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In this paper we show that firms might get an additional strategic benefit from using marginal-cost-reducing investments in conjunction with a managerial incentive scheme. While both these instruments allow firms to \aggressively" participate in product market competition, we show that they act as strategic substitutes or complements depending on whether they are chosen simultaneously or sequentially under complete information. Given that the use of such instruments is inseparably linked with a Prisoner's Dilemma kind of situation, our analysis shows a way to mitigate such effects, through their simultaneous use.

Keywords: Strategic delegation, Cost-Reducing Investment, Strategic Substitutes, Strategic Complements, Subgame Perfection.

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Investment Choice with Managerial Incentive Schemes

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In this paper we show that firms might get an additional strategic benefit from using marginal-cost-reducing investments in conjunction with a managerial incentive scheme. While both these instruments allow firms to "aggressively"participate in product market competition, we show that they act as strategic substitutes or complements depending on whether they are chosen simultaneously or sequentially under complete information. Given that the use of such instruments is inseparably linked with a Prisoner's Dilemma kind of situation, our analysis shows a way to mitigate such effects, through their simultaneous use.

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1 Introduction

In the strategic delegation literature there are several studies which show that the R&D incentives of managerial firms are different from those of entrepreneurial ones. However, these models consider direct strategic effects of investments only. We show, inter alia, that investments made by firms could have an (additional) indirect strategic benefit, which has so far, not been studied. To avail of such benefits, the owners of these firms need to use an additional instrument, which is known to fetch a strategic advantage. We use managerial incentives as the second instrument, which has been shown to lead to stronger (weaker) product market competition if the goods are strategic substitutes (complements) (Fershtman and Judd (FJ, 1987))[1].

Since investment choices made by a firm serve multiple purposes, we assume that such decisions are made by the owners rather than by their agents. Our assumption is backed by a general perception in the literature that though business-unit managers take day-to-day tactical decisions, major decisions such as capacity increases, major capital investments, performance appraisal and selection of managerial incentive schemes are taken by owners and directors (Sengul et. al. (2012), Collis and Montgomery (2005), Holmstrom and Costa (1986)). Using a quantity competition model with demand uncertainty, we then pose the following research questions:

1. is the level of investment higher or lower in a setup in which two instruments are used rather than one?
2. does the answer to the above question depend of the timing of these choices?
3. how does the choice of the managerial incentive scheme vary as the level of investment changes?
4. what is the optimal sequencing of these instruments from the perspective of the owners?
5. does the answer to the above question change when the focus shifts to welfare maximization?

[1]The use of such a second instrument is in fact a dominant strategy for the incumbent firms.

Given that both these instruments are known to effectuate stronger product market competition in a quantity-competition setting, it is not immediately obvious whether they would behave as complements or substitutes.

Our paper follows the literature which shows that firms use a variety of means to compete more aggressively in the product market. These strategies are referred to as “top-dog strategies”by Fudenberg and Tirole (1984) and include inter alia, R&D (Brander and Spencer (1983)), debt obligation (Brander and Lewis (1986)), sticky prices (Fershtman and Kamien (1987)), franchise systems (Esther Gal-Or (1995)) and capacity constraints (Spence (1977, 1979) and Dixit (1980)). However, as all players attempt to push their opponents into a less aggressive position, it results in a situation similar to a Prisoner's Dilemma in which, equilibrium payoffs are Pareto-dominated by an outcome in which such strategies are not used.

The genesis of the literature on strategic delegation is credited to FJ (1987), Sklivas (1987) and Vickers (1985). (footnote: For an exhaustive review of the literature on strategic delegation, please refer to Sengul et. al. (2012)). Vickers (1985) and FJ (1987) show how oligopolistic owners can increase profits by offering managers an incentive scheme which is based on profits as well as sales. In the case of Bertrand-price competition with differentiated products, FJ (1987) shows that owners incentivize managers to set high prices. They do so by overcompensating the agents and set negative weights on sales. The literature on investments made to reduce marginal costs is older and shows that these investments allow firms to compete more vigorously and leads rivals either to compete less vigorously or to exit the industry altogether (Spence (1979), Dixit (1980), Eaton and Lipsey (1981)). Papers which focus on the strategic incentives of investment build on the seminar work of Fudenberg and Tirole (1984) and Bulow et. al. (1985). They argue that players make their long-term decisions while considering their impact on the investment decisions of their rivals. Depending on whether such investments are “soft”or “aggressive”and the nature of product market competition, firms invest more or less than they would have in the absence of such strategic effects.

We are not the first to analyze a setup in which firms use two instruments. Clayton (2009) for instance, studies a model in which firms use both leverage and investment in a quantity competition game, and shows that debt with limited liability lowers the incentive to invest. To construct our argument, we first solve an extensive form game in which two owners simultaneously choose investments in the first period and compete in quantities, in the second. We show that the investments made in this setting have two components, one part which reduces costs and another which provides a strategic benefit. We then show that the equilibrium collapses once firms have the option to strategically delegate the output decision to a manager (in an intervening period). This leads to a setup in which the duopolists choose investments and managerial incentives.

We first consider the setup where investment and managerial incentives are chosen sequentially. In this case, each firm's owner is aware of the investment level of both the firms when she decides the managerial incentive. This leads to an added strategic advantage of signalling to be tough by choosing a higher level of investment, as it reduces (increases) the weight assigned to sales by the rival firm (itself) in the second period. The investment made in the first period therefore has an (additional) effect on the managerial incentives chosen in period 2, which in turn affect output in the last period. This is apart from the direct impact of investment on the corresponding outputs. Thus, in the three-stage game, there is an additional strategic advantage which is brought in through such sequential play.

In an alternative setting, the investment choices made by the firms in the first period are not made public in the second. This essentially reduces the game into one with two-periods. In the first period incentives and investment are simultaneously chosen by the owner and output decisions are made by the manager in the second. Since investment choices are no longer "visible" to the players when they choose managerial incentives, the former can no longer be indirectly used to push an opponent into a disadvantageous position. For this reason, investment in the simultaneous setup is lower than that in the sequential one. The workings of our model are therefore different from that of previous studies[2] as it allows the investment choices made by the owner to indirectly impact product market competition through the managerial incentive scheme. This mechanism was hitherto unavailable in prior studies as investment choices were made after the selection of the managerial incentive scheme.

We find that investment in the sequential choice setting is higher than that of the benchmark model (I0), it is lower than I0 in the simultaneous case (Propositions 1 and 2). Hence it is not the case that managerial firms unambiguously invest more (or less) than their entrepreneurial counterparts.

