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Hippu Salk Kristle Nathan and Srijit Mishra

Indira Gandhi Institute of Development Research, Mumbai
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Hippu Salk Kristle Nathan and Srijit Mishra
Indira Gandhi Institute of Development Research (IGIDR)
General Arun Kumar Vaidya Marg
Goregaon (E), Mumbai- 400065, INDIA
Email (corresponding author): srijit@igidr.ac.in

Abstract

The study revisits the properties of Group Differential (GD) measures and extends it to include monotonicity and policy sensitivity axioms. Imposing level sensitivity, which indicates that a given gap is worse off at higher (lower) level of attainment (failure), the study concurs that â€˜simple differenceâ€™ and â€˜simple ratioâ€™ are the most basic GD measures for attainment and failure indicators respectively. It proposes two new measures, one each for attainment and failure, which have certain advantages from a policy implication perspective. An empirical illustration has been provided by taking two indicators from millennium development goals for different regions of the world.

Keywords: Level sensitivity, Monotonicity, Policy sensitivity, Simple difference, Simple ratio

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Hippu Salk Kristle Nathan is a Post-Doctoral fellow at the National Institute of Advanced Studies, Bangalore.

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Hippu Salk Kristle Nathan[†] and Srijit Mishra[‡]

Indira Gandhi Institute of Development Research (IGIDR),
General A K Vaidya Marg,
Goregaon (E), Mumbai- 400065, India

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[†] Post-Doctoral fellow at the National Institute of Advanced Studies, Bangalore; Email: hsknathan@nias.iisc.ernet.in.

[‡] Corresponding author, Associate Professor at the Indira Gandhi Institute of Development Research, Mumbai; Email: srijit@igidr.ac.in.

Group Differential for Attainment and Failure

1. Introduction

Group differential is an important class of measures to know the difference between two groups in attainment (or failure) indicators.[1] This differential can be expressed by simple difference or simple ratio or can be modified further. Whether simple or modified, the measure needs to satisfy certain properties. Mishra and Subramanian (2006) have introduced two axioms on level sensitivity, difference-based level sensitivity (DBLS) and ratio-based level sensitivity (RBLS). These axioms indicate that for an attainment (failure) indicator a given hiatus between two groups should acquire a greater salience the higher (lower) the level at which the hiatus arises.[2] They discuss three existing and a fourth new measure of group differential, which were later refined and enriched by addition of the axiom of normalization in a related paper (Mishra 2008).

This paper will add to the above-mentioned literature in two ways. First, this will treat attainment and failure indicators differently. A separate discussion is essential because for an attainment indicator the DBLS axiom is a stricter condition whereas for a failure indicator the RBLS axiom is a stricter condition. The other rationale for treating attainment and failure differently comes from the larger policy motive. When progress is interpreted as movement away from the state of 'no development' or origin, evaluation is done in terms of attainment. Life expectancy, income, or the composite quality of life indices like HDI are such attainment

[1] There are attainment indicators like literacy rate and immunization rate and failure indicators like infant mortality rate (IMR), maternal mortality rate (MMR) and death rate. An attainment indicator can be converted as failure by taking its inverse, like when literacy rate is replaced with illiteracy rate.

[2] The level sensitivity axioms have similarity with transfer sensitivity property of poverty indices , which requires a worse off person in a given income configuration to have greater weight on the income short-fall than that of a better off person (Sen 1976; Foster 1984; Kakwani 1993), negativity or equal addition property of inequality decomposition rule which says the contribution of an income source to inequality should be negative if all the individuals receive identical positive income from that source (Morduch and Sicular 2002; Paul 2004).

indicators. Also, at certain times it is important to analyse progress in terms of reductions in deprivation through the use of failure indicators, particularly when there is a defined goal to be achieved. Poverty estimates are a case in point. So, both the choice and treatment of attainment and failure indicators need careful consideration. In a recent work, Easterly (2008) while arguing that the indicators related to Millennium Development Goals (MDGs) has been unfair to Africa, has shown how the interpretation reverses when an attainment indicator is considered in place of failure. For instance, the failure indicator of people without access to clean water if converted to an attainment indicator of those with access to clean water gives opposite impression on Africa's progress.

Another contribution of this paper is that we have added the axioms of Monotonicy and Policy Sensitivity and have proposed new measures. The axiomatic properties are discussed over an attainment indicator in the main text and that of the failure indicator is given in Appendix 1. An empirical illustration has been provided with proportion of birth attended by skilled personnel and infant mortality rate for different regions of the world as indicators of attainment and failure respectively.

