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Measuring Human Development Index: The old, the new and the elegant

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Abstract

The Human Development Index (HDI) is calculated using normalized indicators from three dimensions- health, education, and standard of living (or income). This paper evaluates three aggregation methods of computing HDI using a set of axioms. The old measure of HDI taking a linear average of the three dimensions satisfies monotonicity, anonymity, and normalization (or MAN) axioms. The current geometric mean approach additionally satisfies the axioms of uniformity, which penalizes unbalanced or skewed development across dimensions. We propose an alternative measure, where HDI is the additive inverse of the distance from the ideal. This measure, in addition to the above-mentioned axioms, also satisfies shortfall sensitivity (the emphasis on the neglected dimension should be at least in proportion to the shortfall) and hiatus sensitivity to level (higher overall attainment must simultaneously lead to reduction in gap across dimensions). An acronym of these axioms is MANUSH, which incidentally means human in some of the South Asians languages and the alphabets can also be rearranged to denote HUMANS. Using Minkowski distance function we also give an alpha-class of measures, special cases of which turn out to be the old linear averaging method (alpha=1) and our proposed displaced ideal measure (alpha=2) and when alpha>=2 then these class of measures also satisfy the MANUSH axioms.

Keywords: Displaced ideal, Euclidean distance, Geometric mean, Hiatus sensitivity to level, Linear averaging, MANUSH, Minkowski distance, Shortfall sensitivity, Uniform development

JEL Code: D63, I31, O15

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Measuring Human Development Index: The old, the new and the elegant[1]

Srijit Mishra, Hippu Salk Kristle Nathan[2]

Abstract

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1. Introduction

In the human development paradigm the emphasis is on human beings as *ends* in themselves and not so much as *means* of development.[3] Further, the ends can be in multiple dimensions. It is in this context that Mahbubul Haq, the founder of *Human Development Reports* (HDR),[4] considers one-dimensionality as the most serious drawback of the income-based measures. This led to the birth of the Human Development Index (HDI), see Haq (1995, chapter 4). The measurement of HDI has evolved over time and contributed to the policy discourse.[5]

The calculation of HDI involves three dimensions–health (h), education (e), and the ability to achieve a decent standard of living, represented by income (y). The performances of each country in these three dimensions are normalized such that $0 \leq h,e,y \leq 1$,[6] and then aggregated to get the composite HDI. Prior to 2010, linear averaging (LA) across three dimensions was used as an aggregation method to obtain HDI, $(h+e+y)/3$; we denote this as HDI_{LA}. In 2010, this aggregation method was changed to the geometric mean (GM), $(h \times e \times y)^{1/3}$. We denote this as HDI_{GM}. In this paper, we propose an alternative aggregation method, which is the additive inverse of the distance from the ideal.[7] Following Zeleny (1982), we refer to this as the displaced ideal (DI) method and denote this as HDI_{DI}.[8]

[3] For discussions on this see Streeten *et al.* (1981), Sen (1989, 1997, 1999 and 2000), Desai (1991), Streeten (1994), and Haq (1995) among others.
[4] The human development report is being published annually since 1990 and serves as a cornerstone in terms of philosophy as well as an approach of the United Nations Development Programme (UNDP).
[5] For discussions on birth, evolution, and critique of measurement methodology of HDI and its policy discourse, see Anand and Sen, 1993, 1995, 1997, 2000, and 2003; Haq, 1995; Lüchters and Menkhoff, 1996; Hicks, 1997; Noorbakhsh, 1998; Sen, 2000; Fukuda-Parr *et al*., 2003; Jehan, 2003; Raworth and Stewart, 2003; Ranis *et al*., 2006; Grimm *et al.*, 2008; UNDP, 2010; and Klugman *et al*. (2011) among others.
[6] The normalization used: Index=(actual-minimum)/(maximum-minimum).
[7] The ideal corresponds to the maximum values for all the three dimensions as posited by UNDP for HDI calculation. In this sense, ideal indicates complete attainment.
[8] The two HDI measures discussed here also turn out to be special cases of a class of HDI measures (see section 5) which are based on the Minkowski distance function. The human poverty indices (HPI-1 and HPI-2) by UNDP also use similar methods. An alternative measure of HDI has been proposed by Chatterjee (2005) to capture the inequality in achievement across population groups. Moreover, a different class of HDI measures based on Atkinson's Index was attempted by Foster *et al*. (2005) to capture inequality both under each dimension of HDI and across dimensions.

As a first step, this paper evaluates the above-mentioned three aggregation methods. While evaluating these, it does not look into the rationale behind choosing of the different dimensions and how they are measured, scaled, weighed, and normalized. These are important issues, but beyond the scope of this current exercise. Rather, we take these as given or common for all the aggregation methods and then evaluate the methods using a set of axioms, namely, *monotonicity*, *anonymity*, *normalization*, *uniformity*, *shortfall sensitivity* and *hiatus sensitivity to level* with the acronym MANUSH.

In the second step, we propose a class of measures, HDI_{α}, based on the Minkowski distance function.[9] Both HDI_{LA} (where α=1) and HDI_{DI} (where α=2) turn out to be special cases of this class of measures. We also show that HDI_{α} (for $\alpha \geq 2$) satisfy MANUSH.

The three different aggregation methods are discussed in Section 2. The MANUSH axioms of HDI measure are elaborated in section 3. On the basis of these axioms, the LA, GM, and DI methods of aggregation are compared in Section 4. In section 5, HDI_{α} class of measure are proposed. Concluding remarks are given in Section 6.

2 The three methods of aggregation

2.1 Linear Averaging

The LA method applied to any set of parameters has an underlying assumption that the parameters are perfectly substitutable. The perfect substitutability assumption means that a differential improvement (or increment) in one indicator at any value can be substituted or neutralized by an equal differential decline (or decrement) in another indicator at any other value. This assumption is understandable when used in the case of same parameters like finding the average height of students in a class, or, when production of rice in different plots of land are added to compute yield per unit of land. Thus, LA essentially makes the thinking

[9] In a recent paper Subramanian (2006) has used the Minkowski distance function to the Foster *et al*. (1984) class of poverty measures. The *Human Development Reports* have also been using a similar method to calculate Human Poverty Indices (Anand and Sen 1997). Foster *et al*. (2005) propose a different class of HDI measures based on Atkinson's index.

one dimensional wherein different dimensions are treated as same or similar parameters, which in principle are perfectly substitutable. By using LA in the construction of HDI, it is assumed that health, education, and income are perfectly substitutable. Mathematically,

$$HDI_{LA}=1/3(h+e+y). \quad (1)$$

In the three dimensional space (h, e, y), one will have inclined triangular iso-HDI_{LA} planes indicating same HDI_{LA} values. The corresponding locus in two dimension will be 45^0inclined (or backward hatched) lines. For presentation convenience and without loss of generality, the iso-HDI_{LA} plot for a two-dimensional space of health and education has been given in Fig. 1.