[2]See Zhang and Zhang (1997), Kopel and Reigler (2006, 2008), and Mitrokostas and Petrakis (2014).

Following this result we show that while the weights assigned to sales in either the simultaneous or sequential setup is higher than that of the FJ model, the owner stresses sales to a larger extent in the sequential case (Proposition 3). The instruments therefore, act as substitutes when they are chosen together but are complements in the sequential setup. Our results indicate that net profits earned when the instruments are chosen simultaneously dominate those when they are decided sequentially (Proposition 4). Hence, it is in the interest of both firms not to divulge their investment decisions to their rivals. This result is however reversed when we compare welfare across the two scenarios (Proposition 7).

2 The Model

We assume that there are two firms, each of which has two players: an owner, who maximizes the firm's profit, and a manager, who executes the decision of the owner and makes the output decision. We consider two settings, in one of which the instruments are chosen sequentially, and in the other one, simultaneously. In the sequential setup, the owners simultaneously choose an investment level to reduce marginal cost of production in the first period. In the second period they simultaneously choose a managerial incentive scheme such that the individual rationality constraint of the manager is satisfied. The investment levels chosen in the first period, therefore, affect the choices of managerial incentives. In the simultaneous setup, investment and managerial incentives are chosen at the same time in period one. Finally, in the third (second) period the managers choose quantities *à* la Cournot in the sequential (simultaneous) choice model.

We assume both the managers are risk neutral. We restrict ourselves with the linear incentive structure as in Fershtman and Judd [8]. The incentive structure is a linear combination of the profit (gross profit) and total revenue given by

$$O_i = (1-\alpha_i)\Pi_i + \alpha_i TR_i$$

where $0 \leq \alpha_i < 1$. The manager's remuneration is given by

$$A_i + B_i O_i$$

The constants $A_i \geq 0, B_i > 0$ are chosen such that in equilibrium the manager get's at least her outside option, $\overline{W}_i$ in expectation,

$$A_i + B_i \int_a O_i f(a) d(a) \geq \overline{W}_i.$$

Since B_i is strictly positive, the risk neutral manager seeks to maximize O_i. Emphasizing partly on total revenue is fairly standard, as firms do write contract with employees to maximize sales (also perhaps the bonus is given as a function of sales and profit). Other incentive contracts can be written but this is a tractable form to influence output in period 3, to maximize profit. The focus of this paper is to show that investment levels change when optimal output of the third stage can be influenced by other instruments. Therefore, we assume a tractable incentive structure.

The firms face a linear inverse demand function given by

$$a - b(q_i + q_j).$$

The intercept a, is a random variable with distribution function $F(.)$ over the range $[\underline{\beta}, \overline{\beta}]$. For our purpose, we will be interested in the expected value of a, denoted by $\overline{a}$. The uncertainty in demand is resolved before the third period, following which rival managers choose their corresponding levels of output. Thus, one justification for having managers can be that owners maximize profit but do not get involved in the day to day working of the firm. The manager takes into account the actual demand realized and chooses the optimal quantities. In the absence of uncertainty, the owners can write a contract explicitly stating the quantity to be chosen to the managers. This would take away the strategic advantage, as the rival manager's choice would no longer be a function of the third period quantity, i.e., each manager's choice would have no effect on the choice of the other.

In this case, each firm has two types of fixed costs and one variable cost. The fixed costs are wages paid to the manager and investments made to reduce the marginal cost, while the variable cost component is represented by the marginal cost, $c(I)$. We maintain the following basic assumptions.

Assumption 1: The marginal cost $c(I)$, is a mapping $c : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ and satisfies $c' < 0$, $c'' > 0$. Further, we assume that $\underline{\beta} > c(0)$.

Assumption 2: We define $\mu : \mathbb{R}_+ \rightarrow \mathbb{R}$ as $\mu(I) = (\overline{a} - c(I_i))\, c'(I_i)$, and assume $\mu(I)$ is an increasing function.

Assumption 1 ensures that in all states, the variable profit for both firms is positive, while assumption 2 ensures that the necessary second order conditions hold while solving for the optimal investment with or without managerial incentives.

Using these assumptions we solve for the *subgame perfect Nash equilibrium (SPNE)*. We proceed to the third stage and compute the Nash equilibrium given the investment levels, the managerial incentives of both firms, and the realized demand. The manager maximizes her incentive as follows:

$$\max_{q_i} V^i = \max_{q_i}(a - b(q_i + q_j) - (1 - \alpha_i)c(I_i))q_i$$

The first order condition is given as:

$$V_i^i = a - 2bq_i^* - bq_j - (1 - \alpha_i)c(I_i) = 0$$

The second order condition holds as

$$V_{ii}^i = -2b < 0; \; V_{ij}^i = -b < 0$$

Since, $V_{ii}^i < V_{ij}^i < 0$ the best response functions are downward slopping, with slope greater than -1, for both $i, j = 1, 2.$ as in a standard model.

The optimal quantity for $i, j = 1, 2,\ i \neq j$, is given by,

$$q_i^* = \frac{a - 2(1 - \alpha_i)c(I_i) + (1 - \alpha_j)c(I_j)}{3b}.$$

Any increase in own managerial incentives and investment affects the optimal quantity choices of the third stage. Both these have similar effect on the third period, reducing the effective marginal cost for the manager. Lower effective marginal cost translates to an aggressive behavior of the firm, which in turn would reduce the equilibrium quantity of the rival firm since best responses are downward slopping. Thus we get the following comparative static for managerial incentives and investment levels.

$$\begin{aligned}
\frac{\partial q_i^*}{\partial \alpha_i} &= \frac{2c(I_i)}{3b}; \frac{\partial q_i^*}{\partial \alpha_j} = -\frac{c(I_j)}{3b} \\
\frac{\partial q_i^*}{\partial I_i} &= -\frac{2(1 - \alpha_i)c'(I_i)}{3b} > 0; \frac{\partial q_i^*}{\partial I_j} = \frac{(1 - \alpha_j)c'(I_j)}{3b} < 0.
\end{aligned}$$

For any increase in investment level or managerial incentives, the increase in own output outweighs the decrease in rival's output, since, the best response functions are downward slopping with slope greater than -1. Therefore the total output increases with any increase in managerial incentives or investment levels for either firm. We now analyze the optimal investments without any managerial incentives.