2. Axiomatic Properties of Group Differential

Consider a socio economic attainment indicator, $I_{js} \in [0,1]$; 0=no attainment and 1=full attainment for j^{th} group ($j=a,b$), under situation s ($s=A,B$). Without loss of generality, given a situation s let group a be considered to be at higher attainment level than b, $I_{as}>I_{bs}$ and given a group j situation A is at least as good as B so that $I_{jA} \geq I_{jB}$. Following are a number of intuitive properties that a measure of group differential, d or $d(I_{as},I_{bs})$ should satisfy.

Normalization: The measure of group differential should lie between zero and unity, $d \in [0,1]$ such that 0=no group-differential and 1=maximum group-differential. At no group-differential both the groups have the equal level of attainment; at maximum group-differential one of the groups has no attainment and the other group is at full attainment.

Monotonicity: The measure of group differential should be such that it is higher (lower) if one of the groups is at a particular level of attainment and the other changes so that the absolute gap increases (decreases). Mathematically, $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$ when $I_{aA}>I_{aB}$ and $I_{bA}=I_{bB}$. Two corollaries of monotonicity are axioms of minimality and maximality.

Minimality: The measure of group differential should be greater than zero if there is some group differential. Mathematically, $d>0$ if $(I_{as}-I_{bs})>0$.

Maximality: The measure of group differential should be lower than unity if the group-differential is less than the maximum. Mathematically, $d<1$ if $(I_{as}-I_{bs})<1$.

Difference based level sensitivity (DBLS): The measure of group differential should be such that it is more pronounced if the difference level persists at a higher level of attainment. Mathematically, if $I_{aA}-I_{bA}\geq I_{aB}-I_{bB}=h$; $h>0$, then the DBLS axiom requires that $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$. DBLS is weakly satisfied if for $I_{aA}-I_{bA}=I_{aB}-I_{bB}$ one gets $d(I_{aA},I_{bA})=d(I_{aB},I_{bB})$.

Ratio based level sensitivity (RBLS): The measure of group differential should be such that it is more pronounced if the ratio level persists at a higher level of attainment. Mathematically, if $I_{aA}/I_{bA}\geq I_{aB}/I_{bB}=k$; $k>1$, then the RBLS axiom requires that $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$. RBLS is weakly satisfied if for $I_{aA}/I_{bA}=I_{aB}/I_{bB}$ one gets $d(I_{aA},I_{bA})=d(I_{aB},I_{bB})$.

For attainment, DBLS is the stricter condition than RBLS, as when $I_{aA}-I_{bA}=I_{aB}-I_{bB}$, $I_{aA}/I_{bA}<I_{aB}/I_{bB}$. On the contrary, for failure indicators, RBLS is stricter.

Policy sensitivity: A measure of group differential should be such that it is at least as high as the absolute gap between the indicators. It is important from policy perspective as a measure lower than this gap could give the impression that the differences are not that serious, whereas if the measure is greater than this gap would induce sensitivity during policy intervention.[3] Mathematically, policy sensitivity axiom requires $d(I_{as},I_{bs})\geq(I_{as}-I_{bs})$.

3. Measures of Group Differential

3.1 Measure of group differential for attainment indicators

A measure of group-differential is simple difference,

$$d_1 = I_a - I_b \tag{1}$$

It satisfies axiom of normalization, monotonicity with its corollaries and RBLS whereas DBLS and policy sensitivity are satisfied in a weak sense only. Mishra and Subramanian (2006) discuss this in the context of failure indicators and indicate that it violates the axioms of DBLS and RBLS. It is a poor differential measure of failure, but it is a powerful measure for attainment.

A modified version of simple difference is,

$$d_2 = I_a^{\alpha} - I_b^{\alpha}; \quad \alpha > 1 \tag{2}$$

This satisfies axiom of normalization, monotonicity with its corollaries and the two level sensitivity axioms (DBLS and RBLS). However, it satisfies policy sensitivity axiom in a conditional sense when $\left(I_a^{\alpha} - I_b^{\alpha}\right) > \left(I_a - I_b\right)$.[4] Moreover, the value of group-differential is dependent on the subjective choice of α. In the limiting sense, when $\alpha = 1$ one can get $d_2 = d_1$.

An alternative measure that we propose is,

$$d_3 = \frac{I_a - I_b}{1 - \lambda I_b}; \quad \lambda \in (0,1) \tag{3}$$

This satisfies all axioms. Higher is the value of λ higher is the salience for a given hiatus,

[3] A similar argument was made by Subramanian (2004) while extending externality adjusted literacy measurement of Basu and Foster (1998).