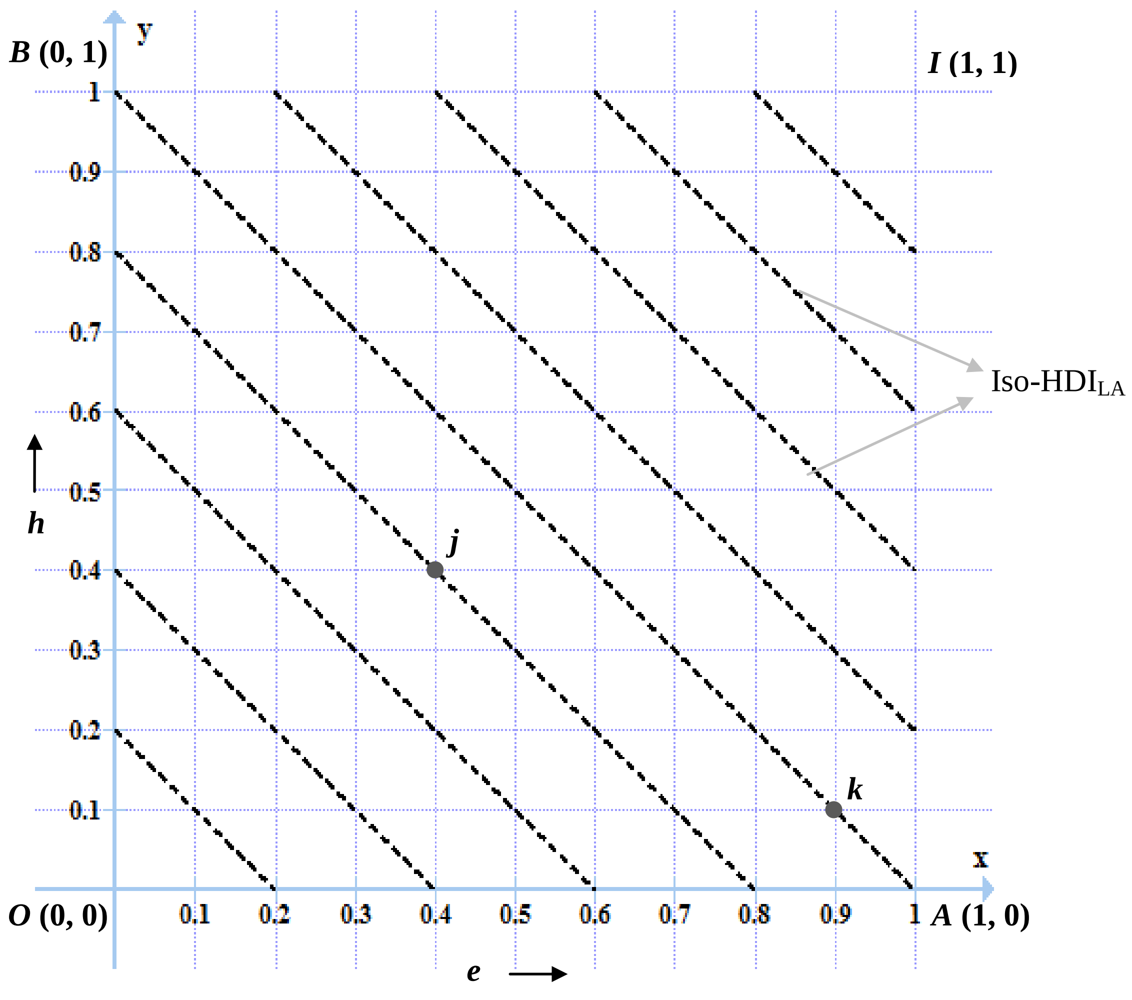

Fig. 1. *Iso-HDI_{LA} in a two-dimensional space*

Figure 1 shows HDI space *OAIB* with origin, *O* (0, 0), presenting both education, *e*, and health, *h*, are at their minimum, and ideal, *I* (1, 1) where both the indicators are at their maximum.[10] Any random country will occupy a point in the space *OAIB*. The locus of the points having same HDI_{LA} measure is indicated through the iso-HDI_{LA} lines. It is apparent that *j* (0.4,0.4) is lower than *k* (0.9,0.1) in terms of HDI_{LA}.

2.2 Geometric mean

The LA method of aggregation which implies perfect substitutability was criticized in literature for not being appropriate (Desai, 1991; Hopkins, 1991; Palazzi and Lauri, 1998; Sagar and Najam, 1998; Raworth and Stewart, 2003; Mishra and Nathan, 2008; Nathan *et al*., 2008; Herrero *et al*., 2010a). Perfect substitutability means, "that no matter how bad the health state is, it can be compensated with further education or additional income, *at a constant rate*, which is not very natural" (Herrero *et al*., 2010a: 4). According to Sagar and Najam (1998: 251), masking of trade-offs between various dimensions suggests that "a reductionist view of human development is completely contrary to the UNDP's own definition." Acknowledging this limitation, in the 20th anniversary edition of human development report (UNDP, 2010), the aggregation method shifted to geometric mean (GM). Mathematically,

$$HDI_{GM}=(h\times e\times y)^{1/3} \tag{2}$$

Geometric mean does not allow for perfect substitutability, gives higher importance to the dimension having lower performance, and penalizes unbalanced development (Gidwitz *et al*., 2010; Herrero *et al*., 2010b; Kovacevic and Aguña, 2010).

[10] In a three-dimensional HDI space, ideal, *I*, implies maximum attainment in all the dimensions (*h*=1, *e*=1, *y*=1). Noorbakhsh (1998) had used the concept of ideal for the country with maximum standardized score and suggested calculating a distance from the ideal. This would be in line with the annual maximum/minimum used in the measure of HDI then. Subsequently, as indicated in Dutta e*t al*.(1997) and Panigrahi and Sivaramkrishna (2002), the global maximum/minimum has been used in each dimension. UNDP (2010) has retained the fixing of the global minimum, but has kept the maximum open-ended, as it would not affect the ordinal representation across countries under the geometric mean method that it has since used to calculate HDI.

In the three dimensional space (h, e, y), one will have hyperbolic iso-HDI_{GM} surfaces indicating same HDI_{GM} values, the corresponding locus in two dimension will be rectangular hyperbola lines in the positive quadrant. For presentation convenience and without loss of generality, the iso-HDI_{GM} plot for a two-dimensional space of health and education has been given in Fig. 2.

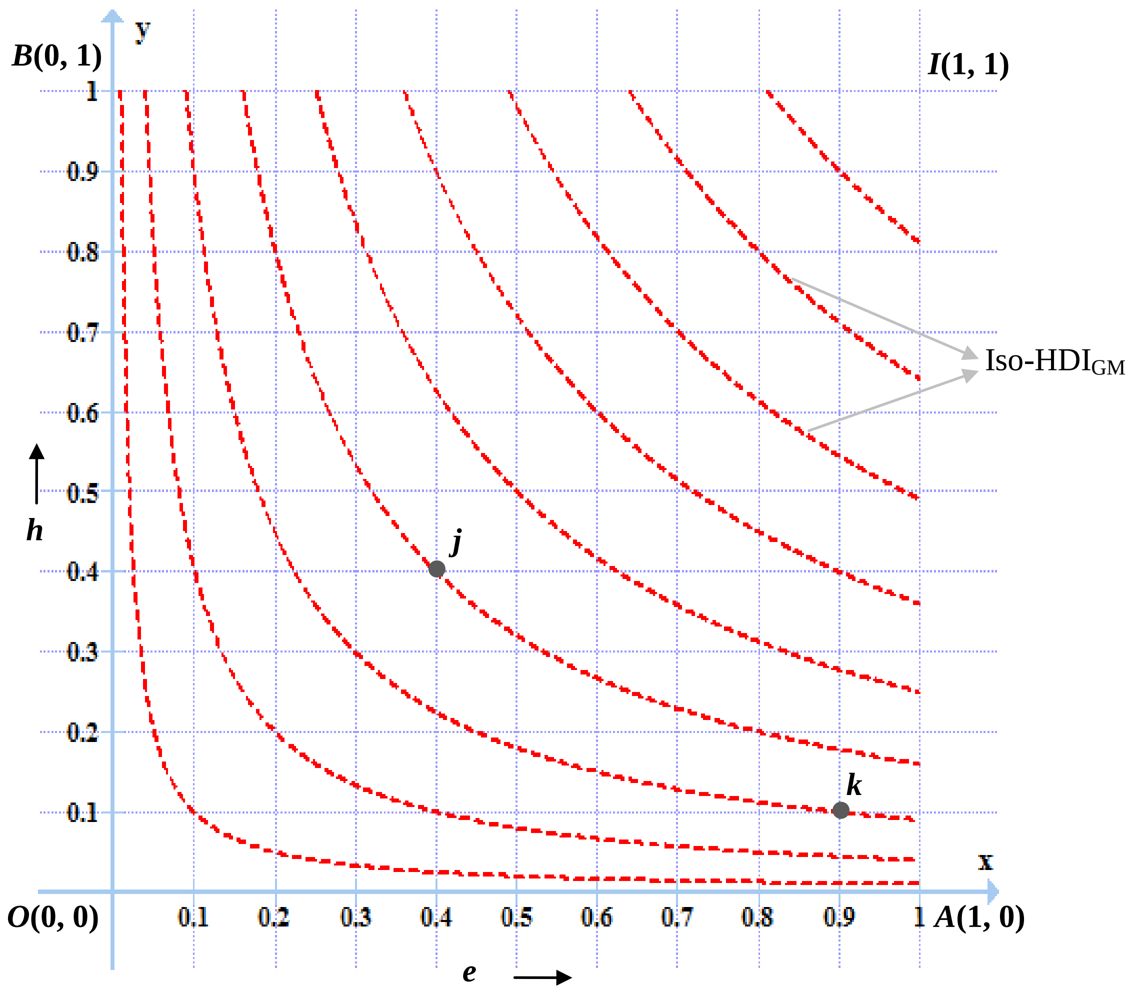

Fig. 2. *Iso-HDI_{GM} in a two-dimensional space*

Figure 2 shows the HDI space *OAIB* where *O* and *I* represent origin and ideal, respectively, as in Fig. 1. The locus of the points having same HDI_{GM} measure is indicated through the iso-HDI_{GM} lines. Unlike the case of linear average, *j* (0.4,0.4) is higher than *k* (0.9,0.1) in terms of HDI_{GM}.

