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Abstract

This paper examines the implications of network externalities on equilibrium outcomes in a differentiated products duopoly under strategic managerial delegation through relative performance based incentive contracts. It shows that Miller and Pazgal (2001)'s equivalence result does not go through in the presence of network externalities. Instead, Singh and Vives (1984)'s rankings of equilibrium outcomes under Cournot and Bertrand hold true under relative performance based delegation contracts as well, if there are network externalities. However, when firms can choose whether to compete in price or in quantity, there are two pure strategy Nash equilibria and one mixed strategy Nash equilibrium. Interestingly, in pure strategy Nash equilibria asymmetric competition occurs, where a firm competes in price and its rival firm competes in quantity. Further, the mixed strategy Nash equilibrium probability of a firm to compete in terms of price increases with the strength of network effects and is always greater than the probability to compete in terms of price.

Keywords: Asymmetric competition, Price competition, Network externalities, Quantity competition, Relative performance contract, Strategic delegation

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1 Introduction

In their path breaking papers, Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) (henceforth VFJS) argue that owners can gain strategic advantage by having managers with distorted objectives to take decisions on their behalf, since distortions in manager's objective in one firm alters rival firms' decisions in its favour. In particular, considering a two stage delegation game where a profit maximizing owner chooses a publicly observable incentive scheme for her manager based on a linear combination of own firm's profit and sales before the managers compete in an oligopoly game, they show that it is optimal for the owners to induce their managers to be more (less) aggressive in the product market in the case of Cournot type quantity (Bertrand type price) competition compared to that under no delegation. Thus, in equilibrium, each owner offers a sales-oriented incentive scheme (penalizes sales maximization) and earns lower (higher) profit under Cournot type quantity (Bertrand type price) competition in the case of delegation compared to that in the case of no delegation. A corollary of this

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result is that strategic managerial delegation leads to lower differences between equilibrium outcomes under alternative modes of product market competition compared to that under no delegation. Building on these results, Miller and Pazgal (2001) demonstrate that the equilibrium outcomes are not sensitive to the mode of product market competition under relative performance based incentive contracts, which allow the owners to exercise greater control over their managers than that in the case of VFJS's incentive contracts. In other words, under relative performance based incentive contracts, the equilibrium price, output and profit of each firm remain the same irrespective of whether there is Bertrand type price competition or Cournot type quantity competition or firms compete in terms of different strategic variables. To obtain these results, however, they ignore the effects of network externalities by assuming *non-network goods*, that is, goods are such that there are no consumption/network externalities.

In reality there are many products for which utility derived by a particular consumer of the good increases with the number of other users of that good (i.e., with total sales of the good). For example, a typical consumer's utility from using a telephone increases with the number of other telephone users. Similar is the case with softwares. Also, for many consumer durable goods, utility of a consumer depends on the availability and quality of post-sale services, which is likely to increase with the total volume of sales of that good. In such scenarios, consumers' expectations regarding market size and, thus, their willingness to pay for the product can be influenced by publicly observable managerial incentive scheme, which can serve as an instrument to commit to deviate from standard profit maximizing output. It implies that managerial delegation can have an additional effect, other than strategic effect as in VFJS, on firms' profits in the presence of positive consumption externalities. Therefore, the questions are as follows. Does the equivalence result of Miller and Pazgal (2001) hold true in the case of network goods as well? How does the equilibrium look like in the presence of network externalities, when firms can choose whether to compete in terms quantity or in terms of price? This paper attempts to answer these questions.

Considering that the owner of each firm offers relative performance based managerial incentive scheme as in Miller and Pazgal (2001), this paper shows that in the presence of network externalities Cournot type quantity competition leads to lower outputs, higher prices, higher profits, lower consumer surplus and lower social welfare compared to that under Bertrand type price competition. Moreover, equilibrium outcomes under asymmetric competition, i.e., when a firm competes in terms of price and its rival firm competes in terms of quantity, differ from both Cournot equilibrium outcomes and Bertrand equilibrium outcomes. In other words, Miller and Pazgal (2001)'s equivalence result does not hold true in the presence of network externalities. The reason is both direct and indirect (via incentive parameters) effects of network externalities on equilibrium price/quantity are sensitive to firms' strategic variables for product market competition.

This paper also demonstrates that, in the presence of network externalities, the ranking of incentive equilibrium profits under different modes of competition is such that there are two pure strategy Nash equilibria and one mixed strategy Nash equilibrium of the larger game, which allows for endogenous determination of firm's strategic variables for product market competition. Interestingly, pure strategy Nash equilibria involve asymmetric competition between the two firms. And, the mixed strategy Nash equilibrium probability of a firm to compete in terms of price is higher in the case of stronger network effects. Further, it shows that both consumer surplus and social welfare are higher (lower) under asymmetric competition compared to that under Cournot type quantity (Bertrand type price) competition, in network goods duopoly with delegation.

In a seminal paper, Singh and Vives (1984) demonstrate that, in differentiated duopoly without delegation and network externalities, (i) Bertrand type price competition leads to lower prices, higher quantities, lower profits and higher social welfare than that under Cournot type quantity competition, and (ii) given the choice, firms will set quantities rather than prices. Clearly, analysis of this paper reveals that, while Singh and Vives (1984)'s first result go through even in the case of network goods duopoly with delegation, their second result is significantly modified in the present context.

We note here that, other than Miller and Pazgal (2001), several authors have investigated robustness of Singh and Vives (1984)'s results by considering different scenarios. For example, Lambertini (1997) extends the analysis to a repeated duopoly game, Qiu (1997) considers a two stage differentiated duopoly game and allows for cost-reducing R&D choice of firms prior to product market competition, Dastidar (1997) examines the implications of sharing rule and cost asymmetry in a homogeneous product market, Hackner (2000) considers vertical product differentiation in a n-firm oligopoly, Lopez and Nayor (2004) and Fanti and Meccheri (2011) examine the role of labour unions, Zanchettin (2006) relaxes the assumption of positive primary outputs and allows for cost asymmetry, Pal (2010b) examines the implications of firms' choice of technology adoption without imposing the assumption of positive primary outputs, Ghosh and Mitra (2010), Matsumura and Ogawa (2012) and Scrimitore (2013) considers the case of mixed duopoly with a public firm, Choi (2012) examines the role of labour unions in the context of mixed duopoly, so on and so forth. However, implications of network externalities have not been examined in this stream of literature.

On the other hand, starting with VFJS, the literature on strategic managerial delegation has been enriched by many studies, which offers useful insights to understand a variety of issues: such as, implications of wage bargaining (Szymanski, 1994), limit pricing in the case of sequential entry (Church and Ware, 1996), trade policy (Das, 1997), divisionalization decisions of firms (Gonzlez-Maestre, 2000), effects of public firm on internal organization of firms in oligopoly (White, 2001), mergers policy (Gonzalez-Maestre and Lopez-Cunat, 2001; Ziss, 2001), cartel stability (Lambertini and Trombetta, 2002), environmental damage control (Barcena-Ruiz and Garzon, 2002; Pal, 2012), relative performance of alternative managerial incentive schemes (Jansen et al., 2007), implications of mixed ownership (Saha, 2009), delegation of multiple decisions and implications of semi-collusion (Pal, 2010a), etc. However, to the best of our knowledge, issues pertaining to network goods industries have largely been ignored in this literature. Very recently, Hoernig (2012) and Bhattacharjee and Pal (2013) have examined the implications of network externalities on incentive equilibrium, by considering VFJS's incentive schemes for managers, under Bertrand type price competition and under Cournot type quantity competition, respectively. Nonetheless, they do not attempt to examine possible implications of network externalities on (a) rankings of equilibrium outcomes under different modes of product market competition and (b) endogenous determination of strategic variables for product market competition. This paper attempts to fill these gaps.

