. Now, since firm 2 is a profit maximizing private firm, it maximizes the sum of its own profit and $s\theta$ proportion of firm 1's profit.[10] Therefore, the objective function of firm 2 can be written as $O_2 = \pi_2 + s\theta\pi_1$.

The stages of the game involved are as follows.

Stage 1: The government decides the level of privatization (θ).

Stage 2: Firm 2 chooses the value of the cross-ownership parameter (s).

Stage 3: Firm 1 and firm 2 simultaneously and independently choose their outputs q_1 and q_2, respectively.

Note that, in the absence of cross-ownership ($s = 0$), if the two firms have identical CRS

[8] Alternatively, following Fershtman (1990), if we consider that the private partner(s) and the public sector bargain over the quantity of output to be produced, where bargaining powers are determined by respective share holdings, qualitative results of this analysis go through. The reason is the formulations of Fershtman (1990) and Matsumura (1998) lead to comparable objective functions of the partially privatized firm (Kumar and Saha, 2008; Saha, 2009)

[9] The specified objective function of the (partially) privatized firm is in line with the existing mixed oligopoly literature. It is also likely to be plausible in many real life situations, even if there is cross-ownership. This is because, in case of cross-ownership, legal framework and institutional factors are likely to restrict the rival private firm to manipulate the objective function of the partially privatized firm further.

[10] We assume, for simplicity, that the price of share of firm 1 is exogenously determined and we normalize that to be zero.

technologies ($c_1 = c_2$ and $d = 0$) and products are homogeneous ($\gamma = 1$), the public firm drives the private firm out of market and full nationalization ($\theta = 0$) is socially optimal.[11] Full nationalization is socially optimal in case of completely unrelated products ($\gamma = 0$) as well.[12]. In the intermediate case, i.e. when products are differentiated ($0 < \gamma < 1$), partial privatization is socially optimal under Cournot type quantity competition, because industry profit increases more than the decrease in consumer surplus up to a certain level of privatization (Fujiwara, 2007). On the other hand, in case of homogeneous products Cournot duopoly, partial privatization is socially optimal provided that production technologies exhibit DRS (Matsumura, 1998). This is because, due to increase in privatization up to a certain level, private firms' output rises at the expense of public firm's output, which results in cost-saving and, thus, leads to sufficient increase in industry profit to over compensate the associated loss in consumer surplus. If the public firm is relatively less efficient, the same mechanism works in case of CRS technologies as well. Therefore, for (partial) privatization to be socially optimal in the absence of cross-ownership ($s = 0$), at least one of the following three conditions need to be satisfied: (a) firms have DRS technology of production ($d > 0$), (b) products are differentiated ($0 < \gamma < 1$) and (c) public firm is less efficient than the private firm ($c_1 > c_2$). It is easy to understand that the socially optimal level of (partial) privatization would crucially depend on the demand and cost characteristics. It implies that cross-ownership may not be equally attractive to the private firm always. Also, the possibility of cross-ownership may have differential impact on optimal privatization and vice-versa. In order to keep the analysis tractable and focused, we consider the above three scenarios separately. It helps us to understand the implications of demand and cost conditions more clearly.[13]

[11] Since, the public firm's objective is to maximize social welfare, it produces up to the level where price is equal to marginal cost and serves the entire market.

[12] Since, in that case, public firm is the sole producer in its market

[13] If we allow for more than one of the three conditions to be satisfied at the same time, qualitative results of this paper remain valid. However, that makes the analysis complex.

3 Homogeneous products mixed duopoly

Let us first consider that the two firms produce homogeneous goods ($\gamma = 1$) using identical DRS technologies ($c_1 = c_2$ and $d > 0$). To obtain the sub-game perfect Nash equilibrium, we solve the game by standard backward induction method. Now, from the first order conditions of the maximization problems of firm 1 and firm 2 in stage 3, $\underset{q_1}{Max}\, O_1(q_1, q_2; \theta, s)$ and $\underset{q_2}{Max}\, O_2(q_1, q_2; \theta, s)$ respectively, we get the quantity reaction functions of firm 1 ($RF1$) and firm 2 ($RF2$) as follows.

$$q_1 = \frac{A - c - q_2}{1 + d + \theta} \tag{RF1}$$

$$q_2 = \frac{A - c - (1 + s\theta)q_1}{2 + d} \tag{RF2}$$

It is easy to check that, if the level of privatization increases, in $q_1 - q_2$ plane (a) the reaction function of firm 1 rotates inward and (b) firm 2's reaction function also rotates inward in case of cross-ownership. It implies that the output shifting effect of privatization is dampened in the presence of cross-ownership. Moreover, given any level of privatization, the point of equilibrium slides down along the firm 1's reaction function, in $q_1 - q_2$ plane, due to increase in cross-ownership. Clearly, the effects of privatization and cross-ownership on firms outputs are opposite in nature. It indicates that the possibility of cross-ownership may adversely affect the prospect of privatization.

Now, solving $RF1$ and $RF2$ we get the stage 3 equilibrium outputs $q_1(\theta, s)$ and $q_2(\theta, s)$ of firm 1 and firm 2, respectively. Let us denote the corresponding equilibrium price, profit of firm 1, profit of firm 2, consumer surplus and social welfare by $p(\theta, s)$, $\pi_1(\theta, s)$, $\pi_2(\theta, s)$, $CS(\theta, s)$ and $SW(\theta, s)$, respectively. See Appendix 1 for the expressions of $q_1(.)$, $q_2(.)$, $p(.)$, $\pi_1(.)$, $\pi_2(.)$, $CS(.)$ and $SW(.)$.

It is straightforward to check that, for any given level of privatization ($\theta > 0$), partially privatized firm's output is increasing, but private firm's output is decreasing, in extent of cross-ownership: $\frac{\partial q_1(\theta,s)}{\partial s} > 0$, but $\frac{\partial q_2(\theta,s)}{\partial s} < 0$.[14] The reason is higher stake of firm 2 in firm 1 induces firm 2 to be less aggressive in the product market, which, in turn, leads

[14] Needless to mention here that, if $\theta = 0$, there can not be any cross-ownership.

to lower output of firm 2 and higher output of firm 1. Moreover, absolute value of the marginal effect of cross-ownership on firm 2's output is greater than that on firm 1's output: $(\frac{\partial[q_1(\theta,s)+q_2(\theta,s)]}{\partial s} < 0)$. As a result, consumer surplus as well as firm 2's profit decreases, while firm 1's profit increases, with the increase in cross-ownership: $\frac{\partial CS(\theta,s)}{\partial s} < 0$ and $\frac{\partial \pi_2(\theta,s)}{\partial s} < 0$, but $\frac{\partial \pi_1(\theta,s)}{\partial s} > 0$. The net effect of cross-ownership on industry profit is ambiguous. It turns out that negative effects of cross-ownership on consumer surplus and firm 2's profit together dominate its positive effect on firm 1's profit. Therefore, social welfare decreases with the increase in cross-ownership: $\frac{\partial SW(\theta,s)}{\partial s} < 0$. Clearly, cross-ownership is not desirable from social welfare point of view. We summarize these comparative statics results in Lemma 1.

Lemma 1: *In case of homogeneous products mixed duopoly with identical DRS technologies, for any given level of privatization ($\theta > 0$), we have the following.*
(a) $\frac{\partial q_2(\theta,s)}{\partial s} < 0$, $\frac{\partial q_1(\theta,s)}{\partial s} > 0$, $\frac{\partial[q_1(\theta,s)+q_2(\theta,s)]}{\partial s} < 0$ *and* $\frac{\partial CS(\theta,s)}{\partial s} < 0$.
(b) $\frac{\partial \pi_1(\theta,s)}{\partial s} > 0$ *and* $\frac{\partial \pi_2(\theta,s)}{\partial s} < 0$. *(c)* $\frac{\partial SW(\theta,s)}{\partial s} < 0$.