2.3 Displaced Ideal

The DI method is based the on the concept that the better system should have less distance from ideal (Zeleny 1982).[11] Additive inverse of the normalized Euclidean distance from the ideal gives

$$HDI_{DI} = 1-(\sqrt{((1-h)^2+(1-e)^2+(1-y)^2)}/\sqrt{3}) \quad (3)$$

where $\sqrt{((1-h)^2+(1-e)^2+(1-y)^2)}$ is the Euclidean distance from the ideal. Dividing the same with $\sqrt{3}$ normalizes it in the three-dimensional space. Thus, for country *j*, the lower the distance from ideal, the higher is HDI_{DI}.

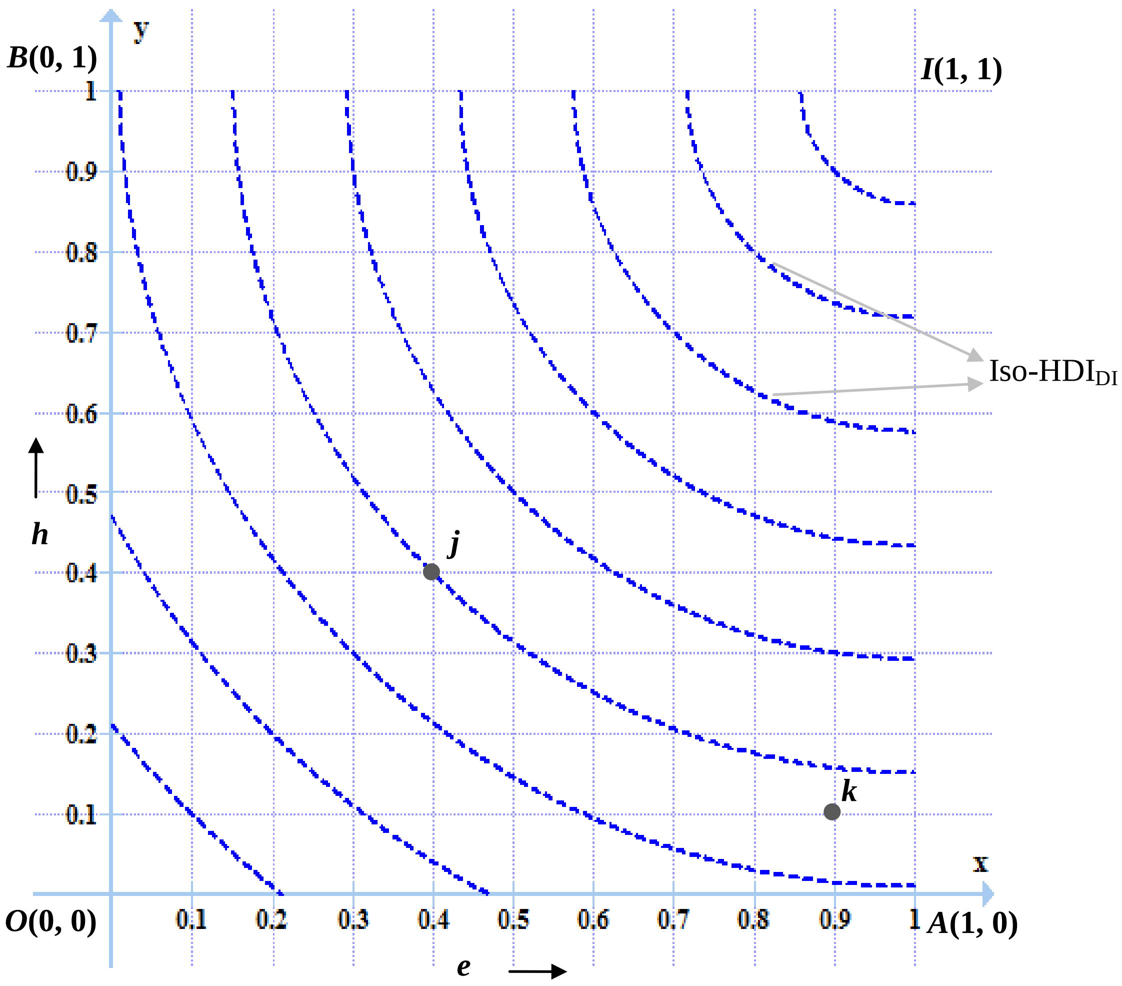

Fig. 3. *Iso-HDI_{DI} in a two-dimensional space*

[11] As discussed earlier, full attainment indicates maximum in each dimension (*h*=1, *e*=1, *y*=1) and depends on how each of these are computed. We reiterate that these computations are important, but consider them as given for the current exercise.

In the three-dimensional space, iso-HDI_{DI} surfaces indicating same HDI_{DI} values will be concentric quarter spheres with their centre at ideal. The corresponding locus in two dimensions will be concentric quarter circles. For presentation convenience and without loss of generality, the iso-HDI_{DI} plot for a two-dimensional space of health and education has been given in Fig. 3. The two points, *j* and *k*, representing two countries are the same as in Figures 1 and 2. The ranks between *j* and *k*, as in the case of geometric mean, have reversed when compared with the linear averaging method.

3. The MANUSH axioms

This section presents a number of intuitive properties that a measure of HDI should satisfy. They are as follows.

Monotonicity (Axiom M): A measure of HDI should be greater (lower) if the index value in one dimension is greater (lower) with indices value remaining constant in all the other dimensions. With two countries *j* and *k*, this would mean that if indices value remain the same in two dimensions (say, health and education such that $h_j=h_k$ and $e_j=e_k$) and different in the third dimension of income, $y_j \neq y_k$, then $HDI_j \gtreqless HDI_k$ iff $y_j \gtreqless y_k$.

Anonymity (Axiom A):A measure of HDI should be indifferent to swapping of values across dimensions. With two countries *j* and *k*, this would mean that $HDI_j=HDI_k$ if values are interchanged across two dimensions (say, health and education such that $h_j=e_k$ and $h_k=e_j$) and remains the same in the third dimension of income, $y_j=y_k$. This axiom implies a symmetry condition. This is not to be interpreted to indicate that one dimension can be replaced or substituted by another.[12]

Normalization (Axiom N): A measure of HDI should have a minimum and a maximum, HDI $\in$ [0,1]. At its minimum, HDI=0 indicates no development in all the three

[12] When dimensions of HDI have different weights, the swapping has to be by appropriately weight-adjusted.

dimensions (h=0, e=0, y=0); and at its maximum, HDI=1 indicates complete attainment in all the dimensions (h=1, e=1, y=1). Alternatively, in a three-dimensional Cartesian space, the two positions refer to the origin, O, and ideal, I, respectively.[13]

Uniformity (Axiom U): A measure of HDI should be such that for a given mean of indices value across dimensions, μ, a greater (lower) dispersion across dimensions, σ, should indicate a lower (greater) value. For two countries j and k, if $\mu_j=\mu_k$ and $\sigma_j \gtreqless \sigma_k$ then $\text{HDI}_j \lesseqgtr \text{HDI}_k$. This is in line with the notion of human development that each dimension is intrinsic (Sen 1999); and hence they cannot be complete substitutes to each other. So, this axiom connotes balanced or uniform development across dimensions.[14]

Shortfall sensitivity (Axiom S): A measure of HDI should indicate that improvement on the neglected dimension should be at least in proportion to the shortfall. For instance, in a country if the three dimensions of HDI have values as h=0.2, e=0.6, and y=0.8 (indicating that shortfalls are 0.8, 0.4, and 0.2, respectively) then the future course of action should give an emphasis on health that is at least twice more than education and four times more than income, while the emphasis on education should be at least twice more than income. An exacting situation under this will be to give entire emphasis to the most neglected dimension till it becomes equal to the dimension that is ordered just above it. And then both these dimensions will be given entire emphasis shared equally till they reach to the dimension that is ordered above them, and then all the three dimensions get equal emphasis. This is leximin ordering that can be considered equivalent to the Rawlsian scenario. Thus, without loss of generality, if $h<e<y$ such that $(1-h)=\beta(1-e)=\delta(1-y)$ where $(\beta, \delta)>1$ and Δ denotes positive change then shortfall sensitivity can be denoted as