The rest of the paper is organized as follows. Section 2 describes the model. It also characterizes and compares equilibrium outcomes under alternative modes of product market competition. Section 3 examines the equilibrium of the extended game that allows for endogenous determination of modes of product market competition. Section 4 concludes.

2 The Model

We consider that there are two firms producing imperfectly substitutable goods that have positive consumption externalities. In other words, we consider that firms produce differentiated network goods. Each firm incurs constant marginal (average) cost of production c. Following Hoernig (2012) and Bhattacharjee and Pal (2013), we consider that the utility function of the representative consumer is given by

$$\begin{aligned} U(x_1, x_2; y_1, y_2) =& m + \frac{\alpha(x_1 + x_2)}{1-\beta} - \frac{x_1^2 + 2\beta x_1 x_2 + x_2^2}{2(1-\beta^2)} \\ &+ n[\frac{(y_1 + \beta y_2)x_1 + (y_2 + \beta y_1)x_2}{1-\beta^2} - \frac{y_1^2 + 2\beta y_1 y_2 + y_2^2}{2(1-\beta^2)}]; \end{aligned}$$

where m denotes the consumption of all other goods measured in terms of money, x_i denotes the quantity of the good produced by firm i $(= 1, 2)$, y_i denotes the consumers' expectation regarding firm i's total sales, and $\alpha > 0$, $\beta \in (0, 1)$ and $n \in [0, 1)$ are preference parameters. Lower value of β corresponds to higher degree of product differentiation. Note that marginal utility of good i increases with y_i: $\frac{\partial}{\partial y_i}\left[\frac{\partial U}{\partial x_i}\right] = \frac{n}{1-\beta^2} > 0$, $i = 1,\ 2$. It implies that there is positive consumption externality. Also, we can say that higher value of the parameter n indicates stronger network effects, since the rate of increase in marginal utility due to increase in y_i is positively related to the parameter n. $n = 0$ corresponds to the case of usual non-network goods. Also, note that, since the two goods are imperfect substitutes, marginal utility of good i increases with y_j, but at a lower rate than that due to increase in y_i: $0 < \frac{\partial}{\partial y_j}\left[\frac{\partial U}{\partial x_i}\right] = \frac{n\beta}{1-\beta^2} < \frac{n}{1-\beta^2}$, $i, j = 1,\ 2$, $i \neq j$, since $0 \leq \beta < 1$. It is evident that for any given consumption bundle (x_1, x_2) utility reaches the highest level, if consumers' expectations are correct, i.e., if $y_1 = x_1$ and $y_2 = x_2$. We assume that $0 < c < \frac{\alpha}{1-\beta}$, which ensures that equilibrium quantities and prices are always positive.

Given the utility function of the representative consumer as mentioned above, the inverse demand function for good i can be written as follows.[2]

$$p_i = \frac{\alpha\ (1+\beta) - x_i - \beta\, x_j}{1-\beta^2} + \frac{n\ (y_i + \beta\, y_j)}{1-\beta^2},\ \ i, j = 1,\ 2,\ i \neq j; \tag{1a}$$

[2]Similar demand functions can be derived by considering continuum of buyers with heterogeneous preferences.

where p_i is the price of good i. Note that the inverse demand function is quite similar to that used by Miller and Pazgal (2001), except the last term that captures network effects. The corresponding direct demand function is given by

$$x_i = \alpha + ny_i - p_i + \beta p_j,\ \ i, j = 1,\ 2,\ i \neq j. \tag{1b}$$

Clearly, network externalities enter additively in demand functions and shift demand curves outward without changing their slopes, as in Economides (1996) and Hoernig (2012).

Now let us consider the delegation game where in the first stage owners of each firm simultaneously and independently decide whether to compete by setting price or by setting quantity. Next, in the second stage each owner simultaneously and independently design a relative performance based incentive scheme, which is a linear combination of own firm's profit and the rival firm's profit, for her manager and delegates the task to set the price or quantity, depending on the mode of product market competition chosen in the first stage, so that own firm's profit is maximized. Finally, in the third stage managers are engaged in the product market competition.

Let λ_i be the weight on the rival firm j's profit in the firm i's manager's incentive scheme, which is decided by the owner of firm i in the second stage ($i = 1,\ 2;\ i \neq j$). Following the literature on strategic managerial delegation, we also assume that mangers are risk neutral and the market for managers is perfectly competitive. Then, in the third stage, the objective function of the manager of firm i can be written as follows.

$$O_i = \pi_i + \lambda_i \pi_j,\ \ i, j = 1,\ 2,\ i \neq j, \tag{2}$$

where $\pi_i = (p_i - c)x_i$ is the profit of firm i. We do not impose any restriction on the value of the incentive parameter λ_i. It is easy to observe that, if the incentive parameter set by a firm is positive (negative), her manager is rewarded (penalized) for the rival firm's profit and, thus, the manager behaves less (more) aggressively in the product market compared to that in the case of no delegation.

We solve this game by the standard backward induction method. In order to do so, first we consider each possible type of product market competition that can arise from the owners' decisions in the first stage as given, and compare the equilibrium outcomes. That is, we analyze the following four cases: (a) Bertrand type price competition - (price, price) game, (b) Cournot type quantity competition - (quantity, quantity) game, (c) firm 1 sets price and firm 2 sets quantity - (price, quantity) game, and (d) firm 1 sets quantity and firm 2 sets price - (quantity, price) game, where the first (second) entry in each parenthesis denotes firm 1's (firm 2's) strategic variable for product market competition in the third stage. Next, we examine the owners' optimal choices of the mode of product market competition in the first stage.

2.1 Bertrand type price competition

Let us first consider that in the first stage owners of each firm decides to compete in terms of price. In this case the problem of firm i's manager in the third stage is $\underset{p_i}{Max}\ O_i(p_i, p_j; \lambda_i, y_i, y_j)$, which yields his price reaction function as follows.[3]

$$p_i = \frac{\alpha + c(1 - \beta\,\lambda_i) + n\,y_i + \beta\,p_j\ (1+\lambda_i)}{2}\quad, i, j = 1,\, 2,\, i \neq j, \tag{3}$$

It is easy to check that p_i is positively related to the incentive parameter λ_i.[4] That is, if the owner of firm i decreases the weight on the rival firm's profit in her manager's incentive scheme, for any given p_j, the manager of firm i behaves more aggressively in the product

[3] We mention here that, given the parametric configurations, second order conditions for maximization are satisfied in all the stages of the game involved, and in each of the possible scenarios.

[4] $\frac{\partial p_i}{\partial \lambda_i} = \frac{\beta(p_j - c)}{2} \geq 0$, since $p_j \geq c$.

market by undercutting its price in order to capture greater market share. Interestingly, the manager of firm i perceives that prices are strategic substitutes (complements), if her incentive parameter λ_i is less (greater) than -1.[5] That is, by designing the incentive scheme appropriately, owner of a firm can even induce her manager to increase its price in response to a reduction in rival's price. Also note that, for any given incentive parameter and rival's price, the higher the consumers' expectations regarding firm i's total sale (y_i), the higher the price set by firm i's manager. It implies that the strength of network effects (n) and consumers' expectations are likely to play important roles in determining equilibrium prices and incentive parameters.

