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Abstract

Conditions for stability in an open economy dynamic stochastic general equilibrium model adapted to a dualistic labor market (SOEME) are the same as for a mature economy. But the introduction of monetary policy transmission lags makes it deviate from the Taylor Principle. Under rational expectation a policy rule is unstable, but under adaptive expectations traditional stabilization gives a determinate path, with weights on the objective of less than unity. Estimation of a Taylor rule for India and optimization in the SOEME model itself, all confirm the low weights. The results imply that under rational expectations optimization is better than following a rule. If backward looking-behavior dominates, however, a policy rule can prevent overshooting and instability. Economy-specific rigidities must inform policy design, and the appropriate design will change as the economy develops.

Keywords: DSGE; emerging market; rigidities; stability; optimization; Taylor rule

JEL Code: E26, E52

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Stability and Transitions in Emerging Market Policy Rules

I. Introduction

We examine stability and determinacy in a dynamic stochastic general equilibrium (DSGE) model for a small open economy (SOE), adapted to an emerging market (SOEME) with two types of consumer-workers. One group has high and the other low productivity[1]. Stability results have the same structure as in the SOE.

In the calibrated SOEME model, however, some transmission lags and some degree of backward-looking behavior are necessary to reproduce data moments for an emerging market (EM).With these changes, policy optimization solving for macro variables as a function of expected future value delivers stability and determinacy at lower output cost. Coefficients of the policy reaction function are low. But a monetary rule requires unrealistic weights for stability. Therefore discretionary optimization is more effective than following a monetary policy rule if agents are forward-looking.

A rule-based traditional stabilization where agents solve current variables as a function of past data turns out to require low weights on both inflation and the output gap for stability. The weights fall with a rise in the share of forward-looking behavior. A rule can be followed, if backward-looking behavior dominates, but its coefficients should not exceed unity. Then it contributes to stability by preventing over-or under-shooting of policy rates.

In an EM with delayed transmission of policy rates, stability turns out to be very sensitive to the share of forward-looking behavior. This suggests volatility can be high if policy rules are followed unless weights are kept low.

[1] See Goyal (2011), who extends Galí and Monacelli (2005) to such an emerging market.

Such a policy rule contributes to stability since it prevents over or under-shooting due to lagged effect of policy.

These results imply the coefficients of a policy rule in an EM should change as monetary transmission matures and behavior becomes more forward-looking. During transition, discretionary optimization may offer more flexibility, and the Taylor Principle that the weight on inflation in a policy rule must exceed unity does not hold. The results may apply more generally than to EMs alone since other economies also have various types of frictions.

The practice of monetary policy in India is consistent with these results. The CB's reaction function in the calibrated SOEME, and a monetary policy rule estimated with Indian data both show response coefficients much smaller than unity. Thus, during the estimation period, policy avoided the instability that the Taylor Principle could entail in Indian conditions.

The structure of the paper is as follows: Section II places the SOEME stability issues in the relevant literature; Section III presents the basic SOEME model; Section IV derives the implications of stability for a policy rule; Section V develops the adaptive expectations case; Section VI presents an estimated monetary policy rule, before Section VII concludes.

II. Literature Review

Models with forward-looking behavior can have nominal explosions. Sargent and Wallace (1975) demonstrated that with rational expectations inflation is indeterminate under an interest rate instrument. Since there is no effective nominal anchor inflation can take many values. But McCallum (1981) later showed such indeterminacy only occurs if the Central Bank (CB) places no weight on inflation in its response. The CB response to inflation, or its targeting of nominal money stock, can provide the nominal anchor required to fix the price level or inflation. In addition, other rigidities and lags may serve to anchor inflation (Friedman, 1990).

It is shown analytically, in the canonical NKE forward-looking model (Woodford, 2003) that a CB response to inflation above target exceeding unity selects the unique saddle-stable inflation path thus ensuring determinacy. Since there is a unique stable path, rational expectations converge to this.
It ensures stability since it rules out explosive nominal paths of self-fulfilling inflation expectations. If for each one-percent increase in inflation, the central bank raises the nominal interest rate by more than one percentage point (Taylor 1993, pp. 202) the policy rate adjusts more than one-to-one with inflation. This is known as the Taylor Principle and eventually implies positive real interest rates.

Evans and Honkapohja (2003) make the point that such a policy rule works because it conditions the policy response not only on inflation but also on individuals' expectations. It contributes to stability since outcomes depend on these expectations.

This literature justifies a Taylor type monetary policy rule, with a weight on inflation greater than unity, showing it can perform as well as discretionary optimization. A rule can also be justified as a credible commitment, preventing opportunistic behavior that results in an inflation bias. In emerging democracies where inflation hurts the poor and loses votes, however, CBs may not have an inflation bias (Goyal, 2007) [2].

But, further work, surveyed in Galí et. al (2004) shows monetary policy rules are fragile and sensitive to the assumptions of the model. Such rules can be a

[2] Clarida et. al. (1999) point out, however, in a discretionary optimum also, even though the CB reoptimizes every period, it may not have an incentive to deviate and create surprise inflation, and the private sector recognizes this.

source of instability. A large literature on monetary policy rules under adaptive learning followed seminal work by Evans and Honkapohja (2003). It generally shows rules that generate determinate equilibrium also lead to expectational stability (E-stability) under learning (Bullard and Mitra, 2002), so that expectations are not perfectly rational. But again regions of determinacy are sensitive to model assumptions and parameters.

