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Abstract

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Keywords: Probabilistic intellectual property rights; Infringement; Damage rules; Import competition; Trade policy

JEL Code: O34; L13; K40; F13

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Probabilistic Patents, Alternative Damage Rules and Optimal Trade Policy

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1 Introduction

When a firm infringes on another's patent, the patent holder generally seeks remedies from the courts. Courts may provide remedies to the patent holder which can be civil, criminal, or both. Civil remedies in the case of intellectual property rights (IPR) generally are damages and injunctions. Criminal remedies generally involve imprisonment of the wrongdoer. This paper makes an attempt to understand the competition and welfare implications of the often used civil remedies, the Lost Profits (LP) damages, or the Unjust Enrichment (UE) damages and its relevance to trade policies. In particular we examine the optimal trade policy under different damage rules from the point of view of a country in which a foreign firm, which holds the patent, exports and faces competition from a domestic firm. Probabilistic nature of patent enforcement is taken into account while drawing the policy conclusions. The paper thus bridges the gap between the literature on damages in cases of patent infringement, when patents are probabilistic, and literature on trade policies with respect to IPR.

Many scholars have examined the importance of cross country patent protection policies and its impact on international trade. For instance, increasing patent protection is found to be positively associated with bilateral manufacturing imports in both small and large developing economies (Maskus and Penubarti, 1995). Canadian manufacturing exports tend to go to those countries which provide high patent safeguards (Rafiquzzaman, 2002). However the interrelations of damage rules and trade policy is not examined in the existing literature and this is potentially the most important aspect in the international legal provisions on protection of IPR because it could provide direct policy repercussions for countries that host or import from foreign firms which bring IPR. This paper delves into the optimal trade policy with respect to damage rules with the aim of providing policy recommendations.

Patents are assumed to be probabilistic, following the existing literature (Amir et al. (2014), Choi (2009), Anton and Yao (2007), Henry and Turner (2010), Lemley and Shapiro (2005), Shapiro (2003), Ayres and Klemperer (1999) and Allison and Lemley (1998)). The probabilistic nature of patent rights means that patents can be invalidated if contested in a court of law. According to the estimates by Allison and Lemley (1998), 46% of all litigated patents were found to be invalid in a set of 299 patents litigated in the United States (US) during 1989-1996. Miller (2013) examines 980 litigated patents between 2000 and 2010 from the US, and estimates that approximately 28% of these would be deemed invalid. It is clear that patents are not fixed or always certain, that is, they can be declared invalid by the courts with a positive probability. Thus we incorporate the uncertainty of patents validation in our set up by considering probabilistic patents.

Most jurisdictions across the globe provide for damages in the case of patent infringement. Reitzig et. al. (2007) analyzes the potential damages and their potential benefits to firms who file suit against the infringers of patents. They map international indemnification rules in a theoretical framework, discussing damage rules. Cotter (2013) performs a comparative study of damage rules prevalent in various parts of the world. He studies the US, the United Kingdom (UK), Canada, Australia, Continental Europe, Japan and China. It has been found that the lost profits, unjust enrichment, reasonable royalty damages and injunctions are the general remedies in the case of patent infringement. A brief description of these remedies is given below.

Injunctions: When a court provides injunctive relief, it generally inhibits the infringer to indulge in production or selling of the infringed good. Injuctions are a primary remedy for most jurisdictions. The patentee usually gets damages in addition to injunctions.

Lost Profits Damages: The LP damage rule provides for reinstatement of the patentee to the position she would be in if the infringement did not take place. In other words, the LP method provides damages to the patentee apropos loss of any profit due to infringement. Most jurisdictions provide for the LP damages. For example, Article 102 (1) of the Patent Act of Japan provides that "the amount of damage sustained…may be presumed to be the amount of profit per unit of articles which would have been sold by the patentee…if there had been no such act of infringement, multiplied by the quantity…of articles assigned by the infringer…." In the US, Section 284 of the Patent Act entitles the patentee to recover damages adequate to compensate for the infringement. Courts have generally provided for lost profit or lost sales due or price erosion. Similar legal provision for LP damages exist in other jurisdictions, like Section 139 of Patent Act of Germany, Section 59 of Patents Act of the UK, Article 65 of the Chinese Patent Law, and Section 108 of the Patents Act in India.

Unjust Enrichment Damages: Some jurisdictions provide for the profits made by the infringer to be paid as damages to the patentee. In Japan, Article 102 (2) of the Patent Act states that "where a patentee or an exclusive licensee claims against an infringer compensation for damage sustained as a result of the intentional or negligent infringement of the patent right or exclusive license, and the infringer earned profits from the act of infringement, the amount of profits earned by the infringer shall be presumed to be the amount of damage sustained by the patentee or exclusive licensee." In the UK, Section 61(1), a claim may be made to the infringer for an "account of profits" derived by the infringer. Section 55 of the Canadian Patent Act states that "a person who infringes a patent is liable to the patentee and to all persons claiming under the patentee for all damage sustained by the patentee or by any such person, after the grant of the patent, by reason of the infringement." Section 122(1) of Australian Patent Act provides for damages at the option of the plaintiff, either damages or an account of profits.

Reasonable Royalty: Another popular damage system used by courts has been the Reasonable Royalty (RR). RR has been used by courts in most jurisdictions. When either lost profits or unjust enrichment cannot be proved in court, reasonable royalty is used. In Japan, prior to 1999 Patent Act article 102(2) authorized courts to award RR in "the amount the patentee or exclusive licensee normally would have been entitled to receive for the working of the patented invention as the amount of damage sustained." This required courts to rely on previous licensees granted by the patentee or the generally accepted license fees existing in the relevant industry. However many critics argued for modifications in the law and the word "normally" was removed after 1999 amendment to the Act (article 102(3)).

Even though RR has been used extensively in practice, the idea of RR has been questioned in the literature. There is a logical inconsistency in the calculation of RR. Courts usually determine RR by analyzing a hypothetical negotiation between the parties, if both had been reasonable in their negotiation. The inconsistency arises because the hypothetical ex-ante negotiation is supposed to take place before the uncertainty about the rights is resolved, and the question of damages arises only after the invalidity of those rights (Schankerman and Scotchmer (2001)). In the case of probabilistic patents, the concept of reasonable royalty lacks consistency (Choi (2009)). The hypothetical ex-ante negotiation is supposed to take place before the uncertainty about the patent is resolved, while the damage is payable only if the patent is found to be valid in court. Thus the "reasonable" royalty is an impossible requirement in case of probabilistic patents. We do not analyse RR in our present model in view of the inconsistencies described above.

