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Abstract

This paper examines statistical reliability of univariate filters for estimation of trend in leading indicators of cyclical changes. For this purpose, three measures are used: mean square error for quantitative accuracy, minimum revisions with additional data for statistical accuracy and directional accuracy to capture property of signaling cyclical movements. Our focus is on the widely used Hodrick-Prescott and Henderson filters and their generalizations to splines and RKHS(Reproducing Kernel Hilbert Spaces) embedding respectively. Comparison of trend fitted by the filters is illustrated with Indian and US Industrial production data and a simulated data series. We find that although Henderson smoothers based on RKHS preform better than classical filter, they are not better than spline based methods on the selected criterion for Indian macroeconomic time series. Overall findings suggest that in cases when penalized splines converge in quasi real time, they are better than HP filter on the three criterion.

Keywords: Hodrick-Presscott filter, Penalized splines, Henderson smoothers in RKHS, end-of-sample reliability, leading indicators

JEL Code: C32, E37

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1 Introduction

Trend estimates are used to calculate deviations of an economic indicator from its potential level. This deviation from trend is referred to as the "gap" and is the cyclical component which is useful in forecasting as a leading indicator of cyclical changes in the economy. Aim of this study is to compare and find a reliable method of estimating trend at the end of the sample which has the property of accurately detecting direction of change and is consistent in terms of minimum revision error as the estimate is recomputed at given time point when new data becomes available. Current economic analysis requires computation of estimates in the real time, which implies using only ex-ante data.

Commonly used filters for trend estimation like Hodrick-Prescott filter (HP hereafter) are biased at the end of the sample as they are basically symmetric filters and work as a centered moving average in the middle of the sample but are truncated at start and end of the sample. Suboptimality of HP filter in this regard has been documented in studies by Kaiser and Maravall (1999), Mise, Kim, and Newbold (2005), St-Amant and Norden (1997). Studies by Orphanides and Norden (2002) and Watson (2007) have demonstrated the unreliability of trend estimates in real time across alternative detrending methods and potentially high costs of policy decisions based on them.

Therefore before moving onto the discussion of alternative methods of trend estimation, we discuss the inherent limitations of filters due to finite length of data and explain why it occurs.

Improving reliability of end of sample estimates can take two approaches : Extrapolate the series and apply symmetric filters or use one-sided/asymmetric filters. Given the problem of truncated weighing pattern in two sided filters, filters which use asymmetric weights at beginning and end of sample are viable alternatives to explore. In this regard, Henderson filters have been widely used in economic applications. Another area in which we can improve is in the application of HP filter, which is commonly used symmetric filter, by making the selection of smoothing parameter more objective.

An important aspect to be emphasized in the comparison of symmetric and asymmetric filter is in terms of quantitative accuracy and degree of smoothing necessary for detection of turning points. A filter designed to detect turning points must ideally be a zero phase shift filter. Asymmetric filters can minimize the mean square of filter but cause a phase shift. Although symmetric filters have inherent limitation of higher mean square error at the boundary points relative to interior data, they have the property of being zero shift filters which allows them to be theoretically better for timely detection of turning points. Thus the two goals of filtering in terms of accuracy and lag in turning point detection involve a tradeoff and filters have to separately evaluated on these two criterion.

With this context, we compare commonly used HP and Henderson filters and their extensions for the purpose of end of sample reliability on the various metrics. HP filter can be generalized to penalized splines (Paige and Trindade, 2010) which allows for objective choice of smoothing parameter based on data characteristics. The historically used Henderson filters have been embedded in Reproducing Kernel Hilbert Space (RKHS) in a recent set of papers by Dagum and Bianconcini (2008) and Dagum and Giannerini (2006) who have found them to have better noise suppression and signal properties than the classical Henderson filters implemented in X-12 Census Bureau software. We thus apply this set of filters to the important macroeconomic series and illustrate the results with time series of Industrial Production in India and US, US bank credit and a simulated dataset to find how each filter works in terms of metrics for statistical reliability of leading indicators of cyclical change.

The paper is organized as follows: Section 2 discusses unreliability of HP filter at the end of sample and inherent limitation of symmetric filter in this regard. We then look at alternative methods in Section 3 which briefly summarizes extensions of HP filters to splines and discusses Henderson smoothers in RKHS. Section 4 describes the measures used for evaluating accuracy and consistency of the estimates. Section 5 presents tables and graphs for selected filters and illustrates performance of these filters with Indian and US macroeconomic time series. Conclusions are gathered in Section 6.

2 Limitations of Symmetric Filters for Current Economic Analysis

HP filter is criticized in the literature for unreliability at the end of sample and spurious cycles, for example (Canova, 1998; Pedersen, 2001). Before we explore alternative methods for trend estimation, it is important to understand the source of error and inherent limitations of filtering with a finite dataset. In particular, we discuss the end of sample properties of HP filter and highlight reasons for high uncertainty of trend estimates for current economic analysis.

2.1 Hodrick Presscott Filter

Hodrick and Prescott (1997) based their filter on the assumption that the growth component of aggregate economic series varies smoothly over time. It can be thought of an an approximation of high pass filter which extracts trend of about 8 to 9 yrs.

This filter decomposes time series into cyclic and growth components i.e. trend + cycle or $y_t = \tau_t + c_t$. To recover these two components from the data, the following optimization problem is solved

$$\min_{\tau^t}[\sum_{t=1}^{T}(y_t-\tau_t)^2+\boldsymbol{\lambda}\sum_{t=2}^{T-1}[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})]^2] \tag{1}$$

λ imposes penalty on changes in trend's slope i.e. parameter λ penalizes acceleration in the trend relative to the business cycle component and determines the degree of smoothing.

Gain function [1] which characterizes the HP filter is given by

$$G(w) = \frac{4\lambda(1-cosw)^2}{1+4\lambda(1-cosw)^2}$$

As deduced by King and Rebelo (1993), this filter is equivalent to a two sided infinite moving average symmetric filter with time varying coefficients. It does not induce phase shift and removes unit root components up to fourth order.

For finite sample approximation of HP filter, the weights are derived using an alternative way to avoid loss of observations at the end of the sample. As discussed in detail by Mills (2003), the

[1] Technical terms for filters have been explained in the appendix to this paper.

optimization problem in (1) is written in terms of system of equations to be solved

$$c_t = y_t - \tau_t = \lambda(\tau_{t-2} - 4\tau_{t-1} + 6\tau_t - 4\tau_{t+1} + \tau_{t+2})$$

with modifications for end of the sample as this expression cannot be used at t = 1, 2, T-1 and T. Using matrix notation, this set of equations can be written as $c = \lambda\boldsymbol{\Gamma}\boldsymbol{\tau}$ where the matrix $\boldsymbol{\Gamma}$ is given by

$$\begin{pmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 & & \cdots & & & 0 \\ -2 & 5 & -4 & 1 & 0 & 0 & 0 & & \cdots & & & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & & \cdots & & & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & & \cdots & & & 0 \\ \vdots & & & & & & & & & & & \vdots \\ 0 & & \cdots & & & & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & & \cdots & & & & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & & \cdots & & & & 0 & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & & \cdots & & & & 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{pmatrix}$$

The non-symmetric nature of the matrix of weights induces a phase shift near the end points of data and distorts the gain function. Note that as we move closer to the center of the data, these phase shifts disappear and gains become more similar to that of an ideal filter. Therefore, approximation of HP filter for finite sample is optimal mid-sample but not at the end of the sample.