Now, for any given value of the cross-ownership parameter s, higher level of privatization induces firm 1 to be less aggressive in the product market, since increase in level of privatization turns the focus of the (partially) privatized firm further away from social welfare maximization. Therefore, firm 1's output is decreasing in level of privatization: $\frac{\partial q_1(\theta,s)}{\partial \theta} < 0$. Again, a reduction in firm 1's output is supposed to induce firm 2 to produce higher amount, since $q_1(.)$ and $q_2(.)$ are strategic substitutes. However, cross-ownership adds twists to this mechanism. Upon inspection we find that $\frac{\partial q_2(\theta,s)}{\partial \theta} > 0$, if and only if $s < \frac{1}{1+d}$. Nonetheless, total output and, thus, consumer surplus always decrease due to an increase in the level of privatization $(\frac{\partial[q_1(\theta,s)+q_2(\theta,s)]}{\partial \theta} < 0$ and $\frac{\partial CS(\theta,s)}{\partial \theta} < 0)$, as in case of no cross-ownership. However, profit of firm 2 and industry profit are always increasing in level of privatization: $\frac{\partial \pi_1(\theta,s)}{\partial \theta} > 0$ and $\frac{\partial[\pi_1(\theta,s)+\pi_2(\theta,s)]}{\partial \theta} > 0$. Clearly, whether (partial) privatization is socially desirable or not depends on the relative magnitudes of the marginal effects of privatization on consumer surplus and industry profit, which further depends on the extent of cross-ownership.

Lemma 2: *For any given level of cross-ownership (s), in case of homogeneous products mixed duopoly with identical DRS technologies, we have the following.*
(a) $\frac{\partial q_2(\theta,s)}{\partial\theta} < 0$*, if* $s > \frac{1}{1+d}$*. Alternatively, if* $s < \frac{1}{1+d}$*,* $\frac{\partial q_2(\theta,s)}{\partial\theta} > 0$*.*
(b) $\frac{\partial q_1(\theta,s)}{\partial\theta} < 0$*,* $\frac{\partial[q_1(\theta,s)+q_2(\theta,s)]}{\partial\theta} < 0$ *and* $\frac{\partial CS(\theta,s)}{\partial\theta} < 0$*, for all* $s \in [0,1]$*.*
(c) $\frac{\partial\pi_2(\theta,s)}{\partial\theta} > 0$ *and* $\frac{\partial[\pi_1(\theta,s)+\pi_2(\theta,s)]}{\partial\theta} > 0$*, for all* $s \in [0,1]$*.*

It is now evident that, unless the extent of cross-ownership (s) is less than a critical level ($\frac{1}{1+d}$), outputs of both public and private firms are decreasing in level of privatization.[15] It implies that, in case of DRS technologies, privatization need not necessarily lead to cost saving, since output shifting effect of privatization from public firm to private firm may cease to exist in the presence of cross-ownership. Also, note that, if there is no cross-ownership ($s = 0$), firm 2's output can never be decreasing in the level of privatization in case of DRS technologies. These are new insights.

Proposition 1: *Privatization of the public firm does not necessarily lead to an increase in private firm's output, unlike as in case of no cross-ownership, when production technologies exhibit decreasing returns to scale.*

Before we proceed to solve the game further, let us first consider two extreme scenarios: (i) there is no possibility of cross-ownership ($s = 0$) and (ii) both firms are profit maximizing private firms ($\theta = 1$). In the first scenario, the extent of cross-ownership is exogenously determined to be zero, which may be possible due to institutional and legal framework or because of some other exogenous factors. In contrast, in the second scenario, the level of privatization is exogenously determined and the public firm is assumed to be fully privatized. We examine the socially optimal level of privatization and the equilibrium cross-ownership in the first and second scenario, respectively, in order to understand the implication of endogenous determination of both the level of privatization and the extent of cross-ownership.

[15] Note that $s > \frac{1}{1+d}$ is plausible for any positive value of d, since (a) $d > 0$ implies that $\frac{1}{1+d} < 1$ and (b) $0 \leq s \leq 1$.

Privatization in absence of cross-ownership: It is straightforward to check that, if there is no cross-ownership ($s = 0$), social welfare $SW(\theta)$ ($=SW(\theta, 0)$) is maximum at $\theta = \frac{d}{1+d(3+d)} = \theta_{0,(h,d)}$, where subscripts 0, h and d denote 'no cross-ownership', 'homogeneous products' and 'DRS technologies', respectively.[16] Clearly, $0 < \theta_{0,(h,d)} < 1$, since $d > 0$. That is, if there is no cross-ownership, partial privatization of the public firm is socially optimal, when firms produce homogeneous goods with identical DRS technologies. This result is well documented in the literature, as mentioned in Section 1. Also, note that the socially optimal level of privatization, $\theta_{0,(h,d)}$, first increases and then decreases with the increase in d, which is the rate of increase in marginal cost of production.[17] This is because, the difference between marginal costs of production of the (fully)public firm and the private firm, evaluated at their respective equilibrium outputs, is concave in the cost parameter d. Therefore, the extent of inefficiency in a mixed duopoly with a fully nationalized firm increases with the increase in d up to a certain level, thereafter the extent of inefficiency is decreasing in d. Thus, higher (lower) value of d calls for a higher level of privatization of the public firm unless d exceeds (falls below) a certain level.[18]

Lemma 3: *In case of homogeneous products mixed duopoly with identical DRS technologies, partial privatization of the public firm is socially optimal and the optimal level of privatization is given by* $\theta_{0,(h,d)} = \frac{d}{1+d(3+d)}$, *if the possibility of cross-ownership does not exist.*

Cross-ownership in private duopoly: Now, lets examine the equilibrium cross-ownership, when both are profit maximizing private firms, i.e., when $\theta = 1$. In order to compare the equilibrium cross-ownership in this scenario with that in case of mixed duopoly, we consider only one way cross-ownership. Without any loss of generality, we assume that firm 2 owns s ($0 \leq s \leq 1$) proportion of firm 1. We denote the equi-

[16] $SW(\theta, 0)$ is obtained by substituting $s = 0$ in the expression for $SW(\theta, s)$, which is provided in Appendix 1.

[17] $\frac{\partial \theta_{0,(h,d)}}{\partial d} = \frac{1-d^2}{(1+d\,(3+d))^2} = 0 \Rightarrow d = 1$. Moreover, $\frac{\partial^2 \theta_{0,(h,d)}}{\partial d^2} = -\frac{2\left(3+3\,d-d^3\right)}{(1+d\,(3+d))^3} < 0$, if $d < 2.1038$.

[18] $\{\frac{\partial C_1}{\partial q_1}\mid_{q_1(\theta=0)}\} - \{\frac{\partial C_2}{\partial q_2}\mid_{q_2(\theta=0)}\} = \frac{(A-c)\,d}{1+d\,(3+d)} > 0$ and $\frac{\partial}{\partial d}[\{\frac{\partial C_1}{\partial q_1}\mid_{q_1(\theta=0)}\} - \{\frac{\partial C_2}{\partial q_2}\mid_{q_2(\theta=0)}\}] = \frac{(A-c)\left(1-d^2\right)}{(1+d\,(3+d))^2}$

librium cross-ownership in this scenario by $s_{1,(h,d)}$; where subscripts 1, h and d denote 'full privatization', 'homogeneous products' and 'DRS technologies', respectively. Clearly, $s_{1,(h,d)} = \underset{s\in[0,1]}{Argmax}\, O_2(s)$, where $O_2(s) = O_2(1, s)$.[19] It is easy to check that total profit of firm 2 is monotonically increasing in level of cross-ownership: $\frac{\partial O_2(s)}{\partial s} > 0$, for all $s \in [0, 1]$. The intuition is as follows. Higher cross-ownership makes firm 2 less aggressive in the product market, which results in higher profit of firm 1, but lower profit of firm 2: $\frac{\partial \pi_1(s)}{\partial s} > 0$) and $\frac{\partial \pi_2(s)}{\partial s} < 0$, where $\pi_1(s) = \pi_1(1, s)$ and $\pi_2(s) = \pi_2(1, s)$. Moreover, due to an increase in cross-ownership, firm 1's profit increases sufficiently so that firm 2 is over compensated for the loss in its own profit ($\pi_2(s)$) by the gain due to higher profit-share from firm 1.[20] Therefore, it is optimal for firm 2 to own firm 1 fully: $s_{1,(h,d)} = 1$. In other words, firm 2 has unilateral incentive to obtain the maximum possible stake in firm 1.