Anton and Yao (2007) consider process patents in a Cournot competition game. It is argued that the LP doctrine does not deter entry. In case of process patents the patent can be infringed without resulting in any loss of the patent holder by choosing the quantity consistent with no infringement. However infringement may not be possible in case of product patents. Choi (2009) examine LP and UE rules in a Cournot duopoly game and finds that the LP rule protects the patent holder better compared to the UE rule when both patent holder and the imitator are equally efficient. Ex-post innovation, both the damage rules exhibit the same welfare and output as when both patent holder and imitator are equally efficient whereas the LP rule exhibits a higher welfare under a linear demand when the patent holder is more efficient.

Henry and Turner (2010) examine price competition between a spatially differentiated product patentee and an imitator expecting probabilistic damage payment. They find that the LP rule may deter infringement and may exhibit the highest innovation incentives. The UE rule on the other hand exhibit low innovation incentives. Our focus is not on the innovation incentives but on the optimal trade policies based on the damage rules.

In this paper, we construct a model of import competition where a foreign firm owns a patent and a potential competitor is located in the home country. With this set-up, we compare the implications of the LP and UE rule of damages on competition and welfare. This paper is built on Anton and Yao (2007), Choi (2009), Henry and Turner (2010). The objective of all these papers has broadly been to compare damage rules in terms of their effects on competition whereas our objective in this paper is to examine optimal trade policies when different damage systems are in place.

We demonstrate two reversal results. First, a shift from the regime of trade policy interventions to the free trade regime results in reversal of different stakeholder's preferences over alternative liability doctrines to assess infringement damages. In the free trade regime, while the patentee prefers the LP rule, the infringer's payoff and consumers' surplus are higher under the UE rule compared to any convex combination of these two damage rules and, thus, given the choice, the government of the home country would enforce the UE rule. In contrast, in the regime of trade policy intervention, the government of the home country always prefers the LP rule, which protects the infringer but at the expense of consumers and the patentee. Second, a change in the liability doctrine in place from the LP rule to the UE rule reverses the optimal trade policy of the home country from levying a tariff on imports to import subsidization, unless the patent in weak. In the case of a weak patent, imposition of import tariff is the optimal policy the regardless of the liability doctrine in place, though the 'lost profit' rule calls for a higher rate of import tariff than that under the 'unjust enrichment' rule. Clearly, the optimal trade policy depends on both the damage rule and the strength of the patent, and stakeholders' preferences over alternative damage rules depend on whether there is any trade policy intervention or not .

2 The Model

We consider a 'two-country two-supplier world' in which a product-patent holder (firm 1) is located in the foreign country (F) and a potential competitor/infringer (firm 2) is located in the home country (H). Alternatively, we can consider that firm 1 is the patentee of a specific technology, which is essential to produce the good, and no other firm can compete without infringing the patent. In the case of infringement these two firms produce homogeneous goods and engage in Cournot quantity competition in country H's market, whereas firm 1 enjoys absolute monopoly power in the case of no infringement. It is assumed, for simplicity, that there is no demand for the product in country F.

$C_2(q_2)$ and $C_1(q_1)$ denote, respectively, the cost function of firm 2 in the case of infringement and the cost function of firm 1, where q_i $(i = 1, 2)$ is the quantity of firm i's output. The inverse market demand function for the product is given by $p = p(Q)$, where $Q = q_1 + q_2$ is aggregate output in the market. We make the following assumptions regarding demand and cost functions.

Assumption 1: $p'(Q) < 0$ and $p''(Q) \leq 0$ at all Q.
Assumption 2: $C_i(0) = 0$, $0 \leq C_1'(q_1) \leq C_2'(q_2)$ and $C_1''(q_1) = C_2''(q_2) = 0$

The first assumption is the standard regulatory assumption. It implies that the inverse market demand function is downward sloping and (weakly) concave in Q. From Assumption 2, which is a simplifying assumption, it follows that (a) there is no sunk cost of production, (b) production technologies exhibit constant returns to scale and (c) the patent holder is at least as efficient as the infringer. Assumptions 1 and Assumption 2 together ensure that, in absence of any tax/subsidy, industry profit under duopoly is less than that under monopoly: $\pi_1(q_1, q_2) + \pi_2(q_1, q_2) < \pi_1^M$, where $\pi_i(q_1, q_2)$ denotes duopoly profit of firm i ($i = 1, 2$) corresponding to any plausible combination (q_1, q_2) of duopoly outputs and π_1^M denotes monopoly profit of firm 1.

Following Amir et al. (2014), Choi (2009), Anton and Yao (2007), Lemley and Shapiro (2005), Shapiro (2003), Ayres and Klemperer (1999) and Allison and Lemley (1998), among others, we consider that enforcement of the intellectual property right (IPR) is uncertain. An act of patent infringement can be proved in the court of law with probability α ($0 < \alpha < 1$), which is assumed to be common knowledge. If an act of infringement is proved, the court of law penalizes the infringer following a predetermined damage rule.

Two damage rules have been widely considered in the existing literature on IPR – the 'unjust enrichment' (UE) damage rule and the 'lost profit' (LP) damage rule. According to the UE damage rule, the infringer is required to give up any extra profit earned through infringement as penalty to the patent holder. In contrast, as per the LP damage rule, the infringer needs to pay the amount of decrease in patent holder's profit caused by infringement as penalty to the patent holder. That is, if patent infringement is proved in the court, firm 2 is liable to pay the following amount of money as penalty to firm 1.

$$D = \begin{cases} D^{UE} = \pi_2(q_1, q_2), & \text{when } UE \text{ damage rule is followed} \\ D^{LP} = \pi_1^M - \pi_1(q_1, q_2), & \text{when } LP \text{ damage rule is followed} \end{cases} \qquad (1)$$

Note that the following damage function ($\mathrm{D^{Gen}}$) encompasses UE and LP damage rules as special cases.

$$D^{Gen} = \theta\, D^{UE} + (1 - \theta) D^{LP}, \text{where } 0 \le \theta \le 1 \qquad (2)$$

Clearly, (a) if $\theta = 1$, $D^{Gen} = D^{UE}$ and (b) if $\theta = 0$, $D^{Gen} = D^{LP}$. For intermediate values of the parameter θ, $0 < \theta < 1$, penalty for infringement corresponding to the damage function

D^{Gen} is a linear combination of penalties under UE and LP damage rules. Thus, D^{Gen} is a general form of the damage function.

We characterize the optimal damage rule from the perspectives of various stake holders, viz., the patentee, the infringer, the government of country H and the government of country F, under two alternative trade regimes, separately. First, we consider the case of free trade regime, which serves as the benchmark for this analysis. Next, we examine the implications of unilateral trade policy interventions by the government of country H on the choice of damage rule.

3 Free Trade Regime

We begin with the scenario in which there is no policy intervention in the market. Note that, if firm 2 does not infringe the patent, it stays out of the market and firm 1 obtains monopoly profit π_1^M. In that case the issue of optimal damage rule becomes irrelevant. However, if firm 2 infringes the patent, firm 1 and firm 2 engage themselves in simultaneous move quantity competition in the product market and, subsequently, patent litigation takes place. Thus, for any given damage rule and considering that firm 2 always infringes the patent, stages of the game involved are as follows.