2.1 The Benchmark Case: No Managerial Incentives

The benchmark case is the optimal investment under no managerial incentives, that is, α_i is set to zero. We thus, consider a two period model, with quantity competition in the second stage and investment choice in the first stage. We denote the optimal quantities of the second stage with no managerial incentives as q_i^0, and q_j^0. The second stage optimization exercise of the manager remains the same. Without managerial incentives, wages are a function of just profits in each state. In the first stage the owner maximizes the expected profit net of investment and wages paid to the manager.

$$\begin{aligned}\max_{I_i} Y^i &= \max_{I_i} \int (a - b(q_i^0 + q_j^0) - c(I_i))q_i^0 f(a)da - I_i \\ &\quad -A_i - B_i \int_a (a - b(q_i^0 + q_j^0) - c(I_i)q_i^0) f(a)d(a)_i\end{aligned}$$

subject to

$$A_i + B_i \int_a (a - b(q_i^0 + q_j^0) - c(I_i)q_i^0) f(a)d(a) \geq \overline{W}_i$$

with

$$q_i^0 = \frac{a - 2c(I_i) + c(I_j)}{3b}$$

$i, j = 1, 2,\ i \neq j$. From the principle of optimality, the inequality constraint will be binding. Thus, the objective function simplifies to:

$$\max_{I_i} Y^i = \max_{I_i} \left\{ \int (a - b(q_i^0 + q_j^0) - c(I_i))q_i^0 f(a)da - I_i - \overline{W}_i \right\}.$$

The manager is paid exactly her outside option.

The first order condition is

$$Y_{I_i}^i = (\overline{a} - 2b\overline{q}_i^0 - b\overline{q}_j^0 - c(I_i))\frac{dq_i^0}{dI_i} - b\overline{q}_i^0 \frac{\partial q_j^0}{\partial I_i} - c'(I_i))\overline{q}_i^0 - 1 \tag{1}$$

while the second order condition is given by

$$Y_{I_i I_i}^i = \frac{8c'(I_i^0)c'(I_i^0)}{9b} - \frac{4(\overline{a} - 2c(I_i^0) + c(I_j))}{9b} c''(I_i^0) < 0$$

The optimal investments with no managerial incentives is given by equation 1, where the first term is zero from the equilibrium condition of the product market. The second term is positive, and captures the strategic advantage through rival's reduction in output. Equation 1 simplifies to

$$-\left[\frac{4(\overline{a} - 2c(I_i^0) + c(I_j))}{9b}\right] c'(I_i^0) - 1 = 0$$

The symmetric condition for the duopoly is given by

$$\begin{aligned} -\left[\frac{4(\overline{a} - c(I^0))}{9b}\right] c'(I^0) - 1 = 0 \Rightarrow -\left(\overline{a} - c(I^0)\right) c'(I^0) &= \frac{9b}{4} \\ \Rightarrow \mu(I^0) &= -\frac{9b}{4} \end{aligned} \quad (2)$$

with symmetric quantities for each realization of uncertainty at the second period,

$$q^0 = \frac{a - c(I^0))}{3b}$$

The strategic advantage in terms of reduction in rival's output is of course absent for a monopoly, for which the optimality condition for investment is given by,

$$-c'(I^M)\overline{q}^M - 1 = 0$$

which simplifies to,

$$\begin{aligned} -\left(\overline{a} - c(I^M)\right) c'(I^M) &= 2b \\ \Rightarrow \mu(I^M) &= -2b \end{aligned} \quad (3)$$

with quantity

$$q^M = \frac{\left(a - c(I^M)\right)}{2b}$$

Comparing conditions (2) and (3) the investment under monopoly, I^M, is greater than the symmetric investments of each duopoly, I^0. Thus, even though the monopoly has no strategic advantage in reducing marginal cost, the investment level of the monopolist is much higher. The reason for this is as follows. The per unit profit of the monopolist is very high. To capture the high monopoly profit, the monopoly investment is much higher despite no strategic advantage. Thus, to have a more reasonable comparison, we set the strategic component, that is, $bq_i^0\frac{\partial q_j^0}{\partial I_i}$ to zero in the first order condition 1.

In the absence of any strategic advantage the symmetric investment level is:

$$\begin{aligned} -c'(\widetilde{I})\overline{q}^0 - 1 = 0 \\ \Rightarrow -\left(\overline{a} - c(\widetilde{I})\right) c'(\widetilde{I}) = 3b \\ \Rightarrow \mu(\widetilde{I}) = -3b \end{aligned} \tag{4}$$

Comparing (4) with (3) and (2) we find

$$\widetilde{I} < I^0 < I^M$$

Thus the symmetric optimal investment can be decomposed into two parts: an $\widetilde{I}$ amount is to increase the profit margin per unit of output. Any investment above this, is for strategic advantage. Now a natural question is if the strategic advantage can be gained through another instrument, would the optimal investment fall to the level of $\widetilde{I}$? In the next section we introduce managerial incentives in the intermediary step to capture this strategic advantage. A discussion on whether the optimal investment made in the benchmark model is efficient, is included in the appendix. The symmetric equilibrium price and expected gross profits in the benchmark case are,

$$p^0 = \frac{\overline{a} + 2c(I^0)}{3}; \Pi^0 = \frac{\left(\overline{a} - c(I^0)\right)^2 + \sigma^2}{9b} - \overline{W}.$$

where σ^2 denotes the variance of the random variable a.

We now show that a firm can do better by strategically delegating the output decision to a manager using an incentive scheme which is identical to the one described above. This implies that it will always be in the interest of firm i to deviate and choose an $\alpha_i > 0$, provided such an option opens up in the intervening period. The third-period product market competition therefore involves manager of firm i maximizing $V^i = (a - b(q_i + q_j) - (1 - \alpha_i)c(I_i))q_i$, while the rival manager maximizes $\Pi_j = (a - b(q_i + q_j) - c(I_j))q_j$. The reaction functions for the two firms are given by

$$q_i = \frac{1}{3b}(a + c(I_j) - 2(1 - \alpha_i)c(I_i)); \; q_j = \frac{1}{3b}(a + c(I_i)(1 - \alpha_i) - 2c(I_j)).$$

The maximization problem for firm i therefore is given by

$$\max_{\alpha_i} \int (a - b(q_i(\alpha_i, I_i, I_j) + q_j(\alpha_i, I_i, I_j)) - c(I_i))q_i(\alpha_i, I_i, I_j)f(a)da$$

The associated first order condition is

$$\int ((a - 2bq_i(\alpha_i, I_i, I_j) - bq_j(\alpha_i, I_i, I_j) - c(I_i)\frac{dq_i}{d\alpha_i} - bq_i(\alpha_i, I_i, I_j)\frac{dq_j}{d\alpha_i})f(a)da = 0$$

$$\implies \alpha_i = \frac{1}{4c(I_i)}(\overline{a} + c(I_j) - 2c(I_i)).$$

Since these calculations are done off the equilibrium path, following the choices made in period 1, we can assume that $I_i = I_j$ such that $c(I_i) = c(I_j)$. Hence, $\alpha_i = \frac{1}{4c(I_i)}(\overline{a} - c(I_i))$. Assuming $c(I_i) = k - \sqrt{bI}$ we get $\alpha_i = \frac{1}{4c(I_i)}\frac{9}{7}(\overline{a} - k) > 0$. This is turn implies that the reaction curve for firm i will lie above the reaction curve for the same firm without such delegation, while the reaction curve for firm j stays unchanged. This ensures that there is an incentive for both firms to use the additional instrument of strategic delegation.

