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Abstract

This paper examines the possibility of emergence of incentive equilibrium in the case of monopoly, without relying on agency theory based arguments. It shows that, when there is network effect of consumption, it is optimal for a monopolist to offer sales-oriented incentive scheme to her manager. The extent of sales-orientation of the optimal incentive scheme is higher in the case of stronger network effect. It also shows that both the monopolist and consumers are better off under managerial delegation than in case of no delegation, unlike as in the case of usual oligopoly without network effect.

Keywords: Incentive equilibrium, Managerial delegation, Monopoly, Network effects

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1 Introduction

Managerial delegation is a common phenomenon in modern firms that faces imperfect competition, irrespective of whether firms produce network goods or non-network goods. It is also well documented that profit oriented owners of firms often index managerial compensation to variables other than firm's own profit, e.g. sales, market share, rival firms profit, etc. Several authors have attempted to explain this issue of managerial delegation through distorted incentive schemes so far.

In their seminal papers Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) (henceforth VFJS) offer a novel justification for managerial delegation. In particular, considering usual oligopolistic market structure they demonstrate that it is optimal for profit oriented owners of firms to delegate tasks to their managers by offering them appropriately designed incentive schemes, which make the managers to deviate away from pure profit maximization. The reason for emergence of such incentive equilibrium in oligopoly is that owners can gain strategic advantage by distorting their managers' objectives, since managerial delegation in one firm alters the rival firms' behaviour in its favour. Clearly, such strategic effect does not exist in the case of monopoly. Therefore, managerial delegation in monopolistic firms, which is also widely observed, does not appear

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to be justified on the basis of strategic effect as in VFJS. However, strategic behaviour of firm(s) need not necessarily be limited to oligopolistic market structure. It may be optimal for firm(s) to behave strategically to influence consumers' expectations regarding product quality, availability of after-sale services for consumer durables, number of other users of the product, etc. In such scenarios, managerial delegation may serve as a commitment device for the firm to influence consumers' willingness to pay in its favour. This paper attempts to examine the possibility of emergence of incentive equilibrium under monopoly in such a scenario.

Considering a two stage game between the owner of a monopoly firm, which produces a good that has positive consumption externalities (i.e., network effects), and her manager, in this paper we demonstrate that it is optimal for the monopolist to offer sales-oriented delegation contract to her manager. Moreover, the stronger the network effect, greater the extent of sales-orientation of the optimal delegation contract. The intuition behind these results are as follows. When there is network effect of consumption, publicly observable sales-oriented managerial delegation contract serves as an instrument to enhance consumers' expectations about the market size of the monopolist and, thus, consumers' willingness to pay for the product. This positive effect of delegation on firm's profit via consumers' willingness to pay for the product/service is larger in case of stronger network effect. Clearly, the mechanism behind the emergence of incentive equilibrium in the case of monopoly with network effect is quite different from that under usual oligopolistic market structure.

We also find that both profit of the monopolist and consumers' surplus are higher in the incentive equilibrium compared to that under no delegation, unlike as in the case of usual oligopoly. Such win-win effect of delegation may have some implications for regulation of monopoly in case of network goods industries.

Needless to mention here that there are wide ranges of products and services for which the utility derived by a particular consumer of the good increases with the number of other users (i.e., total sale) of the good. In other words, a large number of products and services are having network effects. Therefore, it seems that the analysis of this paper offer useful insights to understand the issue of separation of ownership and management in a number of real-life cases.

We note here that, in the principal-agent literature, distorted incentives for managers in monopolistic firms is justified by considering asymmetric information between the owner and her manager and their conflicting objectives (Holmstrom, 1977; Harris and Raviv, 1979; Bolton and Dewatripont, 2004). In contrast, results of this paper do not rely on agency problem based arguments. Clearly, this paper identifies a new channel through which managerial delegation can work even in case of monopoly, when there is positive consumption externalities.