Stage 1: Firm 1 and Firm 2 engage in Cournot quantity competition in the product market.

Stage 2: Firm 1 files a lawsuit of patent infringement against firm 2, the court of law pronounces judgment and the dispute is settled in the court.

We solve this game by the Backward Induction Method. For this purpose, let us consider that the damage rule is given by D^{Gen}. Now, as noted before, the probability of the court's judgment to be against the infringer is α $(0 < \alpha < 1)$, i.e., the probability that patent infringement to be proved in the court of law is α. Therefore, optimization problems of firm 1 and firm 2, respectively, in stage 1 of the game can be written as follows.

$$\begin{aligned} \textit{Firm } 1: \quad \operatorname*{Max}_{q_1} O_1 &= \pi_1(q_1, q_2) + \alpha\, D^{Gen} \\ &= (1-\alpha+\alpha\theta)\pi_1(q_1,q_2) + \alpha\theta\, \pi_2(q_1,q_2) + \alpha\,(1-\theta)\pi_1^M \qquad (3) \end{aligned}$$

$$\textit{Firm 2:} \quad \underset{q_2}{\text{Max}}\, O_2 = \pi_2(q_1, q_2) - \alpha\, D^{Gen}$$
$$= (1 - \alpha\theta)\pi_2(q_1, q_2) + \alpha(1 - \theta)\pi_1(q_1, q_2) - \alpha\,(1 - \theta)\pi_1^M \qquad (4)$$

It follows that in the case of UE damage rule ($\theta = 1$), firm 1 attaches a positive weight on its rival's profit while choosing its output q_1, but firm 2 does not do so. That is, in the case of UE damage rule, firm 1 behaves in a collusive manner in the product market, but firm 2 does not. In contrast, in the case of LP damage rule ($\theta = 0$), firm 2 behaves in a collusive manner, but firm 1 does not. In other words, in the case of UE (LP) damage rule firm 1 (firm 2) behaves less aggressively in the product market compared to that in absence of any IPR ($\alpha = 0$), while the extent of its rival's aggressiveness in the product market remains the same regardless of whether there is any IPR involved or not. If $0 < \theta < 1$, each firm behaves less aggressively in the product market compared to that in the case of no IPR. The higher the value of θ, the lower (higher) the extent of firm 1's (firm 2's) aggressiveness in the product market.[1]

Further, note that $\frac{\partial^2 O_1}{\partial q_2 \partial q_1} = (1 - \alpha + \alpha\theta)[p''(Q)q_1 + p'(Q)] + \alpha\theta[p''(Q)q_2 + p'(Q)] < 0$ and $\frac{\partial^2 O_2}{\partial q_1 \partial q_2} = (1 - \alpha\theta)[p''(Q)q_2 + p'(Q)] + \alpha(1 - \theta)[p''(Q)q_1 + p'(Q)] < 0$ for all $\theta \in [0, 1]$, by Assumption 1. That is, each firm regards q_1 and q_2 as strategic substitutes. Given effective marginal (average) costs of firms, when strategic variables are strategic substitutes, more (less) aggressive behavior by a firm than its rival results in higher (lower) profit of that firm (Bulow et al., 1985). It implies that, given firms' effective marginal (average) costs, the higher value of θ leads to lower output and profit of firm 1 than firm 2 in the equilibrium. Below we demonstrate this result formally.

Considering that demand and cost parameters are such that both firms operate in the market, in stage 1 the equilibrium outputs $q_1^F(\theta)$ and $q_2^F(\theta)$, where superscript 'F' indicates free trade regime, are given by the solution of the following system of first order conditions.

$$\frac{\partial O_1}{\partial q_1} = (1 - \alpha + \alpha\theta)\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} + \alpha\theta\frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = 0 \qquad (5)$$

[1] Note that $\underset{q_1}{\text{ArgMax}}\, O_1 \equiv \underset{q_1}{\text{ArgMax}}\,(\pi_1 + \eta_1\pi_2)$ and $\underset{q_2}{\text{ArgMax}}\, O_2 \equiv \underset{q_2}{\text{ArgMax}}\,(\pi_2 + \eta_2\pi_1)$, where $\eta_1 = \frac{\alpha\theta}{1 - \alpha + \alpha\theta}$ and $\eta_2 = \frac{\alpha(1-\theta)}{1 - \alpha\theta}$. Clearly, $\eta_1(\theta = 0) = 0$ and $\frac{\partial\eta_1}{\partial\theta} > 0\ \forall\, \theta \in [0, 1]$; $\eta_2(\theta = 1) = 0$ and $\frac{\partial\eta_2}{\partial\theta} < 0\ \forall\, \theta \in [0, 1]$.

$$\frac{\partial O_2}{\partial q_2} = (1-\alpha\theta)\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} + \alpha(1-\theta)\frac{\partial \pi_1(q_1, q_2)}{\partial q_2} = 0 \tag{6}$$

Second order conditions for maximization and the stability condition are satisfied (see Appendix for proofs).

From equations (5) and (6), it is evident that $\frac{\partial O_1}{\partial q_1}\Big|_{\theta=0} = 0 \Rightarrow \frac{\partial \pi_1(q_1,q_2)}{\partial q_1} = 0$ and $\frac{\partial O_2}{\partial q_2}\Big|_{\theta=1} = 0 \Rightarrow \frac{\partial \pi_2(q_1,q_2)}{\partial q_2} = 0$, which implies that in the case of LP (UE) damage rule firm 1's (firm 2's) reaction function remains the same as that in the case of standard Cournot competition in absence of any IPR. However, $\frac{\partial O_2}{\partial q_2}\Big|_{\theta=0} = 0 \Rightarrow \frac{\partial \pi_2(q_1,q_2)}{\partial q_2} = -\alpha p'(Q)q_1 > 0$ and $\frac{\partial O_1}{\partial q_1}\Big|_{\theta=1} = 0 \Rightarrow \frac{\partial \pi_1(q_1,q_2)}{\partial q_1} = -\alpha p'(Q)q_2 > 0$, which implies that, for any given q_1 (q_2), firm 2 (firm 1) sets a lower output in the case of LP (UE) damage rule than that in the case of standard Cournot competition in absence of any IPR. Overall, it suggests that in the equilibrium under LP (UE) damage rule firm 1 (firm 2) sets a higher output and firm 2 (firm 1) sets a lower output compared to those in the case of no IPR. Now, from comparative static analysis with respect to the parameter θ we obtain the following Lemmas.

Lemma 1: $\frac{\partial q_1^F(\theta)}{\partial \theta} < 0$, $\frac{\partial q_2^F(\theta)}{\partial \theta} > 0$ *and* $\frac{\partial [q_1^F(\theta) + q_2^F(\theta)]}{\partial \theta} \geq 0 \; \forall \; \theta \in [0, 1]$, *where the sign of equality holds in the case of* $C_1' = C_2'$. *It implies that* $q_{1,LP}^F > q_{1,UE}^F$, $q_{2,LP}^F < q_{2,UE}^F$ *and* $q_{1,LP}^F + q_{2,LP}^F \leq q_{1,UE}^F + q_{2,UE}^F$.