Following Katz and Shapiro (1985) and Hoernig (2012), we consider that consumers' expectations satisfy 'rational expectations' conditions and, thus, we assume that $y_1 = x_1$ and $y_2 = x_2$ hold true in equilibrium. Now, solving the reaction functions of the managers, together with the conditions $y_1 = x_1$ and $y_2 = x_2$, we get equilibrium prices $p_1 = p_1(\lambda_1, \lambda_2)$ and $p_2 = p_2(\lambda_1, \lambda_2)$ in the third stage.[6] Substituting these equilibrium prices in the profit expressions we get $\pi_i = \pi_i(\lambda_i, \lambda_j)$; $i, j = 1, 2$ and $i \neq j$. Therefore, in the second stage of the game, the problem of the owner of firm i $(= 1, 2)$ can be written as $\underset{\lambda_i}{Max}\ \pi_i(\lambda_i, \lambda_j)$. Solving these second stage problems, we get the equilibrium incentive parameters under Bertrand type price competition as follows.

$$\lambda_1^B = \lambda_2^B = \lambda^B = \frac{\beta^2 - (2-n)\, n}{(1-n)\,(2-n-\beta)\,\beta} \tag{4}$$

It is evident that in absence of network externalities the optimal incentive parameter is positive: $\lambda^B\ |_{n=0} = \frac{\beta^2}{(2-\beta)\,\beta} > 0$. That is, if there is no network externalities, it is optimal for each owner to induce her manager to be less aggressive than that in the case of no

[5] $\frac{\partial}{\partial p_j}[\frac{\partial O_i}{\partial p_i}] = \beta(1+\lambda_i) < (>)\, 0$, if $\lambda_i < (>) -1$.

[6] $p_i(\lambda_i, \lambda_j) = \frac{\alpha + c\,(1-n)}{2-n-\beta} + \frac{(1-n)\,\{\alpha - c\,(1-\beta)\}\,\beta\,\{(2-n)\,\lambda_i + \beta\,\lambda_j + (1-n)\,\beta\,\lambda_i\,\lambda_j\}}{(2-n-\beta)\,[(2-n)^2 - \beta^2 - (1-n)\,\beta^2\,\{\lambda_i + \lambda_j + (1-n)\,\lambda_i\,\lambda_j\}]}$, $i, j = 1, 2$, $i \neq j$. Clearly $p_i(\lambda_i, \lambda_j)$ is well defined for all $\beta \in (0, 1)$ and $n \in [0, 1)$.

delegation (Miller and Pazgal, 2001). Also, note that the optimal incentive parameter is decreasing in the strength of network effects: $\frac{\partial \lambda^B}{\partial n} = -\frac{(1-\beta)\left(4-4\,n+n^2+2\,\beta-2\,n\,\beta-\beta^2\right)}{(1-n)^2\,(2-n-\beta)^2\,\beta} < 0$ for all $n \in [0, 1)$. Clearly, whether it is optimal for the owners to induce their managers to be less aggressive (i.e. $\lambda^B > 0$) or more aggressive (i.e. $\lambda^B < 0$) that depends on the strength of network effects. It turns out that, if $n < (>)1 - \sqrt{1-\beta^2} = \hat{n}^B$, $\lambda^B > (<)0$. It implies that, if network effects are sufficiently strong, strategic effect of managerial delegation is dominated by network effects and owners find it optimal to induce their managers to be more aggressive in the product market even in the case of Bertrand type price competition. By considering VFJS type incentive schemes and price competition, Hoernig (2012) also shows similar relationship between the strength of network effects and optimal response of the owners. We summarize these results in Lemma 1.

Lemma 1: *In the case of Bertrand type price competition with managerial delegation we have the following.*

(a) The optimal incentive parameter chosen by the owner of each firm is given by $\lambda^B = \frac{\beta^2-(2-n)\,n}{(1-n)\,(2-n-\beta)\,\beta}$.

(b) The stronger the network effects (n), the lower the optimal incentive parameter.

(c) If the strength of network effects is greater (smaller) than the critical level $\hat{n}^B$, *in equilibrium owners induce their managers to be more (less) aggressive in the product market compared to that in the case of no delegation; where* $\hat{n}^B = 1 - \sqrt{1-\beta^2}$, $0 < \hat{n}^B < 1\ \forall \beta \in (0, 1)$.

It is interesting to note that $n > \frac{1}{2\,(1+\beta)}[(1+\beta)(4-\beta) - 2 - \sqrt{4 + (\beta+\beta^2)(4-7\beta+\beta^2)}] = n_0^B > \hat{n}^B \Rightarrow \lambda^B < -1$. Also, we have seen that prices are strategic substitutes when incentive parameters are less than minus one. Thus, if network effects are very strong

$(n > n_0^B)$, prices (p_1, p_2) are strategic substitutes. Needless to mention here that such possibility never arises if we consider standard non-network goods oligopoly or network goods oligopoly without delegation, ceteris paribus. Also note that, since $n_0^B > \hat{n}^B$, it is not necessary to have prices as strategic substitutes for the optimal incentive parameter to be negative.

Corollary 1: *In Bertrand type price competition between network goods producing managerial firms, prices are strategic complements (substitutes) if the strength of network effects is less (greater) than n_0^B.*

Now, substituting the expression for optimal incentive parameters from Lemma 1 in the expressions for prices, quantities, profits, consumers' surplus and social welfare, we get the following.

Lemma 2: *In the case of Bertrand type price competition with managerial delegation, the equilibrium prices, quantities, profits, consumers' surplus and social welfare are, respectively, as follows.*

$$p_1^B = p_2^B = p^B = \frac{\alpha\ (2-n-\beta) + c\ (1-\beta)\ (2-n+\beta)}{2\ (2-n)\ (1-\beta)},$$

$$x_1^B = x_2^B = x^B = \frac{\{\alpha - c\ (1-\beta)\}\ (2-n+\beta)}{2\ (2-3n+n^2)},$$

$$\pi_1^B = \pi_2^B = \pi^B = \frac{\{\alpha - c\ (1-\beta)\}^2\ (2-n-\beta)\ (2-n+\beta)}{4\ (1-n)\ (2-n)^2\ (1-\beta)},$$

$$CS^B = \frac{\{\alpha - c\ (1-\beta)\}^2\ (2-n+\beta)^2}{4\ (1-n)\ (2-n)^2\ (1-\beta)}\ \textit{and}$$

$$SW^B = \frac{\{\alpha - c\ (1-\beta)\}^2\ (6-3n-\beta)\ (2-n+\beta)}{4\ (1-n)\ (2-n)^2\ (1-\beta)}.$$

Clearly, $\frac{\partial p^B}{\partial n} < 0$ and $\frac{\partial x^B}{\partial n} > 0$. However, equilibrium quantity increases more than proportionately in the present context, since the strength of network effects has both direct and indirect (via price) positive effects on equilibrium quantity. As a result, equilibrium profits are higher in the case of stronger network effects: $\frac{\partial \pi^B}{\partial n} > 0$. Therefore, the strength of network effects has positive impact on both consumer surplus and social welfare: $\frac{\partial CS^B}{\partial n} > 0$ and $\frac{\partial SW^B}{\partial n} > 0$.