How does our model fit in and contribute to the literature? Like the learning literature, we explore non-rational expectations, but we also consider the case where the share of backward-looking behavior is large enough to analyze a traditional stabilization, rather than rational expectations equilibrium. The learning literature stays with the latter. Thus we also consider a pure backward-looking equilibrium.

Our results support the general finding of fragility and context-specificity of policy rules and of the Taylor principle. But show this in a context of a dualistic EM not yet addressed in the literature. In Galí et. al (2004) a share of rule-of-thumb consumers implies demand rises with output, requiring a higher weight on inflation for stability. Our model differs in that subsistence consumption is given exogenously and so demand does not rise with output. Our aggregate demand equation therefore differs from theirs.

The literature has also explored the role of inertia and lags. Mitra and Bullard (2007) add interest rate stabilization or a lagged interest rate term in the policy rule and show this raises the region of determinacy and learning. Duffy and Xiao (2007) have a similar result. We have such an interest stabilization term, but the rational expectations equilibrium is still indeterminate in our model if the Taylor Principle is followed.

Evans and McGough (2005) have lagged inflation and output terms in the AS and AD curves respectively and find these do not affect the indeterminacy result, which continues to prevail. New regions of explosiveness are added. We have a lagged inflation term, but add a lagged interest rate term in the AD to capture market segmentation that delays policy transmission. The literature does not, to our knowledge, as yet explore the effect of such a term. We find this combination of lags does erode the existence of unique rational expectations equilibria.

Another new result is a high share of backward-looking behavior can justify a policy rule with low weights. But the presence of some forward-looking behavior makes the equilibrium determinate for some parameter combinations leading to a unique equilibrium.

Our result therefore offers a response to Cochrane's (2011) criticism of the NKE determinacy result that it requires unrealistic CB behavior on out of equilibrium paths. On such explosive paths, higher inflation requires CBs to raise expected inflation even more in order to switch expectations to the unique path as against the old Keynesian stabilizing logic of raising interest rates to reduce demand and therefore inflation. On an explosive path, for example, real rates can fall with inflation, and create more inflation as money supply rises under an interest rate rule. Such expected policy reactions cannot apply in a stabilizing solution based on past behavior. But there is a unique stable path, so the ad hoc charge that applies to old Keynesian stabilization is not valid. Instability occurs, however, with large policy responses due to overshooting of policy rates in the presence of lags. This is known as instrument instability.

The above argument also gives a justification for the empirical estimation of a policy rule. Cochrane (2011) argues a monetary policy rule is not identified under rational expectations. Since variables jump to a unique path, the adjustment process cannot be estimated. What are estimated are correlations in shocks that shift inflation to a unique equilibrium. The criticism does not hold for past adjustment paths with smoothing.

Since the objective is to explore stability in the specific context of a dualistic EM with transmission lags and draw implications for policy, we do not analyze different types of policy rules, and other issues, which are already examined in the literature (Bullard and Mitra 2002).

After a brief description of the SOEME model, we turn to stability results.

III. A Small Open Emerging Market Model

A microfounded dynamic stochastic general equilibrium (DSGE) model for a small open emerging market is used to derive the aggregate demand (AD) (2) and aggregate supply (AS) (3)[3]. The central bank (CB) minimizes a loss function (1), based on consumers' welfare and a desire for smoothing, subject to (2) and (3). The loss function is a weighted average of output, inflation and interest rate deviations from equilibrium values:

$$L = q_x x_t^2 + q_\pi \pi_t^2 + q_i i_t^2 \tag{1}$$

The last captures smoothing preferences that prevent large changes in the policy rate, where i_t is the riskless nominal interest rate. The first term is the output gap $x_t = y_t - \bar{y}_t$, and the second term, inflation, can be either consumer price inflation $\pi_t \equiv p_t - p_{t-1}$ (where price $p_t \equiv \log P_t$) or domestic inflation $\pi_{H,t}$. Lower case letters are logs of the respective variables. Table 1 explains the parameters and gives their calibrated values.

[3] The derivations are available in Goyal (2011).

The AD and AS are derived from forward-looking consumer and firm optimization respectively, in a dualistic structure with two types R and P of consumer-workers. The R types, with population share η, 0< η <1, are able to smooth consumption at international levels in perfect capital markets. P types are assumed to be at a subsistence consumption c_P, financed by transfers from R types mediated by the government. This is assumed to change exogenously.

Table 1: Benchmark calibrations		
Baseline Calibrations		
Degree of price stickiness	θ	0.75
Price response to output	φ	0.25
Labour supply elasticity of P type	φ_P	0.01
Labour supply elasticity of R type	φ_R	0.6
Elasticity of substitution between differentiated goods	ε	6
Steady state real interest rate or natural interest rate	ρ or $\bar{i}$	0.01
Variations in the natural interest rate due to temporary shocks	$\overline{rr}$	0.01 $\pm$
Degree of openness	α	0.3
Proportion of the R type	η	0.4
The intertemporal elasticity of substitution of the R type	$1/\sigma_R$	1
The intertemporal elasticity of substitution of the P type	$1/\sigma_P$	0
Consumption of the P type	C_p	0.2
Consumption of the R type	C_R	1
Share of backward looking price setting	γ_b	0.2
Share of forward looking price setting	γ_f	0.8
Weight of output in the CB's loss function	q_y	0.7
Weight of inflation in the CB's loss function	q_π	2
Weight of the interest rate in the CB's loss function	q_i	1
Implied parameters		
Weighted average elasticity of substitution	$1/\sigma_D$	0.58
Discount factor	β	0.99
Weighted average consumption level	C	0.75
Log deviation from world output	κ	0.1
Philips curve parameter	λ	0.24
Steady state real interest rate, discount rate	ρ	0.01
Labour supply elasticity	$1/\varphi$	4
Shocks		
Persistence of natural rate shock	ρ^r	0.75
Persistence of cost-push shock	ρ^c	0
Standard deviation of natural rate shock	σ_ε^r	0.01
Standard deviation of cost-push shock	σ_ε^c	0.2