Lemma 4: *In case of homogeneous products private duopoly with identical DRS technologies, full cross-ownership emerges in the equilibrium:* $s_{1,(h,d)} = 1$.

Endogenous privatization and cross-ownership: Finally, we turn to solve the remaining part of the game. That is, we now allow both the level of privatization (θ) and cross-ownership (s) to be endogenously determined, considering the sequential move game as discussed in Section 2. The problem of firm 2 in stage 2 of the game can be written as follows.

$$\underset{s}{Max}\, O_2(s;\theta) = O_2(q_1, q_2, s; \theta),$$

subject to the constraints

(a) $q_1 = q_1(\theta, s)$

(b) $q_2 = q_2(\theta, s)$ and

(c) $0 \leq s \leq 1$.

As noted before, in case of private duopoly, it is optimal for a firm to own its rival firm

[19] $O_2(1, s)$ is obtained by substituting $\theta = 1$ in $[\pi_2(\theta, s) + s\theta\pi_1(\theta, s)]$, where $\pi_1(\theta, s)$ and $\pi_2(\theta, s)$ are as in Appendix 1.

[20] We illustrate this mechanism further while analyzing cross-ownership in case of mixed duopoly.

fully. However, in case of mixed duopoly, output orientation of the partially privatized rival firm may reduce the gain from cross-ownership of the private firm significantly. To illustrate it further, note that the marginal effect of cross-ownership on firm 2's payoff can be expressed as follows.

$$\frac{\partial O_2(.)}{\partial s} = \underset{(-)}{\frac{\partial \pi_2(.)}{\partial s}} + \underset{(+)}{s\theta \frac{\partial \pi_1(.)}{\partial s}} + \underset{(+)}{\theta \pi_1(.)}$$

The first term is the marginal effect of cross-ownership on firm 2's own profit, which is negative. The marginal effect of cross-ownership on the gain of firm 2 via its share in firm 1's profit is the sum of the second and the third term, which are positive. It is straightforward to check that, if $\theta = 1$, $s\theta \frac{\partial \pi_1(.)}{\partial s} + \theta \pi_1(.) > \mid \frac{\partial \pi_2(.)}{\partial s} \mid \Rightarrow \frac{\partial O_2(.)}{\partial s} > 0$. That is, in case of private duopoly positive effects together dominate the negative effect of cross-ownership on firm's payoff. However, for $\theta < 1$, $\frac{\partial O_2(.)}{\partial s}$ may or may not be positive. In other words, in case of mixed duopoly cross-ownership need not necessarily be profitable, unlike as in case of private duopoly.

Now, solving the above problem, we get the equilibrium cross-ownership, $s_{h,d}(\theta)$, for any given level of privatization as follows. (See Appendix 2 for more details.)

$$s_{h,d}(\theta) = \begin{cases} 0, \text{ if } d < 0.205569 \text{ and } 0 < \theta < \theta_1 \\ \hat{s}_{h,d} = \frac{d\,\{d\,(2+d)^2-1\}+d\,(2+d)\,(5+3\,d)\,\theta+2\,(1+d)\,(2+d)\,\theta^2}{d\,(3+d)\,\theta}, \text{ if } d < 0.205569 \text{ and } \theta_1 \leq \theta < \theta_2 \\ 1, \text{ if } d \geq 0.205569 \text{ or } \theta \geq \theta_2; \end{cases} \quad (1)$$

where $0 < \theta_1 < \theta_2 < 1$, if $d < 0.205569$.

Clearly, when the rate of increase in marginal cost is low, i.e., the cost function is not sufficiently convex, the level of privatization needs to be greater than a critical level (θ_1) in order to induce firm 2 to own any stake in the partially privatized firm. Moreover, in this case, it is optimal for firm 2 to own the entire privatized portion of firm 1 only if the level of privatization is sufficiently high ($\theta \geq \theta_2$); otherwise, in the intermediate range of privatization ($\theta_1 < \theta < \theta_2$), optimal cross-ownership is less than one. We depict this relation in Figure 1, where the thick dashed curve denotes the optimal cross-ownership. In

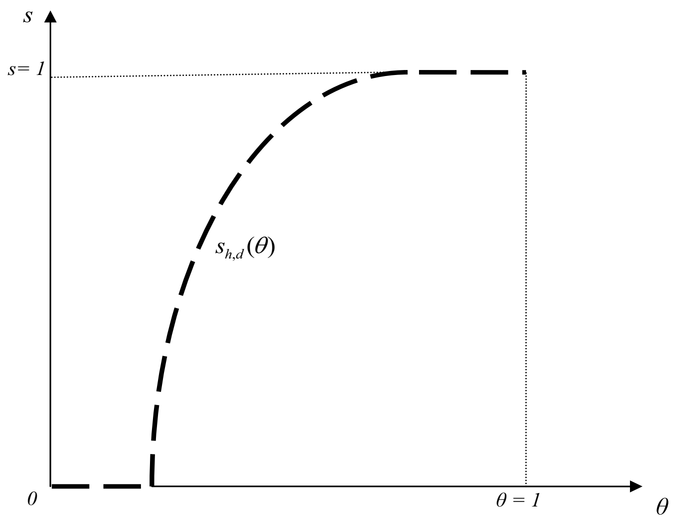

Figure 1: Level of privatization and optimal cross-ownership when d is low

contrast, if the rate of increase in marginal cost due to increase in production is high, it is optimal for firm 2 to own the entire privatized portion of firm 1, irrespective of the level of privatization. The intuition behind this result is as follows. If d increases, output of the private firm falls short of the partially privatized firm by a lesser amount.[21] Therefore, when d is high, output reduction due to cross-ownership has large impact on product market competition, which in turn makes the positive effect of cross-ownership on firm 2's payoff stronger than the negative effect even if the level of privatization is low. On the other hand, if d is low, output of the private firm falls short of the partially privatized firm by a large amount unless the level of privatization is sufficiently high. Therefore, when d is low, cross-ownership is profitable to the private firm only if the level of privatization is greater than a critical level.

Proposition 2: *In case of homogeneous products mixed duopoly with DRS technologies, cross-ownership is not always profitable to the private firm unlike as in case of private duopoly. Full cross-ownership is optimal for any level of privatization of the public firm only if marginal cost of production increases at a high rate due to increase in production.*

[21] $\frac{\partial}{\partial d}[q_1(\theta,s) - q_2(\theta,s)] \mid_{s=0} = -\frac{(A-c)\,(1-\theta)\,(3+2\,d+\theta)}{(1+2\,\theta+d\,(3+d+\theta))^2} < 0$

Otherwise, optimal cross-ownership crucially depends on the level of privatization. In the latter case, the level of privatization must be (a) greater than a minimum level for equilibrium cross-ownership to be positive and (b) sufficiently high for full cross-ownership to be optimal.

The above proposition indicates that the prospect of cross-ownership can be much lower in case of mixed duopoly than that in case of private duopoly. Clearly, this result is likely to have implications to competition policies that aim to prevent collusion among firms and promote competition.