2.2 Three-period Model (Sequential Case)

In this section we analyze how the optimal investment level changes in the presence of managerial incentives. Managerial incentives, like investment, reduces the effective marginal cost in the product competition stage. When effective marginal cost is lowered, the optimal quantity of period three increases, thereby reducing rival's optimal quantity. Reduction in rival output increases prices ceteris paribus, which is gainful for the firm. This strategic advantage of signalling to be aggressive, is identical for both investments and managerial incentives. Hence, the owner views these two as instruments to influence product market behavior in the same way. However, even though these two instruments affect the product market competition in the same way, to the owners these two instruments might be strategic complements or substitutes depending on whether the instruments are chosen simultaneously or sequentially.

We first consider the case where the two instruments are chosen sequentially. Thus, before choosing the managerial incentive both the owners know the rival's firm investment level. Consequently, investment has an additional role: a higher investment not only signals an aggressive behavior directly by reducing marginal cost in the product market, but also affects the managerial incentives of the second period. As shown next, a higher investment increases own managerial incentives while reducing the rival's. A high investment signifies an aggressive managerial incentive choice, which reduces rival's managerial incentives as the best responses of managerial incentives are also negatively slopped.

In the second period given the investment levels, I_i, and I_j, and the rival's incentive α_j, the owner of firm i maximizes gross profit less managerial wages to choose the optimal incentive scheme. The second period optimization problem is given by[3]:

$$\max_{\alpha_i} W^i = \max_{\alpha_i} \left\{ \int \left(\left(a - b(q_i^* + q_j^* - c(I_i)\right) q_i^* \right) f(a)da - \overline{W}_i \right\}$$

where

$$q_i^* = \frac{a - 2(1-\alpha_i)c(I_i) + (1-\alpha_j)c(I_j)}{3b}$$

$i, j = 1, 2$, $i \neq j$. The first order condition for $i = 1, 2$ is given as

$$W^i_{\alpha_i} = \int \left((a - 2bq_i^* - bq_j^* - c(I_i)) \frac{dq_i}{d\alpha_i} - bq_i \frac{\partial q_j}{\partial \alpha_i} \right) f(a)da = 0 \qquad (5)$$

Simplifying,

$$\overline{a} + 4(1-\alpha_i)c(I_i) + (1-\alpha_j)c(I_j) - 6c(I_i) = 0$$

The second order condition is given by $W^i_{\alpha_i\alpha_i} = -4c(I_i) < 0$. At $\alpha = 0$ the first term of the expression 5 is zero from the equilibrium condition of the third stage. The second term captures the strategic benefit of a higher managerial incentives and is positive. Consequently since the second order condition is satisfied, the optimal investment scheme is positive. Particularly, the optimal incentive is given by

$$\alpha_i^* = \frac{\overline{a} - 3c(I_i) + 2c(I_j)}{5c(I_i)}$$

The comparative statics of the optimal managerial incentives with respect to own and rival investment is given by:

$$\frac{d\alpha_i}{dI_i} = \frac{-\left(\overline{a} + 2c(I_j)\right) c'(I_i)}{5c(I_i)c(I_i)} > 0, \quad \frac{d\alpha_i}{dI_j} = \frac{2c'(I_j)}{5c(I_i)} < 0.$$

[3]Like in the benchmark case, the expected managerial wages are set at the outside option of the manager.

Since $\overline{a} - 2c(I_i) > 0$, the own effect dominates the cross effect for any realization of rival's investment.

Now in the first period the owners choose the optimal level of investment to maximize net profit:

$$\max_{I_i} Y^i = \max_{I_i} \int \left((a - b(q_i^* + q_j^*) - c(I_i))q_i^* - I_i\right) f(a)da$$

where

$$q_i^* = \frac{a - 2(1-\alpha_i^*)c(I_i) + (1-\alpha_j^*)c(I_j)}{3b}$$
$$\alpha_i^* = \frac{\overline{a} - 3c(I_i) + 2c(I_j)}{5c(I_i)}$$

given investments I_i, and I_j, $i \neq j$. The first order condition is

$$Y^i_{I_i} = \left[\frac{dq_i^*}{d\alpha_i}\frac{d\alpha_i^*}{dI_i} + \frac{dq_i^*}{d\alpha_j}\frac{d\alpha_j^*}{dI_i} + \frac{dq_i^*}{dI_i}\right] \int \left(a - 2bq_i^* - bq_j^* - c(I_i)\right) f(a)da(a)$$
$$- \left[\frac{dq_j^*}{d\alpha_i}\frac{d\alpha_i^*}{dI_i} + \frac{dq_j^*}{d\alpha_j}\frac{d\alpha_j^*}{dI_i} + \frac{dq_j^*}{dI_i}\right] \int bq_i^* f(a)da(a) - c'(I_i) \int q_i^* f(a)da(a) - 1 = 0$$
$$\Rightarrow -\alpha_i^* c(I_i) \left[\frac{dq_i^*}{d\alpha_i}\frac{d\alpha_i^*}{dI_i} + \frac{dq_i^*}{d\alpha_j}\frac{d\alpha_j^*}{dI_i} + \frac{dq_i^*}{dI_i}\right] - \left[\frac{dq_j^*}{d\alpha_i}\frac{d\alpha_i^*}{dI_i} + \frac{dq_j^*}{d\alpha_j}\frac{d\alpha_j^*}{dI_i} + \frac{dq_j^*}{dI_i}\right] \int bq_i^* f(a)da(a)$$
$$-c'(I_i) \int q_i^* f(a)da(a) - 1 = 0$$