Proof: See Appendix.

Lemma 1 states that higher value of θ leads to lower output of firm 1, but higher output of firm 2. Moreover, increase in firm 2's output due to increase in θ is at least as large as the corresponding decrease in firm 1's output and, thus, industry output increases or remains unchanged. Since LP damage rule corresponds to $\theta = 0$ and UE damage rule corresponds to $\theta = 1$, we can say that, in the equilibrium under free trade regime, the patentee (infringer) produces more output in the case of LP (UE) damage rule; but LP damage rule leads to lower industry output than that in the case of UE damage rule as long as the patentee is more efficient.

Lemma 2: $\frac{\partial O_1^F(\theta)}{\partial \theta} < 0$ *and* $\frac{\partial O_2^F(\theta)}{\partial \theta} > 0 \;\; \forall\, \theta \in [0,1]$. *It implies that* $O_{1,LP}^F > O_{1,UE}^F$, $O_{2,LP}^F < O_{2,UE}^F$.

Proof: See Appendix.

Lemma 2 states that the higher the value of θ, the lower (greater) the payoff of firm 1 (firm 2). Clearly, firm 1's (firm 2's) payoff is lower (higher) in the case of UE damage rule compared to that in the case of LP damage rule. Therefor, the following proposition is immediate.

Proposition 1: *In the regime of free trade, the patentee prefers the `lost profit' damage rule the most, while the infringer prefers the 'unjust enrichment' damage rule the most, over any convex combination of the 'lost profit' damage rule and the 'unjust enrichment' damage rule.*

Clearly, there is a conflict of preferences between the patentee and the infringer regarding the damage rule. Proposition 1 further reinforces the result of Choi (2009) that LP damage rule protects the patent holder better and the infringer prefers UE damage rule to LP damage rule.

Finally, let us turn to answer the following questions. Which damage rule benefits the consumers the most? Given the choice, will the social planner of country H enforce the patentee's most preferred damage rule? Note that, for any given damage rule, in the regime of free trade the equilibrium consumers' surplus and social welfare of country H are given by $CS^F(\theta) = \int_0^{Q^F(\theta)} p(Q)dQ - p\big(Q^F(\theta)\big)Q^F(\theta)$ and $SW^F(\theta) = CS^F(\theta) + O_2^F(\theta)$, respectively, where $Q^F(\theta) = q_1^F(\theta) + q_2^F(\theta)$ and $\theta \in [0,1]$. Differentiating $CS^F(\theta)$ and $SW^F(\theta)$ with respect to θ, we obtain the following.

Lemma 3: $\frac{\partial CS^F(\theta)}{\partial \theta} \geq 0$, *where the sign of equality holds in the case of* $C_1' = C_2'$ *and* $\frac{\partial SW^F(\theta)}{\partial \theta} > 0 \;\forall\, \theta \in [0,1]$. *It implies that* $CS_{LP}^F \leq CS_{UE}^F$ *and* $SW_{LP}^F < SW_{UE}^F$.

Proof: See Appendix.

Lemma 3 states that in the regime of free trade the equilibrium social welfare is higher in the case of higher value of θ. The same is true for the equilibrium consumers' surplus as well, unless both the patentee and the infringer are equally efficient. In the later case, the equilibrium consumers' surplus is invariant to the type of damage rule. Now, since $\theta = 0$ ($\theta = 1$) corresponds to LP (UE) damage rule, we can say that social welfare in the equilibrium under free trade regime is higher in the case of UE damage rule than that in the case of LP damage rule. Clearly, the preference of the social planner of country H is aligned with the preference of the infringer, *not* with the preference of the patentee.

Proposition 2: *In the regime of free trade, given the choice, the government of the home country would always enforce the 'unjust enrichment' damage rule to be followed in the court of law, which best protects interests of consumers and the infringer at the cost of the patentee.*

Clearly, there are conflicts of interests with respect to the damage rule, not only between the patentee and the potential infringer, but also between the patentee and the government of the home country in which the product is sold. Interestingly, such conflicts of interests exist regardless of the strength of the patent (parameterized by α).

4 Trade Policy Intervention

In this section we consider a scenario of unilateral trade policy intervention by the importing country H. For simplicity we assume that (a) the exporting country F does not intervene in the market and (b) import tariff is the only policy instrument available to the government of country H. Let t ($\gtreqless 0$) denote the per unit tariff on imports. Needless to mention here that a negative value of 't' implies import subsidization and $t = 0$ corresponds to the case of free trade.

Let us consider that the government of country H imposes tariff on imports at the rate t in order to maximize country H's social welfare (SW_H), which is the sum of consumers' surplus, firm 2's net profit and tariff revenue. In this case, for any given damage rule, stages of the game involved are as follows, and we solve this game by Backward Induction Method.

Stage 1: The government of country H imposes per unit import tariff t.

Stage 2: Firm 2 decides whether to infringe the patent or not. If infringement does not take place, firm 1 produces monopoly output and the game ends. Otherwise, if firm 2 infringes the patent, Cournot quantity competition between firm 1 and firm 2 takes place in the product market.

Stage 3: Firm 1 files a lawsuit of patent infringement against firm 2, the court of law pronounces judgment and the dispute is settled in the court.

When firm 2 decides not to infringe the patent, the problem of firm 1 in stage 2 of the game can be written as follows.

$$\underset{q_1}{\text{Max}}\, \tilde{\pi}_1 = p(q_1)q_1 - C_1(q_1) - t\, q_1 \tag{7}$$

The first order condition of the above problem, $\frac{\partial \tilde{\pi}_1}{\partial q_1} = p'(q_1)q_1 + p(q_1) - C_1'(q_1) - t = 0$, yields the monopoly output of firm 1, $q_1 = q_1^M(t)$, for any given rate of import tariff t. The higher the rate of import tariff, the lower the equilibrium monopoly output of firm 1: $\frac{\partial q_1^M(t)}{\partial t} = -\frac{\frac{\partial}{\partial t}[\frac{\partial \tilde{\pi}_1}{\partial q_1}]}{\frac{\partial^2 \tilde{\pi}_1}{\partial q_1^2}} = \frac{1}{\frac{\partial^2 \tilde{\pi}_1}{\partial q_1^2}} = \frac{1}{p''(q_1)q_1 + 2p'(q_1)} < 0$, by Assumption 1 and Assumption 2. Now, in stage 1, the problem of the government of country H can be written as follows.

$$\underset{t}{\text{Max}}\, SW = \int_0^{q_1} p(q_1)dq_1 - p(q_1)q_1 + t\, q_1 \tag{8}$$

Let $t = t^{M,R}$ denotes the solution of problem (8), where superscripts 'M'and 'R' indicate monopoly and regulated trade, respectively. Note that, $\frac{dSW}{dt} = \frac{\partial SW}{\partial q_1}\frac{\partial q_1}{\partial t} + \frac{\partial SW}{\partial t} = [t - p'(q_1)q_1]\frac{\partial q_1}{\partial t} + q_1 = \frac{t + p''q_1^2 + p'q_1}{p''q_1 + 2p'}$. Clearly, $\left.\frac{dSW}{dt}\right|_{t=0} > 0$. It implies that $t^{M,R} > 0$, i.e., country H imposes tariff on imports in the equilibrium under monopoly. Substituting $t = t^{M,R}$ in the expression for profit of firm 1, we get the equilibrium monopoly profit of firm 1 in the regime of trade policy intervention $\pi_1 = \pi_1^{M,R}$.