2.2 Cournot type quantity competition

Let us now consider that there is Cournot type quantity competition in the product market. That is, in the first stage each of the two owners choose 'quantity' as the strategic variable for product market interaction, and in the third stage managers engage in simultaneous move quantity competition. Thus, we can write the problem of the manager of firm i in the third stage as $\underset{x_i}{Max}\ O_i(x_i, x_j; \lambda_i, y_i, y_j),\ i, j = 1, 2; i \neq j$. From the first order condition for maximization we get the quantity reaction function of the manager of firm i as follows.

$$x_i = \frac{\alpha\ (1+\beta) - c\ (1-\beta^2) + n\,(y_i + \beta\, y_j) - \beta\, x_j\ (1+\lambda_i)}{2},\ i, j = 1,\ 2,\ i \neq j \qquad (5)$$

Clearly, for any given quantity choice of the rival firm, (a) the owner of a firm can induce her manager to choose lower quantity by rewarding her for rival firm's profit, i.e., by choosing greater value of the incentive parameter, (b) the strength of network effects has positive impact on a manager's optimal quantity choice, (c) consumers' expectations regarding a firm's own sales as well as regarding the rival firm's sales positively affect a manager's optimal quantity choice and (d) the impact of consumers' expectations of a firm's own sales is larger, compared to that of the rival firm's sales, on that firm's manager's optimal quantity choice. Manager of a firm perceives quantities, x_1 and x_2, as strategic substitutes (complements), if her incentive parameter is greater (less) than minus one.

From the quantity reaction functions of the managers and the 'rational expectations conditions' ($y_1 = x_1$ and $y_2 = x_2$), we get the third stage equilibrium quantities as functions of the incentive parameters: $x_1 = x_1(\lambda_1, \lambda_2)$ and $x_2 = x_2(\lambda_1, \lambda_2)$. Substituting the third stage equilibrium quantities in the profit expressions, we get $\pi_1 = \pi_1(\lambda_1, \lambda_2)$ and $\pi_2 = \pi_2(\lambda_1, \lambda_2)$. Solving the second stage problems of the owners, $\underset{\lambda_i}{Max}\ \pi_i(\lambda_i, \lambda_j)$, $i, j = 1, 2$ & $i \neq j$, we get the optimal incentive parameters under Cournot type quantity competition as follows.

$$\lambda_1^C = \lambda_2^C = \lambda^C = -\frac{(2-n)\ n + (1-n)^2\ \beta^2}{(2-n)\beta + (1-n)\ \beta^2} \tag{6}$$

It is easy to check that stronger network effects lead to lower value of the optimal incentive parameter in this case as well: $\frac{\partial \lambda^C}{\partial n} = 1 - \frac{1}{\beta} - \frac{2}{(2-n+(1-n)\,\beta)^2} < 0$. However, unlike as in the case of Bertrand type price competition, the optimal incentive parameter under Cournot type quantity competition is always negative: $\lambda^C < 0\ \forall\, n \in [0, 1)$. It implies that, under quantity competition, owners penalize their managers for rival firm's profit to a greater extent due to network effects. In other words, the presence of network externalities enhance the owners' incentives to induce their managers to be more aggressive quantity competitors than that in absence of network externalities. This result and the underlying mechanisms are similar to that in the case of VFJS type incentive scheme (Bhattacharjee and Pal, 2013).

Lemma 3: *In the case of Cournot type quantity competition with managerial delegation, the equilibrium incentive parameter,* $\lambda^C = -\frac{(2-n)\,n+(1-n)^2\,\beta^2}{(2-n)\beta+(1-n)\,\beta^2}$, *is always negative and decreasing in the strength of network effects.*

Also, note that (a) $\lambda^C < -1$, if $n > \frac{1}{2} + \frac{1}{2\,(1-\beta)}\left[1 - \sqrt{\frac{4-8\,\beta+5\,\beta^2+\beta^3}{(1+\beta)}}\right] = n_0^C$ and (b) from equation (5) it follows that, if $\lambda_i > -1$ ($\lambda_i < -1$), manager of firm i perceives that x_i, x_j are strategic substitutes (complements). It implies that, if the strength of network effects is greater than a critical value ($n > n_0^C$), quantities (x_1, x_2) are strategic complements. In other words, whether quantities are strategic substitutes or strategic complements that depends on the strength of network effects.

Corollary 2: *In network goods Cournot duopoly with managerial delegation, quantities are strategic substitutes (complements) if the strength of network effects is less (greater) than the critical level n_0^C.*

From Corollary 1 and Corollary 2, we can say that network effects play crucial role in determining the nature of the strategic variables for product market interactions in oligopoly under managerial delegation. The possibility of prices (quantities) being strategic substitutes (complements) in the case of Bertrand (Cournot) type price (quantity) competition cannot be ruled out in the presence of network effects, though there are symmetric firms and demand functions are linear. Further, upon inspection we find that $0 < n_0^B < n_0^C < 1$. It implies that strategic substitutability of quantities (prices) does not necessarily imply that prices (quantities) are strategic complements.

Lemma 4: *The equilibrium prices, quantities, profits, consumers' surplus and social welfare in the case of network goods producing Cournot duopoly with managerial delegation* *are, respectively, as follows.*

$$p_1^C = p_2^C = p^C = \frac{\alpha \left\{2 - n\ (1-\beta) - \beta\right\} + c\ (1-\beta)\ \left\{2 - n + (1-n)\ \beta\right\}}{2\ (2-n)\ (1-\beta)},$$

$$x_1^C = x_2^C = x^C = \frac{\left\{\alpha - c\ (1-\beta)\right\} \left\{2 - n + (1-n)\ \beta\right\}}{2\ (1-n)\ (2-n)},$$

$$\pi_1^C = \pi_2^C = \pi^C = \frac{\left\{\alpha - c\ (1-\beta)\right\}^2 \left\{2 - n\ (1-\beta) - \beta\right\} \left\{2 - n + (1-n)\ \beta\right\}}{4\ (1-n)\ (2-n)^2\ (1-\beta)},$$

$$CS^C = \frac{\left\{\alpha - c\ (1-\beta)\right\}^2 \left\{2 - n + (1-n)\ \beta\right\}^2}{4\ (1-n)\ (2-n)^2\ (1-\beta)} \ \textit{and}$$

$$SW^C = \frac{\left\{\alpha - c\ (1-\beta)\right\}^2 \left\{6 - n\ (3-\beta) - \beta\right\} \left\{2 - n + (1-n)\ \beta\right\}}{4\ (1-n)\ (2-n)^2\ (1-\beta)}.$$

It is straightforward to check that, unlike as in the case of Bertrand type price competition, both the equilibrium quantity and the equilibrium price of each firm increase with the strength of network effects in the present scenario: $\frac{\partial x^C}{\partial n} > 0$ and $\frac{\partial p^C}{\partial n} > 0$. The reason is, higher outputs in the presence of stronger network effects enhance consumers' willingness to pay, which dominates the negative effect of increase in output on price. As a result, equilibrium profits, consumers surplus and social welfare are higher in the case of stronger network effects: $\frac{\partial \pi^C}{\partial n} > 0$, $\frac{\partial CS^C}{\partial n} > 0$ and $\frac{\partial SW^C}{\partial n} > 0$.

Now, comparing the equilibrium outcomes corresponding to the case of no network externalities ($n = 0$) under Bertrand type price competition with that under Cournot type quantity competition, from Lemma 2 and Lemma 4, we get $x^B = x^C$, $p^B = p^C$, $\pi^B = \pi^C$, $CS^B = CS^C$ and $SW^B = SW^C$, as in Miller and Pazgal (2001). The intuition behind this equivalence result is as follows. In absence of network externalities, each owner has unilateral incentive to distort her manager's objective sufficiently, but in opposite directions under alternative modes of product market competition - induce the manager to be more (less) aggressive under quantity (price) competition compared to that in the case of no delegation. And, owners can exercise sufficient control over managers' behaviour by appropriately designing relative performance based incentive contracts.