The labour supply elasticities of the P types are higher than the R types and for the P type intertemporal elasticity of consumption is zero.

The dynamic AD equation for the SOEME is:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_D}(i_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) \qquad (2)$$

Where:

$$\overline{rr}_t = \rho - \sigma_D \Gamma(1-\rho_a)a_t - \sigma_D(1-\eta+\Phi)E_t\{\Delta c_{P,\ t+1}\} + \sigma_D(\Theta-\Psi)E_t\{\Delta y^*_{t+1}\}$$

$$\Theta = \alpha(\varpi-\eta),\ \ d = \frac{1}{\sigma_D+\varphi},\ \ \Gamma = \frac{(1+\varphi)}{\sigma_D+\varphi},\ \Psi = \eta(\sigma-\sigma_D)d,$$

$$\sigma_D = \frac{\sigma_R}{(\eta(1-\alpha)+\varpi\alpha)},\ \ \Phi = d((1-\eta)(\sigma-\sigma_D)),$$

$$\varpi = \sigma_R + (1-\alpha)(\sigma_R - 1)$$

The steady-state natural interest rate, ρ, is defined as the equilibrium real rate, consistent with a zero or target rate of inflation, when prices are fully flexible. It is also the time discount rate since $\rho \equiv \beta^{-1} - 1 = -\log\beta$ where β is the discount factor. Shocks that change the natural rate open an output gap and affect inflation. The term $\overline{rr}_t$ that enters the AD therefore captures deviation of the natural rate from its steady-state value. The deviation occurs due to real disturbances that change natural output; $\overline{rr}_t$ rises for any temporary demand shock and falls for any temporary supply shock. Optimal policy requires insulating the output gap from these shocks, so that the CB's interest rate instrument should move in step with the natural rate. Thus the CB would accommodate positive supply shocks that raise the natural output by lowering interest rates. It would offset positive demand shocks that raise output above its potential by raising interest rates. Full stabilization at the current natural output implies that $x_t = \pi_{H,t} = 0$, $y_t = \overline{y}_t$ and $r_t = \overline{rr}_t$.

In an EM a change in c_P is an additional shock. A fall requires reduction in the policy rate, since it increases willingness to work of P type workers. The distance from the world consumption level also rises. The parameters of the other shock terms also differ from those for the SOE. Since productivity shocks, a_t, can be more persistent in EMs that are in transition stages of upgrading technologies, they change the natural rate less. A temporary shock to c_P turns out to have the largest effect on the natural rate (Goyal 2009).

The dynamic AS is:

$$\pi_{H,t} = \gamma_f \beta E_t \{\pi_{H,t+1}\} + \kappa_D x_t + \gamma_b \pi_{H,t-1} \qquad \gamma_f + \gamma_b = 1 \tag{3}$$

The response of inflation to the output gap is $\kappa_D = \lambda(\sigma_D + \varphi)$. Since both empirical estimations and the dominance of administered pricing in an EM suggest that past inflation affects current inflation, the AS (3) has a positive γ_b as the share of lagged, and γ_f as the share of forward-looking inflation.

Marginal cost at its steady-state level, when prices are perfectly flexible, defines the natural output $\overline{y_t}$. But the world output level is the final steady-state for a SOEME. Low productivity, poor infrastructure and other distortions keep the natural output in an EM below world levels. Convergence to world output levels for all the SOEME population is part of the process of development. Goyal (2011) systematically compares the differences in behavior and outcomes for the SOE and SOEME. As η approaches unity the EM becomes developed and the SOEME converges to the Galí and Monacelli (2005) (GM) type SOE with per capita consumption reaching the normalized world level of unity.

In the next section we analyze the stability properties of the SOEME. In NKE-SOE models a Taylor-type policy rule imposes stability in the rational expectations solution. An equivalent rule can be derived for a basic SOEME model. Since the two models have a similar structure, they differ only in parameter values. The policy response coefficient to inflation turns out to exceed unity in both models.

IV. What Stability Implies for a Policy Rule

As in the SOE system, the SOEME system comprising (2) and (3) is unstable under forward-looking optimization solving for endogenous variables as a function of expected future values. To see this, substitute dynamic AD (2) in the AS[4] (3) to write the AS as a function of x_{t+1}. The two equations then become:

$$x_t = E_t\{x_{t+1}\} + \sigma_D^{-1} E_t\{\pi_{H,\,t+1}\} \tag{4}$$

$$\pi_{H,t} = \kappa_D E_t\{x_{t+1}\} + \left(\beta + \sigma_D^{-1}\kappa_D\right) E_t\{\pi_{H,\,t+1}\} \tag{5}$$

In matrix form they are:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_o \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \quad \text{and} \quad A_o = \begin{bmatrix} 1 & \sigma_D^{-1} \\ \kappa_D & \beta + \kappa\sigma_D^{-1} \end{bmatrix} \tag{6}$$

Since the determinant and trace of the coefficient matrix A_o are both greater than zero, the system is unstable. There is local indeterminacy. Sunspot explosions can occur.