St-Amant and Norden (1997) give some more insights to understand HP filter estimates for most recent observations. Since HP filter is like a two sided moving average in the middle to the sample, each observation gets about 6% weightage but at the end of sample, its a one sided average and last observation gets 20% of the weight. Clearly the distortion in the weighting pattern causes HP filter to be more variable at the end of sample. Basically, the optimization problem is a tradeoff between deviations from trend and smoothness which means that in face of a transitory shock, the filter is reluctant to change the trend as the penalty term would require raising the trend before the shock and lowering it afterwards. At the end of sample, latter penalty is absent. Note that second term in the optimization problem sums from $t = 2$ rather than $t = 1$ upto $t = T - 1$ which implies that trend estimated at the end of sample will be more affected by a transitory shock.

Kaiser and Maravall (1999) give substantial empirical evidence that HP filter is inefficient at the end of sample using extensive Monte Carlo simulations. To overcome this defect, they extend series by forward and backward forecasts using ARIMA models. Mise, Kim, and Newbold (2005) further investigate the HP filter at the endpoints and confirms these findings through simulations and find mean square error at time T to be much higher than at $T - 20$. The study also shows that degree of unreliability and reduction in revision errors depends on value of the smoothing parameter.

We can therefore conclude that HP filter optimally applies to infinitely long time series or to center of a series of typical length but not for the recent observations which are of interest for decision purposes. However this problem is not restricted to HP filter but is an inherent limitation of symmetric filters in general. To further understand why the symmetric filters are inefficient at the end of sample, we summarize key aspects of applying two sided filters.

Symmetry of filter implies that the most recent signal cannot be estimated without extrapolating the series with forecasts and backcasts. There is a substantial revision in estimates as new data becomes available. It is only when more data becomes available that the trend cycle decomposition at a given time point is stabilized.

Chatfield (2003) notes that a symmetric filter will always have an *end effects* problem as the filter with say k weights can only be calculated for t = k+1 to t = N-k. This end effects problem has been noted and discussed in Kendall, Stuart, and Ord (1983, Sec.46.11) and is particularly important in forecasting studies which requires smoothed values for the most recent time point. Solution is to either project the smoothed values or use an asymmetric filter that uses only present and past values of the series.

1. Problem of truncated weighing pattern

Why is Mean Square Error(MSE hereafter) of symmetric filter higher for the boundary of the sample ? Peña, Tiao, and Tsay (2001) argues that the weighting pattern becomes deformed towards the boundary as there are not enough observations to apply to the filter. As the size of local neighborhood shrinks towards the boundary, the bias part of the MSE will be lower in the boundary area as compared to the interior area. On the other hand, variance part will increase since fewer observations are included and more recent observations get more than optimal weight.

2. Problem with using forecast augmented series

One way to use symmetric filter efficiently is to augment the series with forecasts and backcasts and then apply the filter. This method is criticized as the trend estimates will depend on how good the forecasts are and another problem is that we do not know how the filter processes the error in the forecast or impact of these estimates when used for further analysis.

Mohr (2005) states that stochastic model of HP filter can be used to forecast the series where trend is modeled as a second order random walk and says that the there is no end point problem if new data that arrives comply with implicit forecast of HP filter. Therefore the end point problem exists because the stochastic model underlying the filter is a weak representation of the data generating process.

Another way is to extend the series by using reflection of the series at either end. Pollock (1999) claims that if the series is strongly trended, then it will increase or decrease the estimates relative to that obtained through linear extrapolation and therefore not a viable approach.

3. Problem of Spurious Cycles and Slutzky effect

The finite approximation of filter can also cause *leakage* i.e. spurious cycles that may lead to false turning points and inaccurate estimates of the length of cycles in the data. The use of HP filter is highly criticized in studies like Pedersen (2001), Harvey and Jaeger (1993), Cogley and Nason (1995) and Canova (1994), Canova (1998) on account of its distortionary effects.

One of the reasons for spurious cycles is the Slutzky-Yule effect which refers to the spurious periodicity seen in output when averaging and differencing procedures applied to a random series induces sinusoidal variation in the data i.e. cycles are induced in the data due to method used for filtering.

4. Effect of Finite data length on Approximation of Filters

Koopmans (1974, pg.177) gives mathematical reasoning for effect of finite data length on the filtered estimates and we present a summary of the key ideas.

When applying this ideal filter to a finite segment of a time series X(t), $x_1, x_2, ...x_N$, we can extend the series by putting $x_t = 0$ for $t \leq 0$ and $t \geq N+1$. The resulting output is not the same as the characteristics of filter are based on assumption that the entire input is being used. Another way to express a finite data sample is by using an indicator function $I_{[1,N]}$ as a data window and comparing the results of applying the filter to $X(t)$ and $I_{[1,N]}(t)X(t)$ say $U(t)$ and $Y(t)$ respectively i.e. $U(t)$ is the result of applying ideal filter and $Y(t)$ is result of applying it to finite data length.

The *frequency response function* of a low pass filter with cutoff frequency f_0 is 1 for $|f| \leq f_0$ and 0 otherwise. This corresponds to weights

$$a_k(f) = \begin{cases} \frac{\sin f_0 k}{\pi k} & k = \pm 1, \pm 2, \dots \\ \frac{f_o}{\pi} & k = 0 \end{cases}$$

Measure of deviation from the ideal can be expressed as $E[U(t) - Y(t)]^2$ and using spectral representation in the frequency domain, bound on the error in terms of the weight of the filter can then be obtained as

$$E[U(t) - Y(t)]^2 \leq 2\pi M \sum {a_k}^2$$

It is to be noted that magnitude of the error depends on data length N and on how rapidly the filter weights tend to 0 with increase in the lag index k. Typically larger weights are concentrated around k = 0.

The following quote from this study summarizes the impact of finite data length on filtering time series data.

> The most compelling reason for not using ideal low pass filter is that the rate of decrease of filter weights is very slow. Thus the actual outputs of these filters will differ substantially from the ideal. Put in another way, any attempt to explain the outputs of these filters to a finite length of data by the characteristics of ideal filter will be in error. The magnitude of error which can be estimated from equation will be uncomfortably large for all values of t. Thus it is desirable to work with low pass filters which are less than ideal but for which actual filter characteristics are more closely described over given range of t values by the transfer function of the corresponding linear filter.

Thus, we acknowledge that there will be errors in filtered series at the end of sample. This is helpful while taking economic decisions and take into account the uncertainty associated with the computation using any method. We note the limitations of symmetric filters in terms of higher MSE for real time trend extraction and explore an asymmetric filter which may overcome the limitation of distorted weighting pattern at the end of sample.

3 Alternative Methods of Trend Estimation

3.1 Penalized and Smoothing Splines

Spline functions are widely used for smoothing data as exposited in classic work by Wahba (1990). We motivate the use of spline functions for smoothing and filtering purposes by seeing it as a generalization of HP filter. Nonparametric methods of splines or piecewise continuous functions for smoothing univariate time series data open up variety of data driven methods for selecting the smoothing parameter which implies that instead of relying on trial and error or a fixed number, we have an objective way of choosing correct amount of smoothing in the data.