We also note that starting with VFJS, the literature on managerial delegation has been enriched by many studies, which examines the implications of managerial delegation to various issues.[2] However, these studies have primarily focused on oligopolistic competition in markets for non-network goods. The issue of network externalities of consumption has not received much attention in this stream of literature. In a very recent paper, Hoernig (2012) attempts to examine, through an example, the implications of network effects on

[2]For example, wage bargaining (Szymanski, 1994), sequential entry (Church and Ware, 1996), trade policy (Das, 1997), delegation to bureaucrats (Basu et al., 1997), divisionalization (Gonzlez-Maestre, 2000), mixed oligopoly (White, 2001), equivalence of price and quantity competition (Miller and Pazgal, 2001), mergers (Gonzalez-Maestre and Lopez-Cunat, 2001; Ziss, 2001), cartel stability (Lambertini and Trombetta, 2002), environmental damage control (Barcena-Ruiz and Garzon, 2002; Pal, 2012), choice of incentive scheme (Jansen et al., 2007), mixed ownership (Saha, 2009), cooperative managerial delegation and R&D (Pal, 2010), to name a few.

optimal incentive contract in the case of Bertrand type price competition. However, none of these papers offer any insight to understand the issue of managerial delegation in the case of monopoly.

The rest of the paper proceeds as follows. Section 2 describes the model and characterizes the equilibrium under monopoly. It also illustrates the results through an example. Section 3 provides concluding remarks. All proofs are relegated to Appendix.

2 Delegation under monopoly

Let us consider that there is only one firm in the market producing a good that has network effects. The utility function of the representative consumer is assumed to be quasi-linear: $U(x;y) = m + u(x;y)$, where x denotes the quantity of the good produced by the monopolist, y denotes the consumers' expectation about the market size (sales) of the monopolist, and m denotes the consumption of all other goods measured in terms of money. Therefore, the inverse demand function faced by the monopolist is given by $p = \frac{\partial U}{\partial x} = p(x;y)$. The assumptions imposed on the utility function and the corresponding demand curve are summarized below.

Assumption 1: The inverse demand function $p = p(x;y)$ is downward sloping and (weakly) concave in x: $\frac{\partial p}{\partial x} < 0$ and $\frac{\partial^2 p}{\partial x^2} \leq 0$.

Assumption 2: (a) Marginal utility of the good produced by the monopolist increases with the increase in consumers' expectation about the total consumption of the good: $\frac{\partial^2 U}{\partial y \partial x} > 0$. That is, there is positive network effect of consumption of good x. (b) $|\frac{\partial p}{\partial x}| > |\frac{\partial p}{\partial y}|$.

Assumption 3: (a) $\frac{\partial U}{\partial y} = 0$ if $y = x$; $\frac{\partial^2 U}{\partial y^2} < 0$. (b) $\frac{\partial^2 p}{\partial y \partial x} = \frac{\partial^2 p}{\partial x \partial y} = 0$

The first assumption is the standard regulatory assumption. From the Assumption 2(a), it follows that $\frac{\partial p}{\partial y} > 0$, i.e., consumers' willingness to pay increases due to increase in their expectations about the monopolist's market size. Clearly, higher value of $\frac{\partial p}{\partial y}$ indicates stronger network effect. Assumption 2(b) ensures that there is an upper-bound of the strength of network effect. Assumption 3(a) implies that, for each level of consumption x, correct expectations lead to highest utility. We impose Assumption 3(b) for simplicity.

The cost function of the monopolist is assumed to be linear in output, $C = C(x)$, where $\frac{\partial C}{\partial x} > 0$ and $\frac{\partial^2 C}{\partial x^2} = 0$. We consider that the monopolist delegates the task to take decisions concerning output/price to her risk-neutral manager by offering a publicly observable incentive scheme, which is a linear combination of profit and sales revenues as in Fershtman and Judd (1987), to the manager. Therefore, given the incentive scheme, the manager will maximize $O = \lambda\pi + (1-\lambda)\, p\, x$, where π is the profit of the firm and λ is the incentive parameter set by the monopolist. Clearly, if $\lambda < 1$, the owner gives incentive to her manager for sales maximization. Alternatively, if $\lambda > 1$, the manager is penalized for sales maximization. Thus, we can say that the incentive contract is sales-oriented (profit-oriented), if the incentive parameter λ is less (greater) than one. Needless to mention here that $\lambda = 1$ corresponds to the case of a pure profit maximizing monopolist without managerial delegation.

Note that we can re-write the incentive scheme as $O = px - \lambda C(x)$. It implies that marginal cost of production is perceived to be lower (greater) than its actual level ($\frac{\partial C}{\partial x}$) by the manager, if the value of the incentive parameter λ is less (greater) than one. Therefore, sales-oriented (profit-oriented) incentive scheme is likely to induce the manager to be more (less) aggressive in the product market compared to that of a pure profit maximizing monopolist without delegation.