Further simplifying from the equilibrium conditions of the first two stages, the first order condition collapses to,

$$Y^i_{I_i} = \left[-\alpha_i c(I_i)\frac{dq_i^*}{d\alpha_j} - \int bq_i^* f(a)da(a)\frac{dq_j^*}{d\alpha_j}\right]\frac{d\alpha_j^*}{dI_i} - c'(I_i) \int q_i^* f(a)da(a) - 1 = 0 \tag{6}$$

which gives us

$$Y^i_{I_i} = -\frac{12}{25b}\left[\overline{a} - 3c(I_i^*) + 2c(I_j^*)\right] c'(I_i^*) - 1.$$

The second order condition is given by

$$Y^i_{I_i I_i} = \frac{36}{25b} c'(I_i^*)c'(I_i^*) - \frac{12}{25b}\left(\overline{a} - 3c(I_i^*) + 2c(I_j^*)\right) c''(I_i^*).$$

In the sequential setup, the investment made by firm i, I_i has an additional effect on α_i,α_j, which in turn affects q_i, q_j. This is apart from the direct impact of the investment I_i on q_i, q_j. The indirect strategic impact is captured by the first term in equation (6), which shows that I_i first reduces α_j, which then leads to an increase in own output, q_i. The direct strategic benefit captured through aggressive behavior of the manager leading to a reduction in rival's quantity is captured by

$$\frac{dq_i^*}{dI_i}\int\left(a-2bq_i^*-bq_j^*-c(I_i)\right)f(a)da(a)-\frac{dq_j^*}{dI_i}\int bq_i^*f(a)da(a)$$
$$-c'(I_i)\int q_i^*f(a)da(a)-1=0$$

which is similar to equation (1) of the benchmark model. The crucial part of the indirect effect is that firms can use investment made in period 1 to signal to it's rivals that it is a strong type. In the next proposition we show that the optimal investment in the sequential setting, I^*, will be higher than in the benchmark case.

Proposition 1 *When managerial incentives and investments are chosen sequentially, under assumptions 1 and 2, the symmetric optimal investment,* $I^* > I^0$.

Proof. With $I_1^* = I_2^* = I^*$, the symmetric managerial incentives and quantities of second and third period is given by

$$\alpha^*=\frac{\overline{a}-c(I)}{5c(I)};q^*=\frac{2(a-c(I_i))}{5b}$$

Thus, the optimal investment satisfies,

$$-\left(\overline{a}-c(I^*)\right)c'(I^*)=\frac{25b}{12}$$
$$\Rightarrow \mu(I^*)=-\frac{25b}{12} \tag{7}$$

Comparing with (2), (3) and (4), it follows that, $\widetilde{I} < I^0 < I^* < I^M$. ■

Therefore, investment and managerial incentives act as *strategic complements* when investment can influence the rival's managerial incentive choice.

The market price for each state a, and symmetric expected gross and net profits are

$$p^* = \frac{a + 4c(I^*)}{5}, \ \Pi^* = \frac{2\left[(\overline{a} - c(I^*))^2 + \sigma^2\right]}{25b} - \overline{W},$$
$$\Pi_N^* = \frac{2\left[(\overline{a} - c(I^*))^2 + \sigma^2\right]}{25b} - \overline{W} - I^*.$$

2.3 Two-period Model (Simultaneous Case)

Next, we consider the case where the owners choose the managerial incentives and investment levels simultaneously. Since both the instruments have similar effects on the product market competition, the two are strategic substitutes. The first period problem is

$$\max_{I_i,\alpha_i} Y^i = \max_{I_i,\alpha_i} \int \left((\overline{a} - b(q_i^{**} + q_j^{**}) - c(I_i))q_i^{**} - I_i\right) f(a)da$$

where ex-post realization of the random variable,

$$q_i^{**} = \frac{a - 2(1-\alpha_i^{**})c(I_i) + (1-\alpha_j^{**})c(I_j)}{3b}.$$

The first order conditions are

$$Y_{I_i}^i = \frac{dq_i^{**}}{dI_i} \int \left(a - 2bq_i^{**} - bq_j^{**} - c(I_i)\right) f(a)d(a) - \frac{dq_j^{**}}{dI_i} \int bq_i^{**} f(a)d(a)$$
$$- c'(I_i) \int q_i^{**} f(a)d(a) - 1 = 0$$

$$Y_{\alpha_i}^i = \frac{dq_i^{**}}{d\alpha_i} \int \left(a - 2bq_i^{**} - bq_j^{**} - c(I_i)\right) f(a)d(a) - \frac{dq_j^{**}}{d\alpha_i} \int bq_i^{**} f(a)d(a) = 0$$

Further simplifying the first order conditions we get

$$Y_{I_i}^i = (1-\alpha_i)c'(I_i) \int \left(\left(a - 2bq_i^{**} - bq_j^{**} - c(I_i)\right)\frac{2}{3b} - bq_i^{**}\frac{1}{b}\right) f(a)d(a)$$
$$-c'(I_i) \int q_i^{**} f(a)d(a) - 1 = 0 \tag{8}$$

$$Y_{\alpha_i}^i = c(I_i) \int \left(\left(a - 2bq_i^{**} - bq_j^{**} - c(I_i)\right)\frac{2}{3b} - bq_i^{**}\frac{1}{b}\right) f(a)d(a) = 0 \tag{9}$$

Since $(1-\alpha_i)c'(I_i)$, $c(I_i) > 0$, at α_i^*, $i = 1, 2$, the first order condition (8) collapses to,

$$Y_{I_i}^i = c'(I_i^{**}) \int q_i^{**} f(a) d(a) - 1 = 0$$

Since, the optimal managerial incentives are

$$\alpha_i^{**} = \frac{\overline{a} - 3c(I_i) + 2c(I_j)}{5c(I_i)}$$

consequently for each realization of a,

$$q_i^{**} = \frac{2(a - 3c(I_i) + 2c(I_j))}{5b}$$

the first order condition further simplifies to

$$Y_{I_i}^i = \frac{2(\overline{a} - 3c(I_i^{**}) + 2c(I_j^{**}))}{5b} c'(I_i^{**}) - 1 = 0$$

In this case, the firm benefits from investment through a lower marginal cost only. However, the investment doesn't fall to the benchmark case without strategic advantage $(\widetilde{I})$, but is lower than the investment without managerial incentives.

Proposition 2 *Under assumptions 1 and 2,* $\widetilde{I} < I^{**} < I^0$.