HP filter can be seen as a special case of penalized spline smoothing as demonstrated in the paper by Paige and Trindade (2010). They establish link between HP filter and Penalized splines by proving that HP filter is a linear penalized spline model with knots placed at all data points except first and last point and uncorrelated residuals. Infact, it is stated as a theorem that HP filter is exactly equivalent to (is a special case of) the solution obtained from a penalized spline of degree 1, equispaced knots at points $2, \ldots, n-1$ and independent errors.

Other important papers which discuss filtering with penalized splines are Krivobokova and Kauermann (2007), Eilers and Marx (1996) and Kauermann, Krivobokova, and Semmler (2011) where they use penalized splines for filtering time series based on mixed model representation and Restricted Maximum Likelihood(REML hereafter) selection of λ . The empirical findings with US macroeconomic data suggest that residuals or the cyclical component derived using penalized splines have better properties in terms of correlation and cross correlation compared to HP and band pass filters.

Thus, instead of using HP filter routinely with λ as 1600 or any arbitrary value, we can use data driven methods for filtering time series which are easily applicable and also allow flexibility in adjusting the metrics and methods for smoothness based on the nature of data and research question. Penalized splines are therefore considered as viable alternative to HP filter which have the advantage of using a data driven choice of smoothing parameter and takes serial correlation in the data into account.

3.1.1 Generalizing HP filter's optimization problem

Consider the HP filter with solves the following optimization problem to extract trend and cycle, $y_t = \tau_t + c_t$ and examine the equation in terms of interpretation of each component.

$$\min_{\tau^t}[\sum_{t=1}^{T}(y_t - \tau_t)^2 + \boldsymbol{\lambda}\sum_{t=2}^{T-1}[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \tag{2}$$

The first term in this equation is a measuring fidelity of estimates to the data and the second term is using second difference as a metric for amount of smoothing which is controlled by parameter λ . Therefore, the objective function for smoothing can be expressed as follows and various metrics can be chosen for each feature.

$$\text{(measure of fit)} + \lambda \text{ (measure of roughness)}$$

Smoothing Splines

For the general problem of smoothing univariate data using nonparametric methods, suppose the responses $y_1, y_2, \ldots y_n$ be observed at design points $t_1 < \cdots < t_n$ following the regression model

$$y_i = f(t_i) + \epsilon_i, \qquad i = 1, 2, \ldots\ldots, n$$

x where $f(t_i)$ is an unknown function and $\epsilon_1, \ldots, \epsilon_n$ are zero mean uncorrelated random errors.

Minimizing the residual sum of squares(RSS),

$$RSS(f) = \sum_{i=1}^{n}(y_i - f(t_i))^2$$

is a starting point for linear regression with $f(t)$ being a straight line but is inefficient in most situations. For a middle path between curves of constant slopes and interpolation, one way is to penalize functions whose slopes vary too rapidly. The rate of change of slope of a function g is given by g'' or the second derivative. Thus, an overall measure of change in slope of potential fitted function is given by $J(g) = \int_{t_1}^{t_n} g''(t)^2\,dt$.

As elucidated in Silverman (1985), the least squares problem of the first part can be made zero by using an interpolating function which is not statistically admissible as it will be too fluctuating. To derive a estimates of a curve τ or the trend , we can either restrict the curve to some parametric class (like a degree m polynomial) or we can add a "*roughness penalty*". A measure of rapid local variations in a curve can be given by a roughness penalty such as integrated squared second derivative.Therefore the modified estimation criterion which incorporates a penalty for irregularity is

$$RSS(g) + \lambda J(g), \qquad \lambda \geq 0$$

which is minimized over all functions with twice continuous derivatives. Adding a penalty to RSS criterion means given equal RSS, prefer one with less average curvature. The smoothing parameter λ can be seen as a handle on flexibility or curvature as $\lambda = \infty$ implies choosing a straight line and $\lambda = 0$ corresponds to interpolation. λ represents exchange between residual error and local variation. Objective function on adding a penalty on degree of curvature can be written as

$$L(m, \lambda) \equiv \frac{1}{n} \sum_{i=1}^{n} (y_i - f(t_i))^2 + \lambda \int_{t_1}^{t_n} g''(t)^2\,dt \tag{3}$$

Solution to this optimization problem is a function or curve which gives the best compromise between smoothness and goodness of fit and called a **smoothing spline**. Remarkably, it has been shown mathematically that regardless of initial data, all solutions to (3) take the form of *piecewise cubic polynomials* which are continuous. The boundaries between pieces are located at the original data points and referred to as *knots* of the spline.

In a more general extension of objective function (3) where penalty term is an integral of m^{th} derivative and $n \geq m$, the solution is a natural polynomial spline of order 2m and degree 2m-1 with knots at the design points. In fact, the smoothing spline becomes a polynomial of order m outside of $[t_1, t_n]$ and thus satisfies *natural boundary conditions*

$$g^{(m+j)}(t_1) = g^{(m+j)}(t_n) = 0, \quad j = 0, \ldots, m-1$$

from which the term natural spline is derived. This implies that for the case m=2 illustrated above, f_λ is a piecewise cubic polynomial with two continuous derivatives that is linear outside of $[t_1, t_n]$. Natural splines therefore have the beneficial feature that the usually high variance near the boundary points in case of polynomial functions is minimized. In practice however, they may overfit and are thus used with caution.

3.1.2 Penalized Splines Regression Model

To use penalized splines for extracting trend in a univariate dataset, we can consider the function $f(t_i)$ as an approximation of the mean or true conditional expectation which can be defined as the trend or the long term variation. Residuals ϵ_{t_i} can be interpreted as the cyclical component and short term variations. We will present a general formulation of penalized spline regression and illustrate the basic ideas. A complete discussion can be found in Hastie and Tibshirani (1990) and Paige and Trindade (2010).

For a time series y(t) , $y_1, y_2, \ldots, y_n$, consider the model

$$y_i = \mu(t_i) + \epsilon_i, \qquad i = 1, 2, ..., n$$

To extract trend from this model using splines, we represent the function $\mu(t_i)$ as a linear combination of a rich flexible set of spline basis functions $B(t)$with knots τ_i chosen over the observed time points $t_1, \ldots, t_n$ and a vector of coefficients θ. i.e.

$$\mu(t) = B(t)\theta$$

. A simple choice of basis functions for *penalized splines* are truncated polynomials and splines or piecewise continuous functions which can be written as

$$B(t) = 1, t, t^2, \ldots, t^q, (t-\tau_1)_+^q, \ldots, (t-\tau_k)_+^q$$

where q is the maximum degree of polynomials,$(x)_+ = x$ for $x > 0$ and 0 otherwise. The knots $\tau_1, \ldots, \tau_k$ are chosen equidistantly at say every fifth observation or at quantiles and $k \ll n$ and covering the range of time points t. Number of knots chosen varies with the spline basis and sometimes all time points may be taken as knots.