We consider that stages of the game involved are as follows. In Stage 1, the owner decides the incentive parameter so that profit is maximized, which is publicly observable.

In Stage 2, the manager chooses the quantity to maximize his incentive scheme, given the incentive parameter. We solve this game by the backward induction method.

Following Katz and Shapiro (1985), we impose the additional 'rational expectations' condition $y = x$ to find the Stage 2 equilibrium outcomes. Now, the first order condition of the manager's maximization problem, $\underset{x}{Max}\ O(x; y|\lambda)$, is given by $\frac{\partial O}{\partial x} = p + \frac{\partial p}{\partial x}x - \lambda\frac{\partial C}{\partial x} = 0$.[3] Thus, the equilibrium output in Stage 2 is determined by the following two equations.

$$\frac{\partial O}{\partial x} = p + \frac{\partial p}{\partial x}x - \lambda\frac{\partial C}{\partial x} = 0 \tag{1a}$$

$$y = x \tag{1b}$$

Lemma 1: *Lower value of the incentive parameter leads to higher equilibrium output in Stage 2:* $\frac{dx}{d\lambda} < 0$.

Proof: See Appendix.

In other words, greater sales-orientation (profit-orientation) of the incentive scheme offered by the owner to her manager results in higher (lower) output. Thus, if the monopolist offers a sales-oriented (profit-oriented) incentive scheme to her manager, the equilibrium output will be greater (less) than the standard monopoly output under no-delegation.

Now, in Stage 1, the problem of the owner can be written as follows.

$$\begin{cases} \underset{\lambda}{Max}\ \pi = px - C(x) \\ \text{subject to the constraints} \\ \qquad (1a) \text{ and } (1b). \end{cases} \tag{2}$$

[3]The second order condition of maximization is satisfied for any value of λ, since $\frac{\partial^2 O}{\partial x^2} = 2\frac{\partial p}{\partial x} + x\frac{\partial^2 p}{\partial x^2} < 0$ by Assumption 1.

Note that we can decompose the total effect of delegation on firm's profit as follows.

$$\frac{d\pi}{d\lambda} = \frac{\partial\pi}{\partial x}\frac{dx}{d\lambda} + \frac{\partial\pi}{\partial y}\frac{\partial y}{\partial x}\frac{dx}{d\lambda} \tag{3}$$

While the first term in the right-hand-side of (3) is the direct effect of delegation on firm's profit, the second term is the indirect effect of delegation on firm's profit. The indirect effect works through the impact of delegation on consumers' expectations due to network effects, which is an additional effect of delegation on firm's profit in the present context.

It is easy to check that the direct effect of a marginal change in incentive parameter (λ) from one is zero. That is, in absence of network effect, managerial delegation is not profitable to the monopolist. However, in the presence of network effect, a decrease in incentive parameter (λ) from one leads to an increase in output (x), which enhances consumers' expectation about the market size (y) of the firm and that, in turn, is likely to increases firm's profit. Therefore, due to the existence of such indirect effect of delegation through consumers' expectations in the case of network effects, it is likely to be profitable to the monopolist to offer sales-oriented incentive scheme to the manager. Now, solving the problem (3), we get the following.

Proposition 1: *When there is network effect, the equilibrium incentive parameter set by the monopolist is given by* $\lambda^* = 1 - \left[\frac{x\frac{\partial p}{\partial y}}{\frac{\partial C}{\partial x}}\right] < 1$, *implying that it is optimal for the monopolist to offer sales-oriented delegation contract to her manager.*

Proof: See Appendix.

From Proposition 1, it is easy to observe that the optimal incentive parameter is (a) decreasing in strength of network effect ($\frac{\partial p}{\partial y}$) and (b) increasing in marginal cost of production. Therefore, λ^* can even be negative, if marginal cost of production is sufficiently low and/or network effect is sufficiently strong. In other words, the monopolist may find it optimal to penalize her manager for profit maximization, if marginal cost of production is sufficiently low or network effect is sufficiently strong.