[4] For instance, for $\alpha=2$, this condition is satisfied for $(I_a+I_b)>1$.

$\partial d_3/\partial\lambda>0$. For mathematical simplicity and to have a moderate salience one can keep $\lambda=0.5$ for empirical calculations. In the limiting sense, when $\lambda=0$, one can get $d_3=d_1$. We argue in favor of d_3 because of its axiomatic advantages. Thus, we have the following proposition.

Proposition 1 *For a two group scenario, there exists, d_3, a group differential measure for attainment indicators that satisfies the axioms of normalization, monotonicity, level sensitivity and policy sensitivity.*

3.2 Measure of group differential for failure indicators

Simple ratio is a powerful differential measure for failure indicators, I'. Mishra and Subramnian (2006) initially proposed the indicator to be $I_b^{'}/I_a^{'}$. To satisfy normalization, it can be modified to

$$d_1^{'} = 1 - \frac{I_b^{'}}{I_a^{'}} \quad (4)$$

This satisfies axiom of normalization, monotonicity with its corollaries (except when $I_b^{'}=0$), DBLS and policy sensitivity, whereas RBLS is satisfied in a weak sense only.

A modified version of simple ratio is,

$$d_2^{'} = 1 - \frac{I_b^{'\gamma}}{I_a^{'\delta}}; \quad \gamma \geq \delta > 0, \gamma = \delta \neq 1 \quad (5)$$

This satisfies the DBLS and RBLS axioms, but fails in normalization when $I_a^{'}= I_b^{'}$,[5] and monotonicity when $I_b^{'}=0$. It satisfies policy sensitivity axiom in a conditional sense when $\left(1 - \frac{I_b^{'\gamma}}{I_a^{'\delta}}\right) > \left(I_a^{'} - I_b^{'}\right)$.[6] Further, this measure is dependent on the subjectivity of γ and δ.

If one takes $\gamma=\delta=1$ in Equation 5, then $d_2^{'}=d_1^{'}$.

[5] At $I_a^{'}=I_b^{'}$, $d_2^{'}=1-I_a^{'(\gamma-\delta)}>0$. This means that when there is no hiatus between the two sub-groups the group-differential measure gives a positive value.

[6] For $\gamma>\delta>1$, this condition is always satisfied

An alternative measures that we propose is,

$$d_3^{'} = \left(1 - \frac{I_b^{'}}{I_a^{'}}\right)\left(1 - \mu I_b^{'}\right); \quad \mu \in (0,1] \tag{6}$$

This satisfies axioms of normalization, monotonicity with its corollaries (except when $I_b^{'}=0$) and the level sensitivity axioms (DBLS and RBLS). Lower is the value of μ higher is the salience for a given hiatus, $\partial d_3^{'}/\partial\mu<0$. For mathematical simplicity and to have a moderate salience one can keep $\mu=0.5$ for empirical calculations. In the limiting sense of $\mu=0$, one can get $d_3^{'}=d_1^{'}$.

The measure satisfies policy sensitivity axiom if $(I_a^{'}+\mu I_b^{'})<1$ or $\mu<((1-I_a^{'})/I_b^{'})$. The limiting value of μ for different values of $I_a^{'}$ and $I_b^{'}$ is given in Appendix 2. For all failure values expect $I_a^{'}=1$, policy sensitivity is satisfied for certain values of μ. For lower failure values ($I_a^{'}<0{,}5$; note that by assumption $I_b^{'}<I_a^{'}$), the axiom is satisfied for all values of μ. For $\mu=0.5$, policy sensitivity holds when $I_a^{'}$ is less than two-thirds. This condition is likely to be met for all practical purposes. For higher values of $I_a^{'}$ one may suggest the use of lower values of μ or the use of the attainment indicator that can be obtained by taking the inverse of the failure indicator. For instance, if we are comparing the gender differences in mortality across population subgroups suffering from different chronic diseases where mortality could range from 50-90% then one can either use $\mu=0.1$,[7] or it may be pertinent to consider an analysis of survival rate. We argue in favor of $d_3^{'}$ because of it satisfies all axioms, including policy sensitivity, for practical purposes. Now, we give the following proposition.

Proposition 2 *For a two group scenario, there exists, $d_3^{'}$, a group differential measure for failure indicators that satisfies the axioms of normalization, monotonicity (except when $I_b^{'}=0$), level sensitivity and policy sensitivity (except when $I_a^{'}=1$).*

[7] For an $I_a^{'}$ value of 99%, a μ value of 0.01 would suffice.