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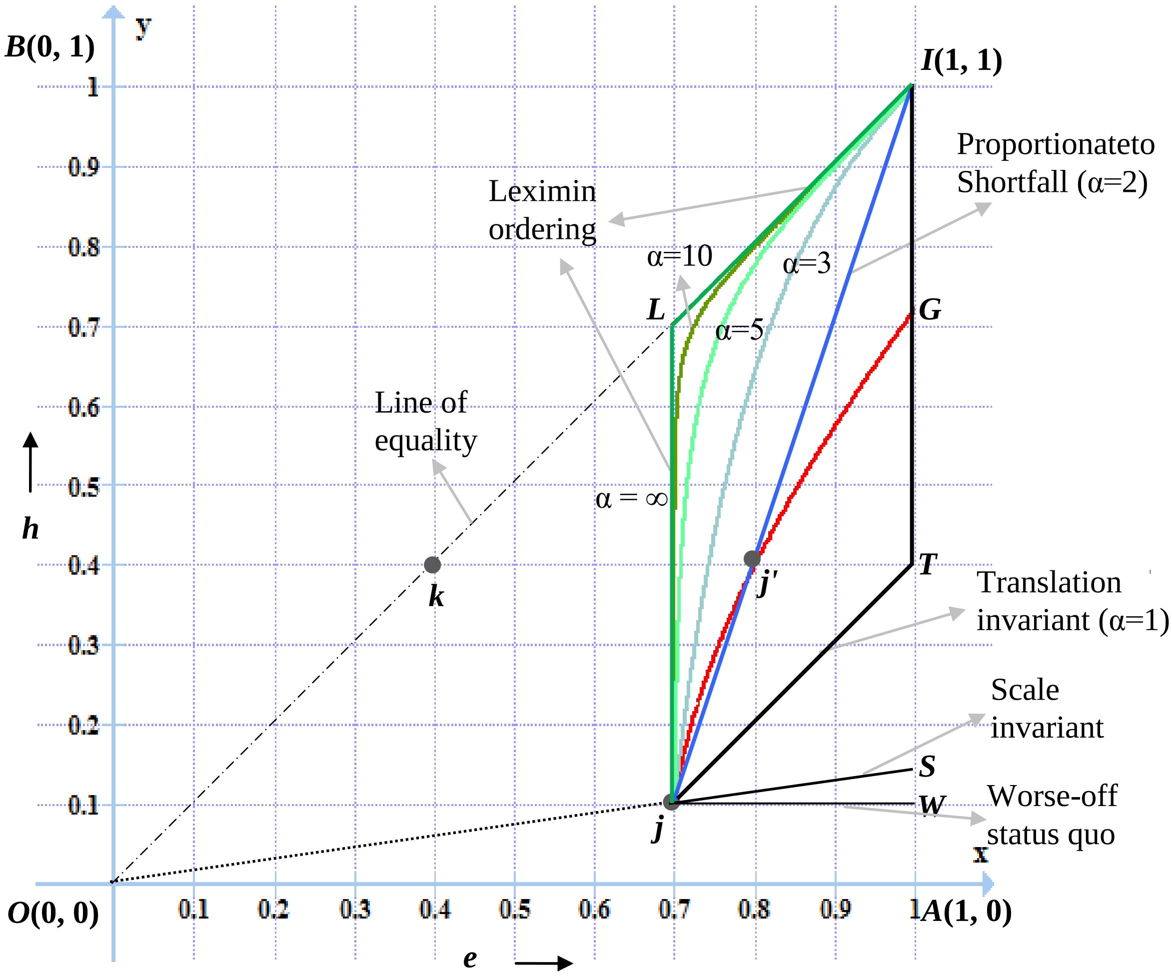
[13] The origin and ideal would depend how each of the indices are measured, scaled, weighed, and normalized. However, as indicated earlier, these are given to us. Also see notes 7, 10 and 11 above.

[14] Uniformity axiom should not be confused with zero substitutability or complete complementarity across dimensions. Suppose, from a uniform value in all dimensions, there is an increase in one dimension with all other dimensions remaining constant. Zero substitutability would not consider this as an improvement in HDI; whereas uniformity axiom simply says that any improvement in HDI from a uniform position will be maximized when the increase is shared equally by all dimensions.

$$\Delta h \geq \beta \Delta e \geq \delta \Delta y. \qquad (4)$$

The exacting case for shortfall sensitivity is leximin ordering (i.e., Δe=0, Δy=0 till $(h+\Delta h) \leq e$; then Δy=0 and $(h+\Delta h)=(e+\Delta e)$ till $(h+\Delta h) \leq y$; finally $(h+\Delta h)=(e+\Delta e)=(y+\Delta y)$). Shortfall sensitivity is weakly satisfied when $(\Delta h=\beta\Delta e=\delta\Delta y)$, i.e., the equal proportion to shortfall case.



Note: *OI* denotes the line of equality. For the given position *j*, the area *jIL* satisfies shortfall sensitivity. *jL* and *LI* indicate the leximin ordering case or Rawlsian scenario which corresponds to the path under $HDI_{\alpha=\infty}$. The optimal path under HDI_{α}; α=3, 5 and 10 are also given. *jI* indicates the proportion to shortfall case which corresponds to the optimal path under HDI_{DI} (or $HDI_{\alpha=2}$). *jG* indicates the optimal path under HDI_{GM}. *jT* indicates the translation invariant case which corresponds to the optimal path under HDI_{LA} (or $HDI_{\alpha=1}$). *jS* is scale invariant where future attainment across dimensions continue to be in the same proportion. *jW* indicates that the dimension that is worse-off maintains a status quo in an absolute sense.

Fig. 4. *Shortfall sensitivity*

For country j in Fig. 4, with a two-dimensional space of health and education, shortfall sensitivity is feasible in the area *jIL*. The movement from j to I along line segments *jL* and *LI* indicate the leximin ordering case whereas the line *jI* denotes equal proportion to shortfall case.

Hiatus sensitivity to level (Axiom H): A measure of HDI should be such that the same gap (or hiatus) across dimensions should be considered worse off as the attainment increases.[15] In other words, for a given gap, g, of indices values across dimensions (by gap, we imply that the deviations of h, e, and y from mean, μ, are constants),[16] a measure of HDI should be such that its deviation from its uniform development situation (i.e., when all the dimensions have equal values) will be greater (lower) for a greater (lower) μ. For two countries j and k, if $g_j = g_k$ and $\mu_j \gtreqless \mu_k$ then $\Delta \text{HDI}_j \gtreqless \Delta \text{HDI}_k$ (Δ refers to the change from the corresponding uniform development situation). This is in line with development with equity. For any development constituting more than one dimension, higher overall attainment must simultaneously lead to a reduction in gap across dimensions. It supports the view that "concern with inequality increases as a society gets prosperous since the society can 'afford' to be inequality conscious" (Sen, 1997: 36).

The above set of axioms, namely, monotonocity, anonymity, normalization, uniformity, shortfall sensitivity, and hiatus sensitivity to level are collectively referred to with the acronym of MANUSH.

4. Axiomatic Comparison among LA, GM, and DI methods

The three methods of calculating HDI, viz., LA, GM, and DI, satisfy the axioms of monotonicity, anonymity, and normalization. The GM and DI methods satisfy the axiom of

[15] This is similar to the level sensitivity axiom in the context of group differential (Mishra and Subramanian, 2006; Mishra, 2008; Nathan and Mishra, 2013).

[16] These constants can be interchanged across dimensions. For instance, considering two countries j and k with two dimensions h and e, if $h_j - \mu_j = 0.2$ and $e_j - \mu_j = -0.2$ then either $h_k - \mu_k = 0.2$ and $e_k - \mu_k = -0.2$ or $h_k - \mu_k = -0.2$ and $e_k - \mu_k = 0.2$.

uniformity. The axioms of shortfall sensitivity and hiatus sensitivity to level are satisfied by the DI method alone. Let us elaborate.

Monotonicity: This axiom is satisfied for all the three methods. For two countries j and k if the value in one dimension is higher for one, with the other dimensions being the same, say, $h_j > h_k$, while $e_j = e_k$, and $y_j = y_k$, then equations (1), (2), and (3) show $\text{HDI}_{\text{LA}j} > \text{HDI}_{\text{LA}k}$, $\text{HDI}_{\text{GM}j} > \text{HDI}_{\text{GM}k}$, and $\text{HDI}_{\text{DI}j} > \text{HDI}_{\text{DI}k}$, respectively.