Woodford's (2001) result was that interest rate rules lead to indeterminacy of the rational expectations consistent price level only if the path of the short-term policy rate is exogenous. In particular, determinacy requires the Taylor Principle to be satisfied, since it implies the policy rate reacts to inflation.

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[4] We consider the simplest version of the SOEME so γ_f is taken as equal to 1.

We derive the equivalent stability condition for the simplified SOEME. A Taylor-type policy rule whereby the interest rate is raised if there is domestic inflation or if the output gap is positive is:

$$i_t = \overline{rr}_t + \phi_\pi \pi_{H,t} + \phi_x x_t \qquad (7)$$

Substituting for i_t minus its equilibrium value from the policy rule (7) into (2), transforming π_{Ht} into $\pi_{t,}$ substituting for $\pi_{t,}$ then substituting for x_t, with i_t substituted in it, in (3), we get:

$$(\sigma_D + \phi_x + \phi_\pi \kappa_D) x_t = \sigma_D E_t \{x_{t+1}\} + (1 - \phi_\pi \beta) - \phi_\pi \beta E_t \{\pi_{t+1}\} \qquad (8)$$

$$(\sigma_D + \phi_x + \phi_\pi \kappa_D) \pi_t = \kappa_D \sigma_D E_t \{x_{t+1}\} + (\kappa_D + \beta(\sigma_D + \phi_x)) E_t \{\pi_{t+1}\} \qquad (9)$$

The AD and AS (2) and (3) are transformed to (8) and (9), as required for a rational expectation solution, and written in matrix form:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \qquad (10)$$

where

$$A_T = \Omega \begin{bmatrix} \sigma_D & 1 - \beta\phi_\pi \\ \sigma_D \kappa_D & \kappa_D + \beta(\sigma_D + \phi_x) \end{bmatrix} \text{and}\, \Omega = \frac{1}{\sigma_D + \phi_x + \kappa_D \phi_\Pi}$$

The stability condition[5] for a unique non-explosive solution, to which the forward-looking variables jump, is $\kappa_D(\phi_\pi - 1) + (1 - \beta)\phi_x > 0$. A policy response to inflation that exceeds unity is sufficient to ensure stability. The result is qualitatively similar as for the SOE in GM, although the coefficient values are different. GM's κ becomes κ_D in the SOEME; $\overline{rr}_t$ is also different, subject to

[5] The stability condition for a two equation difference system is determinant A > 0, and determinant A+ trace A>-1 when the system is written in the form $z_t = E(z_{t+1}) + \ldots\ldots\ldots$. (see Woodford, 2003).

larger shocks[6]. The price equation (3) has the implication that in the steady-state a rise in inflation raises the output gap by $\frac{(1-\beta)}{\kappa_D}$. Then $\phi_\pi + \frac{(1-\beta)}{\kappa_D}\phi_x$ gives the long-run response of the policy rate to a persistent rise in inflation. The Taylor principle, which the stability condition satisfies, says this response must exceed unity. In the SOEME, since $\kappa_D > \kappa$ and is large[7] the permanent rise in policy rate is driven more by ϕ_π.

Calibrating the SOEME model required a lagged term in the policy variable to capture slow monetary policy transmission through segmented financial markets[8]. In addition, only a proportion $\gamma_f < 1$ of firms set prices in a forward-looking manner. The model equation (11) and (12), with these additions, turned out to be stable under discretionary optimization and sensitivity analysis conducted with the parameters as given in Table 1, even where weights on inflation were low in the loss function and in the derived policy reaction function.

The latter gives the final weight on the CB objectives in the calibrations after the constraints, subject to which the optimization is done, are substituted in the policy objective function. A CB reaction function may include more variables than a Taylor rule, but the two are related since a monetary policy rule normally gives the CBs response to current or forecasted macro variables. Given the CB's objective function used, the arguments of the derived reaction function are the same as our estimated monetary policy rule, and differ from the standard Taylor rule in including a lagged interest rate term.

[6]There can be many reasons for higher shocks to the real interest rate in EMs. Another factor affecting EM interest rates is country risk. Neumeyer and Perri (2005) reproduce EM business cycle stylized facts by introducing large shocks to the real interest rate due to changes in such risk.
[7] In the calibrated SOEME for the values given in Table 1 $\kappa_D = 0.47$ so $(1-\beta)/\kappa_D = 0.02$
[8] This differs from different types of lags due to inertia and stabilization objectives analysed in the literature. See, for example Evans and McGough (2004).

$$x_{t+1} = x_t + \frac{1}{\sigma_D}\left(0.2r_{t-1} + 0.8r_t - E_t\pi_{H,t+1} - \overline{rr_t}\right) \qquad (11)$$

$$\pi_{H,t+1} = \frac{1}{\gamma_f\beta}\pi_{H,t} - \frac{\lambda}{\gamma_f\beta}(\sigma_D + \phi)x_t - \frac{\gamma_b}{\beta\gamma_f}\pi_{H,t-1} \qquad (12)$$

Since a positive smoothing parameter q_i in the CB loss function reduces the policy response to inflation, some weight on π is required for stability. With $q_i = 0$ even no weight on inflation generates stable outcomes. For example, if $q_i = 1$ outcomes are indeterminate with $q_x = 0$ and q_π less than 1; they are also indeterminate with $q_x = 0.07$ if q_π less than 0.9; but if $q_i = 0$ and $q_x = 0.07$ outcomes are determinate even with $q_\pi = 0$. In the estimated reaction functions with varying parameter values in Goyal (2011, Table 4) the weights on inflation range from 4.28 to 0.0091. The lags in the system, and other structural aspects, may be contributing to stability even with a low policy reaction to inflation.