Corollary 1: *The extent of sales-orientation of the optimal incentive scheme offered by the monopolist to her manager is higher in the case of stronger network effect. If the network effect is sufficiently strong, the monopolist may even penalize profit maximization in equilibrium, ceteris paribus.*

It is evident from Proposition 1 that the equilibrium profit of the monopolist is greater under managerial delegation than that in case of no delegation. Also, since the equilibrium incentive parameter is less than one, Lemma 1 implies that the equilibrium output under delegation is greater than that under no delegation. Now, since consumers' correctly anticipate the market size (sales) of the firm (i.e., $y = x$) in equilibrium, irrespective of whether there is delegation or not, the equilibrium price under delegation is less than that under no delegation.[4] Therefore, the equilibrium consumer surplus under delegation is greater than that under no delegation. Clearly, the equilibrium social welfare also improves due to delegation.

Corollary 2: *In the case of network effects, managerial delegation by a monopolist leads to higher profit, higher output, lower price, higher consumer surplus and higher social welfare in equilibrium compared to that under no delegation.*

Note that in the case of usual oligopoly without network effects, managerial delegation leads to either (a) higher profits and lower consumer surplus (under price competition) or (b) lower profits and higher consumer surplus (under quantity competition), in equilibrium

[4]Note that $\frac{dp}{d\lambda} = \frac{\partial p}{\partial x}\frac{\partial x}{\partial \lambda} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial \lambda} = \left[\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right]\frac{\partial x}{\partial \lambda} > 0$, since $\left(\frac{\partial p}{\partial x} < 0 \text{ and } |\frac{\partial p}{\partial x}| > |\frac{\partial p}{\partial y}|\right) \Rightarrow \left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) < 0$. So, a decrease in λ from one leads to fall in price.

(VFJS). It seems to be interesting to note that, in the case of monopoly with network effect, the equilibrium incentive contract leads to a win-win situation for the monopolist and the consumers. Not only the monopolist is better off in the incentive equilibrium than in usual monopoly equilibrium without delegation, the extent of inefficiency due to monopoly is also less in the incentive equilibrium.

2.1 An example

Let the utility function of the representative consumer be $U = m + \left(\alpha x - \frac{x^2 - 2nxy - ny^2}{2}\right)$, where higher value of the parameter $n \in [0, 1)$ indicates stronger network effects. Thus, the inverse demand function faced by the monopolist is $p = \alpha - x + ny$. Let the cost function of the monopolist be $C = cx$, $0 < c < \alpha$. The incentive scheme and the stages of the game involved remain the same as before. Then, in Stage 2, the first order condition of the manager's maximization problem is given by $\alpha - 2x + ny - \lambda c = 0$. From this first order condition and the 'rationality' condition $y = x$, we obtain the Stage 2 equilibrium output and price: $x(\lambda) = \frac{\alpha - \lambda c}{2-n}$ and $p(\lambda) = \frac{\alpha + \lambda(1-n)c}{2-n}$. Clearly, the Stage 2 equilibrium output is a decreasing function of the incentive parameter (λ), as in Lemma 1.

Now, substituting the Stage 2 equilibrium output and price into the profit expression of the monopolist, we get $\pi = \frac{[\alpha - c\,(2-n\,(1-\lambda)-\lambda)]\,(\alpha - c\,\lambda)}{(2-n)^2} = \pi(\lambda)$. Now, solving the problem of the monopolist in Stage 1, $\underset{\lambda}{Max}\ \pi(\lambda)$, we obtain the equilibrium incentive parameter $\lambda^* = 1 - \frac{n\,(\alpha - c)}{2\,c\,(1-n)}$.[5] It is evident that $\lambda^* < 1\ \forall\ n \in (0, 1)$, which implies that it is profitable for the monopolist to delegate the task to decide output/price to her manager by offering the manager a sales-oriented incentive scheme in the presence of network effect. On the otherhand, if there is no network effect ($n = 0$), no managerial delegation is optimal for the monopolist. Also, note that $\frac{\partial \lambda^*}{\partial n} = -\frac{(\alpha - c)}{2\,c\,(n-1)^2} < 0$. That is, stronger is the network effect,

[5]Note that $\frac{d^2\pi(\lambda)}{d\lambda^2} = \frac{-2\,c^2\,(1-n)}{(2-n)^2} < 0$. So, the socond order condition for maximization is satisfied.

greater is sales-orientation of the optimal incentive scheme. Further, if network effect is sufficiently strong ($n > \frac{2c}{c+\alpha}$), the optimal incentive scheme is such that the manager is penalized for profit maximization.