Anonymity: The three methods of aggregation satisfy anonymity. From the mathematical expressions of these three methods, (1), (2), and (3) one can find HDI_{LA}, HDI_{GM}, and HDI_{DI} are symmetric in h, e, and y. Hence, HDI under these methods does not change to swapping of values across dimensions.

Normalization: In all the three methods, the countries are bounded by the minimum, $\text{HDI}_{\text{LA}} = \text{HDI}_{\text{GM}} = \text{HDI}_{\text{DI}} = 0$ at the origin, O (h=0,e=0,y=0); and the maximum, $\text{HDI}_{\text{LA}} == \text{HDI}_{\text{GM}} = \text{HDI}_{\text{DI}} = 1$ at the ideal I (h=1,e=1,y= 1). Hence, they satisfy normalization.

Uniformity: Both the GM and DI methods satisfy this, while LA fails. For two countries j and k, if $\mu_j = \mu_k$ and $\sigma_j > \sigma_k$ then $\text{HDI}_{\text{GM}j} < \text{HDI}_{\text{GM}k}$ and $\text{HDI}_{\text{DI}j} < \text{HDI}_{\text{DI}k}$, but $\text{HDI}_{\text{LA}j} = \text{HDI}_{\text{LA}k}$. For a given mean, the LA method is independent of the deviation from the mean. This makes HDI_{LA} perfectly substitutable, which is "one of the most serious criticisms of the linear aggregation formula" (UNDP, 2010: 216). Linear averaging also enables HDI_{LA} to be subgroup decomposable. However, it should not be seen as an advantage as this implies that the subgroup having lower HDI values can be perfectly substitutable by subgroups having higher HDI values. Additionally, our proposed method can also be made subgroup decomposable by considering the s subgroup's share of contribution as $n_s v_s / \sum n_s v_s$, where n_s is the subgroup's population share and v_s is the subgroup's value of HDI computed independently.

For a given mean, the geometric mean (HDI_{GM}) and the additive inverse of the distance from the ideal (HDI_{DI}) are maximized if and only if all the three dimensions have the same values (proof is given in Appendix 1). Hence, they satisfy the uniformity axiom. Now, let us state the following proposition.

Proposition 1: A measure of HDI cannot satisfy perfect substitutability and uniformity simultaneously.

Proof: A measure of HDI satisfying perfectly substitutability would not change for a given μ even if σ changes. On the contrary, a measure of HDI satisfying uniformity demands the measure to have a lower (higher) value as σ increases (decreases) with μ remaining constant.

Shortfall sensitivity: In order to determine whether the improvement in HDI is shortfall sensitive, one needs to find an optimal path for future progress, where a given increment in HDI can be achieved with a minimal movement. This corresponds to minimizing the Euclidean distance between the current and incremental positions for a given increment in HDI.

For the LA method, the optimal paths are perpendicular to the iso-HDI_{LA} lines. This will imply same increment in all dimensions (for the position *j* in Fig. 4, movement along the line *jT* indicates this). It is nothing but the translation invariant case ($\Delta h=\Delta e=\Delta y$). This way, under LA, the future emphasis is same irrespective of the current variations in the attainment across dimensions. It does not impose greater emphasis on the dimensions that are neglected. Hence, it does not satisfy the shortfall sensitivity axiom.

For the GM method, the equation for the optimal path is given in Appendix 2. If the initial position is (h,e,y), incremental position is (h',e',y') and without loss of generality if $h'=\gamma e'=\theta y'$ then $\Delta h=\gamma^{-1}\Delta e=\theta^{-1}\Delta y$, where $(\gamma, \theta)<1$ and Δ refers to the change from initial to incremental position. In this sense, the emphases across dimensions are sensitive to multiplicative inverse of attainment and not to shortfall, which is additive inverse to attainment. For instance, if h=0.1 and e=0.7 (refers to point *j* in Fig. 4) then the GM method indicates that the emphasis on health, to begin with, should be seven times more than education. However, as the attainment increases the starting emphases would change in proportion to that. This results in the optimal path being *jG* in Fig. 4. The *j'G* segment of the path is outside the area *jIL*, which indicates that beyond *j'* (at *j'* the optimal path for GM intersects the proportionate to shortfall line *jI*) it does not impose greater emphasis on the dimensions that are neglected. Hence, GM fails shortfall sensitivity.

For the DI method, the optimal paths are the lines joining initial position and the ideal (see line *jI* in Fig. 4). Here, the emphases across dimensions turn out to be in proportion to the shortfall throughout the path. Hence, the shortfall sensitivity axiom is satisfied under DI. To be precise, it satisfies shortfall sensitivity weakly. The proof is given in Appendix 3.

Going beyond the bounds under this specific axiom under discussion, one can state that the 'extreme left' and 'extreme right' increments are that of the leximin ordering and the worse-off status quo in an absolute sense (like leximax ordering, *jW* in Fig. 4),[17] respectively. For greater equity, as one moves up from the worse-off status quo, the improvements in sequence are that of scale invariance (the improvement across dimension should be in the same proportion as in their current position, *jS* in Fig. 4), translation invariance (associated with the linear averaging method where the increments across dimensions are equal, *jT* in Fig. 4), the optimal path associated with the geometric mean method (*jG* in Fig. 4), equal proportion to shortfall (associated with the displaced ideal method, *jI* in Fig. 4), and ending with extreme left situation of leximin ordering, i.e., Rawlsian scenario (*jL* and *LI* line

[17] The worst-off status quo in an absolute sense implies that any additional increments in future will entirely focus on the dimension that has the highest attainment and once it reaches its maximum then only will it shift focus to the dimension that is the next highest and so on. If two or more dimensions have equal attainments then the additional increments will also be equal across these dimensions.

segments in Fig.4). For a country on the line of equality, say *k*, all the above-mentioned situations will coincide with *kI* (see Fig. 4).

Hiatus sensitivity to level: The LA method fails to satisfy this axiom, as there is no deviation of HDI_{LA} values from the uniform development situation at all levels. For a given gap, the deviation of HDI from its uniform development situation is a decreasing function of mean for the GM method while it is an increasing function of mean for the DI method. The proof is given in Appendix 4. This means that GM fails whereas DI satisfies hiatus sensitivity to level. The GM method not satisfying this axiom also means that it penalizes greater proportionate deviation of the given gap from uniform development when average attainment increases. This is obvious because the proportionate deviation for a given gap is higher at a lower level of average attainment. This gives us the following proposition.

Proposition 2: A measure of HDI cannot satisfy hiatus sensitivity to level and also penalize greater proportionate deviation of a given gap from uniform development together.

From the above discussion the following results emerge. The HDI_{LA} method satisfies the axioms of MAN (monotonicity, anonymity, normalization). In addition to these axioms, the axiom of uniformity is also satisfied by HDI_{GM}. The HDI_{DI} method satisfies all the aforementioned axioms including shortfall sensitivity (weakly) and hiatus sensitivity to level. Based on this, we state the following proposition.

Proposition 3: There exists a human development index measure HDI_{DI} that satisfies the MANUSH axioms – monotonicity, anonymity, normalization, uniformity, shortfall sensitivity (weakly) and hiatus sensitivity to level.

Thus, HDI_{DI} measure has some axiomatic advantages over the current HDI_{GM} measure. Nevertheless one must mention that an advantage of the GM method is that the ranking of countries are scale independent to changes in the maximum value for each variable, which is used for normalizing the dimension-specific indicator. However this advantage would not come in the way of our proposed method if one followed the pre-2010 practice of fixing the maximum, in a normative sense, as a goalpost.

The use of an open-ended maximum, amenable under the GM method, also raises some concerns. First, the 1980-2010 observations showed that the maximum income was for United Arab Emirates (UAE) in 1980, which no country has ever reached; UAE too has not been able to reach again in the period under consideration. Thus, that observation was a historical-accident and may not indicate a scenario that others ought to emulate and attain in the near future. Second, the change in defining maximum meant that compared to 2009, the computations in 2010 had the maximum for the income dimension increased by about two-and-a-half times (per capita gross national income at purchasing power parity US$ terms in 2005 prices increased from 40,000 to more than 100,000) indicating that countries having per capita income more than 40,000 US$ will now be able to add the excess income as attainments to their valuation of HDI and this will favour the very high income countries. A related third concern is that with this shift the shortfall for income has increased and thus increments from income have become more important relative to other dimensions. Fourth, a changing maximum in an advantaged dimension would mean further neglect of a neglected dimension. A rightward shift of the ideal point (h=1, e=1, y=1) extends the optimal path for GM (along *jG* in Fig. 4) to reach *G*, thereby postponing the move along the vertical segment *GI* to focus on the neglected dimension. Lastly, while conceding that the HDI calculation compared across countries have ordinal relevance, there is merit in an analysis of trends for a specific country or a group of countries over time, as has been carried out by Nathan and Mishra (2010) and UNDP (2010). It is here that our proposed axiomatic advantages gain further importance. It goes without saying that such an analysis should be complemented with an understanding of the state of affairs in health, education and standard of living.