4. Empirical Illustration

For empirical illustration, indicators of Million Development Goals (MDGs) related to child and maternal health (Goals 4 and 5 respectively) have been used. The two indicators that have been used are proportion of birth attended by skilled personnel (an attainment indicator) and infant mortality rate (IMR, a failure indicator). Table 1 shows data for different regions of the world for 1990 and 2005.

Table 1: Birth attended by skilled health staff and infant mortality rate for different regions of the world, 1990 and 2005

Regions	Births attended by skilled health staff (in fraction)		Infant mortality rate (in fraction)	
	1990	2005	1990	2005
World	0.50	0.65	0.063	0.049
High income	–	0.99	0.010	0.006
Low & middle income	0.46	0.63	0.069	0.053
East Asia & Pacific	0.48	0.89	0.042	0.025
Europe & Central Asia	–	0.95	0.041	0.024
Latin America & Caribbean	0.72	0.89	0.044	0.024
Middle East & North Africa	0.47	0.81	0.058	0.034
South Asia	0.32	0.42	0.087	0.062
Sub-Saharan Africa	–	0.45	0.108	0.091

Source: World Bank (2010)
Note: The indicators are scaled to 0-1 range. The regional groups are according to World Bank classification and constitute only developing countries.

From the data given in Table 1, some cases are discussed to demonstrate the various group-differential measures. Table 2 gives the differential measures for the attainment indicator of proportion of births attended by skilled health staff. Case 1 takes the average of Low & Middle income countries and compares it with the better performing Latin American & Caribbean. It shows that with increase in levels of attainment between 1990 and 2005 the simple difference between these two groups remains the same, the d_1 measure. Difference-based level sensitivity, captured through the measures of d_2 and d_3, show that the same hiatus at a higher level have higher salience. Case 2 takes the sub-groups of Middle East & North

Africa and South Asia with both the regions showing less than 50% attainment in 1990. The calculations show the policy sensitivity advantage of d_3 over d_2. As indicated earlier, when average attainment is less than 50% then for specific values of α and β, the differential measure d_2 turns out to be less than the simple difference and thereby may give an impression of a lower gap between the groups considered.

Table 2 Group differential measures for attainment indicator (proportion of births attended by skilled health staff)

Cases and Groups information	Situations	I_a	I_b	Group differential measures		
				d_1	d_2	d_3
Case 1: $I_{aA} - I_{bA} \approx I_{aB} - I_{bB}$ *a*: Latin American & Caribbean *b*: Low & middle income countries	*A*: 1990	0.72	0.46	0.26	0.31	0.34
	B: 2005	0.89	0.63	0.26	0.40	0.38
Case 2: $d_2 < d_1$ *a*: Middle East and North Africa *b*: South Asia	*A*: 1990	0.47	0.32	0.15	0.120	0.179

Note: The differential measures d_1, d_2, and d_3 are as discussed in the text; d_2 has been computed for $\alpha=\beta=2$ in Case 1 and $\alpha=2$ and $\beta=2.01$ in Case 2; d_3 has been computed for $\lambda=0.5$.

Table 3 Group differential measures for failure indicator (infant mortality rate)

Cases and Groups information	Situations	I_a'	I_b'	Group differential measures		
				d_1'	d_2'	d_3'
Case 1': $I_{bA}'/I_{aA}' \approx I_{bB}'/I_{aB}'$ *a*: Europe & Central Asia *b*: High income	*A*: 1990	0.040	0.010	0.750	0.842	0.746
	B: 2005	0.024	0.006	0.750	0.850	0.748
Case 2': $I_{aA}' = I_{bA}'$ *a*: Latin America & Caribbean *b*: Europe & Central Asia	*A*: 2005	0.024	0.024	0	0.311	0
Case 3': $I_{bA}' < I_{aA}'$ & $I_{bB}' > I_{aB}'$ *a*: Latin American & Caribbean *b*: East Asia & Pacific	*A*: 1990	0.044	0.042	0.045	0.305	0.044
	B: 2005	0.024	0.025	0.040^{r}	0.339^{r}	0.040^{r}

Note: The differential measures d_1', d_2', and d_3' are as discussed in the text; d_2' is computed for $\gamma=1.1$ and $\delta=1$; d_3' is computed for $\mu=0.5$. The actual value of $I_a'=0.041$ for Europe & Central Asia in 1990; it was reduced to 0.040 to get equal ratio situations. With actual values the differential measures will be 0.756, 0.846 and 0.752 respectively. In Case 3', there is a rank reversal between the two sub-groups, and hence, the differential measures are indicated with the superscript 'r'.