Now, note that, when firm 2 infringes the patent, for any given damage rule and rate of import tariff, stage 2 problems of firm 1 and firm 2, respectively, can be written as follows.

$$\underset{q_1}{\text{Max}}\ \tilde{O}_1 = (1-\alpha+\alpha\theta)\pi_1(q_1,q_2) + \alpha\theta\,\pi_2(q_1,q_2) + \alpha\,(1-\theta)\pi_1^{M,R} - \ (1-\alpha+\alpha\theta)tq_1, \qquad (9)$$

$$\underset{q_2}{\text{Max}}\ \tilde{O}_2 = (1-\alpha\theta)\pi_2(q_1,q_2) + \alpha(1-\theta)\pi_1(q_1,q_2) - \alpha\,(1-\theta)\pi_1^{M,R} - \alpha(1-\theta)tq_1; \qquad (10)$$

where $\pi_1(q_1,q_2) = p(Q)q_1 - \ C_1(q_1)$ and $\pi_2(q_1,q_2) = p(Q)q_2 - \ C_2(q_2)$.

Considering interior solution, stage 2 equilibrium outputs $q_1^R(t;\ \theta)$ and $q_2^R(t;\theta)$ are given by the following first order conditions.

$$\frac{\partial\tilde{O}_1}{\partial q_1} = (1-\alpha+\alpha\theta)\frac{\partial\pi_1(q_1,q_2)}{\partial q_1} + \ \alpha\theta\frac{\partial\pi_2(q_1,q_2)}{\partial q_1} - (1-\alpha+\alpha\theta)t = 0 \qquad (11)$$

$$\frac{\partial\tilde{O}_2}{\partial q_2} = (1-\alpha\theta)\frac{\partial\pi_2(q_1,q_2)}{\partial q_2} + \ \alpha(1-\theta)\frac{\partial\pi_1(q_1,q_2)}{\partial q_2} = 0 \qquad (12)$$

From (11) and (12) it is easy to check that $\frac{\partial q_1^R(t;\,\theta)}{\partial t} < 0$, $\frac{\partial q_2^R(t;\,\theta)}{\partial t} > 0$ and $\frac{\partial[q_1^R(t;\,\theta)\,+\,q_2^R(t;\,\theta)]}{\partial t} < 0$, as in the case of standard Cournot competition without any IPR.

Finally, in stage 1 the problem of the government of country H can be written as follows.

$$\underset{t}{\text{Max}}\, SW = \left[\int_0^Q p(Q)dQ - \ p(Q)Q\right] + \tilde{O}_2(q_1,q_2,t;\theta) + \ t\,q_1, \qquad (13)$$

subject to the constraints

$q_1 = q_1^R(t;\ \theta)$ and $q_2 = q_2^R(t;\ \theta)$

Solving problem (13), we get the equilibrium rate of import tariff $t^R(\theta)$. However, it turns out to be fairly complicated to ascertain the sign of $\frac{\partial t^R(\theta)}{\partial\theta}$, or to compare optimum tariff rates under LP and UE damage rules, in the general setup. Nonetheless, it indicates that the optimum rate of import tariff would vary with the type of damage rule (parameterized by θ). That is, there is an additional channel, via the rate of import tariff, through which the type of damage rule affects equilibrium payoffs of firms and social welfare of country H. To illustrate it further, let us consider the following example.

4.1 An Example

Assume that (a) both firms have the same marginal cost of production, c, which is normalized to be zero, and (b) the market demand function is given by $p = A - q_1 - q_2$, where $A > 0$. Then, in the case of no infringement, the equilibrium tariff rate and firm 1's monopoly output and profit are, respectively, $t^{M,R} = \frac{A}{3} > 0$, $q_1^{M,R} = \frac{A}{3}$ and $\pi_1^{M,R} = \frac{A^2}{9}$.

In the case of patent infringement by firm 2, assuming interior solution, stage 2 equilibrium outputs of firm 1 and firm 2 are, respectively, as follows.

$$q_1^R(t;\ \theta) = \frac{(1-\alpha\theta)\{A(1-\alpha) - 2t(1-\alpha+\alpha\theta)\}}{(3-\alpha)(1-\alpha)}$$

$$q_2^R(t;\ \theta) = \frac{(1-\alpha+\alpha\theta)\{A(1-\alpha) + t(1+\alpha-2\alpha\theta)\}}{(3-\alpha)(1-\alpha)}$$

It is easy to check that both $q_1^R(t;\ \theta)$ and $q_2^R(t;\ \theta)$ are positive, if $\underline{t} < t < \bar{t}$, where $\underline{t} = -\frac{A\,(1-\alpha)}{1+\alpha-2\alpha\theta} < 0$ and $\bar{t} = \frac{A\,(1-\alpha)}{2(1-\alpha+\alpha\theta)} > 0$. Otherwise, if $t < \underline{t}$ ($t > \bar{t}$), firm 2 (firm 1) ceases to exist in the market. Further, it can be verified that, for all $\theta \in [0,1]$, (a) $\frac{\partial q_1^R(t;\,\theta)}{\partial\theta} < 0$ and $\frac{\partial q_2^R(t;\,\theta)}{\partial\theta} > 0$, if $\underline{t} < t < \bar{t}$; and (b) $\frac{\partial[q_1^R(t;\,\theta) + q_2^R(t;\,\theta)]}{\partial\theta} < (>)0$, if $0 < t < \bar{t}$ ($\underline{t} < t < 0$). That is, in the case of interior solution under patent infringement, the higher value of the parameter θ leads to lower output of the patentee and higher output of the infringer, but industry output may fall or rise depending on whether import is taxed or subsidized. Now, the problem of the government of country H in stage 1 of the game can be written as follows.

$$\operatorname*{Max}_{t} SW = \frac{[q_1^R(t;\ \theta)\ +\ q_2^R(t;\ \theta)]^2}{2} + \tilde{O}_2(q_1^R(t;\ \theta), q_2^R(t;\ \theta), t; \theta) +\ t\, q_2^R(t;\ \theta) \tag{14}$$