Finally, it is straightforward to check that $\pi^* = \frac{(\alpha-c)^2}{4(1-n)} > \frac{(\alpha-c)^2}{(2-n)^2} = \pi^{ND}$, $x^* = \frac{\alpha-c}{2-2n} > \frac{\alpha-c}{2-n} = x^{ND}$, $p^* = \frac{\alpha+c}{2} < \frac{\alpha+c(1-n)}{2-n} = p^{ND}$, $CS^* = \frac{(\alpha-c)^2}{8(1-n)} > \frac{(1-n)(\alpha-c)^2}{2(2-n)^2} = CS^{ND}$ and $SW^* = \frac{3(\alpha-c)^2}{8(1-n)} > \frac{(3-n)(\alpha-c)^2}{2(2-n)^2} = SW^{ND}$, where superscript ND denotes monopoly equilibrium under no delegation, CS denotes consumer surplus and SW denotes social welfare. Also, note that $\frac{d}{dn}\left[\pi^* - \pi^{ND}\right] = \frac{n^2(\alpha-c)^2}{4(1-n)(2-n)^2} > 0$. That is, stronger is the network effect, greater is the incentive for the monopolist to opt for managerial delegation.

3 Concluding remarks

In this paper we have explored the possibility of managerial delegation in a monopoly firm. We have demonstrated that it is optimal for a monopolist to delegate tasks by offering a sales-oriented incentive scheme to her manager, when there is network effect. The extent of sales-orientation of the optimal incentive scheme increases with the strength of network effect. In fact, if the network effect is sufficiently strong, the manager may even be penalized for profits. On the other hand, as is well known, if there is no network effect, it is never optimal for a monopolist to opt for managerial delegation, unless there is agency problem.

As mentioned before, it is well established in the literature that incentive equilibrium emerges in the case of usual oligopolistic market structure, because strategic managerial delegation in a firm changes rival firms' behaviour. On the other hand, in the case of monopoly with network effect, managerial delegation works through its impact on consumers' willingness to pay for the product. It implies that, apart from strategic effect, managerial delegation will have an additional effect on firm's profit through changes in consumers' willingness to pay in the case of oligopoly with network effects. This additional effect of managerial delegation due to network effects is likely to have implications to the equilibrium outcomes in oligopoly.

Note that, when there is only strategic effect, in the case of Betrand type price (Cournot type quantity) competition each owner has unilateral incentive to induce her manager to be less (more) aggressive in the product market by offering profit-oriented (sales-oriented) incentive scheme (Fershtman and Judd, 1987). However, network effects provide incentives to be more aggressive in the product market. Clearly, strategic effect and network effect of managerial delegation work in opposite directions (the same direction) in the case of price (quantity) competition. Therefore, in the case of price competition, whether sales-oriented incentive scheme or profit-oriented incentive scheme or no delegation will emerge as the equilibrium that depends on relative strengths of strategic effect and network effect, as argued in Hoernig (2012). However, in the case of quantity competition, qualitative nature of the incentive equilibrium does not appear to be affected due to network effects. Rather, the equilibrium incentive scheme seems to be even more sales-oriented due to network effect in the case of quantity competition. Therefore, the extent of modification of equilibrium outcomes, due to network effects, in models with managerial delegation under oligopoly is likely to depend on the nature of strategic variables.

Appendix

Proof of Lemma 1

Differentiating (1a) and (1b) with respect to λ we get,

$$\frac{\partial^2 O}{\partial x^2}\frac{dx}{d\lambda} + \frac{\partial^2 O}{\partial y \partial x}\frac{dy}{d\lambda} + \frac{\partial^2 O}{\partial \lambda \partial x} = 0$$
$$\text{and } \frac{dx}{d\lambda} - \frac{dy}{d\lambda} + 0 = 0.$$