Defining the coefficient vector $\theta = [\beta, u]^T = [\beta_0, \dots \beta_p, u_1, \dots u_k]^T$ leads to representation of the mean or trend function as

$$\mu(x_i) = \beta_0 + \beta_1 t_i + \beta_q t_i^q + \sum_{k=1}^{K} u_k (t_i - \tau_k)_+^q$$

which can be written in matrix form as

$$y \equiv X\beta + Zu + \epsilon \equiv B\theta + \epsilon \tag{4}$$

where $B = [X, Z]$ and the matrices X and Z correspond to $X(t) = 1, t, t^2, \dots, t^q$ and $Z(t) = (t - \tau_1)_+^q), \dots, (t - \tau_k)_+^q$

The model is fitted via Penalized Least Squares i.e.

$$min(\|(y - X\beta - Zu)^2 + \lambda \|(u)\|^2) \quad \text{where} \quad \|(v)\| = \sqrt{(}v^T v)$$

Representation of data in (4) is in the form of semiparametric or generalized additive model which contains both smooth functionals (nonparametric) and ordinary linear (parametric) components. e.g. $y_i = \beta_0 + \beta_1 x_1 i + f(x_2 i) + \epsilon_i$. A penalized spline can be represented as a linear mixed model by treating both u and ϵ as random effects and thus maximum likelihood as a method of estimation becomes possible. Representation of penalized spline as a linear mixed model also allows meaningful expressions of β and u as best linear unbiased predictors. Notably, the smoothing parameter is expressible as ratio of variance components in this framework.

Choice of Basis Functions and Knots Popular choices of basis functions include cubic splines, truncated polynomials and B-spline functions. P-splines combine B-splines with difference penalties which have the advantage that they do not have boundary effects (Eilers and Marx, 1996).

Regarding number and selection of knots, it has been shown in Ruppert, Wand, and Carroll (2003) that choice of knots does not play an influential role in the resulting penalized fit as long as number of knots, K is large e.g K = min(n/4, 40) (Krivobokova and Kauermann, 2007). This is important feature for making this procedure easier to apply and for comparative analysis when using variety of series.

Smoothing Parameter Selection with Correlated Errors

Efficacy of any nonparametric method of smoothing depends critically on the choice of smoothing parameter, referred to as λ for notational convenience. We would not expect the same value of λ to work for every dataset. The shape and smoothness of estimated function depends to a large extent on the specific value of λ which can vary with selection criterion or penalty function specified. It has been noted in Wahba (1990) that the optimal value of smoothing parameter depends upon the sample size, the noise variance and the smoothness of the true curve. Choice of smoothing parameter must therefore be dictated by the nature of the data.

Cross Validation for λ in univariate spline and kernel smoothing makes the assumption that the errors are i.i.d. However when dealing with time series data, the curve estimates can be adversely affected if the errors are autocorrelated but treated as if they were independent and can lead to overfitting.

Opsomer, Wang, and Yang (2001) give simulation evidence that nonparametric regression techniques are sensitive to presence of correlation in the errors and highlight the importance of good choice of λ for smoothing. Altman (1990) shows that the standard techniques of selecting λ in kernel smoothing work properly when error are correlated but only when correlations are sufficiently short term. Approaches for spline and kernel regression in context of dependent data are discussed in detail in Kohn, Schimek, and Smith (2000).

Most recently, Krivobokova and Kauermann (2007) show that a maximum likelihood based choice of smoothing parameter is robust to moderate misspecification of correlation structure in the data. Using both theory and simulations, two λ selectors, MSE based Akaike Information Criterion(AIC) and REML estimate are compared for penalized spline smoothing modeled as linear mixed model. Smoothing parameter selection based on REML is found to be better alternative for correlated errors and works even when the correlation is not known or specified. This finding has also been supported by Paige and Trindade (2010).

Another advantage of REML method is that when applied in mixed model formulation of Penalized splines, it estimates λ as a ratio of variance components which is the actual interpretation of λ in the HP filter and λ as the inverse of *signal to noise ratio*. We therefore use this technique in the empirical work.

3.2 Henderson Filters

Henderson Filters are based on local cubic polynomials modified to correct for end point distortions. They have been widely used in official statistics and part of the X-11, X-12 software of US Census Bureau. Framework for obtaining filter weights has been developed by Gray and Thomson (1997) which can adapt the filter weights according to degree of smoothness, fidelity, length of moving average and end of sample properties required. Henderson filters work well as trend-cycle turning point detectors and relevant for short term trend.

Note that Henderson filter for symmetric and asymmetric are calculated separately and it applies asymmetric filters at the beginning and end of sample. The asymmetric Henderson smoothers currently in use were developed on the basis of minimizing mean squared revisions between final (applying symmetric filter) and preliminary estimates(applying asymmetric filter). They have been shown to have unwanted ripples or false turning points in a study by Dagum (1996) and improved by augmenting series with ARIMA forecasts and removing outliers and extreme values to reduce the noise entering in the filtering process.

3.2.1 Henderson Smoothers in RKHS

A recent of studies by Dagum and Giannerini (2006), Dagum and Bianconcini (2013) introduce a new set of trend-cycle filters as an embedding of Henderson filters in RKHS

> RKHS is a Hilbert space characterized by a kernel that reproduces via an inner product, every function of the space or, equivalently, a Hilbert space of real valued functions with the property that every point evaluation functional is bounded and linear.

The authors introduce RKHS representation of Henderson smoothers with particular emphasis on the asymmetric ones applied to most recent observations. The gain functions of kernel based filters are shown to have much better properties of signal passing and noise suppression. Papers by Bianconcini and Quenneville (2010) and Dagum and Bianconcini (2008) and Dagum and Giannerini (2006) empirically show that Henderson smoothers in kernel hierarchy give much better results for current economic analysis.

Therefore, we choose these set of asymmetric filters to assess how they compare with spline based filters on the metrics chosen for selected macroeconomic series.

4 Criterion for Assessing Reliability of Cycle Estimates

Selection of gap estimates to be used as leading indicator of cyclical changes with focus on accuracy of estimates using data available upto the time of computation is based on the following criteria (1) Minimize error of estimates (2) Minimum revision error as estimates are recomputed with additional data (3) Timely and correct detection of direction of change or turning points.

4.1 Measures of Quantitative Accuracy

Mean square forecast error is the standard measure of predictive performance over the forecasting horizon. As trend estimates at time T are one-step ahead forecasts for time T+1 in the context of time series, we can compare them in terms of quantitative measures of ME (Mean Error), MAE (Mean Absolute Error) and RMSE(Root Mean Square Error).

4.2 Measures of Statistical Reliability

Study by Orphanides and Norden (2002) defines *final* and *quasi Real Estimates* for separating source of error in output gap estimates due to "data revisions" (changes in published data) and "statistical revisions" due to method of detrending. As also emphasized in study by Camba-Mendez and Rodriguez-Palenzuela (2003), most important requirement for end of sample reliability of gap estimates is to have temporal consistency i.e. revision error between sequentially estimated measures and finally estimated measures must be minimum.

In a sample of size T, final estimate $y^F_{t|T}$ at time t is computed using all the data available and quasi estimate $y^Q_{t|t}$ at time t is obtained using only data upto time t i.e without using future observations. Therefore, difference between final and quasi estimates is only due to more data being available for the final estimates.

Other terms which can be used for quasi real time estimates are *recursive* or concurrent or backward looking estimates and work similar to Asymmetric filters in the sense that both do not use future observations for calculations at given time. For notational clarity, we will be referring to them in this thesis as *quasi estimates*.

Following methodology used in key studies like Kaiser and Maravall (1999), Dagum and Luati (2000) and Mohr (2005), we create sequences of data for computing quasi estimates at each time point. Estimation of cycle using symmetric filters at the middle of the sample gives final estimates while at the end of sample, the filter is truncated and hence leads to revision error. To compute error in estimates as new data becomes available, the performance of the filter using full sample is compared with the quasi estimates in the middle of the sample where they are relatively accurate.