Proof. With $I_1^{**} = I_2^{**} = I^{**}$, the symmetric managerial incentives and quantities of second and third period is given by

$$\alpha^{**} = \frac{\overline{a} - c(I^{**})}{5c(I^{**})}; q^{**} = \frac{2(\overline{a} - c(I^{**}))}{5b}$$

Thus, the optimal investment satisfies,

$$-\left(\overline{a} - c(I^{**})\right) c'(I^{**}) = \frac{5b}{2}$$

$$\Rightarrow \mu(I^{**}) = -\frac{5b}{2} \tag{10}$$

Comparing (10) with (2), (3) and (4), we get, $\widetilde{I} < I^{**} < I^0 < I^M$. ■

We now compare optimal investment levels in the simultaneous and sequential settings.

Proposition 3 *Under assumptions 1 and 2, (i) $I^* > I^{**}$ and (ii) $\alpha^* > \alpha^{**} > \frac{\overline{a}-c(0)}{5c(0)}$.*

Proof. (i) Follows from equations (7), (10) and assumption 2. (ii) Since the optimum $\alpha(I) = \frac{\overline{a}-c(I)}{5c(I)}$, it follows that $\frac{\partial \alpha}{\partial I} > 0$. $\therefore \alpha^* > \alpha^{**} > \frac{\overline{a}-c(0)}{5c(0)}$, where $\frac{\overline{a}-c(0)}{5c(0)}$ is the optimum weight assigned to sales in the Fershtman and Judd (1987) setup. ■

In the simultaneous setup, the owner has no information about the realized value of rival's investment level while choosing the incentive structure. The firm cannot credibly signal to be tough and influence the rival's choice in the second stage by choosing a higher investment level. The indirect benefit is therefore, absent. On the other hand, managerial incentives act as a substitute instrument, through which owners can compete vigorously in the product market. Consequently optimal investment is lower than the benchmark model with no managerial incentives; however, it is not as low as the investment without incentives and strategic effect. That is because a higher output increases profit per unit, and thus the direct benefit of investment increases. The market price for each state a, and symmetric expected gross and net profits are

$$p^{**} = \frac{a - 4c(I^{**}))}{5b}; \Pi^{**} = \frac{2\left[(\overline{a} - c(I^{**}))^2 + \sigma^2\right]}{25b} - \overline{W};$$
$$\Pi_N^{**} = \frac{2\left[(\overline{a} - c(I^{**}))^2 + \sigma^2\right]}{25b} - \overline{W} - I^{**}.$$

Next we compare the optimal quantities for these two variants with respect to the benchmark case. It is straight forward to conclude that the gross profit of the simultaneous case is less than the sequential case, $\Pi^{**} < \Pi^*$, since $I^{**} < I^*$. However the next proposition shows that the inequality is reversed for the net profits, that is $\Pi_N^{**} > \Pi_N^*$. For the remainder of our discussion, we assume that $\overline{W}$ is close to zero.

Proposition 4 *Under assumptions 1 and 2, the optimal quantities satisfy $q^* > q^{**}, q^0 < q^*$, and $q^{**} > q^0$, if $a > 6c(I^{**}) - 5c(I^0)$. Also, $\Pi_N^{**} > \Pi_N^*$.*

Proof. The optimal quantities with managerial incentives satisfy the following relation

$$q^{**} = \frac{2(a - c(I^{**}))}{5b} < q^* = \frac{2(a - c(I^*))}{5b}, \text{ since } I^{**} < I^*$$

The optimal quantity without managerial incentives, $q^0 < q^*$. To compare q^0 with q^{**}, if $a > 6c(I^{**}) - 5c(I^0)$ then

$$q^0 = \frac{(a - c(I^0))}{3b} < q^{**} = \frac{2(a - c(I^{**}))}{5b}.$$

For the second part, we define a function $g : \mathbb{R}_+ \to \mathbb{R}$ as

$$g(I) = \frac{2\left(\overline{a} - c(I)\right)^2}{25b} - I$$

where

$$g'(I) = -\frac{4\left(\overline{a} - c(I)\right)c'(I)}{25b} - 1 = 0 \Rightarrow \mu(I) = -\frac{25b}{4}$$

Since μ is an increasing function, it follows that g achieves a unique maxima at

$$\mu(I^{\max}) = -\frac{25b}{4}.$$

Using assumption 2 we get $I^{\max} < I^{**} < I^*$, implying $g'(I)$ is decreasing in the range $(I^{\max}, I^*]$, which gives us the result. ■

Thus, when the owners of each firm have full information of the rival's investment before choosing the managerial incentive structure, their profit falls. Consequently, both owners would choose not to divulge or seek information about the other's investment levels. The following proposition ranks the expected net profit from the sequential setup with the one earned from the benchmark model.

Proposition 5 *Under assumptions 1 and 2,* $\Pi_N^0(I^0) > \Pi_N^*(I^*)$.

Proof. Expected net profit in the benchmark model is given by

$$\Pi_N^O = \frac{(\overline{a} - c(I^0))^2 + \sigma^2}{9b} - I^0$$

The expected net profit of the simultaneous or sequential model is given by

$$\Pi_N = \frac{2(\overline{a} - c(I))^2 + 2\sigma^2}{25b} - I, \text{ where } I = I^*, I^{**}.$$

Since $\frac{\sigma^2}{9b} > \frac{2\sigma^2}{25b}$ it is enough to show that $\widetilde{\Pi}^0(I^0) = \frac{(\overline{a}-c(I^0))^2}{9b} - I^0 > \frac{2(\overline{a}-c(I^*))^2}{25b} - I^* = \widetilde{\Pi}(I^*)$. Since both expressions are functions of I, we write

$$\widetilde{\Pi}^0(I) - \widetilde{\Pi}(I) = (\overline{a} - c(I))^2(\frac{1}{9b} - \frac{2}{25b}) > 0.$$