Solving the above, we get the equilibrium rate of import tariff as follows.

$$t^R = \frac{A(1-\alpha)(3-5\alpha\theta+\alpha^2\theta)}{(1-\alpha+\alpha\theta)(9-\alpha-10\alpha\theta+2\alpha^2\theta)} \tag{15}$$

Clearly, $\underline{t} < t^R < \bar{t}$ for all $\alpha \in [0,1]$ and $\theta \in [0,1]$, i.e., the optimum tariff rate is such that both firms produce positive outputs in the equilibrium. Upon inspection we find that $t^R < 0$

if $\left(\frac{5-\sqrt{13}}{2}\right) < \alpha < 1$ and $\frac{3}{\alpha(5-\alpha)} < \theta \leq 1$; otherwise, $t^R \geq 0$.[2] Interestingly, in absence of IPR (i.e., when $\alpha = 0$), $t^R = t^{M,R} = \frac{A}{3} > 0$. Further, note that $\frac{\partial t^R}{\partial \theta} < 0$ for all $\alpha \in (0, 1)$ and $\theta \in [0, 1]$. Therefore, the following Proposition is immediate.

Proposition 3: *Optimal rate of import tariff crucially depends on both the strength of the patent and the type of the damage rule in place. In the case of linear demand function and symmetric firms with constant marginal cost of production, the following is true.*

(a) If the patent is strong ($\hat{\alpha} < \alpha < 1$), it is optimal for the importing country to impose a tariff on imports under `lost profit' damage rule, but import subsidization is optimal under 'unjust enrichment' damage rule, where $\hat{\alpha} = \frac{5-\sqrt{13}}{2}$.

(b) If the patent is weak ($0 \leq \alpha < \hat{\alpha}$), imposition of import tariff is optimal regardless of the damage rule, but 'lost profit' damage rule calls for a higher rate of import tariff than that under 'unjust enrichment' damage rule.

Now, substituting the equilibrium rate of import tariff t^R from (15) in the expressions for firms' outputs and payoffs, consumers' surplus and social welfare, we get the equilibrium output of each firms, payoff of each firm, consumers' surplus and social welfare. Lemma 4 and Lemma 5 reports these equilibrium outcomes under 'unjust enrichment' damage rule and 'lost profit' damage rule, respectively.

Lemma 4: *Under 'unjust enrichment' damage rule, when the demand function is linear and firms are symmetric with constant marginal costs of production, the equilibrium output of each firm, payoff of each firm, tariff rate, tariff revenue, consumers' surplus and social welfare are, respectively, as follows.*

$q^R_{1,UE} = \frac{A}{9-2\alpha}$, $q^R_{2,UE} = \frac{A(4-\alpha)}{9-2\alpha}$, $O^R_{1,UE} = \frac{A^2(1+(5-\alpha)(4-\alpha)\alpha)}{(9-2\alpha)^2}$, $O^R_{2,UE} = \frac{A^2(4-\alpha)^2(1-\alpha)}{(9-2\alpha)^2}$,

$t^R_{UE} = \frac{A(3-(5-\alpha)\alpha)}{9-2\alpha}$, $T^R_{UE} = \frac{A^2(3-(5-\alpha)\alpha)}{(9-2\alpha)^2}$, $CS^R_{UE} = \frac{A^2(5-\alpha)^2}{2(9-2\alpha)^2}$ and $SW^R_{UE} = \frac{A^2(7-(6-\alpha)\alpha)}{18-4\alpha}$.

[2] $t^R > 0$, if $\left(0 \leq \alpha < \frac{1}{2}(5-\sqrt{13})\ and\ 0 \leq \theta \leq 1\right)$ or $\left(\alpha = \frac{1}{2}(5-\sqrt{13})\ and\ 0 \leq \theta < 1\right)$ or $\left(\frac{1}{2}(5-\sqrt{13}) < \alpha < 1\ and\ 0 \leq \theta < \frac{3}{\alpha(5-\alpha)}\right)$

Lemma 5: *Under 'lost profit' damage rule, when the demand function is linear and firms are symmetric with constant marginal costs of production, the equilibrium output of each firm, payoff of each firm, tariff rate, tariff revenue, consumers' surplus and social welfare are, respectively, as follows.*

$q^R_{1,LP} = \frac{A}{9-\alpha}$, $q^R_{2,LP} = \frac{A(4-\alpha)}{9-\alpha}$, $O^R_{1,LP} = \frac{A^2(9+(12-\alpha)(6-\alpha)\alpha)}{9(9-\alpha)^2}$, $O^R_{2,LP} = \frac{A^2(144-\alpha(108-(18-\alpha)\alpha))}{9(9-\alpha)^2}$,

$t^R_{LP} = \frac{3A}{9-\alpha}$, $T^R_{LP} = \frac{3A^2}{(9-\alpha)^2}$, $CS^R_{LP} = \frac{A^2(5-\alpha)^2}{2(9-\alpha)^2}$ and $SW^R_{LP} = \frac{A^2(3-\alpha)(21-2\alpha)}{18(9-\alpha)}$.

From Lemma 5 and Lemma 6, it follows that $q^R_{1,UE} > q^R_{1,LP}$, $q^R_{2,UE} > q^R_{2,LP}$, $O^R_{1,UE} > O^R_{1,LP}$, $O^R_{2,UE} < O^R_{2,LP}$, $t^R_{UE} < t^R_{LP}$, $T^R_{UE} < T^R_{LP}$, $CS^R_{UE} > CS^R_{LP}$ and $SW^R_{UE} < SW^R_{LP}$.

Proposition 4: *In the regime of trade policy intervention, when the demand function is linear and firms have the same constant marginal cost of production, given the choice the government of the home country would always enforce the 'lost profit' damage rule to be followed in the court of law, which best protects interests of the infringer at the cost of both consumers and the patentee.*

From Proposition 2 and Proposition 4 it is evident that consumers are be better off under 'unjust enrichment' damage rule than under 'lost profit' damage rule regardless of whether there is any trade policy intervention or not. However, preferences of the patentee, the infringer and the benevolent government of the home country over damage rules in the regime of trade policy intervention are reversed from those in the regime of free trade. While the preference of the government of the home country continues to be aligned (in conflict) with the preference of the infringer (patentee) over damage rules even in the regime of trade policy intervention, efficacies of alternative damage rules – 'lost profit' versus 'unjust enrichment' – get altered due to trade policy intervention.

In the above analysis we have considered that the court of law takes into account that the rate of import tariff in the case of monopoly would be different from that in the case of duopoly while calculating the damage corresponding to any given damage rule, which may appear to be a strong assumption. Nonetheless, it can be shown that both Proposition 3 and Proposition 4 go through, if the court of law does not consider any such differences in tariff rates. In other words, results of this analysis remain valid even when monopoly profit of the patentee corresponding to the prevailing rate of import tariff is considered while calculating damage due to infringement.