More precisely, given a sample of monthly time series from March 1994 to March 2013, we set 100 as the minimum sample size for applying a filter and allow atleast 24 observations for the final estimates to stabilize.[2] Then at each time s between 2003 to 2010, we get quasi estimate by applying filter to data from $1994 - s$ and final esitmate at time s by using all data from 1994-2013.

The following statistics can be used for evaluating revision error between final and quasi estimates

$$MAPR = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{\hat{y}_t^F - \hat{y}_t^Q}{\hat{y}_t^Q}\right|$$

where $\hat{y}_t^Q$ is last estimate of trend when series is truncated at time t (Jan2003) and then one observation is sequentially added to the calculations till Jan 2010. $\hat{y}_t^F$ is the corresponding final estimate for each point computed using all the data from 1994-2013. [3]

4.3 Measures of Directional Accuracy

Forecasts of direction of change give information on whether the indicator variable will increase or decrease which is valuable information for current economic decisions. Leading indicators are useful essentially for their ability to predict the direction of change or turning Points.

[2] Minimum sample size of 100 is set on the basis of studies cited above which have taken minimum of 84 observations. Sample of 24 observations are left at the end for final estimates to converge and is based on suggestion by Baxter and King (1999) to drop 12 observations at the end when using HP filter and by Mise, Kim, and Newbold (2005) which find 28 observations to be sufficient.

[3] Adjusted MAPR is computed as $= \frac{1}{N}\sum_{t=1}^{N}\left|\frac{\hat{y}_t^F - \hat{y}_t^Q}{\hat{y}_t^F + \hat{y}_t^Q}\right|$

Application of directional tests to evaluate smoothed estimates are relevant to account for oversmoothing which may delay the detection of turning point in the series or undersmoothing which may lead to false turning points.

Instead of using rules of thumb for choosing turning points, our choice of directional tests as a metric for evaluation is guided by practical requirement of a simple unified procedure to select gap estimates across a variety of time series and methods. Also, we do not require detection of turning points at this step of investigation but only to check how closely the direction of cyclical changes match with the predicted cycle and use it as a criterion for selecting method of gap estimation.

Directional Tests for macroeconomic series were first adapted by Schnader and Stekler (1990) from the Henriksson and Merton (1981) test for market timing. Pesaran and Timmermann (1992) and Pesaran and Timmermann (1994) then built a nonparametric test for evaluating qualitative forecasting performance. Some examples of studies using directional tests for forecast evaluation are Pons (2000), Sinclair, Stekler, and Kitzinger (2010), Ash, Smyth, and Heravi (1998) , Greer (2003) and Tsuchiya (2013). The three most commonly used tests are Chi Square test, Fisher Exact Test and Pesaran Timmermann test. More detailed explanation of the tests and applications can be found in Diebold and Rudebusch (1996) and Mariano (2002).

4.3.1 Pesaran Timmermann Test of Directional Accuracy

Evaluating directional change accuracy based on contingency table is based on dichotomizing changes into up and down, i.e predicting an increase or decrease in the variable. The following table illustrates the main idea.

Table 1: Contingency Table for Actual and Forecast Estimates

	Forecast		
Actual	Up	Down	Subtotal
Up	Alarm	Missed Alarm	Observed Up
Down	False Alarm	Correct Negative	Observed Down
Subtotal	Forecast Up	Forecast Down	Total

The contingency table shows the marginal probabilities as shown in the subtotals and the conditional probabilities for hits and false alarms. A good forecast will have high values along the diagonal i.e. when the forecast is correct.

Null hypothesis of directional tests is that sign of change in a forecast and that in the realization are independent. Rejecting the null implies that forecast is useful predictor of actual change in the variable. Note that the test ignores the size of errors and is based only on the signs of predicted and actual changes i.e small and large errors are treated equally.

Basic idea of constructing the 2x2 contingency table is to have attribute variables based on number of times that actual (x_t) and forecast(y_t) data are both of the same sign. The test is based on the proportion of times that direction of change in x_t is correctly predicted in the sample.

Table 2: Contingency Table for Directional Accuracy

		Forecast	
Signal	$\Delta F > 0$	$\Delta F \leq 0$	Subtotal
Actual			
$\Delta A > 0$	(CP)P_{11} [c]	(FN)P_{12}	N1 [a]
$\Delta A \leq 0$	(FP)P_{21} [d]	(TN)P_{22}	N2 [b]
Subtotal	n1	n2	N

[a] N1 = number of observations when actual change is > 0
[b] N2 = number of observations when actual change is ≤ 0.
[c] CP = number of correct forecasts given that actual change is > 0
[d] FP = number of incorrect forecasts given that actual change is ≤ 0
Similarly, CN and FN stand for correct and false negative.

Pesaran and Timmermann (1992) construct a nonparametric test for correct prediction of direction of change in the indicator series as follows.

Let x_t and y_t denote the indicator and predictor respectively. The following variables are then defined for calculating the test statistic.

$$Y_t = 1 \ \ if \ \ y_t > 0 \text{ and } 0 \text{ otherwise. } p_y = prob(y_t > 0)$$
$$X_t = 1 \ \ if \ \ x_t > 0 \text{ and } 0 \text{ otherwise. } p_x = prob(x_t > 0)$$
$$Z_t = 1 \ \ if \ \ x_t * y_t = 0 \text{ and } 0 \text{ otherwise. } p_z = prob(z_t > 0)$$

Then $\hat{p}_y = \bar{Y}, \hat{p}_x = \bar{X}, \hat{p} = \bar{Z}$ and $\hat{p}_* = \hat{p}_y\hat{p}_x + (1-\hat{p}_y)(1-\hat{p}_x)$ where $\hat{}$ and $\bar{}$ denote estimates and sample means respectively.

Test statistic used by Pesaran and Timmermann (1992) is devised to distinguish between observed sample estimate of probability of forecast with correct sign and estimate of what the estimate will be under the null hypothesis of independence between forecasts and outcomes. i.e. $\hat{p}$ = sample estimate of probability of correct signal forecast and $\hat{p}_*$ is an estimator of its expectations under the null that forecasts and outcomes are independent.

$$s_n^2 = \frac{(\hat{p} - \hat{p}_*)^2}{\text{var}(\hat{p}) - \text{var}(\hat{p}_*)} \rightarrow N(0,1)$$

Pesaran and Timmermann (1992) show that the χ^2 goodness-of-fit statistic based on contingency table and their test are not equivalent and are only asymptotically equivalent in the 2x2 case.

Sign Concordance is a simple measure to calculate proportion of times when there is same sign for cyclical change and forecast. In terms of above numbers, measure for sign concordance as denoted by signC i.e.

$$signC = (CP + CN)/N$$

which is mean(z_t). Pesaran Timmermann test and signC are also applied for final and quasi estimates as done in the study by Camba-Mendez and Rodriguez-Palenzuela (2003) and this is the method that we have also followed in the empirical computations.

5 Empirical Results

We perform comparative analysis of HP filter vs Penalized splines, Henderson Classical and Kernel filters and HP filter vs Henderson Filters to find which method works best for time series in terms of selected critetion.

We illustrate choice of filtering methods based on assessment criteria for Indian macroeconomic data Index of Industrial Production in India and US, and bank credit in US for the period 1994-2013. The series are seasonally adjusted using X-13 ARIMA and log transformed [4].