Since the SOEME model under optimal discretionary policy was stable even with a weight on inflation less than unity, we next derive stability conditions for the more complex calibrated SOEME model under a Taylor-type policy rule.

IV.1. Stability Conditions for the Calibrated SOEME with a Policy Rule

To solve for stability under a policy rule with the calibrated SOEME model the equations (11) and (12) are written in the form (15) and (16), where expectations of future variables affect current variables, and the policy rule (13) with a generic auto-correlated shock term v_t (14) substituted in them.

$$r_t = \rho + \phi_\pi \pi_{H,t} + \phi_x x_t + v_t \qquad (13)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t \qquad (14)$$

$$x_t = \Omega\sigma_D E_t x_{t+1} + \Omega(1-0.8\phi_\pi\gamma_f\beta)E_t\pi_{H,t+1} - \Omega(0.2\phi_\pi + 0.8\phi_\pi\gamma_b)\pi_{H,t-1} - 0.2\phi_x\Omega x_{t-1} - \Omega\rho - 0.8\Omega(\rho_v v_{t-1} + \varepsilon_t) - 0.2\Omega v_{t-1} + \Omega\overline{rr_t} \quad (15)$$

$$\pi_{H,t} = \lambda(\sigma_D + \phi)\Omega\sigma_D E_t x_{t+1} + (\gamma_f\beta + \Omega\lambda(\sigma_D + \phi)(1-0.8\phi_\pi\gamma_f\beta))E_t\pi_{H,t+1} - (\Omega\lambda(\sigma_D + \phi)(0.2\phi_\pi + 0.8\phi_\pi\gamma_b) - \gamma_b)\pi_{H,t-1} - 0.2\phi_x\Omega\lambda(\sigma_D + \phi)x_{t-1} + \lambda\Omega(\sigma_D + \phi)(\rho + 0.8(\rho_v v_{t-1} + \varepsilon) + 0.2v_{t-1} - \overline{rr_t}) \quad (16)$$

Where $$\Omega = \frac{1}{(\sigma_D + 0.8\phi_x + 0.8\lambda(\sigma_D + \phi)\phi_\pi)}$$

They give the following higher order difference equation system:

$$\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} = A\begin{bmatrix} E_t x_{t+1} \\ E_t\pi_{H,t+1} \end{bmatrix} - B\begin{bmatrix} x_{t-1} \\ \pi_{H,t-1} \end{bmatrix} + \Omega C[\tilde{rr}_t - 0.2v_{t-1} - 0.8\rho v_{t-1}] \quad (17)$$

Where

$$A = \begin{bmatrix} \Omega\sigma_D & \Omega(1-0.8\phi_\pi\gamma_f\beta) \\ \lambda(\sigma_D + \phi)\Omega\sigma_D & \gamma_f\beta + \Omega\lambda(\sigma_D + \phi)(1-0.8\phi_\pi\gamma_f\beta) \end{bmatrix}$$

$$B = \begin{bmatrix} 0.2\phi_x\Omega & \Omega(0.2\phi_\pi + 0.8\phi_\pi\gamma_b) \\ 0.2\phi_x\Omega\lambda(\sigma_D + \phi) & \Omega\lambda(\sigma_D + \phi)(0.2\phi_\pi + 0.8\phi_\pi\gamma_b) - \gamma_b \end{bmatrix}$$

Consider a second order difference equation:

$$x_{t+2} + ax_{t+1} + bx_t = c_t$$

The solution is stable[9] iff

$$|a| < 1+b$$

$$b < 1$$

Rewriting (17) as:

[9]Stability is discussed in Blanchard and Kahn (1980), Woodford (2003), Gali (2008). Hoy et al. (2004, pp. 831) list three conditions for the system to be locally unique. These are the two given in the text and a third a – b > -1 which is satisfied trivially.

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{H,t+1} \end{bmatrix} - \frac{1}{A}\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} - \frac{B}{A}\begin{bmatrix} x_{t-1} \\ \pi_{H,t-1} \end{bmatrix} = C \qquad (18)$$

$\frac{1}{A} < 1 - \frac{B}{A}$ *and* $-\frac{B}{A} < 1$ will define the stability of the equation.

The first condition can be written as:

$$1 < A - B \qquad (19)$$

Since,

$$|A| = \Omega \sigma_D \gamma_f \beta$$
$$|B| = -0.2\phi_x \Omega \gamma_b$$

$$1 < A - B = \frac{1}{\Omega} < \sigma_D \gamma_f \beta + 0.2\phi_x \gamma_b \qquad (20)$$

The second condition is:

$$-\frac{B}{A} < 1 \qquad (21)$$

$$1 < \frac{\sigma_D \gamma_f \beta}{0.2\phi_x \gamma_b} \qquad (22)$$

Condition (22) defines a cap for the weight given to the output gap, ϕ_x, but does not impose any constraint on the weight given to inflation deviation, ϕ_π. It gives a high upper bound for ϕ_x.