Table 3 gives the group differential measures for the failure indicator, infant mortality rate. It gives the simple ratio measure of d_1' as well as the ratio-based level sensitive measures of d_2' and d_3'. It was difficult to get cases where the ratio measure was equal. In

Case 1' the two sub-groups are the High income countries and the region of Europe & Central Asia where minor adjustments have been made to give us the result of an equal ration in the two years. Case 2' shows the sub-groups of Latin America & Caribbean and Europe & Central Asia having equal indicator values in 2005. The differential measure $d_2^{'}$ gives a positive value indicating failure of normalization axiom. In Case 3' one takes the sub-groups Latin American & Caribbean and East Asia & Pacific; there is rank reversal in the two years, 1990 and 2005. In such scenarios, group-differential can be computed, but the value should be indicated with a remark to distinguish the rank reversal for ease in interpretation.

5. Conclusions

A basic departure of the current paper is that it proposes different set of group differential measures for attainment and failure indicators. The most basic of group differential measures for these two types of indicators are simple difference and simple ratio respectively. In the past, there have been some modifications of these to address level sensitivity concerns. In this paper, two alternative measures have been proposed (one for attainment and one for failure) that satisfy both the axioms of level-sensitivity and normalization. In addition, they also satisfy two newly proposed axioms of monotonicity and policy sensitivity in all situations for the attainment measure and except for boundary values of the indicator for the failure measure. The advantages of the proposed indicators are empirically demonstrated with two of the millennium development goals, viz., proportion of births attended by skilled health staff (for attainment) and infant mortality rate (for failure).

The present study has dwelt upon the two group case. One has to take note of rank reversals while computing and interpreting results. A possible future exploration is to extend the group differential measure for multiple groups.

Appendix 1

Consider a socio economic failure indicator, $I_{j's}{}' \in[0,1]$; 0=no failure and 1=complete failure for j^{th} group ($j=a,b$), under situation s ($s=A,B$). Without loss of generality, given a situation s let group b be considered to be at lower failure level than a, $I_{as}{}' > I_{bs}{}'$ and given a group j situation A is at least as good as B so that $I_{jA}{}' \leq I_{jB}{}'$. Similar to attainment indicator, following are a number of intuitive properties that a measure of group differential, d or $d(I_{as}{}', I_{bs}{}')$ should satisfy.

Normalization: $d \in [0,1]$ such that 0=no group-differential and 1=highest group-differential.

Monotonicity: $d(I_{aA}{}', I_{bA}{}') > d(I_{aB}{}', I_{bB}{}')$ when $I_{aA}{}' = I_{aB}{}'$ and $I_{bA}{}' < I_{bB}{}'$.

Minimality: $d>0$ if $(I_{as}{}' - I_{bs}{}')>0$.

Maximality: $d<1$ if $(I_{as}{}' - I_{bs}{}')<1$.

Difference based level sensitivity (DBLS): If $I_{aA}{}' - I_{bA}{}' \geq I_{aB}{}' - I_{bB}{}' = h'$; $h'>0$, then $d(I_{aA}{}', I_{bA}{}') > d(I_{aB}{}', I_{bB}{}')$.

Ratio based level sensitivity (RBLS): If $I_{aA}{}' / I_{bA}{}' \geq I_{aB}{}' / I_{bB}{}' = k'$; $k'>1$, then $d(I_{aA}{}', I_{bA}{}') > d(I_{aB}{}', I_{bB}{}')$.

Policy sensitivity: $d(I_{as}{}', I_{bs}{}') \geq (I_{as}{}' - I_{bs}{}')$.

Appendix 2

Upper limiting values of μ for which policy sensitivity is satisfied.

$I_a' \downarrow I_b' \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	All										
0.1	All	All									
0.2	All	All	All								
0.3	All	All	All	All							
0.4	All	All	All	All	All						
0.5	All	All	All	All	All	All					
0.6	All	All	All	All	All	0.80	0.67				
0.7	All	All	All	All	0.75	0.60	0.50	0.43			
0.8	All	All	All	0.67	0.50	0.40	0.33	0.29	0.25		
0.9	All	All	0.50	0.33	0.25	0.20	0.17	0.14	0.13	0.11	
1.0	All	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: 'All' is mentioned in the cases for which policy sensitivity is satisfied for all the values of μ

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