Final and quasi cyclical estimates using each method are plotted for comparison. Following abbreviations of filter names are used to keep the notations simple: HP for Hodrick-Presscott filter, HDF for Henderson filter, SPL for Splines and suffixes refer to the characteristics of the smoothing parameters: the value of smoothing parameter for HP filter and number of terms in Henderson filters.

[4] A minimum of 12 observations are used to calculate the filters.

5.1 Tables and Graphs

5.1.1 Indian Industrial Production

Table 3: IIP: Quantitative and Directional Accuracy Measures

	ME	RMSE	MAE	MAPE	MAPR	adjMAPR	dacTest	p.value	Sign
SPL-Smooth	0.00	0.01	0.01	281.00	358.69	-98.54	7.77	*; 0.0001	0.76
SPL-Penalized	0.00	0.01	0.01	202.35	227.92	11.22	4.98	*; 0.0001	0.67
SPL-cubic	0.00	0.01	0.01	1081.46	182.49	-90.47	5.62	*; 0.0001	0.69
HP-400000	0.01	0.04	0.03	257.21	236.79	14.48	-1.46	0.93	0.45
HP-14400	0.00	0.02	0.01	354.56	144.97	-29.55	6.02	*; 0.0001	0.70
HP-129600	0.01	0.03	0.03	1148.02	330.64	79.03	-0.43	0.67	0.49
HDF-Kernel9	0.02	0.03	0.02	1040.23	1374.67	-90.68	3.16	0.00078	0.57
HDF-Kernel23	0.05	0.06	0.05	3628.24	5353.38	-73.52	2.14	0.016	0.67
HDF-Kernel13	0.04	0.05	0.05	1294.64	1842.71	-201.80	0.57	0.28	0.19
HDF-Classical9	0.02	0.03	0.03	1132.49	1064.01	-150.13	2.53	0.0056	0.37
HDF-Classical23	0.07	0.08	0.07	1032.68	839.76	-141.67	1.42	0.078	0.18
HDF-Classical13	0.04	0.04	0.04	1209.65	1900.40	-75.84	0.58	0.28	0.21

[4]SPL refers to splines and HDF to Henderson filters.The suffixes refer to choice of parameters.

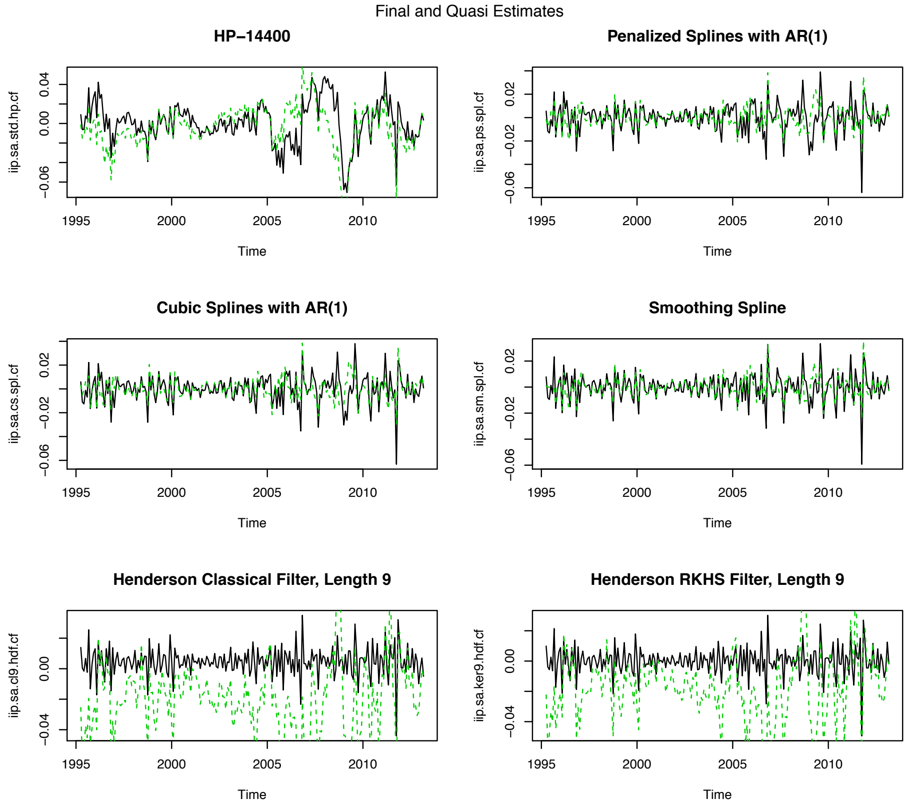


Figure 1: Index of Industrial Production: Final and Quasi Estimates

Table 4: USIIP: Quantitative and Directional Accuracy Measures

	ME	RMSE	MAE	MAPE	MAPR	adjMAPR	dacTest	p.value	Sign
SPL-Smooth	0.00	0.00	0.00	188.84	146.40	-103.77	8.07	*; 0.0001	0.77
SPL-Penalized	0.00	0.00	0.00	203.82	186.17	-14.78	6.99	*; 0.0001	0.74
SPL-cubic	0.00	0.00	0.00	442.78	217.85	-68.65	7.93	*; 0.0001	0.77
HP-400000	0.01	0.03	0.03	157.49	85.79	-96.82	5.08	*; 0.0001	0.67
HP-14400	0.00	0.03	0.02	438.10	538.15	156.12	1.06	0.14	0.54
HP-129600	0.00	0.03	0.03	436.63	189.54	10.90	3.54	0.0002	0.62
HDF-Kernel9	0.01	0.02	0.01	3123.82	3435.06	-26.65	2.46	0.0069	0.58
HDF-Kernel23	0.01	0.04	0.03	2832.83	2449.33	-67.27	1.29	0.099	0.78
HDF-Kernel13	0.03	0.03	0.03	423.89	406.50	-1356.49	-0.07	0.53	0.16
HDF-Classical9	0.01	0.02	0.02	1019.63	1669.26	28.57	0.07	0.47	0.24
HDF-Classical23	0.03	0.05	0.05	2786.69	6153.46	-147.75	0.96	0.17	0.20
HDF-Classical13	0.02	0.03	0.03	356.77	346.17	-2596.19	-0.16	0.56	0.23