5. The HDI$_\alpha$ Class of Measure

Now, let us define an α-class of HDI measures,

$$\text{HDI}_\alpha=\text{M}_\alpha=1-{}_wD_{\alpha I} \tag{5}$$

where

$${}_wD_{\alpha I}=\left(\frac{\sum\left(w_i\left(1-x_i\right)\right)^\alpha}{\sum\left(w_i\right)^\alpha}\right)^{\frac{1}{\alpha}};i=1,\ldots,n,\quad \alpha=[1,\infty) \tag{6}$$

is the normalized Minkowski distance function of order α calculated from the ideal, I, where x_i refers to the normalized indices for n dimensions (i=1,2…n) such that at the origin $x_i=0\ \forall i$ and at the ideal $x_i=1\ \forall i$, and w_i refers to the weights assigned to each dimension. For equal weights, (6) reduces to

$$D_{\alpha I}=\left(\frac{1}{n}\sum\left(1-x_i\right)^\alpha\right)^{\frac{1}{\alpha}};i=1,\ldots,n,\ \alpha=[1,\infty) \tag{7}$$

In the above class of HDI measures, the linear average and displaced ideal methods indicated in (1) and (3), respectively turn out to be special cases. This is suggested in the following proposition.

Proposition 4: There exists an α-class of human development index measures such that for n=3, $\text{HDI}_{\alpha=1}=\text{HDI}_{\text{LA}}$ and $\text{HDI}_{\alpha=2}=\text{HDI}_{\text{DI}}$.

Proof: Substituting (7) in (5) one gets (1) and (3) for α=1 and α=2, respectively.

Another special case is $\text{HDI}_{\alpha=\infty}$, where the human development index measure reduces to the lowest-valued dimension. This corresponds to a situation where the iso-HDI lines can be depicted through right-angled lines. Thus, as α increases from unity to infinity we move from a measure that allows for perfect substitutability to one that allows no substitution across dimensions (Fig. 5).[18]

[18] The similarity of HDI$_\alpha$ class of functions with constant elasticity of substitution (CES) functions is obvious.

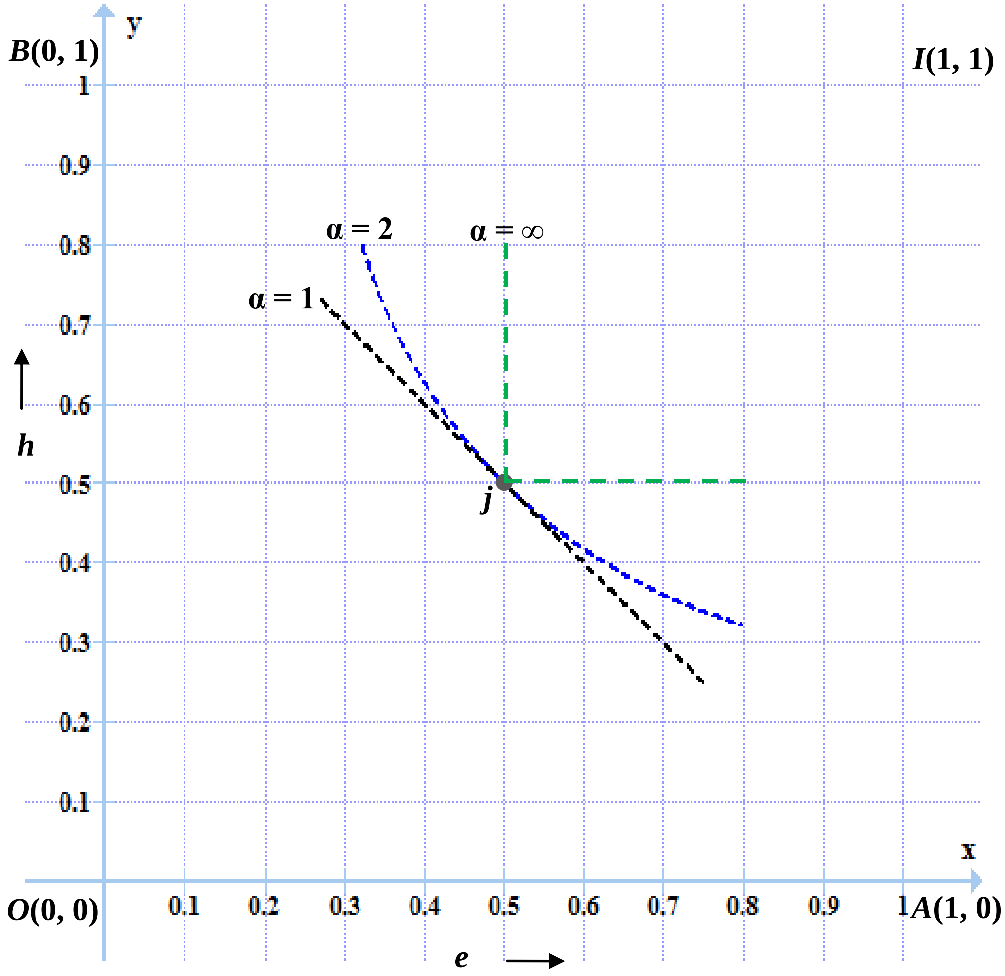


Fig.5. *Class of Measures*

In addition to the above mentioned special cases, it is interesting to note that we have a class of HDI_{α} measures that satisfy the MANUSH axioms. We state that in the following proposition.

Proposition 5: There exists anα-class of human development index measures HDI_{α} such that for $\alpha>2$ the MANUSH axioms are satisfied.

Proof: The proofs for the six axioms are as follows.

M_{α} satisfies montonicity, $\partial M_{\alpha}/\partial x_i>0\forall i$.

It is evident from equations (5) and (7) that M_{α} remains the same if the values of x_i and $x_{i'}$ are swapped ($i\neq i'$). Appropriate adjustments can also be made when weights are not equal. Thus, M_{α} satisfies anonymity.

M_α satisfies normalization, $M_\alpha \in [0,1]$.

For $\alpha>1$, $M_\alpha$maximizes when values are shared equally in all dimensions; hence it satisfies uniformity (see Appendix 1for proof; the proof for $\alpha=2$ is valid for $\alpha>1$).

As discussed earlier shortfall sensitivity is satisfied weakly for $\alpha=2$, but it is satisfied strongly for $\alpha>2$ (a formal proof is given in Appendix 3). The optimal paths of HDI measured in Euclidian distance for various values of $\alpha=1,2,\infty$ are given in Fig. 4 indicating translation invariance, proportionate to shortfall, and leximin ordering, respectively. For values of $\alpha \in (2,\infty)$ the path will be within *jIL* and concave to the line segment *jI* – the optimal paths for $\alpha=3,5$, and 10 are given in Fig. 4.

M_α satisfies hiatus sensitivity to level for $\alpha \geq 2$ (see Appendix 4 for proof).

Thus, one summarizes by stating that HDI_α satisfies MAN axioms when $\alpha=1$, it satisfies the MANUSH axioms when $\alpha \geq 2$. However, a distinguishing feature is that for $\alpha=2$ shortfall sensitivity is satisfied weakly whereas for $\alpha>2$ it is satisfied strongly. In fact, one can state that the penalty for shortfall sensitivity increases as α increases such that at $\alpha=\infty$ the satisfaction of shortfall sensitivity demands a leximin ordering like a Rawlsian scenario. In targeting and policy intervention for specific situations, α may be appropriately increased. For instance, when human immunodeficiency virus/acquired immunodeficiency syndrome (HIV/AIDS) epidemic led to substantive reductions in life expectation in many Sub-Saharan countries it required a much greater emphasis on improving health than just a proportionate shortfall.