Condition (20) also defines cap for ϕ_x and for ϕ_π since the latter enters Ω. The cap for ϕ_π is negative unless ϕ_x is negative, and vice versa. Thus for stability the policy rule imposes a negative weight either on inflation deviation from target or on the output gap. Since the caps derived from condition (20), are lower, it is the operative condition.

The stability condition (22) holds but condition (20) is not satisfied for the parameters in Table 1, so while optimization with moderate weights is stable, the Taylor rule is unstable[10]. Combinations of ϕ_π and ϕ_x for which both the conditions are satisfied with the benchmark calibrations, with (20) holding just at the margin, are ϕ_π = 2 and ϕ_x = -1.5, or ϕ_π = -4.2 and ϕ_x = 1.6. A CB's policy rule is unlikely to have such weights.

IV.2. Response to Shocks

Analytically deriving[11] the response to monetary shocks v_t from the policy rule (13), using the method of undetermined coefficients, gives:

$$\pi_{H,t} = \Theta v_{t-1} \text{ where } \Theta = \frac{\Omega\left(0.2 + 0.8\rho_v\right)}{\frac{\gamma_f \beta 0.2\phi_x \Omega}{\lambda\left(\sigma_D + \phi\right)} - \frac{\gamma_b \Omega\sigma_D + 1}{\lambda\left(\sigma_D + \phi\right)}} \tag{23}$$

$$x_t = \frac{1}{\lambda\left(\sigma_D + \phi\right)}\left(\Theta v_{t-1} - \gamma_b \Theta v_{t-2} - \gamma_f \beta\, \Theta v_t\right) \tag{24}$$

We can now compare the inflation and output responses under the policy rule with the discretionary optimization undertaken in Goyal (2011), using the benchmark parameters of Table 1, and the stable weight combination of ϕ_π = -4.2 and ϕ_x = 1.6. Under the monetary rule, the fall in inflation (0.0065) is less and output (0.0011) is more compared to optimization. Under optimization a natural rate shock raises the policy rate by 0.013 and reduces inflation by 0.01 and output by 0.006 in the first period. For a persistent rise in v_t (monetary tightening) of 0.1 optimization is more effective, but in the same direction as a policy rule. Optimization is not only stable but has a lower output cost.

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[10] In the benchmark optimizing calibrations the upper bound for ϕ_x is 34.1 from condition (22) so this condition is satisfied. However, the lower cap from condition (20), which is not satisfied is -1.5. The cap on ϕ_π, -2.3, is also not satisfied.

[11] Derivations are available on request.

Key results, supported by analysis and calibrations are: policy lags in the SOEME make it stable under forward-looking optimization for a coefficient of inflation in the loss function that differs from unity. But a forward looking policy rule with realistic weights to the function variables in the objective is not stable under rational expectations.

V. Adaptive Expectations and a Policy Rule

Rigidities and volatility in an EM make it difficult to forecast future variables. Policy-makers and agents typically have adaptive rather than model consistent rational expectations. So next we explore the functioning of a policy rule under such behavior, where current variables are solved based on past variables. Future inflation enters both the AD and AS, but since expectations are formed adaptively there is no overall rational expectations solution. Future variables have to be solved as a function of past variables to analyze stability unlike the form of equations (4) and (5). We examine stability in such a case, with our calibrated SOEME AD and AS, adding a policy rule.

The Lucas critique was agents' anticipation of policy will prevent a unique outcome of traditional Keynesian stabilization, where policy makers raise rates to reduce current demand and output. With our structure of monetary transmission however, a policy rule delivers a unique equilibrium. In the calibrated SOEME model, with interest rate lags and some forward-looking behavior, the model solved backwards as a function of past data is unstable. Adding a policy rule delivers a unique stable solution, to which the economy converges. But, just as in the forward-looking optimizing SOEME solution, the weights on inflation and output gap deviations have to be less than unity for stability.

The backward-looking system (11) and (12), with the policy rule (13) and (14) substituted in it, is stable if the following conditions are satisfied:

$$\begin{aligned} 1 &> -A - B \\ 1 &> A - B \end{aligned} \tag{25}$$

Where:

$$|A| = \frac{1}{\gamma_f \beta}\left(\sigma_D + 0.8\phi_x + \lambda(\sigma_D + \phi)\left(0.8\phi_\pi - \frac{2}{\gamma_f \beta} \right) \right) \tag{26}$$

$$|B| = -0.2\frac{\gamma_b}{\gamma_f \beta}\frac{\phi_x}{\sigma_D} \tag{27}$$

These can be solved to obtain floors and caps on the policy rule weights.

The first condition written as, 1 > - A – B, can be used to derive the floors:

$$\phi_x > \left[\frac{2\lambda(\sigma_D + \phi)}{\gamma_f \beta} - \lambda(\sigma_D + \phi)0.8\phi_\pi - \sigma_D - \gamma_f \beta \right] \frac{\sigma_D}{0.8\sigma_D + 0.2\gamma_b} \tag{28}$$

$$\phi_\pi > \left[\frac{2\lambda(\sigma_D + \phi)}{\gamma_f \beta} - \left(0.8 + 0.2\frac{\gamma_b}{\sigma_D} \right)\phi_x - \sigma_D - \gamma_f \beta \right] \frac{1}{\lambda(\sigma_D + \phi)0.8} \tag{29}$$

That is, the weights in any policy rule followed must exceed the above values.