Therefore for each value of I, $\widetilde{\Pi}^0$ lies above $\widetilde{\Pi}$. We now check for the optimum investment which maximizes $\widetilde{\Pi}^0(I^0)$, I^B

$$\frac{d\widetilde{\Pi}^0}{dI} = -\frac{2}{9b}(\overline{a} - c(I))c'(I) - 1 = 0$$

which gives us the first order condition

$$-(\overline{a} - c(I))c'(I) = \frac{9b}{2}. \tag{11}$$

The second order condition is given by

$$\frac{d^2\widetilde{\Pi}^0}{dI^2} = \frac{2}{9b}\left[(c'(I))^2 - (\overline{a} - c(I))c''(I)\right] < 0$$

which follows from assumption 2. Similarly the optimum investment level which maximizes $\widetilde{\Pi}(I)$, $\widehat{I}$ is determined from the following first order condition

$$-(\overline{a} - c(I))c'(I) = \frac{25b}{4} \tag{12}$$

Comparing equations (2), (7), (10), (11) and (12), we can show that $I^* > I^0 > I^{**} > I^B > \widehat{I}$. Also, since $\widetilde{\Pi}(I^0)$ and $\widetilde{\Pi}(I)$ are strictly concave, $\widetilde{\Pi}'(I^0) < 0$ and $\widetilde{\Pi}'(I^*), \widetilde{\Pi}'(I^{**}) < 0$.
Now assume that $\widetilde{\Pi}^0(I^0) < \widetilde{\Pi}(I^*) < \widetilde{\Pi}(I^{**})$. Since $I^* > I^0 > I^{**}$ we have $\widetilde{\Pi}(I^*) < \widetilde{\Pi}^0(I^0) < \widetilde{\Pi}(I^{**})$, which combined with $\widetilde{\Pi}^0(I^0) < \widetilde{\Pi}(I^*)$ gives us $\widetilde{\Pi}(I^0) > \widetilde{\Pi}^0(I^0)$, a contradiction. Hence $\widetilde{\Pi}^0(I^0) > \widetilde{\Pi}(I^*)$. ■

In order to compare the expected net profit from the simultaneous setting (Π_N^{**}) with Π_N^0, we derive the following sufficient condition.

Proposition 6 $\Pi_N^0 > \Pi_N^{**}$ *if* $\frac{9b}{16(c'(I^0))^2} - I^0 > \frac{b}{2(c'(I^{**}))^2} - I^{**}$.

Proof. Since $\frac{\sigma^2}{9b} > \frac{2\sigma^2}{25b}$ it is enough to show that $\Pi_N^0 > \Pi_N^{**}$ if

$$\frac{(\overline{a} - c(I^0))^2}{9b} - I^0 > \frac{2(\overline{a} - c(I^{**}))^2}{25b} - I^{**}$$

Using equations (2) and (10) we can show that the above condition simplifies to $\frac{9b}{16(c'(I^0))^2} - I^0 > \frac{b}{2(c'(I^{**}))^2} - I^{**}$. ■

While the expected net profit from the simultaneous setup is higher than that from the sequential setup, we now show that the (ex-ante) social welfare from the former (TW^*) is lower than that of the latter (TW^{**}).

Proposition 7 $TW^* > TW^{**}$.

Proof. Ex-ante consumer surplus for the sequential case is given as follows

$$CS^* = \frac{1}{2}(a - p^*)(q_1^* + q_2^*) = \frac{8(a - c(I^*))^2}{25b}.$$

Total welfare is given by,

$$TW^* = CS^* + \Pi_N^* = \frac{2(a - c(I^*))^2 + 2\sigma^2}{5b} - \overline{W} - I^*.$$

Similarly ex-ante total welfare for the simultaneous case is given by

$$TW^{**} = \frac{2(a - c(I^{**}))^2 + 2\sigma^2}{5b} - \overline{W} - I^{**}$$

The expression

$$TW(I) = \frac{2(a - c(I))^2 + 2\sigma^2}{5b} - \overline{W} - I$$

is concave and is maximized at

$$TW'(I_{\max}) = -\frac{4(a - c(I))c'(I_{\max})}{5b} - 1 = 0$$

Given assumption 2,

$$I^{**} < I^* < I_{\max}$$

which implies that $TW^{**} < TW^*$. ■

An Example. To illustrate the results obtained so far, we solve the following example. We use the marginal cost function

$$c(I) = k - \sqrt{bI}$$

with $k < \alpha$ which gives us $c'(I) = -\frac{1}{2}\sqrt{\frac{b}{I}} < 0$ and $c''(I) = \frac{1}{4}\sqrt{b}(I)^{-3/2} > 0$. Therefore assumption 1 is satisfied. We then assume $h(I) = (\overline{a} - c(I))c'(I) = (\overline{a} - k + \sqrt{bI})(-\frac{1}{2}\sqrt{\frac{b}{I}})$. Since $h'(I) = (\overline{a} - k)(\frac{1}{4}\sqrt{b}(I)^{-3/2}) > 0$, therefore, assumption 2 is satisfied. Using this function, we get

$$I^M = \frac{(\overline{a} - k)^2}{9b}, I^o = \frac{4(\overline{a} - k)^2}{49b}, \widetilde{I} = \frac{(\overline{a} - k)^2}{25b}, I^* = \frac{36(\overline{a} - k)^2}{361b}, I^{**} = \frac{(\overline{a} - k)^2}{16b}.$$

which implies that $I^M > I^* > I^0 > I^{**} > \widetilde{I}$. The corresponding equilibrium output levels are $q^* = \frac{2(a-k+\frac{6}{19}(\overline{a}-k))}{5b}, q^{**} = \frac{2(a-k+\frac{\overline{a}-k}{4})}{5b}, q^o = \frac{a-k+\frac{2(\overline{a}-k)}{7}}{3b}$ such that for a particular realization of a, $q^* > q^{**} > q^0$. Finally, substituting into the gross profit functions we get

$$\Pi^* = \frac{2(a-k+\frac{6}{19}(\overline{a}-k))^2}{25b}, \ \Pi^{**} = \frac{2(a-k+\frac{1}{4}(\overline{a}-k))^2}{25b} \text{ and}$$
$$\Pi^o = \frac{(a-k+\frac{2}{7}(\overline{a}-k))^2}{9b}.$$

It is straightforward to then show that expected net profit $\Pi^0_N = \frac{5(\overline{a}-k)^2}{49b} + \frac{\sigma^2}{9b} > \Pi^{**}_N = \frac{(\overline{a}-k)^2}{16b} + \frac{2\sigma^2}{25b} > \Pi^*_N = \frac{14(\overline{a}-k)^2}{361b} + \frac{2\sigma^2}{25b}$.

3 Conclusion

Our paper is the first to show how firm owners can derive strategic benefits from using both investment and managerial delegation. Given that the use of such instruments is inextricably linked with a Prisoner's Dilemma kind of situation, our analysis shows a way to mitigate such effects – by simultaneously choosing these instruments.

We also find that the R&D investment made in the setting in which these instruments are used sequentially is higher than the entrepreneurial one, while the corresponding level for the simultaneous setup is lower than the one in the benchmark. Therefore, these instruments, which are known to individually induce stronger product market competition, act as substitutes (complements) in the simultaneous (sequential) case.