However, in the presence of network externalities there is a gain in profits from being aggressive, since such behaviour shifts the demand curve outward, irrespective of the mode of product market competition. Thus, as seen before (Lemma 1 and Lemma 3), in the presence of network externalities it is optimal for the owners to induce their managers to be relatively more aggressive compared to that in absence of network externalities, under both quantity and price competition. It implies that the indirect effect of network externalities, via its impact on incentive parameter, on the equilibrium price (quantity) under price (quantity) competition is negative (positive). On the other hand, in the case of no delegation, both the equilibrium price and quantity increases with the strength of network effects irrespective of the mode of product market competition.[7] It indicates that the direct effect of network externalities on the equilibrium price and quantity are positive under both quantity and price competition. Overall, it turns out that the indirect effect dominates the direct effect of network externalities on the equilibrium price under price competition ($\frac{\partial p^B}{\partial n} < 0$). However, under quantity competition, though the equilibrium quantity increases due to both the direct and the indirect effects of network externalities, the equilibrium price is positively related to the strength of network effects. In other words, demand shifting effect of network externalities pushes the equilibrium price upward, in spite of increase in quantity, under quantity competition ($\frac{\partial p^C}{\partial n} > 0$). Clearly, impacts of network externalities on optimal prices under price and quantity competition are opposite in nature. Also, there is differential impact of network externalities on equilibrium quantities under price and quantity competition: $0 < \frac{\partial x^C}{\partial n} < \frac{\partial x^B}{\partial n}$. Thus, we have $0 < \frac{\partial \pi^B}{\partial n} < \frac{\partial \pi^C}{\partial n}$, $0 <$

[7]In the case of no delegation ($\lambda_1 = \lambda_2 = 0$) we have the following. $\frac{\partial p^B}{\partial n} = \frac{\partial x^B}{\partial n} = \frac{\alpha - c\,(1-\beta)}{(2-n-\beta)^2} > 0$, $\frac{\partial p^C}{\partial n} = \frac{(\alpha - c\,(1-\beta))\,(1+\beta)}{(1-\beta)\,(2-n-(1-n)\,\beta)^2} > 0$ and $\frac{\partial x^C}{\partial n} = \frac{(\alpha - c\,(1-\beta))\,(1+\beta)^2}{(2-n-(1-n)\,\beta)^2} > 0$.

$\frac{\partial CS^C}{\partial n} < \frac{\partial CS^B}{\partial n}$ and $0 < \frac{\partial SW^C}{\partial n} < \frac{\partial SW^B}{\partial n}$. Clearly, the equivalence result of Miller and Pazgal (2001) will not hold true in the case of network goods oligopoly. In other words, unlike as in the case of standard non-network goods oligopoly, in the presence of network externalities it is not possible to achieve equivalent equilibrium outcomes under price and quantity competition.

Comparing the equilibrium outcomes under Bertrand type price competition with that under Cournot type quantity competition, from Lemma 2 and Lemma 4, we get $x^B > x^C$, $p^B < p^C$, $\pi^B < \pi^C$, $CS^B > CS^C$ and $SW^B > SW^C$, for all $n \in (0,1)$.[8] That is, in the presence of network externalities, the well known Singh and Vives (1984)'s rankings of Cournot and Bertrand equilibrium outcomes hold true even in the case of strategic managerial delegation through relative performance based incentive contracts. This is true irrespective of whether quantities (prices) are strategic substitutes or strategic complements.

Proposition 1: *In the presence of network externalities, under strategic managerial delegation through relative performance based contracts, Bertrand type price competition leads to higher output, lower price, lower profit, higher consumers' surplus and higher social welfare in equilibrium compared to that under Cournot type quantity competition.*

[8] $x^B - x^C = \frac{n\{\alpha - c(1-\beta)\}\beta}{2(2-3n+n^2)}$, $p^C - p^B = \frac{n\{\alpha - c(1-\beta)\}\beta}{2(2-n)(1-\beta)}$, $\pi^B - \pi^C = \frac{n\{\alpha - c(1-\beta)\}^2\beta^2}{4(2-3n+n^2)(1-\beta)}$, $CS^B - CS^C = \frac{n\beta(2+\beta)\{\alpha - c(1+\beta)\}^2}{4(2-3n+n^2)(1-\beta)}$ and $SW^B - SW^C = \frac{n\{\alpha + c(1-\beta)\}^2(2-\beta)\beta}{4(2-3n+n^2)(1-\beta)}$ are (a) strictly positive if $n \in (0,1)$ and (b) equal to zero if $n = 0$.

2.3 Asymmetric competition

In this section we consider the scenario in which the manager of one firm competes in terms of quantity, while the manager of the other firm competes in terms of price. That is, in the first stage the owner of one firm chooses quantity and her rival firm's owner chooses price to be their respective managers strategic variable for product market competition. So, the modes of product market competition are asymmetric and, as discussed before, two possibilities arise in this scenario: (price, quantity) game and (quantity, price) game in the third stage, where the first (second) entry in each parenthesis corresponds to firm 1's (firm 2's) strategic variable. Since firms are otherwise identical, the equilibrium outcomes in these two cases would be mirror images of each other. Therefore, it is sufficient to analyze either of these two cases of asymmetric competition.

Without any loss of generality, let us consider that firm 1's manager competes in terms of price, while firm 2's manager competes in terms of quantity, i.e., we consider the (price, quantity) game in the third stage. We can write the problems of firm 1's manager and firm 2's manager as $\underset{p_1}{Max}\ O_1(p_1, x_2; \lambda_1, y_1, y_2)$ and $\underset{x_2}{Max}\ O_2(p_1, x_2; \lambda_2, y_1, y_2)$, respectively.

It is easy to check that $\frac{\partial}{\partial x_2}[\frac{\partial O_1}{\partial p_1}] = -\beta\ (1 - \lambda_1)$ and $\frac{\partial}{\partial p_1}[\frac{\partial O_2}{\partial x_2}] = \beta\ (1 - \lambda_2)$. Therefore, the manager of firm 1 perceives that p_1 and x_2 are strategic substitutes (complements), if his incentive parameter λ_1 is less (greater) than one, whereas the manager of firm 2 perceives that p_1 and x_2 are strategic complements (substitutes), if his incentive parameter λ_2 is less (greater) than one. Now, from the first order conditions of the managers' maximization problems, we get the reaction functions of the managers of firm 1 and firm 2, respectively, as follows.

$$p_1 = \frac{\alpha\ (1+\beta) + c\ (1-\beta^2) + n\ (y_1 + \beta\, y_2) - \beta\, x_2\ (1-\lambda_1)}{2\ (1-\beta^2)} \tag{7}$$

$$x_2 = \frac{\alpha - c\ (1-\beta\,\lambda_2) + n\, y_2 + \beta\, p_1\ (1-\lambda_2)}{2} \tag{8}$$

Clearly, (a) for any given output choice x_2 of the quantity-setting firm, the optimal response price p_1 of the price-setting firm's manager is positively related to his own incentive parameter λ_1, and (b) for any given choice of price p_1, such that $p_1 > c$, of the price-setting firm, the optimal response quantity x_2 of the quantity-setting firm's manager is negatively related to her own incentive parameter λ_2. It implies that the owner of the price-setting (quantity-setting) firm can induce her manager to increase the price (sales) of its product, i.e., to be less (more) aggressive in the product market, by setting higher (lower) value of the incentive parameter while designing her manager's incentive scheme. Also, note that higher consumers' expectations regarding either firm's sales shift the reaction function of price-setting firm's manager outward, as occurs in the case of Cournot type quantity competition. However, consumers' expectation regarding rival firm's sales does not have any direct impact on the quantity-setting firm's manager's reaction function, which is similar to that in the case of Bertrand type price competition.