5. Conclusion

This paper develops a model of import competition between a foreign firm which holds a patent and a potential infringer in the home country. The model is characterized by probabilistic patents. The patentee and the infringer compete in Cournot fashion. The goal is to find policy implications of damage rules protecting patent holder against infringements. The literature on alternative damage rules has focused primarily on their impacts on patent holder and the infringer and the incentives to innovate. Our focus on the other hand has been the trade policy.

The analysis of this paper bridges the gap between the literature on patent infringement with probabilistic patents and international trade. It also highlights the importance of international legal protection of patents in a globalized world. It is shown that optimal policy depends on the damage rule in place and governments may prefer a particular kind of damage rule to protect the domestic firms.

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Appendix: Proofs and Derivations

1. Second order conditions and the stability condition in the regime of free trade

From (3) and (4) we get the following

$$\frac{\partial^2 O_1}{\partial q_1^2} = (1-\alpha+\alpha\theta)\frac{\partial^2 \pi_1}{\partial q_1^2} + \alpha\theta\frac{\partial^2 \pi_2}{\partial q_1^2}$$

$$\frac{\partial^2 O_2}{\partial q_2^2} = \alpha(1-\theta)\frac{\partial^2 \pi_1}{\partial q_2^2} + (1-\alpha\theta)\frac{\partial^2 \pi_2}{\partial q_2^2}$$

Now, $\frac{\partial^2 \pi_1}{\partial q_1^2} = p''q_1 + 2p' < 0$, $\frac{\partial^2 \pi_2}{\partial q_1^2} = p''q_2 < 0$, $\frac{\partial^2 \pi_1}{\partial q_2^2} = p''q_1 < 0$ and $\frac{\partial^2 \pi_2}{\partial q_2^2} = p''q_2 + 2p' < 0$, by Assumption 1 and Assumption 2. We also have $0 < \alpha < 1$ and $0 \leq \theta \leq 1$. Therefore, it follows that $\frac{\partial^2 O_1}{\partial q_1^2} < 0$ and $\frac{\partial^2 O_2}{\partial q_2^2} < 0$, i.e., for each firm the second order condition for maximization is satisfied.

For stability of the market equilibrium we must have $|D| = \begin{vmatrix} \frac{\partial^2 O_1}{\partial q_1^2} & \frac{\partial^2 O_1}{\partial q_2 \partial q_1} \\ \frac{\partial^2 O_2}{\partial q_1 \partial q_2} & \frac{\partial^2 O_2}{\partial q_2^2} \end{vmatrix} > 0.$

Note that

$$\frac{\partial^2 O_1}{\partial q_2 \partial q_1} = (1-\alpha+\alpha\theta)\frac{\partial^2 \pi_1}{\partial q_2 \partial q_1} + \alpha\theta\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} \quad and$$

$$\frac{\partial^2 O_2}{\partial q_1 \partial q_2} = \alpha(1-\theta)\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} + (1-\alpha\theta)\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2}.$$

Therefore, we can write

$$\begin{aligned} |D| = {} & (1-\alpha\theta)(1-\alpha+\alpha\theta)\left[\frac{\partial^2 \pi_1}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_1}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2}\right] \\ & + \alpha(1-\theta)(1-\alpha+\alpha\theta)\left[\frac{\partial^2 \pi_1}{\partial q_1^2}\frac{\partial^2 \pi_1}{\partial q_2^2} - \frac{\partial^2 \pi_1}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}\right] \\ & + \alpha\theta\alpha(1-\theta)\left[\frac{\partial^2 \pi_2}{\partial q_1^2}\frac{\partial^2 \pi_1}{\partial q_2^2} - \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}\right] \\ & + \alpha\theta(1-\alpha\theta)\left[\frac{\partial^2 \pi_2}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2}\right] \end{aligned}$$

Since $\pi_i = p(q_1 + q_2)q_i - C_i(q_i)$ and $C_i'' = 0$, $i = 1, 2$ (by Assumption 2), we get

$$|D| = \big[(1-\alpha\theta)(1-\alpha+\alpha\theta)\{(p''q_1 + p')p' + (p''q_2+p')p' + p'^2\}$$
$$+ \alpha(1-\theta)(1-\alpha+\alpha\theta)\left(-p'^2\right) + \alpha\theta\alpha(1-\theta)\{-p'^2 - p'p''(q_1+q_2)\}$$
$$+ \alpha\theta(1-\alpha\theta)\left(-p'^2\right)\big] = p''p'(q_1+q_2)(1-\alpha) + p'^2(1-\alpha)(3-\alpha) > 0,$$

since $p' < 0$ and $p'' \leq 0$ (by Assumption 1) and $\alpha \in (0,1)$.

□

2. Proof of Lemma 1

Differentiating (5) and (6) with respect to θ, we get the following.

$$\left.\begin{array}{l}\dfrac{\partial^2 O_1}{\partial q_1^2}\dfrac{\partial q_1}{\partial \theta} + \dfrac{\partial^2 O_1}{\partial q_2 \partial q_1}\dfrac{\partial q_2}{\partial \theta} + \dfrac{\partial^2 O_1}{\partial \theta \partial q_1} = 0 \\ \dfrac{\partial^2 O_2}{\partial q_1 \partial q_2}\dfrac{\partial q_1}{\partial \theta} + \dfrac{\partial^2 O_2}{\partial q_2^2}\dfrac{\partial q_2}{\partial \theta} + \dfrac{\partial^2 O_2}{\partial \theta \partial q_2} = 0\end{array}\right\}$$

$$\Rightarrow \begin{pmatrix} \dfrac{\partial^2 O_1}{\partial q_1^2} & \dfrac{\partial^2 O_1}{\partial q_2 \partial q_1} \\ \dfrac{\partial^2 O_2}{\partial q_1 \partial q_2} & \dfrac{\partial^2 O_2}{\partial q_2^2} \end{pmatrix} \begin{pmatrix} \dfrac{\partial q_1}{\partial \theta} \\ \dfrac{\partial q_2}{\partial \theta} \end{pmatrix} = \begin{pmatrix} -\alpha \dfrac{\partial(\pi_1+\pi_2)}{\partial q_1} \\ \alpha \dfrac{\partial(\pi_1+\pi_2)}{\partial q_2} \end{pmatrix},$$

since $\frac{\partial^2 O_1}{\partial \theta \partial q_1} = \alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_1}$ and $\frac{\partial^2 O_2}{\partial \theta \partial q_2} = -\alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_2}$.