The second condition, 1 > A – B, can be used to derive the caps:

$$\phi_x < \left[\gamma_f \beta + \frac{2\lambda(\sigma_D + \phi)}{\gamma_f \beta} - \lambda(\sigma_D + \phi)0.8\phi_\pi - \sigma_D \right] \frac{\sigma_D}{0.8\sigma_D + 0.2\gamma_b} \tag{30}$$

$$\phi_\pi < \left[\gamma_f \beta + \frac{2\lambda(\sigma_D + \phi)}{\gamma_f \beta} - \left(0.8 + \frac{0.2\gamma_b}{\sigma_D} \right)\phi_\pi - \sigma_D \right] \frac{1}{\lambda(\sigma_D + \phi)0.8} \tag{31}$$

That is, the weights in any policy rule followed must be less than the above values.

The cap conditions are the binding ones. The derivation of the cap conditions with respect to γ_f gives:

$$\frac{\delta\phi_x}{\delta\gamma_f} = \left[\beta - \frac{2\lambda(\sigma_D + \phi)}{\beta\gamma_f^2}\right]\frac{\sigma_D}{0.8\sigma_D + 0.2\gamma_b} \tag{32}$$

$$\frac{\delta\phi_\pi}{\delta\gamma_f} = \left[\beta - \frac{2\lambda(\sigma_D + \phi)}{\beta\gamma_f^2}\right]\frac{1}{\lambda(\sigma_D + \phi)0.8} \tag{33}$$

Since the second term in the bracket exceeds unity, these derivatives are negative. So the weights in the policy rule are decreasing in γ_f, and have to be lower, the larger the share of forward-looking behavior. The derivatives with respect to γ_b are also negative but are much smaller. So the derivatives with respect to γ_f, (32) and (33) have a much larger effect on the caps, as the caps change with γ_f.

For the calibrated (Table 1) value of parameters, if $\gamma_f = 0.4$, the caps for ϕ_x and ϕ_π respectively are 0.5 and 0.35. Estimation of γ_f with Indian data gives 64 percent of firms to be forward-looking (Tripathi and Goyal, 2012). For the economy as a whole, including administered prices and informal sectors, γ_f will be lower, so optimal weights in the policy rule should be lower than the values for $\gamma_f = 0.4$. The negative derivative of ϕ_x with respect to ϕ_π (34) from the cap condition (30), and vice versa from the cap condition (31), shows if one of them rises, the other must fall. If γ_f rises to 0.8, the caps fall to 0.27 and 0.05 respectively. But at that level of γ_f the rational expectations solution can be expected to be more relevant than the backward-looking one, since behavior will become more forward-looking.

$$\frac{\delta\phi_x}{\delta\phi_\pi} = -\lambda(\sigma_D + \phi)\frac{0.8\sigma_D}{0.8\sigma_D + 0.2\gamma_b} \qquad (34)$$

In an EM with delayed transmission of policy rates, stability turns out to be very sensitive to the share of forward-looking behavior. This suggests volatility can be high if policy rules are followed unless weights are kept low. Such a policy rule contributes to stability since it prevents over or under-shooting of rates due to lagged effect of policy.

We compare our analytical results on the policy response coefficients consistent with stability in EMs with an estimated Indian monetary policy rule. Most EMs follow cautious policies and estimated Taylor Rules do have low weights as we see for India in the next section.

VI. Estimated Monetary Policy Rule

A monetary policy rule was unstable under a rational expectations solution in the calibrated SOEME. But in the backward-looking solution, stability imposed low caps on both policy rule coefficients. Our results suggest that with different types of lags and rigidities the feedback coefficients required for determinacy can be very different from the Taylor Principle. What do estimated rules show?

There is a large empirical literature estimating the Taylor rule. The original equation was:

$$i_t = \pi_t + r_t^* + \phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_x\left(y_t - \bar{y}_t\right) \qquad (43)$$

Where, π_t^* is the desired rate of inflation, r_t^* is the assumed equilibrium real interest rate, y_t, is the logarithm of real GDP, and $\bar{y}_t$ is the logarithm of potential output, as determined by a linear trend. Taylor proposed setting ϕ_π =

ϕ_x = 0.5. As long as ϕ_π > 0, an increase in inflation of one percentage point would lead the CB to raise the nominal interest rate by 1 + ϕ_π, thus raising the real interest rate. The simple NKE models can imply a very low ϕ_x, since in forward-looking models with demand shocks the feedback to inflation is sufficient to stabilize output[12].

As the empirical Taylor rule literature developed, the estimated equation was simplified. Either the short policy rate was regressed on the deviation of output from potential and of inflation from target, or a constant term was assumed to include a constant inflation target and real interest rate. So the short policy rate was regressed on inflation, on the deviation of output from potential, and a constant capturing the inflation target. A lagged interest rate was included to capture policy smoothing.