Now, from (7), (8) and the 'rational expectations' conditions ($y_1 = x_1$ and $y_2 = x_2$), we obtain the third stage equilibrium outputs, prices and profits as functions of incentive parameters λ_1 and λ_2. Thus, the problem of the owner of firm i in the second stage can be written as $\underset{\lambda_i}{Max}\ \pi_i(\lambda_i, \lambda_j)$; $i, j = 1, 2$ and $i \neq j$. Solving these two problems we get the equilibrium incentive parameters under asymmetric modes of product market competition as follows.

$$\lambda^P = 1 - \frac{(2-n)\,(1+\beta)\,\{\beta + n\,(1-\beta^2)\}}{\beta\,\{2+\beta-n\,(1+\beta+\beta^2)\}} \tag{9}$$

$$\text{and }\ \lambda^Q = 1 - \frac{(2-n)\,(1-\beta)\,\{\beta + n\,(1-\beta-\beta^2)\}}{(1-n)\,\beta\,\{2-\beta-n\,(1-\beta^2)\}}, \tag{10}$$

where superscripts P and Q denote the price-setting firm and the quantity setting firm, respectively, in the case of asymmetric competition. It is easy to check that $\lambda^P < 1$ and $\lambda^Q < 1$ $\forall n \in [0,1)$, i.e, in the case of asymmetric competition the manager of the price-setting (quantity-setting) firm always considers that the strategic variables for product market competition are strategic substitutes (complements). Therefore, the reaction curve of the price-setting (quantity-setting) firm is always negatively (positively) sloped in the p_1x_2-plane. Also note that $\lambda^P < 0$ $\forall n \in [0,1)$, but $\lambda^Q > 0$ unless $n \geq \frac{2-2\beta+\beta^3-\sqrt{4-8\beta+8\beta^3-4\beta^4+\beta^6}}{2(1-\beta)}$. It implies that, under asymmetric competition, it is optimal for the owner of the price-setting firm to induce her manager to be more aggressive in the product market compared to that in the case of no delegation, since (a) in response to price-setting manager's more aggressive behaviour the quantity-setting manager sales lower quantity and (b) for any given price p_1, lower x_2 leads to higher profit π_1 of the price-setting firm. On the other hand, unless network effects are sufficiently strong, the owner of the quantity-setting firm induces her manager to be less aggressive compared to that in the case of no delegation. This is because (a) the price-setting firm's manager sets higher price in response to the quantity-setting firm's less aggressive sales and (b) when network effects are not sufficiently strong, higher p_1 leads to higher profit π_2 of the quantity-setting firm corresponding to any given x_2. Moreover, it can be checked that (a) $\frac{\partial\lambda^P}{\partial n} < 0$ $\forall n \in [0,1)$ and (b) $\frac{\partial\lambda^Q}{\partial n} > 0$, if $0.868517 < \beta < 1$ and $0 \leq n < \frac{2-\beta-\beta^2-\beta^3-\sqrt{(1-2\beta)(1-\beta)\beta(1+\beta)(2-2\beta-\beta^2)}}{1-\beta+\beta^2-\beta^3-\beta^4}$; otherwise, $\frac{\partial\lambda^Q}{\partial n} < 0$. In other words, though stronger network effects always lead to lower incentive parameter set by the price-setting firm's owner in equilibrium, the optimal incentive parameter in the quantity-setting firm can be positively related to the strength of network effects. We summarize these results in Lemma 5.

Lemma 5: *In the case of asymmetric competition in the product market between two network goods producing firms, we have the following.*

- *(a) Optimal incentive parameters in the price-setting firm (λ^P) and in the quantity-setting firm (λ^Q) are as given by equation (9) and equation (10), respectively.*
- *(b) Irrespective of the strength of network effects, the owner of the price-setting firm finds it optimal to induce her manager to be more aggressive in the product market compared to that in the case of pure profit maximization. However, it is optimal for the quantity-setting firm's owner to induce her manager to be less aggressive unless network effects are sufficiently strong.*
- *(c) Optimal incentive parameter for the price-setting (quantity-setting) firm's manager is (need not necessarily be) negatively related to the strength of network effects.*
- *(d) The manager of the price-setting firm perceives that his own price and his rival's quantity are strategic substitutes. However, the manager of the quantity-setting firm perceives that his own quantity and his rival's price are strategic complements.*

Now, substituting the values for optimal incentive parameters λ^P and λ^Q from (9) and (10) in the expressions for prices, quantities, profits, consumers' surplus and social welfare, we get the following.

Lemma 6: *When firms differ in terms of their strategic variables for product market competition in network goods producing duopoly with strategic managerial delegation, the* *equilibrium prices, quantities, profits, consumers' surplus and social welfare are, respectively, as follows.*

$$
\begin{aligned}
p^P =& \frac{\alpha\{2-\beta-n\ (1-\beta^2)\}+c\ (1-\beta)\ \{2+\beta-n(1+\beta^2)\}}{2\ (2-n)\ (1-\beta)}, \\
p^Q =& \frac{\alpha\{2-\beta-n\ (1-\beta)\}+c\ (1-\beta)\ \{2+\beta-n(1+\beta)\}}{2\ (2-n)\ (1-\beta)}, \\
x^P =& \frac{\{\alpha-c\ (1-\beta)\}\ (2-n+\beta)}{2\ (2-3n+n^2)}, \\
x^Q =& \frac{\{\alpha-c\ (1-\beta)\}\{2+\beta-n\ (1+\beta+\beta^2)\}}{2\ (2-3n+n^2)}, \\
\pi^P =& \frac{\{\alpha-c\ (1-\beta)\}^2\{2-\beta-n\ (1-\beta^2)\}\ (2-n+\beta)}{4\ (1-n)\ (2-n)^2\ (1-\beta)}, \\
\pi^Q =& \frac{\{\alpha-c\ (1-\beta)\}^2\{2-\beta-n\ (1-\beta)\}\{2+\beta-n\ (1+\beta+\beta^2)\}}{4\ (1-n)\ (2-n)^2\ (1-\beta)}, \\
CS^A =& \frac{\{\alpha-c\,(1-\beta)\}^2[2(2+\beta)^2-2n\,(2+\beta)\,(2+\beta+\beta^2)+n^2\{2+\beta\,(1+\beta)\,(2+\beta)\}]}{8\,(1-n)\,(2-n)^2\,(1-\beta)}, \text{ and} \\
SW^A =& \frac{\{\alpha-c\,(1-\beta)\}^2[(2-n)\,n\beta^3-(2+2n-n^2)\,\beta^2+(2-n)^2\,(6+2\beta)]}{8\,(1-n)\,(2-n)^2\,(1-\beta)},
\end{aligned}
$$

where superscripts P and Q denote the price-setting firm and the quantity-setting firm, respectively, under asymmetric competition and the superscript A denotes asymmetric competition.

It is evident from Lemma 6 that, if there is no network effect ($n = 0$), under asymmetric competition the equilibrium price, quantity and profit of the price-setting firm are the same as that of the quantity-setting firm. Moreover, if $n = 0$, asymmetric competition yields the same equilibrium outcomes as that under symmetric competition, be that Bertrand type price competition or Cournot type quantity competition. That is, as in Miller and Pazgal (2001), under relative performance based delegation contract equilibrium outcomes do not vary with the mode of product market competition in the case of non-network goods duopoly.