Finally, lets turn to stage 1 of the game, where the government decides the socially optimal level of privatization by correctly anticipating the optimal behavior of firms in subsequent stages of the game. Upon inspection, we find that social welfare is decreasing in level of privatization, whenever there is positive cross-ownership: $\frac{\partial SW(\theta;s)}{\partial \theta} < 0$, if $d \geq 0.205569$ or $\theta > \theta_1$. That is, if the private firm finds it optimal to have any stake in the partially privatized firm, full nationalization of the public firm is socially optimal. This is because cross-ownership dampens the intensity of product market competition, which adversely affects social welfare corresponding to any given level of privatization. Also, note that the output shifting effect of (partial)privatization is less in case of cross-ownership than that in case of no cross-ownership. Overall, the negative effect of cross-ownership dominates the positive effect of (partial)privatization on social welfare. In other words, the purpose of (partial)privatization of the public firm is defeated, if the private firm finds it profitable to have a stake in the (partially)privatized firm.

Note that, if there is no cross-ownership, socially optimal level of privatization is $\theta_{0,(h,d)} = \frac{d}{1+d(3+d)}$ (From Lemma 2). Comparing the values of $\theta_{0,(h,d)}$ and θ_1, we find that $\theta_{0,(h,d)} < \theta_1$, if $d < 0.06116$. So, if $d < 0.06116$, socially optimal level of privatization in case of cross-ownership is the same as that in case of no cross-ownership. Otherwise, if $d \geq 0.06116$, $\theta_{0,(h,d)} \geq \theta_1$ and socially optimal level of privatization is $\theta^*_{h,d} = Max\{\theta_1, 0\}$.[22] We depict the socially optimal level of privatization by the thick curve segments in Figure 2.

[22] $\theta_1 \leq 0$ if $d \geq 0.205569$.

Clearly, the possibility of cross-ownership significantly restricts the scope for privatization, unless the rate of increase in marginal cost (d) is very low.

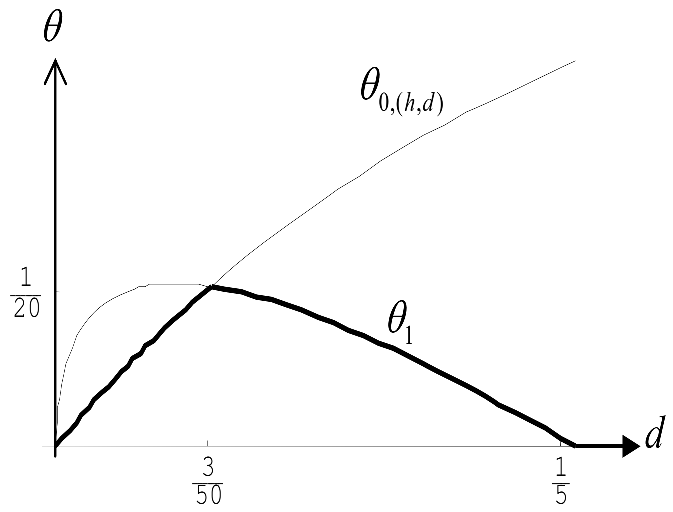

Figure 2: Socially optimal level of privatization

Proposition 3: (a) *In case of homogeneous products mixed duopoly with identical DRS production technologies, (i) full nationalization is socially optimal unless firms' marginal cost of production increases at a low rate with the increase in production, and (ii) partial privatization is socially optimal only if the rate of increase in marginal cost due to increase in production is low. Moreover, in the latter case, the optimal level of privatization is less than that in case of no cross-ownership, if the rate of increase in marginal cost due to increase in production is not* very *low.*
(b) In equilibrium, cross-ownership does not take place, even if the public firm is partially privatized.

The above proposition implies that, if the possibility of cross-ownership in the post privatization era cannot be ruled out by some exogenous legal/institutional factors, mixed duopoly with a fully public firm is more efficient from social welfare point of view, compared to both private duopoly and mixed duopoly with a partially privatized firm, under a wide range of plausible parametric configuration. Clearly, the results of deFraja and Delbono

(1989) and Matsumura (1998) emerge as special cases in the present analysis. Moreover, it is interesting to observe that, when partial privatization is socially optimal, the optimal level of privatization may be less than that in case the possibility of cross-ownership does not exist.

Note that the private firm would like to have a stake in its partially privatized rival firm, only if the stake is large enough. However, in equilibrium, by choosing the level of privatization appropriately the government ensures that the private firm never finds it optimal to opt for cross-ownership. Effectively, this is a negative result on cross-ownership, and as a negative result, this is an interesting finding.

Proposition 3 also indicates that the possibility of cross-ownership in the post privatization regime should be taken into account in order to decide the socially optimal level of privatization of the public firm. The government can prevent cross-ownership by choosing the appropriate level of privatization (nationalization), in absence of any other alternative mechanism to do so. However, whether privatization/nationalization should be used as an instrument to prevent cross-ownership or not is likely to depend on existing institutional and legal framework.

3.1 Relatively less efficient public firm

In this section, we consider the scenario in which the public firm is less efficient compared to its rival private firm. In order to focus on the implications of the efficiency gap between the two firms and to keep the analysis simple, we abstract away from the possibilities of DRS technologies and differentiated products. In other words, we consider that the two firms produce homogeneous products ($\gamma = 1$) with CRS technologies ($d = 0$). We assume that the marginal costs of production of the public firm (firm 1) and the private firm (firm 2) are $c_1 = k$ (> 0) and $c_2 = 0$, respectively. That is, k represents the magnitude of relative inefficiency of the public firm. We assume that $0 < k < \frac{A}{4}$, which ensures that both firms produce positive outputs in equilibrium.

In this scenario, the equilibrium outcomes in stage 3 of the game are as follows. $q_1 =$

$q_{1,k}(\theta, s)$, $q_2 = q_{2,k}(\theta, s)$, $p = p_k(\theta, s)$, $\pi_1 = \pi_{1,k}(\theta, s)$, $\pi_2 = \pi_{2,k}(\theta, s)$, $CS = CS_k(\theta, s)$ and $SW = SW_k(\theta, s)$; where subscript k denotes that the public firm is less efficient. The expressions of the equilibrium outcomes are relegated to Appendix 3.

As before, lets now examine (a) the optimal privatization in case of no cross-ownership and (b) optimal cross-ownership in case of private duopoly, in the present scenario.

Lemma 5: *If the public firm is relatively less efficient and the products are homogeneous, socially optimal privatization in case of no cross-ownership ($s = 0$) is given by,*

$$\theta_{0,(h,k)} = \begin{cases} \frac{k}{A-4k}, & \text{if } k < \frac{A}{5} \\ 1, & \text{if } \frac{A}{5} \leq k < \frac{A}{4}, \end{cases}$$

where the subscripts 0, h and k denote 'no cross-ownership', 'homogeneous products' and 'less efficient public firm', respectively.

Proof: See Appendix 4.

Clearly, in absence of cross-ownership, higher is the relative inefficiency of the public firm, higher is the socially optimal level of privatization. Also, note that, if $k = 0$, i.e., if the two firms are equally efficient, full nationalization is socially optimal. This is because, when efficiency gap is higher, cost saving is higher due to output shifting effect of privatization. On the other hand, when both firms are equally efficient and produce homogeneous goods with CRS technologies, industry profit does not increase sufficiently to over compensate the loss in consumer surplus due to privatization.[23] In fact, when both firms are equally efficient, the public firm produces up to the level where price equals marginal cost, and the private firm ceases to exist.

Now, lets consider that both firms are profit maximizing private firms ($\theta = 1$) and there is possibility of one-way cross-ownership. In this case, the equilibrium cross-ownership is as in Lemma 6.

[23]Since (a) product market competition is more intense in case of homogeneous goods compared to that in case of differentiated goods and (b) there is no cost saving from output shifting from the public firm to the private firm in case of identical CRS technologies.

Lemma 6: *It is optimal for a private firm to fully own its rival private firm, even if the rival firm is relatively inefficient. That is, in case of private duopoly ($\theta = 1$), the equilibrium cross-ownership is $s_{1,(h,k)} = 1$*

Proof: See Appendix 5.