5.1.2 US Industrial Production

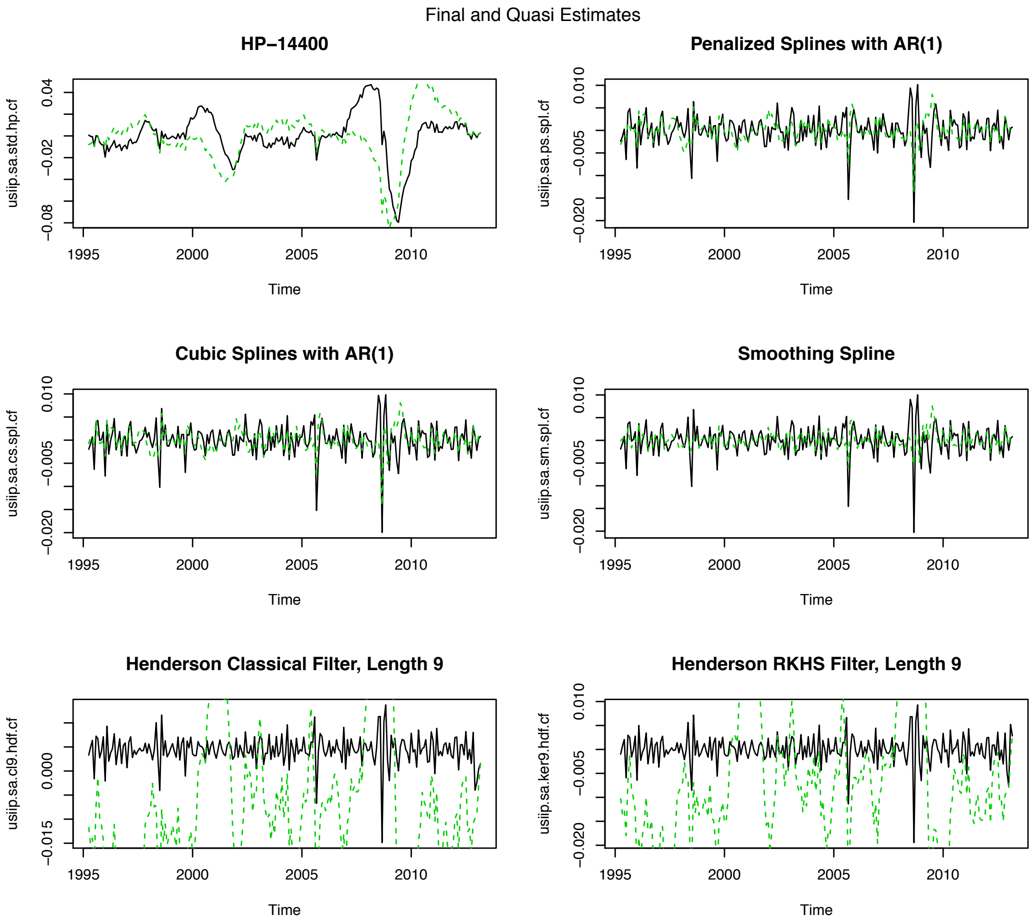
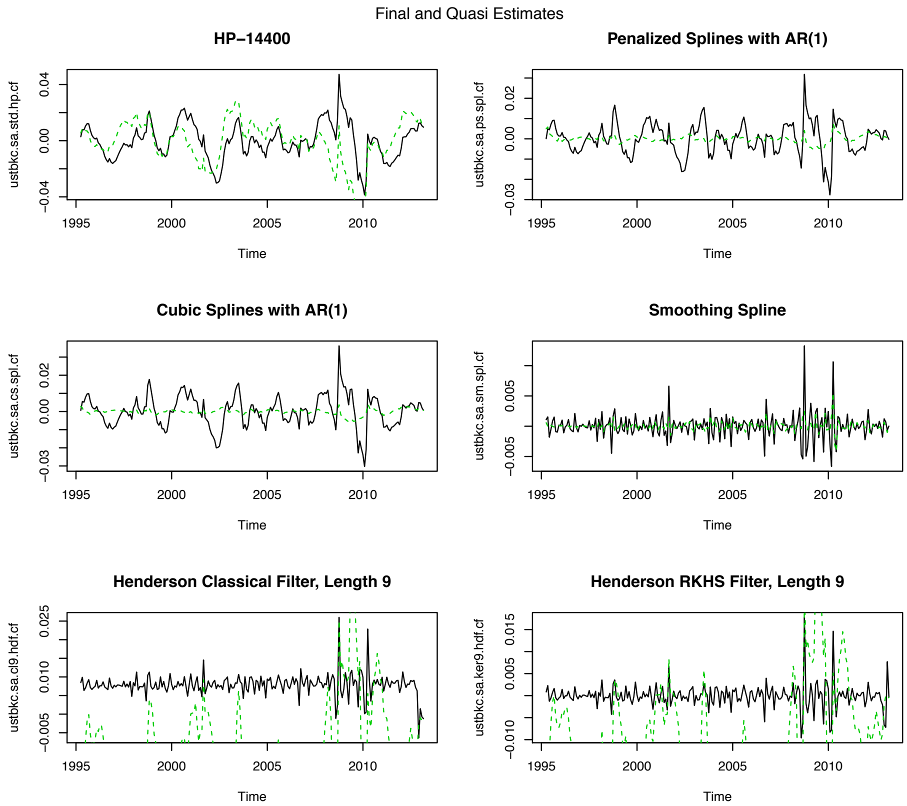


Figure 2: US Industrial Production: Final and Quasi Estimates

Table 5: USTBKC: Quantitative and Directional Accuracy Measures

	ME	RMSE	MAE	MAPE	MAPR	adjMAPR	dacTest	p.value	Sign
SPL-Smooth	0.00	0.00	0.00	189.32	218.18	-51.80	4.23	*; 0.0001	0.64
SPL-Penalized	0.00	0.01	0.01	156.23	123.45	-23.09	3.40	0.00034	0.62
SPL-cubic	0.00	0.01	0.01	135.08	112.57	-11.40	3.46	0.00027	0.62
HP-400000	0.01	0.03	0.02	529.34	378.77	-132.01	6.14	*; 0.0001	0.70
HP-14400	0.00	0.02	0.01	365.65	476.53	-318.56	3.52	0.00021	0.62
HP-129600	0.00	0.03	0.02	279.95	400.88	-314.79	5.45	*; 0.0001	0.68
HDF-Kernel9	0.01	0.02	0.01	4612.18	2838.70	-11.04	0.56	0.29	0.53
HDF-Kernel23	0.03	0.04	0.04	782.47	768.47	-0.27	4.58	*; 0.0001	0.91
HDF-Kernel13	0.05	0.05	0.05	329.02	337.02	-188.88	0.53	0.30	0.07
HDF-Classical9	0.02	0.03	0.02	515.35	821.72	-256.65	0.96	0.17	0.20
HDF-Classical23	0.07	0.07	0.07	224.48	217.63	-277.05	0.82	0.21	0.12
HDF-Classical13	0.03	0.04	0.04	238.80	248.33	-228.35	0.55	0.29	0.14

5.1.3 US Total Bank Credit



[4]Quasi estimates are represented by the dotted lines.

Figure 3: US Total Bank Credit: Final and Quasi Estimates

5.2 Simulated Data

The estimated trends from the filters are illustrated using a simple simulated data which has been used in Kohn, Schimek, and Smith (2000) to illustrate problem with smoothing correlated data. Consider the model $f(x_t) = 1280x_t^4(1-x_t)^4$ where t = 1, ..., 100 and $x_t = t/100$. AR(1) errors with $\rho = 0.3713$ are added to produce a random sample from the model $y_t = f(x_t + e_t)$.