6. Conclusions

This exercise evaluated three methods of aggregation across dimensions for measuring human development index through a set of intuitive axiomatic properties. The linear averaging method satisfied the axioms of monotonicty, anonymity and normalization (or MAN axioms). The geometric mean method, in addition to these three axioms, also satisfied the axiom of uniformity (or MANU axioms). The displaced ideal method (additive inverse of the distance from the ideal) satisfied the above-mentioned four axioms as also the axioms of shortfall sensitivity and hiatus sensitivity to level (or MANUSH axioms). We also proposed an α-class of measures where $\alpha=1$ and $\alpha=2$ turned out to be the linear averaging method and the displaced ideal method, respectively. Further, for the class of measures$\alpha\geq 2$ the MANUSH axioms were satisfied. A higher value of α implies greater shortfall sensitivity.

Our proposed class of measures can be used in different contexts. It can also consider the dimensions as subgroups. Under such an interpretation, the shortfall sensitivity axiom and the related discussions with leximin ordering and worst-off status quo in an absolute sense (or leximax ordering) as two extreme situations representing views that some may ascribe as 'left' and 'right', respectively, assumes importance. To address these extreme positions, many proponents have suggested scale invariance or translation invariance as possible reconciliatory approaches. However, these options would still keep convergence across subgroups at bay. Hence, we suggest that a proportionate to shortfall approach be considered as an intermediary position. Of course, we are aware that implementation at the ground level might be different from this measurement exercise, but nevertheless, this will facilitate our understanding.

The word MANUSH means human in many South Asian Languages such as Assamese, Bengali, Marathi and Sanskrit among others. Besides, MANUSH can be rearranged to HUMANS. Thus, we propose the axiom of MANUSH or HUMANS for a human development index.

Appendices

Appendix 1

For a given sum of indices value in the three dimensions, $c=h+e+y$, we can write

$$y=c-h-e \tag{A1}$$

For proving uniformity axiom for Geometric mean method (HDI_{GM}), let's consider

$$m=(hey)^{1/3} \tag{A2}$$

Applying (A1) in (A2) and differentiating (A2) partially with respect to *h* and *e*, and applying the maximization condition simultaneously,

$$\frac{\partial m}{\partial h}=\frac{1}{3}\left(he(c-h-e)\right)^{-2/3}\left(-he+e(c-h-e)\right)=0 \Rightarrow 2h+e=c \tag{A3}$$

$$\frac{\partial m}{\partial h}=\frac{1}{3}\left(he(c-h-e)\right)^{-2/3}\left(-he+h(c-h-e)\right)=0 \Rightarrow 2e+h=c \tag{A4}$$

solving (A3) and (A4) simultaneously,

$$h=e \tag{A5}$$

substituting (A5) in (A2),

$$m=\left(h^2(c-2h)\right)^{1/3} \tag{A6}$$

Differentiating (A6) with respect to *h* and applying the maximization condition

$$\frac{\partial m}{\partial h}=h^{2/3}\frac{1}{3}(c-2h)^{-2/3}(-2)+(c-2h)^{1/3}\frac{2}{3}h^{-1/3}=0 \Rightarrow h=\frac{c}{3} \tag{A7}$$

From (A1), (A5), and (A7), $h=e=y$. So, HDI_{GM} maximizes when $h=e=y$.

Now, for displaced ideal method, HDI_{DI}, let's consider distance, *d* from the ideal (1,1,1)

$$d^2=(1-h)^2+(1-e)^2+(1-c+h+e)^2 \tag{A8}$$

Differentiating (A8) partially with respect to *h* and *e*, and applying the minimization condition simultaneously (minimization of distance from ideal corresponds to maximization of HDI_{DI}),

$$\frac{\partial(d^2)}{\partial h} = 2(1-h)(-1) + 2(1-c+h+e)(1) = 0 \Rightarrow 2h+e=c \tag{A9}$$

$$\frac{\partial(d^2)}{\partial e} = 2(1-e)(-1) + 2(1-c+h+e)(1) = 0 \Rightarrow 2e+h=c \tag{A10}$$

Solving (A9) and (A10) simultaneously,

$$h=e \tag{A11}$$

Substituting (A11) in (A8),

$$d^2 = 2(1-h)^2 + (1-c+2h)^2 \tag{A12}$$

Differentiating (A12) partially with respect to h and applying the minimization condition

$$\frac{\partial(d^2)}{\partial h} = 4(1-h)(-1) + 2(1-c+2h)(2) = 0 \Rightarrow 12h = 4c \Rightarrow h = \frac{c}{3} \tag{A13}$$

From (A1), (A11), and (A13), $h=e=y$. So, HDI_{DI} maximizes when $h=e=y$.

Appendix 2

If the initial position is (h_1,e_1,y_1) and the next incremental position is a variable point (h,e,y) such that ΔHDI_{GM} is constant, c_1, for all such points,

$$c_1 = (hey)^{1/3} - (h_1e_1y_1)^{1/3} \Rightarrow hey = c_2 \tag{A14}$$

where,

$$c_2 = \left(c_1 + (h_1e_1y_1)^{1/3}\right)^3 \tag{A15}$$

The optimal path corresponds to the incremental position where the distance between the two is least. The distance, d_1, to be minimized,

$$d_1^{\,2} = (h-h_1)^2 + (e-e_1)^2 + (y-y_1)^2 \tag{A16}$$

Applying (A14) in (A16) and differentiating (A16) partially with respect to h and e, and applying the minimization condition simultaneously,

$$\frac{\partial(d_1^{\,2})}{\partial h} = 2(h-h_1) + 2\left(\frac{c_2}{he} - y_1\right)\frac{c_2}{e}\left(-\frac{1}{h^2}\right) = 0 \Rightarrow h(h-h_1) = \frac{c_2}{he}\left(\frac{c_2}{he} - y_1\right) \tag{A17}$$

$$\frac{\partial(d_1{}^2)}{\partial e} = 2(e-e_1) + 2\left(\frac{c_2}{he} - y_1\right)\frac{c_2}{h}\left(-\frac{1}{e^2}\right) = 0 \Rightarrow e(e-e_1) = \frac{c_2}{he}\left(\frac{c_2}{he} - y_1\right) \tag{A18}$$

From, (A17) and (A18);

$$h(h-h_1) = e(e-e_1) \tag{A19}$$

Similarly, proceeding with *h* and *y*;

$$h(h-h_1) = y(y-y_1) \tag{A20}$$

From (A19) and (A20);

$$h(h-h_1) = e(e-e_1) = y(y-y_1) \tag{A21}$$

For a two dimensional case (h, e), the equation for optimal path can be determined by considering infinitesimally small increment and performing integration. From (A19),

$$\frac{\partial h}{\partial e} = \frac{e}{h} \Rightarrow h\partial h = e\partial e \tag{A22}$$

Integrating,

$$\int h\partial h = \int e\partial e \Rightarrow h^2 = e^2 + c_3 \tag{A23}$$

The initial position (h_1, e_1) will be on the optimal path; so $c_3=(h_1{}^2-e_1{}^2)$. Replacing $c_3$in (A23),

$$h^2 = e^2 + h_1{}^2 - e_1{}^2 \tag{A24}$$

Appendix 3

If the initial position is (h_1,e_1,y_1) and the next incremental position is a variable point (h,e,y) such that ΔHDI_α is constant, c_4, for all such points,

$$c_4 = 1 - \left(\frac{(1-h)^\alpha + (1-e)^\alpha + (1-y)^\alpha}{3}\right)^{1/\alpha} - 1 + \left(\frac{(1-h_1)^\alpha + (1-e_1)^\alpha + (1-y_1)^\alpha}{3}\right)^{1/\alpha} \tag{A25}$$

Expressing y in terms of h and e and simplifying,

$$y = 1 - \left(3(c_5 - c_4)^\alpha - (1-h)^\alpha - (1-e)^\alpha\right)^{1/\alpha} \tag{A26}$$

where,

$$c_5 = \left(\frac{(1-h_1)^{\alpha} + (1-e_1)^{\alpha} + (1-y_1)^{\alpha}}{3} \right)^{1/\alpha} \tag{A27}$$