Finally, we turn to examine the relation between cross-ownership and privatization. Solving the problem of firm 2 in stage 2 of the game, we get the equilibrium cross-ownership corresponding to any given level of privatization, $\theta \in [0, 1]$, as follows.

$$s_{(h,k)}(\theta) = \begin{cases} 0, & \text{if } \; 0 < \theta < \hat{\theta}_k \\ 1, & \text{if } \; \hat{\theta}_k \leq \theta \leq 1, \end{cases}$$

where $\hat{\theta}_k \in (0, 1)$ is given by the positive real root of the equation $k + 3\,k\,\theta - 2\,A\,\theta^2 + 6\,k\,\theta^2 - 3\,A\,\theta^3 + 6\,k\,\theta^3 = 0$.

Lemma 7: *When the public firm is relatively less efficient than the private firm, it is optimal for the private firm to fully own the privatized portion of the public firm, if the level of privatization is sufficiently large. Otherwise, no cross-ownership takes place in equilibrium.*

Proof: See Appendix 6.

Note that partial cross-ownership is never optimal in the present scenario, unlike as in case of identical DRS technologies. Also, from Lemma 5 and Lemma 7, we can say that the possibility of cross-ownership is likely to affect the level of privatization, when the efficiency gap between the firms is large. This is because, higher efficiency gap calls for greater level of privatization when the possibility of cross ownership does not exist (Lemma 5) and higher level of privatization induces the private firm to go for full cross-ownership (Lemma 7).

Now, given any level of cross-ownership (s), the government would find it optimal to privatize the public firm, if $\frac{\partial SW_k(\theta,s)}{\partial \theta} \mid_{\theta=0} > 0$, where $SW_k(\theta, s)$ is as given in Appendix 3. Upon inspection, we find that, if $s = 1$, $\frac{\partial SW_k(\theta,s)}{\partial \theta} \mid_{\theta=0} < 0$. It implies that, if there is possibility of full cross-ownership after privatization, full nationalization of the public firm is socially optimal. However, full cross-ownership takes place in equilibrium only when the level of privatization is greater than a critical level, $\theta \geq \hat{\theta}_k$. Also, note that there is no possibility of partial cross-ownership in the present scenario (by Lemma 7). Thus, if $\theta_{0,(h,k)} \geq \hat{\theta}_k$, optimal privatization would be less than that in absence of cross-ownership possibility. It is easy to check that $\theta_{0,(h,k)} \geq \hat{\theta}_k$, if $k \geq \frac{A(9-\sqrt{13})}{34}$. That is, if the efficiency gap is greater than a critical level, possibility of cross-ownership limits the socially optimal level of privatization. Proposition 4 summarize the equilibrium outcomes, when both the level of privatization and cross ownership are endogenously determined, in the present scenario.

Proposition 4: *If the public firm is relatively inefficient than the private firm, cross-ownership does not emerge in equilibrium and partial privatization of the public firm is socially optimal. However, the possibility of cross-ownership limits the optimal level of privatization, if the efficiency gap between the two firms is larger than a critical level.*

From Proposition 3 and Proposition 4, we observe that 'no cross-ownership' emerges as the equilibrium in both the scenarios - identical DRS technologies and asymmetric CRS technologies. Also, in both the scenarios, possibility of cross-ownership limits the socially optimal level of privatization, unless d is very low in case of DRS technologies and k is low in the second scenario. The effect of cross-ownership appears to be stronger in case of identical DRS technologies than that in case of asymmetric CRS technologies. This is because, the possibility of cross-ownership pushes down the optimal level of privatization to zero, when d is sufficiently large, in case of identical DRS technologies; but, such possibility of full nationalization never emerges as the equilibrium in case of asymmetric CRS technologies. It seems to be interesting to extend the present analysis to allow for asymmetric DRS technologies. Nonetheless, the qualitative results of this analysis are likely to go through in case of more general cost functions as well for a wide range of parametric configurations.

4 Differentiated products mixed duopoly

Finally, we consider the case of differentiated products ($0 < \gamma < 1$) and identical CRS technologies ($c_1 = c_2 = c$ and $d = 0$). As before, we solve the game by backward induction method, starting from stage 3. We can write the stage 3 reaction functions of firm 1 and firm 2, respectively, as follows.

$$q_1 = \frac{A - c - \gamma\, q_2}{1 + \theta} \tag{$RF1_\gamma$}$$

$$q_2 = \frac{A - c - \gamma\, q_1 - s\,\gamma\,\theta\, q_1}{2} \tag{$RF2_\gamma$}$$

Note that the above two reaction functions are very similar to the reactions functions, $RF1$ and $RF2$, in case of homogeneous products mixed duopoly with DRS technologies. In both the scenarios, in $q_1 - q_2$ plane, (a) the reaction function of firm 1 rotates inward due to increase in level of privatization, (b) the reaction function of firm 2 also rotates inward due to increase in level of privatization when there is cross-ownership, and (c) only the reaction function function of firm 2 rotates inward, while firm 1's reaction function remains unaltered, with the increase in the extent of cross-ownership. Therefore, the effects of privatization and cross-ownership on equilibrium outputs are going to be very similar in these two scenarios.

Solving the stage 3 problems of the two firms, the equilibrium outputs, prices, profits, consumer surplus and social welfare, for any levels of privatization and cross ownership, are $q_{1,\gamma}(\theta, s)$, $q_{2,\gamma}(\theta, s)$, $p_{1,\gamma}(\theta, s)$, $p_{2,\gamma}(\theta, s)$, $\pi_{1,\gamma}(\theta, s)$, $\pi_{2,\gamma}(\theta, s)$, $CS_\gamma(\theta, s)$ and $SW_\gamma(\theta, s)$, respectively. See Appendix 7 for the expressions of the equilibrium outcomes in stage 3.

It is easy to observe that, given the level of privatization, effects of cross-ownership on equilibrium outcomes are same as in Lemma 1. Underlying mechanisms for comparative static effects of cross-ownership also remain the same as before. However, comparative static effects of privatization on equilibrium outputs are now different. Note that privatization led output shifting from the public firm to the private firm does not lead to cost saving in the present scenario. However, since products are now differentiated, the positive effect of privatization on industry profit is stronger than that in case of homogeneous goods, for any given cost function. As a result, privatization up to a certain level leads to sufficient increase in industry profits to over compensate the loss in consumer surplus, even though firms have identical CRS technologies. It is interesting to note here that, in contrast to Proposition 1, privatization of the public firm always increases the private firm's output in the present scenario: $\frac{\partial q_{2,\gamma}(\theta,s)}{\partial\theta} > 0 \ \forall \ \gamma \in (0,1)$.[24] It implies that the effect of cross-ownership on firm 2's output is always weaker than the effect of privatization, in case of differentiated products. Comparative statics effects on firm 1's output, consumer surplus and profits remain the same as in Lemma 2. Also, it is easy to check that, if firm 1 is fully privatized ($\theta = 1$), it is optimal for firm 2 to fully own the firm 1, as in Lemma 4.