Therefore,

$$\frac{\partial q_1}{\partial \theta} = \frac{1}{|D|}\left\{-\alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_1}\frac{\partial^2 O_2}{\partial q_2^2} - \alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_2}\frac{\partial^2 O_1}{\partial q_2 \partial q_1}\right\},$$

$$\frac{\partial q_2}{\partial \theta} = \frac{1}{|D|}\left\{\alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_2}\frac{\partial^2 O_1}{\partial q_1^2} + \alpha \frac{\partial(\pi_1+\pi_2)}{\partial q_1}\frac{\partial^2 O_2}{\partial q_1 \partial q_2}\right\} \; and$$

$$\frac{\partial(q_1+q_2)}{\partial \theta} = \frac{\alpha}{|D|}\left\{\frac{\partial(\pi_1+\pi_2)}{\partial q_1}\left(\frac{\partial^2 O_2}{\partial q_1 \partial q_2} - \frac{\partial^2 O_2}{\partial q_2^2}\right) + \frac{\partial(\pi_1+\pi_2)}{\partial q_2}\left(\frac{\partial^2 O_1}{\partial q_1^2} - \frac{\partial^2 O_1}{\partial q_2 \partial q_1}\right)\right\}.$$

Now, we have already shown that $|D| > 0$. Also, we have the following.

$$\frac{\partial(\pi_1+\pi_2)}{\partial q_1} = \frac{(1-\alpha)}{(1-\alpha+\alpha\theta)}\frac{\partial \pi_2}{\partial q_1} = \frac{(1-\alpha)\,p'q_2}{(1-\alpha+\alpha\theta)} < 0, \qquad by\ (5);$$

$$\frac{\partial(\pi_1+\pi_2)}{\partial q_2} = \frac{(1-\alpha)}{(1-\alpha\theta)}\frac{\partial \pi_1}{\partial q_2} = \frac{(1-\alpha)\,p'q_1}{(1-\alpha\theta)} < 0, by\ (6);$$

$$\frac{\partial^2 O_2}{\partial q_2^2} = (1-\alpha\theta)(p''q_2 + 2p') + \alpha(1-\theta)p''q_1 < 0;$$

$$\frac{\partial^2 O_1}{\partial q_2 \partial q_1} = (1-\alpha+\alpha\theta)(p''q_1 + p') + \alpha\theta\,(p''q_2 + p') < 0;$$

$$\frac{\partial^2 O_1}{\partial q_1^2} = (1-\alpha+\alpha\theta)(p''q_1 + 2p') + \alpha\theta\, p''q_2 < 0 \text{ and}$$

$$\frac{\partial^2 O_2}{\partial q_1 \partial q_2} = (1-\alpha\theta)(p''q_2 + p') + \alpha(1-\theta)(p''q_1 + p') < 0.$$

Clearly, $\frac{\partial q_1}{\partial \theta} < 0$ and $\frac{\partial q_2}{\partial \theta} > 0$.

Next, it is easy to check that $\frac{\partial^2 O_2}{\partial q_1 \partial q_2} - \frac{\partial^2 O_2}{\partial q_2^2} = (1-\alpha)p'$ and $\frac{\partial^2 O_1}{\partial q_1^2} - \frac{\partial^2 O_1}{\partial q_2 \partial q_1} = (1-\alpha)p'$. Therefore,

$$\frac{\partial(q_1+q_2)}{\partial\theta} = \frac{\alpha(1-\alpha)p'}{|D|}\left\{\frac{\partial(\pi_1+\pi_2)}{\partial q_2} - \frac{\partial(\pi_1+\pi_2)}{\partial q_1}\right\}$$
$$= \frac{\alpha(1-\alpha)p'}{|D|}\{C_1'(q_1) - C_2'(q_2)\} \geq 0,$$

since $C_1'(q_1) \leq C_2'(q_2)$ by Assumption 2, $p' < 0$, $|D| > 0$ and $\alpha \in (0,1)$.

□

3. Proof of Lemma 2

$$\frac{dO_1}{d\theta} = \frac{\partial O_1}{\partial q_1}\frac{\partial q_1}{\partial \theta} + \frac{\partial O_1}{\partial q_2}\frac{\partial q_2}{\partial \theta} + \frac{\partial O_1}{\partial \theta}$$
$$= 0 + \left\{(1-\alpha)\frac{\partial \pi_1}{\partial q_2} + \alpha\theta\frac{\partial(\pi_1+\pi_2)}{\partial q_1}\right\}\frac{\partial q_2}{\partial\theta} + \alpha(\pi_1+\pi_2-\pi_1^M), by\ (5)$$
$$= \left\{(1-\alpha)p'q_1 + \alpha\theta\frac{(1-\alpha)\,p'q_1}{(1-\alpha\theta)}\right\}\frac{\partial q_2}{\partial\theta} + \alpha(\pi_1+\pi_2-\pi_1^M), by\ (6)$$
$$< 0,$$

Since $p' < 0$, $\frac{\partial q_2}{\partial \theta} > 0$ (by Lemma 1) and $\pi_1 + \pi_2 < \pi_1^M$ (by construction), $\alpha \in (0,1)$ and $\theta \in [0,1]$.

$$
\begin{aligned}
\frac{dO_2}{d\theta} &= \frac{\partial O_2}{\partial q_1}\frac{\partial q_1}{\partial \theta} + \frac{\partial O_2}{\partial q_2}\frac{\partial q_2}{\partial \theta} + \frac{\partial O_2}{\partial \theta} \\
&= \left\{\alpha(1-\theta)\frac{\partial \pi_1}{\partial q_1} + (1-\alpha\theta)\frac{\partial \pi_2}{\partial q_1}\right\}\frac{\partial q_1}{\partial \theta} + 0 - \alpha(\pi_1 + \pi_2 - \pi_1^M), by\ (6) \\
&= \left\{\frac{\alpha(1-\theta)(-\alpha\theta)}{(1-\alpha+\alpha\theta)}\frac{\partial \pi_2}{\partial q_1} + (1-\alpha\theta)\frac{\partial \pi_2}{\partial q_1}\right\}\frac{\partial q_1}{\partial \theta} - \alpha(\pi_1 + \pi_2 - \pi_1^M), by\ (5) \\
&= \frac{(1-\alpha)p' q_2}{(1-\alpha+\alpha\theta)}\frac{\partial q_1}{\partial \theta} - \alpha(\pi_1 + \pi_2 - \pi_1^M) \\
&> 0,
\end{aligned}
$$

Since $p' < 0$, $\frac{\partial q_1}{\partial \theta} < 0$ (by Lemma 1) and $\pi_1 + \pi_2 < \pi_1^M$ (by construction), $\alpha \in (0,1)$ and $\theta \in [0,1]$.

□

4. Proof of Lemma 3

a) $\frac{dCS}{d\theta} = \frac{\partial CS}{\partial \theta}\frac{dQ}{d\theta} = \{-p'(Q)Q\}\frac{dQ}{d\theta} \geq 0$, by Assumption 1 and Lemma 1.

b) $\frac{dSW}{d\theta} = \frac{dCS}{d\theta} + \frac{dO_2}{d\theta} > 0$, since we have shown that $\frac{dCS}{d\theta} \geq 0$ and by Lemma 2 we have $\frac{dO_2}{d\theta} > 0$.

□

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