Now, differentiating the equilibrium outcomes with respect to n, we get the following. (a) $\frac{\partial p^Q}{\partial n} > 0$ and $\frac{\partial x^Q}{\partial n} > 0$, which implies that $\frac{\partial \pi^Q}{\partial n} > 0$. (b) $\frac{\partial x^P}{\partial n} > 0$, but $\frac{\partial p^P}{\partial n} < 0$ unless $\beta \geq \frac{1}{2}$. Note that, due to increase in n, both firms' managers' reaction curves shift outward in the $p_1 x_2$-plane, see equations (7) and (8). Moreover, while the extent of the shift of the reaction curve of the quantity-setting firm's manager due to increase in n does not directly depend on the degree of product differentiation, higher degree of product differentiation (i.e., lower β) leads to lower direct impact of n on the intercepts of the price-setting firm's managers' reaction curve. As a result, if $\beta < \frac{1}{2}$, $\frac{\partial p^P}{\partial n} < 0$. Nonetheless, if n increases, the loss in profit of the price-setting firm due to reduction in its price is overcompensated by the gain in profits due to more than proportionate increase in its output. Therefore, we get $\frac{\partial \pi^P}{\partial n} > 0$. Since each firm's equilibrium output as well as profit are higher in the case of stronger network effects, both consumers' surplus and social welfare increase with the strength of network effects: $\frac{\partial CS^A}{\partial n} > 0$, $\frac{\partial SW^A}{\partial n} > 0$.

Further, it is easy to check that $\frac{\partial p^P}{\partial n} < \frac{\partial p^Q}{\partial n}$ and $\frac{\partial x^P}{\partial n} > \frac{\partial x^Q}{\partial n}$, but $\frac{\partial \pi^P}{\partial n} > \frac{\partial \pi^Q}{\partial n}$. That is, due to stronger network effects, the price-setting firm benefits more in terms of profit compared to that of the quantity-setting firm. Clearly, the strength of network effects (n) has differential impact on equilibrium prices, quantities, and profits of the two firms under asymmetric competition. The above three inequalities also imply that $p^P < p^Q$, $x^P > x^Q$ and $\pi^P > \pi^Q$ hold true for all $n \in (0, 1)$, since we have $p^P(n = 0) = p^Q(n = 0)$, $x^P(n = 0) = x^Q(n = 0)$ and $\pi^P(n = 0) = \pi^Q(n = 0)$.

Proposition 2: *Under asymmetric competition between two network goods producing managerial firms, the price-setting firm charges lower price, sales greater quantity of output and earns higher profit compared to that of the quantity-setting firm in equilibrium. Moreover, if network effects become stronger, both output and profit of the price-setting firm increase* *by a larger amount compared to that of the quantity-setting firm.*

Now, comparing the equilibrium outcomes under alternative scenarios, from Lemma 2, Lemma 4 and Lemma 6, we get the following.

Proposition 3: *In the case of network goods duopoly with strategic managerial delegation, equilibrium outcomes under alternative modes of product market competition satisfy the following relations.*

(i) $p^B < p^P < p^Q = p^C$,

(ii) $x^Q < x^C < x^P = x^B$,

(iii) $\pi^B < \pi^Q < \pi^C < \pi^P$,

(iv) $CS^C < CS^A < CS^B$, *and*

(v) $SW^C < SW^A < SW^B$,

where superscripts B, C, A, P *and* Q *denote, respectively, Bertrand type price competition, Cournot type quantity competition, asymmetric competition, price-setting firm in the case of asymmetric competition and quantity-setting firm in the case of asymmetric competition.*

It is interesting to note that the quantity-setting firm under asymmetric competition sales lower quantity, but receives the same price, compared to that under Cournot type quantity competition, which results in lower profit of the quantity-setting firm under asymmetric competition than that under Cournot type quantity competition. Nonetheless, the profit of the quantity-setting firm remains higher than that under Bertrand type price competition. On the other hand, the price-setting firm under asymmetric competition sets a price that is less than the Cournot equilibrium price but greater than the Bertrand equilibrium price; however, it sales the same amount of output as that under Bertrand type price competition. Thus, under asymmetric competition, the equilibrium profit of the price-setting firm is not only higher than that of the quantity-setting firm, it exceeds the equilibrium profit under Cournot type quantity competition. Rankings of equilibrium outputs and profits under alternative modes of product market competition imply that, as compared to Cournot (Bertrand) type quantity (price) competition, asymmetric competition in product market leads to higher (lower) consumers' surplus as well as higher (lower) social welfare in equilibrium, in the presence of network externalities.

3 Optimal choice of the mode of competition

Finally, we turn to answer the following question. Given the choice, which strategic variable - price or quantity - should an owner choose in the presence of network externalities? Note that owners' problem in the first stage can be depicted as the following 2×2 normal-form game, where each of the two owners chooses a strategy from the strategy set $S = \{price,\ quantity\}$.

		Owner of firm 2	
		price	*quantity*
Owner of firm 1	*price*	π^B, π^B	π^P, π^Q
	quantity	π^Q, π^P	π^C, π^C

In the above game matrix, the first (second) entry in each cell is the profit of the owner of firm 1 (firm 2) corresponding to the associated strategy pair of the owners. Needless to mention here that (a) π^B and π^C are as given in Lemma 2 and Lemma 4, respectively, and (b) π^P and π^Q are as given in Lemma 6.

Now, note that we have $\pi^B < \pi^Q < \pi^C < \pi^P$, from Proposition 3(iii). Therefore, if the owner of firm 1 chooses '*price*', it is optimal for firm 2's owner to choose '*quantity*', since $\pi^Q > \pi^B$. Moreover, if the owner of firm 2 chooses '*quantity*', it is optimal for firm 1's owner to choose '*price*', since $\pi^P > \pi^Q$. Therefore, $\{price,\ quantity\}$ is a pure strategy Nash equilibrium of the game. Similarly, it can be argued that $\{quanty,\ price\}$ is also a pure strategy Nash equilibrium of the game, since firms are identical. It is also evident that there is no dominant strategy Nash equilibrium of this game. Therefore, in the case of network goods duopoly with strategic managerial delegation, if one firm chooses to compete in terms of price, it is optimal for the other firm to compete in terms of quantity, and vice-versa. In other words, when firms can choose the mode of product market competition, asymmetric competition emerges as the equilibrium in the present context. This result is in sharp contrast to both Miller and Pazgal (2001) and Singh and Vives (1984). The underlying intuition behind this result is as follows. when a firm competes in price and its rival firm competes in quantity, the price-setting firm faces a price cutter and takes the supply of the rival as given, which puts the price-setting firm in a disadvantageous position compared to that in the case of Cournot type quantity competition. To counteract this negative effect, the owner of the price-setting firm makes her manager more aggressive than she would have done in the case of Cournot type quantity competition. As a result, output of the price-setting firm is higher than that in case both firms compete in quantities. However, due to network effect, price of the price-setting firm does not fall sufficiently to wipe out the gain from being more aggressive. As a result, the price-setting firm earns higher profit than that by setting quantity, when the rival firm sets quantity. On the other hand, if the rival firm sets price, it is better for a firm to set quantity and avoid price war.

Further, it can be easily checked that, other than the two pure strategy Nash equilibria - (*price*, *quantity*) and (*quantity*, *price*), there exits a mixed strategy Nash equilibrium as well. In the mixed strategy Nash equilibrium each owner chooses to compete in terms of price with probability $\rho = \frac{\pi^P - \pi^C}{(\pi^P - \pi^C) + (\pi^Q - \pi^B)} = \frac{1}{2-n}$, $\frac{1}{2} \leq \rho < 1 \; \forall n \in (0, 1)$. Note that, if $n = 0$, $\rho = 1/2$, since in that case $\pi^B = \pi^C = \pi^P = \pi^Q$. Clearly, in the presence of network externalities, price competition is more attractive to firms compared to quantity competition. The probability of an owner to choose price as the strategic variable is higher in the case of stronger network effects.