Now, if there is no cross-ownership, the socially optimal level of privatization of the public firm is given by $\theta_{0,\gamma} = \underset{\theta\in[0,1]}{Argmax}\ SW_\gamma(\theta, s = 0) = \frac{\gamma(1-\gamma)}{4-3\gamma}$. Clearly, $0 < \theta_{0,\gamma} < 1$. That is, in the absence of cross-ownership, partial privatization is socially optimal. Moreover, note that the optimal level of privatization ($\theta_{0,\gamma}$) exhibits inverted U shape with respect to the degree of product differentiation, as demonstrated in Fujiwara (2007). The intuition behind this result is as follows. Note that, if products are completely unrelated ($\gamma = 0$), there is no strategic interaction between the two firms and the private firm's market power is at its maximum level. Thus, privatization of the public firm leads to decrease in social welfare, since decrease in output due to privatization does not have any effect on the private firm's output choice. So, full nationalization is socially optimal, when products are completely unrelated ($\gamma = 0$). On the other extreme scenario, in which products are perfect substitutes ($\gamma = 1$), strategic interaction between the two firms is most strong and the private firm's market power is at its lowest possible level. In that case, privatization leads to sufficient decrease in consumer surplus to more than offset the associated increase in profits. Therefore, full nationalization is socially optimal again in case of perfectly substitute products ($\gamma = 1$). In the intermediate range of γ ($0 < \gamma < 1$), private firm's market power is moderate, which creates the room for industry profit to rise sufficiently to over compensate the loss in consumer surplus up to a certain level of privatization. It

[24] $\frac{\partial q_{2,\gamma}(\theta,s)}{\partial\theta} = \frac{(A-c)\,(1-s)\,(2-\gamma)\,\gamma}{(2\,(1+\theta)-\gamma^2\,(1+s\,\theta))^2} > 0$

is evident that, if the private firm's market power is close either to its maximum possible level or to its minimum possible level, socially optimal level of privatization is close to zero. As the private firm's market power starts decreasing (increasing) from its maximum (minimum) possible level, prospect of privatization increases. Therefore, socially optimal level of privatization reaches its maximum at an intermediate value of γ.

Lets now move to stage 2 of the game. As in Section 3, for any given level of privatization, cross-ownership has two opposing effects on firms 2's payoff: (i) firm 2's own profit ($\pi_{2,\gamma}(.)$) is decreasing in extent of cross-ownership (s), but (ii) the the profit of firm 1 that accrues to firm 2 due to cross-ownership ($s\theta\pi_{1,\gamma}(.)$) increases with the rise in extent of cross-ownership. Clearly, cross-ownership is profitable to firm 2, if the positive effect dominates the negative effect. Solving the firm 2's problem in stage 2, we get the extent of cross-ownership in equilibrium, given the level of privatization, as follows.

$$s_\gamma(\theta) = \begin{cases} 0, \text{ if } 0 \leq \theta \leq \theta_{1,\gamma}, \\ \hat{s}_\gamma = \frac{2}{\gamma^2} + \frac{2\,(2-\gamma)\,\theta}{\gamma^2\,(2-\gamma-\gamma^2)} - \frac{\gamma}{(2+\gamma)\,\theta}, \text{ if } \theta_{1,\gamma} < \theta < \theta_{2,\gamma}, \\ 1, \text{ if } \theta_{2,\gamma} \leq \theta \leq 1; \end{cases} \quad (2)$$

where $0 < \theta_{1,\gamma} < \theta_{2,\gamma} < 1$ for all $\gamma \in (0, 1)$. See Appendix 8 for more details.

It is clear from (1) and (2) that the impact of level of privatization on $s_\gamma(\theta)$ and $s_{h,d}(\theta)$ are very similar, if $d < 0.205569$; except the different critical values of θ in two scenarios. However, unlike $s_{h,d}(\theta)$, $s_\gamma(\theta)$ always depends on the value of θ.

Proposition 5: *(a) In case of differentiated products mixed duopoly, it is not always optimal for the private firm to have as large stake as possible in its rival partially privatized firm, unlike as in cases of (i) private duopoly and (ii) homogeneous products mixed duopoly in which the marginal cost of production increases at a high rate due to increase in output.*

(b) The level of privatization needs to be greater than a threshold level ($\theta_{1,\gamma}$) to have any cross-ownership in equilibrium, and full cross-ownership is never optimal unless the level of privatization is sufficiently large ($1 \geq \theta > \theta_{2,\gamma} > \theta_{1,\gamma} > 0$).

Finally, turning to the problem of the government in stage 1, we observe that $\frac{\partial SW_\gamma(\theta)}{\partial \theta} <$

0, if $s_\gamma(\theta) > 0$. That is, as in Section 3, full nationalization of the public firm is socially optimal, if there is any possibility of cross-ownership after privatization. However, note that $s_\gamma(\theta) > 0$, if $\theta > \theta_{1,\gamma}$, by (2). Now, $\theta_{0,\gamma} \leq \theta_{1,\gamma}$, if $0.851464 \leq \gamma < 1$; otherwise, $\theta_{0,\gamma} > \theta_{1,\gamma}$. Therefore, if products are close substitutes ($0.851464 \leq \gamma$), $\theta_{0,\gamma}$ is the socially optimal level of privatization. But, if the products are sufficiently differentiated ($\gamma < 0.851464$), it is optimal for the government to privatize only up to the level $\theta = \theta_{1,\gamma}$, which is less than $< \theta_{0,\gamma}$). Clearly, cross-ownership does not occur in equilibrium. However, the possibility of cross-ownership restricts the scope for privatization, unless the products are close substitutes. Figure 3 depicts the socially optimal level of privatization, denoted by the thick curve segments, for any given degree of product differentiation. We summarize these results in Proposition 6.

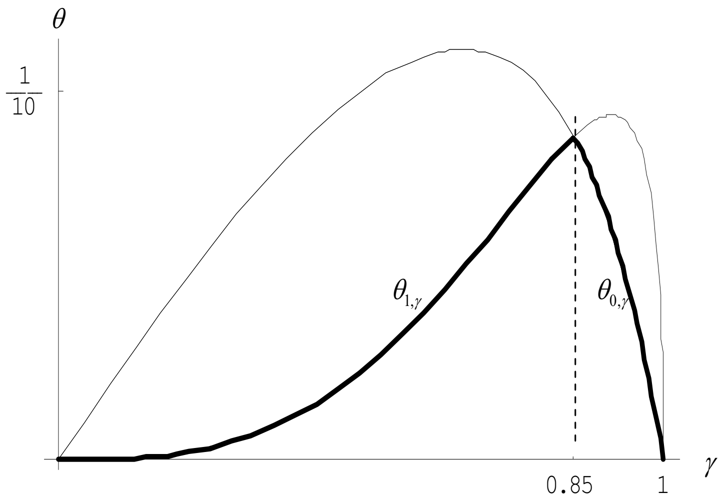

Figure 3: Socially optimal level of privatization and product differentiation

Proposition 6: *(a) Partial privatization of the public firm is socially optimal. (b) Cross-ownership does not occur in equilibrium. However, the possibility of cross-ownership reduces the optimal level of privatization, unless the degree of product differentiation is low.*

From Proposition 3, 4 and 6, we can say that the possibility of cross-ownership would adversely affect the prospect of privatization, if we allow for both (a) DRS technologies and (b) relatively inefficient public firm in a differentiated products mixed duopoly. It is evident that, in such unified framework, socially optimal level of privatization would be less in case the possibility of cross-ownership exists, if products are sufficiently differentiated and cost functions are sufficiently convex (i.e., $d > 0.06116$ is sufficiently large) and/or efficiency gap between the two firms is not very small (i.e., k is not very small). Also, note that (a) when firms produce homogeneous goods identical with DRS technologies, full nationalization of the public firm is socially optimal, if the convexity of the cost function is greater that a critical level ($d > 0.205569$); and (b) possibility of cross-ownership reduces the socially optimal level of privatization in case of CRS technology, if products are not close substitutes ($\gamma < 0.851464$). Therefore, the full nationalization result in Proposition 3(a) is likely to go through in case of differentiated products as well, if products are sufficiently differentiated and the cost function is sufficiently convex. It implies that the qualitative results of this analysis are likely to go through in an unified framework as well.

5 Conclusion

In this paper we have investigated the effects of (a) cross-ownership on socially optimal privatization and (b) level of privatization on equilibrium cross-ownership considering a sequential move game in mixed duopoly. To keep the analysis simple, we have analyzed the implications of increasing marginal costs, relative inefficiency of the public firm and product differentiation to equilibrium outcomes, considering three alternative scenarios. We show that cross-ownership is profitable to the private firm only if the level of privatization of the public firm is sufficiently high. Interestingly, in equilibrium, cross-ownership does not take place even if there is partial privatization. This result is in sharp contrast to that in case of private duopoly. In case of private duopoly, firms have unilateral incentive to have as large stake in the rival firm as possible and full cross-ownership emerges as the equilibrium.