Table 6: SIMN: Quantitative and Directional Accuracy Measures

	ME	RMSE	MAE	MAPE	MAPR	adjMAPR	dacTest	p.value	Sign
SPL-Smooth	0.01	0.55	0.44	340.21	240.51	-89.15	7.64	*; 0.0001	0.76
SPL-Penalized	-0.02	0.46	0.38	272.98	179.17	22.24	11.32	*; 0.0001	0.88
SPL-cubic	-0.06	0.48	0.40	194.97	185.47	59.65	11.19	*; 0.0001	0.88
HP-400000	0.04	1.03	0.90	334.20	404.81	91.35	6.67	*; 0.0001	0.73
HP-14400	-0.05	0.42	0.35	552.21	205.76	21.97	11.46	*; 0.0001	0.89
HP-129600	-0.05	0.81	0.70	426.24	360.81	-13.78	8.16	*; 0.0001	0.78
HDF-Kernel9	-0.01	1.12	0.92	844.42	351.36	-36.00	3.16	0.0008	0.61
HDF-Kernel23	0.05	1.10	0.89	511.26	423.99	88.14	6.98	*; 0.0001	0.74
HDF-Kernel13	0.01	1.19	0.98	6736.44	465.65	-6418.71	4.62	*; 0.0001	0.66
HDF-Classical9	-0.01	1.26	1.04	207841.15	400.33	26.39	3.16	0.0008	0.61
HDF-Classical23	0.08	1.26	1.02	660.06	444.29	17.79	6.84	*; 0.0001	0.73
HDF-Classical13	0.01	1.27	1.05	626.75	532.50	101.37	4.62	*; 0.0001	0.66

[4] Quasi estimates are represented by the dotted lines

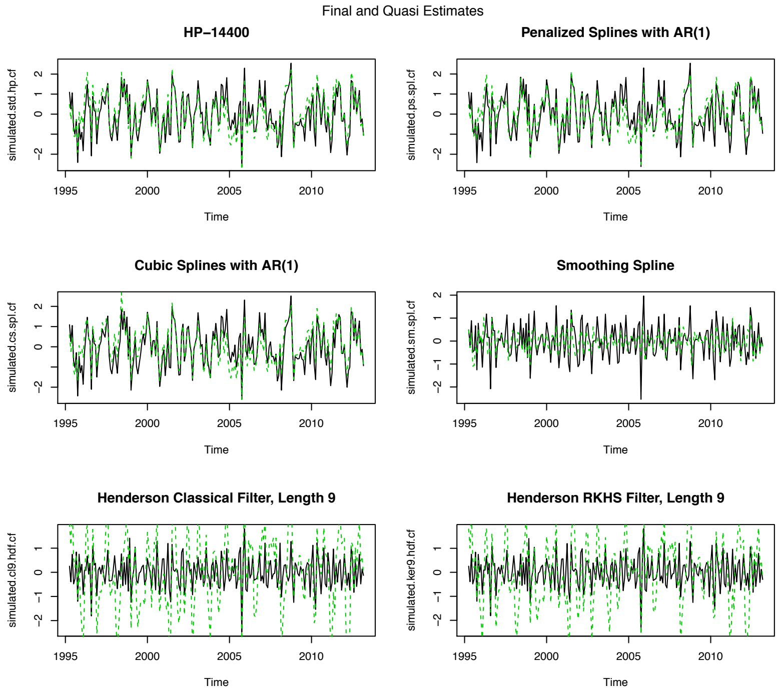


Figure 4: Simulated time series : Final and Quasi Estimates

5.2.1 General Findings

Comparison between Splines and HP filter

Overall penalized splines have been found to be comparatively better in terms of all three criterion for real data series. The benefit of penalized splines fitting is that they take correlation of residuals into account and use a data driven value of smoothing parameter rather than an arbitrary choice of lambda in HP filter. Smoothing splines may also be used in case where the Penalized splines models give singular convergence.

Many studies use a high value of $\lambda = 400000$ which has not been found to be good in terms of the evaluation criterion in comparison to the standard value of 14400. Error statistics for HP filter with high value of λ are low only for simulated series.

Comparison between Splines and Henderson based Filters

Although RKHS based henderson smoothers are found to be lower in MSE and MAPR than classical henderson filters, they have not found to be useful in terms of the three metrics for macroeconomic series used in this study. For the data series used in this study, spline based filters are found to be comparatively better than Henderson based filters on the three criterion. In most cases, only HP and spline methods of filtering reject null hypothesis of Pesaran Timmermann test which implies directional accuracy.

However, there is one main limitation of penalized splines. In case of stock series NFC, M3 and MCWP, the spline models using quasi estimates gives singular convergence when applied in quasi real time which implies that they may not be applicable in some series. This may be due to zero residuals and the matrix not being positive definite.

Comparison of filters using USIIP and USTBKC

As penalized splines for non food credit donot converge for quasi real time, we discuss the results in more detail for US IIP and US Total Bank credit.

For US Industrial production, RMSE and MAPR are comparitively lowest for splines with MAPR being minimum value of 85.79 and 146.40 for HP-400000 and smoothing splines respectively.

On directional accuracy, sign concordance is higher for smoothing splines at 0.77 as compared to 0.67 for HP-400000. Both methods are able to reject the null hypothesis of PT test but smoothing splines has the higher sign concordance at 0.77 compared to 0.67 for HP-400000. HP with standard value of λ does not fair well on all three criterion.

For Henderson based filters, 13 terms based weights perform the best in terms of MAPR but at 356.77 for classical and 423.89 for Kernel filters, it is too high. Also, the p-value of Pesaran Timmermann test is not able to reject the null hypothesis which implies that the filters are not directionally accurate. Also the sign concordance is lowest for classical filters though use kernel weights does increase the sign concordance. However, HDF with 23 terms kernel weights is high in Sign statistic at 0.78 and low in adjMAPR which might be helpful in explaining why they may be useful for assessing build up of financial stress in India.

For US Total bank credit however, HP-400000 has the best sign concordance of 0.7 among splines and HDF-Kernal with 23 terms has a remarkable sign concordance of 0.91 but MAPR for both filters is high. Also, within spline based methods, cubic and penalized splines have the lowest MAPR and sign concordance is 0.64 which is lower but comparable to HP-400000. HP filter with all three values of λ is able to reject PT. Splines however are lowest in RMSE, MAPE and adjMAPR.

Given the statistics and overall results, if quantiative accuracy is important, splines seem to be better but taking directional accuracy into account, explains why many studies have used HP with very high smoothing parameter as filter for detecting credit booms and found the results to be working as good leading indicators of financial crisis and build up of vulnerability.

6 Conclusions

This paper considered alternative methods of estimating trend where deviations from trend or "gap" are used as leading indicators of cyclical changes. Comparison of popular choices of filters in macroeconomics: HP and their generalization to penalized splines and Henderson Filters, with classical and filter weights are evaluated in terms of Mean Square Error, statistical revision error and forecasting directional change.

Results suggest that spline based filters, especially penalized splines generally score better on all three criterion, in particular mean square error which makes them filter of choice to compute potential level of a variable, interpreted as it's value of long term trend. HP filter with high value of smoothing parameter comes close to penalized splines in terms of directional accuracy which explains why it has been found useful in studies on early warning indicators.

Among Henderson filters, RKHS based filters have been found to be an improvement on classical filters. In case of US IIP, 13 term smoothing weights had the best results while for US total bank credit, 23 term RKHS based weights had the highest sign concordance and adjusted MAPR.

Summing up the results, the overall conclusion is that choice of filtering procedure must take the properties of time series into account and metrics most relevant for the analysis must be used as selection criterion.

One limitation of this study is that we have restricted ourselves to the two most commonly known filters. Investigation of other filters and their properties for various applications is therefore recommended. Some of the filtering methods have been reviewed in Alexandrov et al. (2012) and future work can consider their applicability rather than using HP filter as a routine procedure.

Given importance of methods of filtering and smoothing on the results of further analysis, and advances in the filters available, we would further like to explore other filters or their extensions so that trend and cyclical estimates can be computed in a simple way but does not require using HP filter mechanically. This will allow choice of filter according to the purpose and requirements of the analysis.

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