Proposition 4: *When firms can choose whether to compete in terms of price or in terms of quantity, there are two pure strategy Nash equilibria and one mixed strategy Nash equilibrium in network goods duopoly with strategic managerial delegation.*

(a) In each of the two pure strategy Nash equilibria, asymmetric competition occurs - one firm competes in terms of price and the other firm competes in terms of quantity.

(b) In the mixed strategy Nash equilibrium, each firm decides to compete in terms of price with probability $\rho = \frac{1}{2-n}$ *and in terms of quantity with probability* $1 - \rho$; $n \in [0,\, 1)$ *and* $\rho \in [\frac{1}{2},\, 1)$.

4 Conclusion

In this paper we have analyzed the implications of network externalities on equilibrium outcomes under different modes of product market competition in a differentiated duopoly with strategic managerial delegation. In addition, we have characterized the equilibrium of a larger game, which allows for endogenous determination of the mode of product market competition. We have derived several interesting results. First, unlike as in the case of non-network goods a la (Miller and Pazgal, 2001), equivalence of equilibrium outcomes cannot be achieved through relative performance based incentive contracts in the presence of network externalities. Second, the well-known ranking of equilibrium outcomes under Cournot and Bertrand as demonstrated by Singh and Vives (1984) hold true under strategic managerial delegation through relative performance based incentive contracts as well, unless network externalities are absent. Third, asymmetric competition leads to higher (lower) consumers' surplus as well as higher (lower) social welfare than that under Cournot (Bertrand) type quantity (price) competition. Fourth, in contrast to Singh and Vives (1984), Cournot type quantity competition does not emerge as the equilibrium in the case of network goods duopoly, when firms' strategic variables for product market competition are endogenously determined. Rather, there are multiple equilibria of the larger game: two pure strategy Nash equilibria and one mixed strategy Nash equilibrium. In each of the two pure strategy Nash equilibria, asymmetric competition occurs - a firm competes in terms of price but its rival competes in terms quantity. The mixed strategy Nash equilibrium probability of a firm to compete in terms of price is (a) always higher than its probability to compete in terms of quantity and (b) monotonically increasing in the strength of network effects.

References

Barcena-Ruiz, J. C. and Garzon, M. B. (2002). Environmental taxes and strategic delegation. *Spanish Economic Review*, 4(4):301–310.

Bhattacharjee, T. and Pal, R. (2013). *Managerial Delegation under Network Effects*. IGIDR-WP-2013-003, Mumbai, India. http://www.igidr.ac.in/pdf/publication/WP-2013-003.pdf.

Choi, K. (2012). Price and quantity competition in a unionised mixed duopoly: The cases of substitutes and complements. *Australian Economic Papers*, 51(1):1–22.

Church, J. and Ware, R. (1996). Delegation, market share and the limit price in sequential entry models. *International Journal of Industrial Organization*, 14(5):575–609.

Das, S. P. (1997). Strategic managerial delegation and trade policy. *Journal of International Economics*, 43(1-2):173–188.

Dastidar, K. G. (1997). Comparing cournot and bertrand in a homogeneous product market. *Journal of Economic Theory*, 75(1):205–212.

Economides, N. S. (1996). Network externalities, complementarities, and invitations to enter. *European Journal of Political Economy*, 12(2):211–233.

Fanti, L. and Meccheri, N. (2011). The cournot-bertrand profit differential in a differentiated duopoly with unions and labour decreasing returns. *Economics Bulletin*, 31(1):233–244.

Fershtman, C. (1985). Managerial incentives as a strategic variable in duopolistic environment. *International Journal of Industrial Organization*, 3(2):245–253.

Fershtman, C. and Judd, K. L. (1987). Equilibrium incentives in oligopoly. *American Economic Review*, 77(5):927–940.

Ghosh, A. and Mitra, M. (2010). Comparing Bertrand and Cournot in mixed markets. *Economics Letters*, 109(2):72–74.

Gonzalez-Maestre, M. and Lopez-Cunat, J. (2001). Delegation and mergers in oligopoly. *International Journal of Industrial Organization*, 19(8):1263–1279.

Gonzlez-Maestre, M. (2000). Divisionalization and delegation in oligopoly. *Journal of Economics and Management Strategy*, 9(3):321–338.

Hackner, J. (2000). A note on price and quantity competition in differentiated oligopolies. *Journal of Economic Theory*, 93(2):233–239.

Hoernig, S. (2012). Strategic delegation under price competition and network effects. *Economics letters*, 117(2):487–489.

Jansen, T., van Lier, A., and van Witteloostuijn, A. (2007). A note on strategic delegation: The market share case. *International Journal of Industrial Organization*, 25(3):531–539.

Katz, M. and Shapiro, C. (1985). Network externalities, competition and compatibility. *American Economic Review*, 75(3):424–440.

Lambertini, L. (1997). Prisoners' dilemma in duopoly (super)games. *Journal of Economic Theory*, 77(1):181–191.

Lambertini, L. and Trombetta, M. (2002). Delegation and firms' ability to collude. *Journal of Economic Behavior and Organization*, 47(4):359–373.

Lopez, M. C. and Nayor, R. A. (2004). The cournot-bertrand profit differential : A reversal result in a differentiated duopoly with wage bargaining. *European Economic Review*, 48(3):681–696.

Matsumura, T. and Ogawa, A. (2012). Price versus quantity in a mixed duopoly. *Economics Letters*, 116(2):174–177.

Miller, N. H. and Pazgal, A. I. (2001). The equivalence of price and quantity competition with delegation. *RAND Journal of Economics*, 32(2):284–301.

Pal, R. (2010a). Cooperative Managerial Delegation, R&D, and Collusion. *Bulletin of Economic Research*, 62(2):155–169.

Pal, R. (2010b). Technology adoption in a differentiated duopoly: Cournot versus Bertrand. *Research in Economics*, 64(2):128–136.

Pal, R. (2012). Delegation and emission tax in a differentiated oligopoly. *The Manchester School*, 80(6):650–670.

Qiu, L. D. (1997). On the dynamic efficiency of bertrand and cournot equilibria. *Journal of Economic Theory*, 75(1):213–229.

Saha, B. (2009). Mixed ownership in a mixed duopoly with differentiated products. *Journal of Economics*, 98(1):25–43.

Scrimitore, M. (2013). Price or quantity? the strategic choice of subsidized firms in a mixed duopoly. *Economics Letters*, 118(2):337–341.

Singh, N. and Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics*, 15(4):546–554.

Sklivas, S. D. (1987). The strategic choice of managerial incentives. *RAND Journal of Economics*, 18(3):452–458.

Szymanski, S. (1994). Strategic delegation with endogenous costs: A duopoly with wage bargaining. *International Journal of Industrial Organization*, 12(1):105–116.

Vickers, J. (1985). Delegation and the theory of the firm. *Economic Journal*, 95(380a):138–147.

White, M. D. (2001). Managerial incentives and the decision to hire managers in markets with public and private firms. *European Journal of Political Economy*, 17(4):877–896.

Zanchettin, P. (2006). Differentiated duopoly with asymmetric costs. *Journal of Economics and Management Strategy*, 15(4):999–1015.

Ziss, S. (2001). Horizontal mergers and delegation. *International Journal of Industrial Organization*, 19(3-4):471–492.