However, the possibility of cross-ownership adversely affects the prospect of privatization of the public firm for wide ranges of parametric configurations in each of the three scenarios considered in this paper. Unless the cost function is almost linear in case of homogeneous goods or the efficiency gap is very low in case of homogeneous goods or products are close substitutes in case of CRS technology, the socially optimal level of privatization is less than that in case of no cross-ownership. Moreover, it demonstrates that full nationalization is socially optimal, in case of sufficiently convex identical cost functions and homogeneous goods. These results have strong implications to divestment and competition policies.

We also demonstrate that the qualitative results of this analysis are likely to hold true in a more general case, which allows for differentiated products and asymmetric DRS technologies, also. Nonetheless, it seems to be useful to examine the interdependence of cross-ownership and privatization considering general demand and cost functions. This is a limitation of this paper. It seems to be interesting to extend the present analysis to examine the issue of interest group lobbying for privatization and credibility of the government. It might also be interesting to explicitly model the process of privatization in the present context.

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Appendix

Appendix 1: Stage 3 equilibrium outcomes in case of homogeneous products and identical DRS technologies

When the two firms produce homogeneous products using identical DRS technologies, given any θ and s, the equilibrium outputs, price, profits, consumer surplus and social welfare are as follows.

$$\begin{aligned}
q_1(\theta,s) =& \frac{(A-c)(1+d)}{1+3d+d^2+(2+d-s)\theta} \\
q_2(\theta,s) =& \frac{(A-c)(d+\theta-s\theta)}{1+3d+d^2+(2+d-s)\theta} \\
p(\theta,s) =& \frac{A\,(1+d)\,(d+\theta)+c\,(1+2\,d+\theta-s\,\theta)}{1+3\,d+d^2+(2+d-s)\,\theta} \\
\pi_1(\theta,s) =& \frac{(A-c)^2\,(1+d)^2\,(d+2\,\theta)}{2\,(1+3\,d+d^2+(2+d-s)\,\theta)^2} \\
\pi_2(\theta,s) =& \frac{(A-c)^2\,(d+\theta-s\,\theta)\,(2\,\theta+d\,(2+d+\theta+s\,\theta))}{2\,(1+3\,d+d^2+(2+d-s)\,\theta)^2} \\
CS(\theta,s) =& \frac{(A-c)^2\,(1+2\,d+\theta-s\,\theta)^2}{2\,(1+3\,d+d^2+(2+d-s)\,\theta)^2} \\
SW(\theta,s) =& \frac{(A-c)^2 G}{2\,(1+3\,d+d^2+(2+d-s)\,\theta)^2},
\end{aligned}$$

where $G = 2\,d^3+4\,d^2\,(2+\theta)+(1+\theta-s\,\theta)\,(1+3\,\theta-s\,\theta)+d\,(5+12\,\theta-6\,s\,\theta+\theta^2-s^2\,\theta^2)$.

Appendix 2: Equilibrium cross-ownership for any given level of privatization

Solving the firm 2's problem in stage 2, without considering the constraint (c), we get the following.

$$s = \frac{d\,\{d\,(2+d)^2-1\}+d\,(2+d)\,(5+3\,d)\,\theta+2\,(1+d)\,(2+d)\,\theta^2}{d\,(3+d)\,\theta} = \hat{s}_{h,d}, \text{ say.}$$

Now, it is easy to check that, if $d \geq 0.205569$, $\hat{s}_{h,d} > 1$ for all $\theta \in (0,1]$. Otherwise, when $d < 0.205569$, we have the following two cases:

$$(i)\ \hat{s}_{h,d} \leq 0, \text{ if } 0 < \theta < \theta_1$$
$$\text{and } (ii)\ 0 < \hat{s}_{h,d} \leq 1, \text{ if } \theta_1 \leq \theta < \theta_2;$$

where $\theta_1 = \frac{-2-3\,d+\frac{2}{1+d}+\sqrt{\frac{d\,(4+d\,(3+d))\,(2+d\,(5+d))}{(1+d)^2\,(2+d)}}}{4}$ and $\theta_2 = \frac{-1-3\,d+\frac{2}{2+d}+\sqrt{\frac{d\,(16+d\,(1+d)\,(-7+d\,(2+d)))}{(1+d)\,(2+d)^2}}}{4}$. Note that, if $d < 0.205569$, $0 < \theta_1 < \theta_2 < 1$. Therefore, given any level of privatization (θ), optimal cross-ownership is as given in (1).

Appendix 3: Stage 3 equilibrium outcomes in case of less efficient public firm

When the two firms produce homogeneous products using different CRS technologies and the public firm is relatively less efficient, given any θ and s, the equilibrium outputs, price, profits, consumer surplus and social welfare are, respectively,

$$\begin{aligned}
q_{1,k}(\theta,s) =& \frac{A-2h}{1+2\theta-s\theta},\\
q_{2,k}(\theta,s) =& \frac{h+A\theta-As\theta+hs\theta}{1+2\theta-s\theta},\\
p_k(\theta,s) =& \frac{h+A\theta-hs\theta}{1+2\theta-s\theta},\\
\pi_{1,k}(\theta,s) =& \frac{(A-2h)^2\theta}{(1+2\theta-s\theta)^2},\\
\pi_{2,k}(\theta,s) =& \frac{(h+A(1-s)\theta+hs\theta)(A\theta+h(1-s\theta))}{(1+2\theta-s\theta)^2},\\
CS_k(\theta,s) =& \frac{(A-h+(A-As+hs)\theta)^2}{2(1+2\theta-s\theta)^2} \text{ and}\\
SW_k(\theta,s) =& \frac{A^2(-1+(-3+s)\theta)(-1+(-1+s)\theta)+2Ah(-1+\theta(-3+s+s\theta))+h^2(3-\theta(-8+s(2+s\theta)))}{2(1+2\theta-s\theta)^2}.
\end{aligned}$$

Therefore, the payoff of firm 2 is as follows.

$$\begin{aligned}
O_{2,k}(\theta,s) =& \pi_{2,k}(\theta,s)+s\theta\pi_{1,k}(\theta,s)\\
=& \frac{h^2+Ah(2-s)\theta+\left(A^2-Ah(4-s)s+h^2(4-s)s\right)\theta^2}{(1+2\theta-s\theta)^2}.
\end{aligned}$$

Appendix 4: Proof of Lemma 5

The problem of the government can be written as follows.

$$\underset{\theta}{Max}\ SW_k(\theta,s)$$

subject to the constraints

$$s=0 \text{ and}$$

$$0 \le \theta \le 1;$$

where $SW_k(\theta, s)$ is as given in Appendix 3.

Now, it is easy to check that $\underset{\theta}{ArgMax}\ SW_k(\theta, 0) = \frac{k}{A-4k}$.

Since $k < \frac{A}{4}$ by assumption, $\frac{k}{A-4k} > 0$. For $\frac{k}{A-4k} < 1$, we must have $k < \frac{A}{5}$. QED

Appendix 5: Proof of Lemma 6

Suppose that firm 2 decides to own s proportion of its less efficient rival firm, which is also a private firm. In this case, the payoff of firm 2 is $O_2(1, s) = \frac{(A+k)^2+k\,(-5\,A+4\,k)\,s+(A-k)\,k\,s^2}{(3-s)^2}$, which is obtained by substituting $\theta = 1$ in the expression for $O_2(\theta, s)$ as given in Appendix 3. So, the probelem of firm is: $\underset{s}{Max}\ O_2(1, s)$, subject to the constraint $0 \leq s \leq 1$.