The optimal path corresponds to the incremental position where the distance between the two is least. The distance, d_1, to be minimized,

$$d_1^{\,2} = (h-h_1)^2 + (e-e_1)^2 + (y-y_1)^2 \tag{A28}$$

Applying (A26) in (A28) and differentiating (A28) partially with respect to h and e, and applying the minimization condition simultaneously,

$$\frac{\partial (d_1^{\,2})}{\partial h} = 2(h-h_1) + 2\left((1-y_1) - \left(3(c_5-c_4)^{\alpha} - (1-h)^{\alpha} - (1-e)^{\alpha}\right)^{1/\alpha}\right)$$

$$\left(\frac{-1}{\alpha}\right)\left(3(c_5-c_4)^{\alpha} - (1-h)^{\alpha} - (1-e)^{\alpha}\right)^{\frac{1-\alpha}{\alpha}}(-\alpha)(1-h)^{\alpha-1}(-1) = 0$$

$$\Rightarrow \frac{(h-h_1)}{(1-h)^{\alpha-1}} = (y-y_1)(1-y)^{1-\alpha} \tag{A29}$$

$$\frac{\partial (d_1^{\,2})}{\partial e} = 2(e-e_1) + 2\left((1-y_1) - \left(3(c_5-c_4)^{\alpha} - (1-h)^{\alpha} - (1-e)^{\alpha}\right)^{1/\alpha}\right)$$

$$\left(\frac{-1}{\alpha}\right)\left(3(c_5-c_4)^{\alpha} - (1-h)^{\alpha} - (1-e)^{\alpha}\right)^{\frac{1-\alpha}{\alpha}}(-\alpha)(1-e)^{\alpha-1}(-1) = 0$$

$$\Rightarrow \frac{(e-e_1)}{(1-e)^{\alpha-1}} = (y-y_1)(1-y)^{1-\alpha} \tag{A30}$$

From (A29) and (A30)

$$\frac{(h-h_1)}{(1-h)^{\alpha-1}} = \frac{(e-e_1)}{(1-e)^{\alpha-1}} \tag{A31}$$

Similarly, proceeding with h and y;

$$\frac{(h-h_1)}{(1-h)^{\alpha-1}} = \frac{(y-y_1)}{(1-y)^{\alpha-1}} \tag{A32}$$

From (A31) and (A32),

$$\frac{(h-h_1)}{(1-h)^{\alpha-1}} = \frac{(e-e_1)}{(1-e)^{\alpha-1}} = \frac{(y-y_1)}{(1-y)^{\alpha-1}} \tag{A33}$$

For a two dimensional case (h, e), the equation for optimal path can be determined by considering infinitesimally small increment and performing integration. From (A33),

$$\frac{\partial h}{\partial e}=\left(\frac{1-h}{1-e}\right)^{\alpha-1} \tag{A34}$$

For α=1, $\partial h/\partial e=1$; this corresponds to HDI_{LA} case, and the optimal path coincides with translation invariance case (Fig. 4). For α=2, $\partial h/\partial e=(1-h)/(1-e)$; this implies

$$\frac{\partial h}{1-h}=\frac{\partial e}{1-e} \tag{A35}$$

Integrating,

$$\int\frac{\partial h}{1-h}=\int\frac{\partial e}{1-e}\Rightarrow \ln(1-h)=\ln(1-e)+c_6\Rightarrow 1-h=c_7(1-e) \tag{A36}$$

where, c_6 and c_7 are constants. The initial position (h_1, e_1) will be on the optimal path; so $c_7=(1-h_1)/(1-e_1)$. Replacing c_7 in (A36),

$$h=\left(\frac{1-h_1}{1-e_1}\right)e+\frac{h_1-e_1}{1-e_1} \tag{A37}$$

This shows the proportion to shortfall case (Fig. 4) where the slope of the line is a ratio of the shortfalls. For α>2,

$$\frac{\partial h}{(1-h)^{\alpha-1}}=\frac{\partial e}{(1-e)^{\alpha-1}} \tag{A38}$$

Integrating,

$$\int\frac{\partial h}{(1-h)^{\alpha-1}}=\int\frac{\partial e}{(1-e)^{\alpha-1}}\Rightarrow\frac{(1-h)^{2-\alpha}}{2-\alpha}=\frac{(1-e)^{2-\alpha}}{2-\alpha}+c_8 \tag{A39}$$

where, c_8 is constant. The initial position (h_1, e_1) will be on the optimal path; so $c_8=((1-h_1)^{2-\alpha}-(1-e_1)^{2-\alpha})/(2-\alpha)$. Replacing c_8 in (A39) and simplifying,

$$h=1-\left((1-e)^{2-\alpha}+(1-h_1)^{2-\alpha}-(1-e_1)^{2-\alpha}\right)^{\frac{1}{2-\alpha}} \tag{A37}$$

The optimal paths for α=3, 5, and 10 are based on (A37) (see Fig. 4).

Appendix 4

For positions (h, e, y) having same gap from its respective mean, μ (μ=(h+e+y)/3), such that h=μ+c_9, e=μ+c_{10}, and y=μ+c_{11}, where c_9, c_{10}, and c_{11} are constants (given). Let V be the deviation of HDI under GM method from the uniform development situation; V is given as,

$$V = \mu - (hey)^{1/3} = \mu - ((\mu + c_9)(\mu + c_{10})(\mu + c_{11}))^{1/3} \quad \text{(A38)}$$

Differentiating D with respect to μ,

$$\frac{\partial V}{\partial \mu} = 1 - \frac{1}{3}((\mu + c_9)(\mu + c_{10})(\mu + c_{11}))^{-2/3}$$
$$((\mu + c_9)(\mu + c_{10}) + (\mu + c_{10})(\mu + c_{11}) + (\mu + c_{11})(\mu + c_9)) \quad \text{(A39)}$$

Simplifying,

$$\frac{\partial V}{\partial \mu} = 1 - \frac{1}{3}(hey)^{1/3}\frac{(he + ey + yh)}{hey} = 1 - \frac{geometric\ mean\ (h,e,y)}{harmonic\ mean\ (h,e,y)} = 1 - \frac{GM}{HM} \quad \text{(A40)}$$

Since, GM≥HM, ∂V/∂μ≤0, the equality holds good at the line of equality, i.e. when there is no deviation. Equation (A40) proves, under GM method, deviation is a decreasing function of μ.

Next, let V_1 be the deviation of HDI under DI method from the uniform development situation; V_1 is given as,

$$V_1 = \mu - \left(1 - \left(\frac{(1-h)^{\alpha} + (1-e)^{\alpha} + (1-y)^{\alpha}}{3}\right)^{1/\alpha}\right)^{1/3} \quad \text{(A41)}$$

Replacing h=μ+c_9, e=μ+c_{10}, and y=μ+c_{11}; and differentiating D_1 with respect to μ and simplifying,

$$\frac{\partial V_1}{\partial \mu} = 1 - \frac{(1-h)^{\alpha-1} + (1-e)^{\alpha-1} + (1-y)^{\alpha-1}}{3^{\frac{1}{\alpha}}\left((1-h)^{\alpha} + (1-e)^{\alpha} + (1-y)^{\alpha}\right)^{\frac{\alpha-1}{\alpha}}} \quad \text{(A42)}$$

Since, for all p, q, r and n, the following inequality is satisfied.

$$3\left(p^{n+1}+q^{n+1}+r^{n+1}\right)^{n} \geq \left(p^{n}+q^{n}+r^{n}\right)^{n+1} \quad \text{(A43)}$$

Considering p=1-h, q=1-e, and r=1-y and n=α-1one can show

$$3^{\frac{1}{\alpha}}\left((1-h)^{\alpha}+(1-e)^{\alpha}+(1-y)^{\alpha}\right)^{\frac{\alpha-1}{\alpha}} \geq (1-h)^{\alpha-1}+(1-e)^{\alpha-1}+(1-y)^{\alpha-1} \quad \text{(A44)}$$

Hence, from (A42) and (A44) $\partial V_1/\partial\mu \geq 0$ the equality holds good at the line equality, i.e. when there is no deviation. Equation (42) proves, under GM method, deviation is an increasing function of μ. To demonstrate (A43), one can consider the case of α=2; and hence n=1. One can start with,

$$(p-q)^{2}+(q-r)^{2}+(r-p)^{2} \geq 0 \quad \text{(A45)}$$

$$\Rightarrow 2\left(p^{2}+q^{2}+r^{2}\right) \geq 2(pq+qr+rp)$$

$$\Rightarrow 2\left(p^{2}+q^{2}+r^{2}\right)+\left(p^{2}+q^{2}+r^{2}\right) \geq 2(pq+qr+rp)+\left(p^{2}+q^{2}+r^{2}\right)$$

$$\Rightarrow 3\left(p^{2}+q^{2}+r^{2}\right) \geq (p+q+r)^{2}$$

Similarly, one can show (A43) for all α.

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