A similar analysis can be conducted in a differentiated-product price competition game. In this case, the strategic effect of investment will lead to a reduction in the rival's price. With upward rising reaction functions, this in turn, will result in a reduction in the price of the reference firm. Contrary to the quantity competition game in which each firm tries to push its rival into a disadvantageous position, each firm will try to induce softer product market competition by investing less in the price competition game. The optimal investment level without strategic effects $(\widetilde{I})$ will therefore be higher than the level which considers both strategic and cost-reduction effects (I^0). This implies that the strategic use of investment will lead to an outward shift of the reaction functions in the price competition game, in much the same way as the use of managerial delegation as shown by Fershtman and Judd (1987). Both the instruments will therefore work in the same direction by increasing prices. We posit that investment levels will be lower in the case where the instruments are chosen sequentially rather than simultaneously. However, this will be difficult to validate empirically, as the weight assigned to profit in the price competition game is negative (FJ, 1987).

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A Appendix

In this appendix we compare the investment made in the benchmark case with the corresponding investment level that maximizes welfare. We find that investment made in the benchmark case, I^0, is socially efficient if the social planner is unable to merge the two firms and announce a single level of investment.

Proposition 8 $I^0 = I^e$.

Proof. From the product market competition stage we get

$$q_i^0 = \frac{a - 2c(I_i) + c(I_j)}{3b}, i = 1, 2, i \neq j.$$

such that $Q^0 = \frac{2a-c(I_i)-c(I_j)}{3b}$ and $p^0 = \frac{a+c(I_i)+c(I_j)}{3}$. This gives us consumer surplus

$$CS = \frac{1}{2}(a - p^0)Q^0 = \frac{1}{2}\frac{(2a - c(I_i) - c(I_j))^2}{9b} > 0.$$

In order to maximize social welfare, the social planner solves

$$\max_{I_i,I_j}\left[\int((a - b(q_i^0 + q_j^0) - c(I_i))q_i^0 f(a)da - I_i\right.$$
$$\left.+\int((a - b(q_i^0 + q_j^0) - c(I_j))q_j^0 f(a)da - I_j + \frac{1}{2}\int\frac{(2a - c(I_i) - c(I_j))^2}{9b}f(a)da.\right]$$

Payments made to the two managers add up to $2\overline{W}$ and are omitted from this expression. The corresponding FOCs are

$$\begin{aligned}
&(\overline{a} - 2b\overline{q}_i^0 - b\overline{q}_j^0 - c(I_i))\frac{dq_i^0}{dI_i} - b\overline{q}_i^0\frac{dq_j^0}{dI_i} - c'(I_i)\overline{q}_i^0\\
&+(\overline{a} - 2b\overline{q}_j^0 - b\overline{q}_i^0 - c(I_j))\frac{dq_j^0}{dI_i} - b\overline{q}_j^0\frac{dq_i^0}{dI_i}\\
&+(c(I_i) + c(I_j))\frac{1}{9b}c'(I_i) - \frac{2\overline{a}}{9b}c'(I_i) - 1 = 0.
\end{aligned} \tag{13}$$

$$\begin{aligned}
&(\overline{a} - 2b\overline{q}_j^0 - b\overline{q}_i^0 - c(I_j))\frac{dq_j^0}{dI_j} - b\overline{q}_j^0\frac{dq_i^0}{dI_j} - c'(I_j)\overline{q}_j^0\\
&+(\overline{a} - 2b\overline{q}_i^0 - b\overline{q}_j^0 - c(I_i))\frac{dq_i^0}{dI_j} - b\overline{q}_i^0\frac{dq_j^0}{dI_j}\\
&+(c(I_i) + c(I_j))\frac{1}{9b}c'(I_j) - \frac{2\overline{a}}{9b}c'(I_j) - 1 = 0.
\end{aligned} \tag{14}$$

These first order conditions simplify to

$$\begin{aligned}
-c'(I_i)&\left[\frac{4(\overline{a}-2c(I_i)+c(I_j))}{9b}+\frac{-2\overline{a}+4c(I_j)-2c(I_i)}{9b}\right]\\
&+\frac{(c(I_i)+c(I_j))}{9b}c'(I_i)-\frac{2\overline{a}}{9b}c'(I_i)-1=0\\
-c'(I_j)&\left[\frac{4(\overline{a}-2c(I_j)+c(I_i))}{9b}+\frac{-2\overline{a}+4c(I_i)-2c(I_j)}{9b}\right]\\
&+\frac{(c(I_i)+c(I_j))}{9b}c'(I_j)-\frac{2\overline{a}}{9b}c'(I_j)-1=0.
\end{aligned}$$

Which gives us

$$\begin{aligned}
-c'(I_i)\frac{1}{9b}\left[4\overline{a}-11c(I_i)+7c(I_j)\right] &= 1\\
-c'(I_j)\frac{1}{9b}\left[4\overline{a}-11c(I_j)+7c(I_i)\right] &= 1
\end{aligned}$$

Since these conditions are symmetric, we will have $I_i^e = I_j^e = I^e$ which solves

$$\begin{aligned}
-c'(I^e)\frac{1}{9b}\left[4\overline{a}-4c(I^e)\right]=1\\
\Longrightarrow\ -(\overline{a}-c(I^e))c'(I^e)=\frac{9b}{4}.
\end{aligned} \tag{15}$$

Therefore, $I^e = I^0$.

In case the social planner is able to merge the firms, we get, $q^M = \frac{a-c(I)}{2b}, p^M = \frac{a+c(I)}{2}$ and $CS = \frac{1}{2}(a-p^M)q^M = \frac{(a-c(I))^2}{8b}$. The social planner therefore solves

$$\max_I \left[\int ((a-bq^M-c(I))q^M f(a)da + \int \frac{(a-c(I))^2}{8b}f(a)da - I\right]$$

The corresponding first order condition is

$$\left[\overline{a}-2b\overline{q}^M-c(I)\right]\frac{dq^M}{dI}-\overline{q}^M c'(I)+\frac{1}{4b}c'(I)c(I)-\frac{1}{4b}\overline{a}c'(I)-1=0 \tag{16}$$

which simplifies to $-\overline{q}^M c'(I)+\frac{1}{4b}c'(I)(c(I)-\overline{a})=1 \Longrightarrow -(\overline{a}-c(I^e))c'(I^e) = \frac{4b}{3}$. This implies $I^e > I^0$. ■