Therefore,

$$\begin{aligned}
\frac{dx}{d\lambda} &= \frac{\frac{\partial^2 O}{\partial\lambda\partial x}}{-[\frac{\partial^2 O}{\partial x^2} + \frac{\partial^2 O}{\partial y\partial x}]} \\
&= \frac{\frac{\partial C}{\partial x}}{\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) + \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2}x}, \text{ since } \frac{\partial^2 p}{\partial y\partial x} = 0 \text{ (by Assumption 3)} \\
&< 0,
\end{aligned}$$

since (a) $\frac{\partial C}{\partial x} > 0$ (by supposition), (b) $\frac{\partial^2 p}{\partial x^2} \leq 0$, $\frac{\partial p}{\partial x} < 0$ and $|\frac{\partial p}{\partial x}| > |\frac{\partial p}{\partial y}|$ (by Assumption 1 and Assumption 2(b)) and (c) $\left(\frac{\partial p}{\partial x} < 0 \text{ and } |\frac{\partial p}{\partial x}| > |\frac{\partial p}{\partial y}|\right) \Rightarrow \left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) < 0$.

QED.

Proof of Proposition 1

We have the following, from (3).

$$\frac{d\pi}{d\lambda} = \frac{\partial\pi}{\partial x}\frac{dx}{d\lambda} + \frac{\partial\pi}{\partial y}\frac{\partial y}{\partial x}\frac{dx}{d\lambda} = \left[\frac{\partial\pi}{\partial x} + \frac{\partial\pi}{\partial y}\frac{\partial y}{\partial x}\right]\frac{dx}{d\lambda}. \tag{i}$$

Now,

$$\frac{dx}{d\lambda} = \frac{\frac{\partial C}{\partial x}}{\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) + \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2}x} < 0, \text{ by Lemma 1.} \tag{ii}$$

Also, note that

$$\begin{aligned}
\frac{\partial\pi}{\partial x} &= \left[\frac{\partial O}{\partial x} - (1-\lambda)\frac{\partial C}{\partial x}\right], \text{ since } \pi = O - (1-\lambda)C(x). \\
&= \left[-(1-\lambda)\frac{\partial C}{\partial x}\right], \text{ by Eqn. } 1(a). \qquad \text{(iii)}
\end{aligned}$$

Clearly,

$$\frac{\partial\pi}{\partial x}\begin{cases} > 0, & \text{if } \lambda > 1 \\ = 0, & \text{if } \lambda = 1 \\ < 0, & \text{if } \lambda < 1. \end{cases} \tag{iv}$$

From (1b), we get

$$\frac{\partial y}{\partial x} = 1 \tag{v}$$

Since $\pi = px - C(x)$, we get

$$\frac{\partial \pi}{\partial y} = \frac{\partial p}{\partial y} x > 0, \text{ since } \frac{\partial p}{\partial y} > 0 \text{ by Assumption 2(a).} \tag{vi}$$

Therefore,

$$\left[\frac{\partial \pi}{\partial y}\frac{\partial y}{\partial x}\right] > 0 \ \forall \lambda \tag{vii}$$

From (ii),(iv) and (vii), we get

$$\frac{d\pi}{d\lambda} = \left[\frac{\partial \pi}{\partial x} + \frac{\partial \pi}{\partial y}\frac{\partial y}{\partial x}\right]\frac{dx}{d\lambda} < 0, \text{ if } \lambda \geq 1.$$

Therefore, the optimal λ can never be greater than or equal to one.

Now, from (i), (ii), (iii), (v) and (vi), we get the following.

$$\frac{d\pi}{d\lambda} = \left[-(1-\lambda)\frac{\partial C}{\partial x} + \frac{\partial p}{\partial y}x\right]\left[\frac{\frac{\partial C}{\partial x}}{\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) + \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2}x}\right] \tag{viii}$$

From (viii), we get

$$\text{(a)} \quad \frac{d^2\pi}{d\lambda^2} = \frac{\partial C}{\partial x}\left[\frac{\frac{\partial C}{\partial x}}{\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) + \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2}x}\right] < 0$$

$$\text{(b)} \quad \frac{d\pi}{d\lambda} = 0 \Rightarrow \lambda = 1 - \left[\frac{x\frac{\partial p}{\partial y}}{\frac{\partial C}{\partial x}}\right] = \lambda^*$$

Therefore, the optimal incentive parameter is $\lambda^* = 1 - \left[\frac{x\frac{\partial p}{\partial y}}{\frac{\partial C}{\partial x}}\right]$. Since, both $x\frac{\partial p}{\partial y}$ and $\frac{\partial C}{\partial x}$ are positive, $\lambda^* < 1$.

QED.

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