We estimated the latter Taylor rule specification for India to compare its coefficients with our theoretical results, and assess Indian monetary policy. We use data at quarterly frequency from 2000Q2 to 2011Q2. Variables include call or money market rate, GDP, core and headline wholesale price index. All the growth rate and inflation terms are in percentages, following the practice in the literature (Maslowska, 2009). Year-on-year headline inflation is measured as annual percentage change in Wholesale Price Index (WPI). Core inflation is defined as nonfood manufacturing goods inflation, whose share was around 52.2 percent in WPI. All the variables are tested for seasonality. Since analysis of linear plots show that quarterly GDP and WPI series have

[12] The NKE literature calls it the 'divine coincidence' when the CB does not need to take fluctuations in the output gap into account when setting interest rates. While his work supported the Taylor Principle, Woodford’s (2001) differences with the empirical Taylor rule were: First, the welfare theoretic loss function implies the inflation target should be zero in the pure frictionless model. Second, the output gap should be calculated using the natural output, not the past deterministic trend. All the shocks, such as technology, and world income, that affect the natural interest rate in equation (2) affect the natural output. In the SOEME these shocks include consumption of the P-type. See Goyal (2009) for more details on natural output in a SOEME.

multiplicative seasonality, we de-seasonalize the series using the X-12 ARIMA procedure. We estimate trend or target output using the HP filter, and calculate output gap as the percent deviation of real GDP from a target, as originally proposed by Taylor:

y = ((Y-Y*)/Y*)*100

where Y is real GDP (proxied by the industrial production index), and Y*is trend real GDP given by HP filter.

Augmented Dickey-Fuller unit root tests show the variables are stationary. The Durbin Watson test indicates serial correlation and the Breusch-Pagan test shows heteroskedasticity in the error terms. To correct for both autocorrelation and heteroskedasticity, we estimate our equation using ordinary least squares with Newey-West variance-covariance matrix.

The two estimated equations for headline inflation and core inflation (t-values in brackets) are as follows:

(1) Headline inflation

$$\begin{array}{lllll} r_t = & 1.85 & + \ 0.58 r_{t-1} & + \ 0.156 \pi_t & + \ 0.32 y_t \\ & (2.71) & (5.24) & (2.83) & (3.12) \end{array}$$

(2) Core inflation

$$\begin{array}{lllll} r_t = & 2.12 & + \ 0.59 r_{t-1} & + \ 0.126 \pi_t & + \ 0.29 y_t \\ & (2.96) & (5.21) & (2.06) & (2.93) \end{array}$$

These imply the long-run rise in r_t due to a persistent rise in inflation is 0.16 and 0.13 respectively. The long-run response is given by $\phi_\pi + \frac{(1-\beta)}{\kappa_D}\phi_x$

The coefficients are of a similar order of magnitude to the reaction functions estimated in the discretionary optimization and to the cap for ϕ_π in the policy rule with γ_f of 0.5 to 0.6 of the backward-looking case[13]. The results suggest the implicit policy rule Indian policy makers followed in this period was near optimal. The response to both inflation and the output gap was not high, but the weight on the output gap exceeded that on inflation.

VII. Conclusion

The NKE literature shows a response to inflation of above unity can impose stability in optimizing models with rational expectations. Theoretical stability results turn out to be the same in an NKE DSGE model adapted to an open economy EM model with two types of agents to capture heterogeneity in labour markets and consumers. Most estimated EM Taylor rules, including ours in this paper, however, give a coefficient for inflation of much below unity.

Consistent with this, in the calibrated model that has lagged policy rates in the aggregate demand equation, the derived stability condition does not imply the Taylor Principle. A policy rule is unstable in a rational expectations equilibrium. In a backward-looking solution, stability requires low weights on both objectives. As the weight on one rises that on the other should fall. The weight on the output gap exceeds that on inflation deviations. Discretionary forward-looking optimization is also stable. The reaction functions estimated in optimizing simulations and caps from the data driven policy rule are low and consistent with estimated coefficients of Taylor-type rules for India. Discretionary optimization outperforms a policy rule under rational expectations. A policy rule can be followed to the extent backward-looking behavior dominates, but with weights on the arguments of less than unity, it

[13] With the benchmark parameters, ϕ_x = 0.3 and γ_f = 0.6 the cap is 0.11 for ϕ_π.

would implement standard macroeconomic stabilization while preventing overshooting of rates.

Analytical solutions to monetary policy shocks using the calibrated equations, serve as consistency checks, and give results similar to the discretionary optimization. Therefore a key result, supported by analysis, calibration and estimations is: lags and rigidities in the SOEME make it stable for low weights on inflation, in Central Bank loss functions, optimizing reaction functions, as well as a smoothed policy rule.

Outcomes are stable even with a weight of zero on inflation in the loss function when there is no weight on interest rate smoothing, and weights on inflation in estimated reaction functions can be very low. The lags in the system, and other structural aspects, may be contributing to stability even with a low policy reaction to inflation. Such low coefficients may be necessary to prevent instrument instability in the presence of lagged policy transmission when backward-looking behavior dominates. A policy rule delivers a unique saddle stable equilibrium in an adaptive expectation equilibrium. It follows if an EM follows a policy rule it should ensure coefficients are low, until monetary transmission matures and the share of forward-looking behavior rises. During the transition, discretionary optimization may give more flexibilities.

The results suggest, more generally, that the effect of specific rigidities on stability should be more carefully explored, and knowledge of these rigidities can give useful inputs for the design of policy.

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