Now, note that $\frac{\partial O_2(1,s)}{\partial s} = 0 \Rightarrow s = 7 - \frac{2A}{k}$ and $\frac{\partial}{\partial s}[\frac{\partial O_2(1,s)}{\partial s}]\,|_{s=7-\frac{2\,A}{k}} = \frac{k^4}{8\,(A-2\,k)^2} > 0$. That is, $O_2(1, s)$ is minimum at $s = 7 - \frac{2A}{k}$.

Now, $7 - \frac{2A}{k} < 0 \Rightarrow k < \frac{2A}{7}$, which is obvious since $h < \frac{A}{4}$. Therefore, $\frac{\partial O_2(1,s)}{\partial s} > 0$ for all $s \in [0, 1]$. It implies that $O_2(1, s)$ is maximum at $s = 1$, since we must have $0 \leq s \leq 1$. QED

Appendix 6: Proof of Lemma 7 Firm 2's problem in stage 2, for any given level of privatization (θ) can be written as follows.

$$\underset{s}{Max}\ \ O_{2,k}(\theta, s)$$

subject to the constraint

$$0 \leq s \leq 1;$$

where $O_{2,k}(\theta, s)$ is as given in Appendix 3.

Now, it is easy to check that $O_{2,k}(\theta, s)$ is convex in s, and has a minimum at $s = 2 + \frac{1}{\theta} + \left(4 - \frac{2\,A}{k}\right)\,\theta = \underline{s}$, say. Therefore, we have the following.

(a) If $\underline{s} \geq 1$, $s = 0$ is the solution of the above problem.

(b) If $\underline{s} \leq 0$, $s = 1$ is the solution of the above problem.

(c) If $0 < \underline{s} < 1$ and $O_{2,k}(\theta, 0) > O_{2,k}(\theta, 1)$, $s = 0$ is the solution of the above problem.

(d) If $0 < \underline{s} < 1$ and $O_{2,k}(\theta, 1) > O_{2,k}(\theta, 0)$, $s = 1$ is the solution of the above problem.

Therefore, it is sufficient to say that, if $O_{2,k}(\theta, 1) > O_{2,k}(\theta, 0)$, $s = 1$ is optimum; otherwise $s = 0$ is optimum.

Now, $O_{2,k}(\theta,1) > O_{2,k}(\theta,0)$, if $\theta > \hat{\theta}_k$, where $\hat{\theta}_k \in (0,1)$ and is such that $k + 3\,k\,\hat{\theta}_k - 2\,A\,\hat{\theta}_k^2 + 6\,k\,\hat{\theta}_k^2 - 3\,A\,\hat{\theta}_k^3 + 6\,k\,\hat{\theta}_k^3 = 0$. QED

Appendix 7: The equilibrium outcomes in stage 3, when products are differentiated

The decision problem of firm 1 and firm 2, in stage 3, are

$$\underset{q_1}{Max}\; O_1 = \pi_1 + (1-\theta)[\frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + \pi_2]$$
$$\text{and } \underset{q_2}{Max}\; O_2 = \pi_2 + s\theta\pi_1.$$

Solving the above two problems, we get the equilibrium outputs, prices, profits, consumer surplus and social welfare as follows.

$$q_{1,\gamma}(\theta,s) = \frac{(A-c)\;(2-\gamma)}{2\;(1+\theta) - \gamma^2\;(1+s\,\theta)},$$
$$q_{2,\gamma}(\theta,s) = \frac{(A-c)\;(1-\gamma+\theta-s\,\gamma\,\theta)}{2\;(1+\theta) - \gamma^2\;(1+s\,\theta)},$$
$$p_{1,\gamma}(\theta,s) = \frac{A\;(2-\gamma)\;\theta + c\;\left(2-\gamma^2+\gamma\;(1-s\,\gamma)\;\theta\right)}{2\;(1+\theta) - \gamma^2\;(1+s\,\theta)},$$
$$p_{2,\gamma}(\theta,s) = \frac{A\;(1-\gamma+(1+s\;(1-\gamma)\;\gamma)\;\theta) + c\;(1+\theta+\gamma\;(1-\gamma-s\,\theta))}{2\;(1+\theta) - \gamma^2\;(1+s\,\theta)},$$
$$\pi_{1,\gamma}(\theta,s) = \frac{(A-c)^2\,(2-\gamma)^2\,\theta}{(2\;(1+\theta) - \gamma^2\;(1+s\,\theta))^2},$$
$$\pi_{2,\gamma}(\theta,s) = \frac{(A-c)^2\;(1-\gamma+\theta-s\,\gamma\,\theta)\;(1-\gamma+(1+s\;(1-\gamma)\;\gamma)\;\theta)}{(2\;(1+\theta) - \gamma^2\;(1+s\,\theta))^2},$$
$$CS_{\gamma}(\theta,s) = \frac{(A-c)^2\;\left((2-\gamma)^2 + 2\;(2-\gamma)\;\gamma\;(1-\gamma+\theta-s\,\gamma\,\theta) + (1-\gamma+\theta-s\,\gamma\,\theta)^2\right)}{2\,(2\;(1+\theta) - \gamma^2\;(1+s\,\theta))^2},$$
$$SW_{\gamma}(\theta,s) = \frac{(A-c)^2\;\left(7+\theta\;(14+3\,\theta) + 2\,\gamma^3\,(1+s\,\theta)^2 - 2\,\gamma\;(3+\theta\;(5+s+s\,\theta)) - \gamma^2\;(2+s\,\theta\;(4+(2+s)\;\theta))\right)}{2\,(2\;(1+\theta) - \gamma^2\;(1+s\,\theta))^2}.$$

The corresponding payoff of firm 2 is,

$$O_{2,\gamma}(\theta,s) = \frac{(A-c)^2\left((-1+\gamma)^2 + (-1+\gamma)\left(-2+s\,\gamma^2\right)\theta + \left(1+s\,(-1+\gamma)\left(-4+s\,\gamma^2\right)\right)\theta^2\right)}{(-2\,(1+\theta)+\gamma^2\,(1+s\,\theta))^2}.$$

Appendix 8: The stage 2 equilibrium cross-ownership in case of differentiated products

We can write the problem of firm 2 in stage 2 of the game as follows.

$$\underset{s}{Max}\; O_{2,\gamma}(s;\theta) = \frac{(A-c)^2\,[(1-\gamma)^2 + (1-\gamma)\left(2-s\,\gamma^2\right)\theta + \{1+s\,(1-\gamma)\left(4-s\,\gamma^2\right)\}\,\theta^2]}{[2\,(1+\theta) - \gamma^2\,(1+s\,\theta)]^2}$$

$$\text{subject to} \quad 0 \leq s \leq 1$$

From the first order condition of the unconstrained problem, we get $s = \frac{2}{\gamma^2} + \frac{2\,(2-\gamma)\,\theta}{\gamma^2\,(2-\gamma-\gamma^2)} - \frac{\gamma}{(2+\gamma)\,\theta} = \hat{s}_\gamma$, say. It is easy to check that (a) $\hat{s}_\gamma > 0$, if $\theta > \theta_{1,\gamma}$ and (b) $\hat{s}_\gamma < 1$, if $\theta < \theta_{2,\gamma}$; where

$\theta_{1,\gamma} = \frac{4+\sqrt{(1-\gamma)\,(2-\gamma+\gamma^2)\,(2+\gamma-2\,\gamma^2)}}{2\,(2-\gamma)} - \frac{3+\gamma}{2}$

and $\theta_{2,\gamma} = \frac{2-\frac{8}{2-\gamma}+\gamma\,(1+\gamma)\,(2+\gamma)+\sqrt{\frac{(1-\gamma)\,\left(4-6\,\gamma+2\,\gamma^2+\gamma^3\right)\,\left(4+6\,\gamma-\gamma^3-\gamma^4\right)}{(2-\gamma)^2}}}{4}$;

$0 < \theta_{1,\gamma} < \theta_{2,\gamma} < 1$, since $0 < \gamma < 1$. The second order condition for maximization is satisfied in the relevant ranges of parametric values. Therefore, the equilibrium cross-ownership is as given in (2).